Reply to "Comment on 'Light-front Schwinger model at finite temperature' "

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(Received 18 February 2004; published 30 June 2004)

In the preceding comment, Blankleider and Kvinikhidze criticize the form of the thermal propagator used previously by us and propose an alternate thermal propagator for the fermions in the light-front Schwinger model. We show that, within the standard light-front quantization used by us, the thermal propagator for the fermions is unique as presented in that paper.

DOI: 10.1103/PhysRevD.69.128702 PACS number(s): 11.10.Wx, 11.10.Kk, 12.38.Lg

In an earlier paper [1], we studied various questions associated with the light-front Schwinger model at finite temperature where the theory was quantized on the standard light-front surface $\overline{x^0}=x^0+x^1=0$. (We refer the reader to [1] for notation and conventions.) We argued in that paper that one of the components of the fermion field does not thermalize and correspondingly used the real time propagator (only the $++$ component of the propagator and we will suppress the "$i\epsilon$" for simplicity)

$$iS_{+}^{(T)(\mathrm{DZ})}(\overline{p})=-\overline{p}_1\left(\frac{i}{-(2\overline{p}_0+\overline{p}_1)\overline{p}_1}-2\pi n_F(|\overline{p}_0|)\,\delta((2\overline{p}_0+\overline{p}_1)\overline{p}_1)\right),$$

$$iS_{-}^{(T)(\mathrm{DZ})}(\overline{p})=\frac{i}{-\overline{p}_1},\tag{1}$$

where $n_F(|\overline{p}_0|)$ represents the Fermi-Dirac distribution function. This propagator was obtained from the structure of the fermion Lagrangian density of the light-front Schwinger model and was not derived from Eq. (33) in [1], which describes the propagator for a massive fermion in higher dimensions. (The authors in [2] seem to suggest that our propagator was derived from an erroneous limit of that expression.) We did, however, indicate that Eq. (1) can be obtained from Eq. (33) in [1] in a limiting manner. One of the results found in that paper showed that the off-shell thermal n-point photon amplitudes in this theory do not coincide with the ones calculated in the conventionally quantized (equal-time) Schwinger model [3] and we traced the origin of the difference to the fact that one of the fermion components in the light-front Schwinger model is nondynamical in the quantization used and as a result does not thermalize, while both the fermion components in the conventionally quantized theory are dynamical and do thermalize. We note, on the other hand, that all the thermal corrections to the n-point photon amplitudes vanish at zero temperature in both the quantizations and the n-point photon amplitudes do coincide.

In [2] the authors comment that the difference found in [1] has its origin in the use of an erroneously simplified thermal fermion propagator and suggest that the proper thermal propagator for the fermions that should have been used in [1] is of the form (only the $++$ component)

$$iS_{+}^{(T)(\mathrm{BK})}(\overline{p})=-\overline{p}_1\left(\frac{i}{-(2\overline{p}_0+\overline{p}_1)\overline{p}_1}-2\pi n_F(|\overline{p}_0|)\,\delta((2\overline{p}_0+\overline{p}_1)\overline{p}_1)\right),$$

$$iS_{-}^{(T)(\mathrm{BK})}(\overline{p})=(2\overline{p}_0+\overline{p}_1)\left(\frac{i}{-(2\overline{p}_0+\overline{p}_1)\overline{p}_1}-2\pi n_F(|\overline{p}_0|)\,\delta((2\overline{p}_0+\overline{p}_1)\overline{p}_1)\right).\tag{2}$$

They obtain this propagator from Eq. (33) in [1] by setting $m=0$ (and restricting to $1+1$ dimensions). The difference between Eqs. (1) and (2) lies in the thermal part of the propagator $iS_{-}^{(T)}(\overline{p})$; namely, the contention of the authors of [2] is that both components of the fermion field in the light-front Schwinger model should thermalize, even though one of them is nondynamical. In this case, of course, one should not expect any difference from the results of the conventionally quantized theory. The basic issue, therefore, is whether the ψ_- component in the light-front Schwinger model thermalizes in the quantization used in [1].

Given the quantization conditions in a theory, the propagators are, of course, uniquely determined as vacuum expectation values of time ordered products of fields. Therefore, it is not entirely clear from [2] whether the authors find the fermion propagator in [1] to be incorrect within the quantization used or whether their objection is addressed to the quantization used in that paper. We will try to address both these issues in the following.

First, let us discuss the form of the propagator within the quantization used in [1]. There are various ways to see, in both the imaginary time and the real time formalisms, that the ψ_- component in the light-front Schwinger model *does not thermalize* in the standard light-front quantization used in [1]. We briefly discuss the imaginary time formalism before going into the real time formalism. We note that the quadratic part of the fermion Lagrangian density (which is relevant for a discussion of the propagator) for the light-front Schwinger model has the form

$$\mathcal{L}=i\psi_{+}^{\dagger}(2\bar{\partial}_0+\bar{\partial}_1)\psi_{+}-i\psi_{-}^{\dagger}\bar{\partial}_1\psi_{-}. \tag{3}$$

Here ψ_{+},ψ_{-} represent the two chiral components of the theory. The zero temperature propagators of the theory in Eq. (3) have the simple forms

$$iS_{+}^{(0)}(\bar{p})=-\frac{i\bar{p}_1}{-(2\bar{p}_0+\bar{p}_1)\bar{p}_1}=-\frac{i}{-(2\bar{p}_0+\bar{p}_1)},$$

$$iS_{-}^{(0)}(\bar{p})=\frac{i(2\bar{p}_0+\bar{p}_1)}{-(2\bar{p}_0+\bar{p}_1)\bar{p}_1}=\frac{i}{-\bar{p}_1}. \tag{4}$$

In the imaginary time formalism in light-front theories within the quantization used in [1], one obtains the thermal propagators simply by letting [4,5]

$$\bar{p}_0\to(2n+1)i\pi T, \tag{5}$$

where T denotes temperature. This introduces a temperature dependence to $iS_{+}^{(T)}(\bar{p})$ in Eq. (4), while $iS_{-}^{(T)}(\bar{p})$ remains temperature independent since it does not depend on $\bar{p}_0$. This is probably the most direct way to see that the component ψ_{-} does not thermalize in the standard light-front quantization.

Let us next analyze the propagator in the real time formalism. This is best done in the operatorial formalism of thermofield dynamics [6,7]. We note that a Hamiltonian analysis of the theory in Eq. (3) shows that, when quantized on the surface $\bar{x}^0=0$, the only nontrivial anticommutation relation has the form

$$\{\psi_{+}(\bar{x}),\psi_{+}^{\dagger}(\bar{y})\}_{\bar{x}^0=\bar{y}^0}=P^{+}\delta(\bar{x}^1-\bar{y}^1), \tag{6}$$

where P^{+} represents the projection operator for the positive chirality spinors. Since the fermion field ψ_{-} satisfies trivial anticommutation relations, it follows in particular that

$$\{\psi_{-}(\bar{x}),H\}=0, \tag{7}$$

namely, the ψ_{-} component has no time evolution. As a result, the propagator for the ψ_{-} field has the form

$$\begin{aligned}iS_{-}^{(0)}(\bar{x}-\bar{y})&=\langle 0|T(\psi_{-}(\bar{x})\psi_{-}^{\dagger}(\bar{y}))|0\rangle\\&=\langle 0|\psi_{-}(\bar{x})\psi_{-}^{\dagger}(\bar{y})|0\rangle=F(\bar{x}^1-\bar{y}^1),\end{aligned} \tag{8}$$

which is consistent with the form of the zero temperature propagator $iS_{-}^{(0)}$ in Eq. (4).

In going to finite temperature, in thermofield dynamics, one doubles the degrees of freedom (with tilde fields) and obtains a thermal vacuum through a Bogoliubov transformation of the form

$$|0(\beta)\rangle=U(\theta)|0\rangle\otimes|\tilde{0}\rangle, \tag{9}$$

where β represents the inverse temperature in units of the Boltzmann constant. The formally unitary transformation involves both the physical and the tilde fields and has the form

$$U(\theta)=e^{-iQ(\theta)}, \tag{10}$$

with the parameter θ related to the fermion distribution function [6,7]. The finite temperature propagator for the ψ_{-} field is then defined in the standard manner as

$$iS_{-}^{(\beta)}(\bar{x}-\bar{y})=\langle 0(\beta)|T(\psi_{-}(\bar{x})\psi_{-}^{\dagger}(\bar{y}))|0(\beta)\rangle. \tag{11}$$

From Eqs. (8)–(11) as well as the fact that ψ_{-} satisfies trivial anticommutation relations in the standard light-front quantization, it follows that

$$\begin{aligned}iS_{-}^{(\beta)}(\bar{x}-\bar{y})&=\langle\tilde{0}|\otimes\langle 0|[e^{iQ(\theta)}\psi_{-}(\bar{x})\psi_{-}^{\dagger}(\bar{y})e^{-iQ(\theta)}]|0\rangle\otimes|\tilde{0}\rangle\\&=\langle\tilde{0}|\otimes\langle 0|[\psi_{-}(\bar{x})\psi_{-}^{\dagger}(\bar{y})]|0\rangle\otimes|\tilde{0}\rangle\\&=\langle 0|\psi_{-}(\bar{x})\psi_{-}^{\dagger}(\bar{y})|0\rangle=iS_{-}^{(0)}(\bar{x}-\bar{y})\\&=F(\bar{x}^1-\bar{y}^1).\end{aligned} \tag{12}$$

This demonstrates clearly that within the standard light-front quantization used in [1], the fermion field ψ_{-} does not thermalize, and that the unique finite temperature propagator coincides with that at zero temperature, which is the form used in [1].

As we indicated in [1], this form of the propagator can also be obtained from a limit of Eq. (33) (a massive propagator) of that paper. Essentially, this involves looking at the vanishing mass limit of a delta function of the form $(2\bar{p}_0+\bar{p}_1)\,\delta((2\bar{p}_0+\bar{p}_1)\bar{p}_1+m^2)$. If $m=0$, there are two roots for the vanishing of the delta function. Keeping both the roots leads to the propagator in, Eq. (2), which, however, would not be compatible with Eq. (12) and would lead to a nontrivial time dependence (in the coordinate space). Therefore, naively setting $m=0$ in Eq. (33) of [1] would not lead to the proper propagator within the quantization being discussed. The propagator in Eq. (12) [and, therefore, Eq. (1)], on the other hand, can be obtained from a massive theory [Eq. (33) of [1]] only if a particular limiting value is chosen (namely, $|\bar{p}_0|,|\bar{p}_1|\gg m\to 0$) which selects out only the root $(2\bar{p}_0+\bar{p}_1)=0$ of the delta function. As is also noted in [2], the massless limit in light-front theories is subtle; therefore, when necessary one must go back to the basic definitions, as we have just done for the propagator (and as was also done in [1]).

The authors of [2] have also argued how the ψ_{-} field can become dynamical in the nonstandard light-front quantization due to McCartor [8], which involves quantizing the ψ_{+} field on the conventional surface $x^{+}=x^0+x^1=0$ while quantizing the ψ_{-} component on the surface $x^{-}=x^0-x^1=0$. This brings us to the question of whether their objection is really to the quantization used in [1]. It is worth recognizing that a given quantization defines a unique quantum theory, and different quantizations do not yield equivalent quantum theories in general. As McCartor himself has pointed out [9], his nonstandard quantization leads to a vanishing fermion condensate (in the infinite volume limit) which is in disagreement with all the other calculations. The theory quantized on $\overline{x^0} = x^0 + x^1 = 0$, on the other hand, does lead to the condensate [10] (even at finite temperature [1]) which agrees with the results of equal-time quantization. Therefore, it is not clear *a priori* which of the two theories should be called the light-front Schwinger model (if that is the objection being raised by the authors in [2]). It is, of course, an interesting question to see if McCartor's alternative quantization (or a generalization of it) does allow a statistical description (we remind the reader that the conventional light-front quantization does not) and if so whether it leads to the propagator in Eq. (2) at finite temperature. Even if it does, that would not be the appropriate propagator to use in a calculation involving the standard light-front quantization such as in [1]. As we have argued above the propagator used in [1] is the unique propagator within that quantization and leads to the result that at zero temperature the n-point photon amplitudes agree with the calculations using equal-time relations, while at finite temperature, the results are different.

This work was supported in part by U.S. DOE Grant No. DE-FG 02-91ER40685.

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