Scalar-tensor gravity coupled to a global monopole and flat rotation curves

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(Received 16 November 2003; published 18 June 2004)

In a scalar-tensor theory of gravity we consider a global monopole field as a candidate for galactic dark matter. Within the weak gravity approximation we solve the equations of a metric tensor and a scalar field coupled to the monopole and determine the asymptotic structure of a galactic spacetime for large r. In the case of a massless scalar field, we derive a formula for the rotation velocity of stars, which contains an extra constant value in addition to the other known terms, and we discuss its relation to flat rotation curves in the galaxy.

DOI: 10.1103/PhysRevD.69.127502 PACS number(s): 04.50.+h, 14.80.Hv, 95.35.+d, 98.35.Gi

I. INTRODUCTION

The asymptotic flatness of rotation curve (RCs) in the galatic halo suggests the existence of dark matter whose energy density varies as $1/r^2$. Since the global monopole (GM) found by Barriola and Vilenkin [1] has energy density proportional to $1/r^2$, it was suggested by Nucamendi and others [2,3] that the monopole could be the galactic dark matter in spiral galaxies. Even though Harari and Loustó [4] suggested that the monopole core mass is negative and that there are no bound orbits, Nucamendi *et al.* [2] showed that there is an attractive region where bound orbits exist, by the introduction of a nonminimal coupling of gravity to the GM. Banerjee *et al.* [3] noted that the GM in Brans-Dicke theory also exerts gravitational pull on a test particle moving in its spacetime. Therefore it seems important to study in more detail scalar-tensor (ST) theories of gravity coupled to the GM.

About 40 years ago Brans and Dicke [5] introduced a scalar field $\tilde{\varphi}$ instead of the inverse bare gravitational constant G_*^{-1}, for the purpose of generalizing Einstein's general theory of relativity to incorporate Mach's principle. Since then various ST theories of gravity have been studied. A four-dimensional dilaton gravity obtained as a low energy effective theory from strings also has the form of a ST theory, with nontrivial couplings of the dilaton to matter and a possible dilaton potential. When a potential of the scalar field, $\tilde{V}_S(\tilde{\varphi})$, is included as in some models of dilaton gravity, the action of the ST theory of gravity is given by

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{\varphi}\tilde{R}-\frac{\omega}{\tilde{\varphi}}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\varphi}\partial_\nu\tilde{\varphi}-\frac{\tilde{V}_S(\tilde{\varphi})}{\tilde{\varphi}}\right)+S_m \tag{1}$$

with S_m the action for matter fields.

In the so-called Einstein conformal frame [6,7], the ST theory of gravity coupled to the GM is described by the action

$$S=\frac{1}{16\pi G_*}\int d^4x\sqrt{-g}[R-2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-V_D(\varphi)]+S_m[\vec{\Phi},A^2g_{\mu\nu}] \tag{2}$$

with

$$S_m[\vec{\Phi},A^2g_{\mu\nu}]=-\int d^4x\sqrt{-g}\times(\tfrac{1}{2}A^2g^{\mu\nu}\partial_\mu\vec{\Phi}\cdot\partial_\nu\vec{\Phi}+A^4V_M(\vec{\Phi}^2)), \tag{3}$$

where $A^2(\varphi)=1/G_*\tilde{\varphi}$. The above equations are obtained from the action (1) of the ST theory with the GM action S_m, by a conformal transformation [7,8] $\tilde{g}_{\mu\nu}=A^2(\varphi)g_{\mu\nu}$, with

$$\alpha^2\equiv(\partial\ln A/\partial\varphi)^2=1/(2\omega+3). \tag{4}$$

The potential $V_D(\varphi)$ has been defined as $G_*A^4\tilde{\varphi}^{-1}\tilde{V}_S(\tilde{\varphi})$. In the Brans-Dicke theory of gravity $1/(2\omega+3)\equiv\alpha_0^2$ is constant and astronomical constraints for the parameter are given by solar-system experiments as [8,9]

$$\alpha_0^2<0.001. \tag{5}$$

It is well known that the bare gravitational constant G_* is related to the gravitational constant G in Einstein's theory as [5] $G_*=G/(1+\alpha_0^2)$.

Varying the action (2) with respect to the fields, we have equations for the monopole fields $\vec{\Phi}$ and the scalar field φ as well as Einstein's field equations:

$$\frac{1}{\sqrt{-g}}\partial_\mu A^2\sqrt{-g}g^{\mu\nu}\partial_\nu\vec{\Phi}-A^4\frac{\partial V_M}{\partial\vec{\Phi}}=0,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi-\frac{1}{4}\frac{\partial V_D}{\partial\varphi}=-\frac{1}{2}\kappa\frac{\delta S_m}{\delta\varphi}, \tag{6}$$

$$G_{\mu\nu}=\kappa T_{\mu\nu}+2\partial_\mu\varphi\partial_\nu\varphi-\tfrac{1}{2}g_{\mu\nu}[2g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi+V_D(\varphi)],$$

where $\kappa=8\pi G_*$ and the energy-momentum tensor

$$T_{\mu\nu}=-(2/\sqrt{-g})(\delta S_m/\delta g^{\mu\nu}). \tag{7}$$

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In Sec. II, we take the weak gravity approximation of the field equations (6) and we find large-r solutions to the equations. In Sec. III, with these solutions we calculate the rotation velocity of stars in a galactic halo and discuss its possible relation to the flatness of RCs in the galaxy. Section IV contains a summary and discussion.

II. WEAK FIELD APPROXIMATION OF ST GRAVITY COUPLED TO GM AND LARGE-*r* SOLUTIONS

When we consider the potential of a triplet of scalar fields $V_M(\vec{\Phi}^2)=(\lambda/4)(\vec{\Phi}^2-\eta^2)^2$ with a constant η, the global $O(3)$ symmetry is spontaneously broken to $U(1)$ as $|\vec{\Phi}|\to\eta$ [1]. It can be realized with a spherically symmetric hedgehog ansatz for the scalar fields $\vec{\Phi}=f(r)\hat{r}$. In the Einstein conformal frame [6,7] we assume the line element of the spherically symmetric, static spacetime as

$$ds^2_{Ein}\equiv g_{\mu\nu}dX^\mu dX^\nu=-b(r)c(r)dt^2+\frac{dr^2}{b(r)}+r^2d\Omega^2 \tag{8}$$

with $X^\mu=(t,r,\theta,\phi)$ and $d\Omega^2\equiv d\theta^2+\sin^2\theta d\phi^2$.

Taking the weak gravity approximations as

$$b=b_0+\kappa b_1+\kappa^2 b_2\cdots,\quad c=c_0+\kappa c_1+\kappa^2 c_2\cdots, \tag{9}$$

$$\varphi=\varphi_0+\kappa\varphi_1+\kappa^2\varphi_2\cdots,\qquad f=f_0+\kappa f_1+\kappa^2 f_2\cdots$$

($b_0=1$ and $c_0=1$ for the spacetime to be Minkowskian in the $\kappa\to0$ limit [7]), we have the $\mathcal{O}(\kappa^0)$ equation for $f_0(r)$, from Eq. (6),

$$a_0\frac{\partial V_M}{\partial f_0}+\frac{2f_0}{r^2}=f_0''+f_0'\frac{2}{r}+f_0'\frac{a_0'}{a_0}. \tag{10}$$

The function A^2 has been approximated as $A^2(\varphi)=a_0+\kappa a_1+\cdots$. To $\mathcal{O}(\kappa^0)$ Eq. (6) for φ gives

$$\varphi_0''+\varphi_0'(2/r)-\tfrac{1}{4}(\partial V_D/\partial\varphi_0)=0 \tag{11}$$

and

$$(\varphi_0')^2=0,\quad V_D(\varphi_0)=0, \tag{12}$$

whose solutions are

$$\varphi_0=\text{const},\quad \partial V_D/\partial\varphi_0=0. \tag{13}$$

Here and hereafter $\partial V_M/\partial f_0$ means $[\partial V_M/\partial f]_{f=f_0}$, f_0' means df_0/dr, etc., and we calculate all physical quantities up to $\mathcal{O}(1/r^4)$. A series solution of Eq. (10) for f_0 is

$$f_0=\eta(1-\delta^2/r^2-3\delta^4/2r^4), \tag{14}$$

where the size of the monopole core $\delta=1/\sqrt{\lambda}\eta$ [1] and we have taken the constant value $a_0=A^2(\varphi_0)$ as 1, since a_0 can be absorbed by a redefinition of the coordinates X^μ in Eq. (19) [7].

Then from Eq. (6) up to $\mathcal{O}(\kappa^1)$ we have

$$\frac{\partial^2 V_M}{\partial f_0^2}f_1+a_1\frac{\partial V_M}{\partial f_0}+\frac{2f_1}{r^2}$$
$$=f_1''+b_1f_0''+f_0'\left(\frac{1}{2}c_1'+b_1'+\frac{2b_1}{r}+a_1'\right)+\frac{2}{r}f_1' \tag{15}$$

for the GM,

$$\varphi_1''+\frac{2}{r}\varphi_1'-m_D^2\varphi_1=\alpha_0\left\{\frac{1}{2}(f_0')^2+\frac{f_0^2}{r^2}+2V_M(f_0)\right\} \tag{16}$$

for the scalar field φ, and

$$\frac{b_1}{r^2}+\frac{b_1'}{r}=-\frac{1}{2}(f_0')^2-\frac{f_0^2}{r^2}-V_M(f_0), \tag{17}$$

$$\frac{b_1}{r^2}+\frac{b_1'}{r}+\frac{c_1'}{r}=\frac{1}{2}(f_0')^2-\frac{f_0^2}{r^2}-V_M(f_0),$$

$$\frac{b_1'}{r}+\frac{c_1'}{2r}+\frac{b_1''+c_1''}{2}=-\frac{1}{2}(f_0')^2-V_M(f_0)$$

for the metric coefficients. We have parametrized as $\partial^2V_D/\partial\varphi_0^2\equiv 4m_D^2$ and used the relations $\partial A^2/\partial\varphi_0\equiv 2\alpha_0$ and $a_1=2\alpha_0\varphi_1$.

Considering the case of a massless scalar field φ ($m_D=0$), we have the solution

$$\varphi_1=q_0\eta^2[\tfrac{1}{2}\ln(r/r_i)+(\delta^4/6r^4)]-q_0M/16\pi r, \tag{18}$$

where M and r_i are constants of integration, whose physical meanings are given in the next section. Using the solution (18) of φ_1, with b_1 and c_1 easily calculated from Eq. (17), we find the spacetime described by the following metric in the physical frame [7]:

$$\begin{aligned}ds^2&=\tilde{g}_{\mu\nu}dX^\mu dX^\nu=A^2g_{\mu\nu}dX^\mu dX^\nu\\&=\left[1+\frac{8\pi G\eta^2\alpha_0^2}{1+\alpha_0^2}\left(2\ln\frac{r}{r_i}-\frac{M}{4\pi\eta^2 r}+\frac{2\delta^4}{3r^4}\right)\right]\\&\quad\times\left[-\left\{1-\frac{8\pi G\eta^2}{1+\alpha_0^2}\left(\frac{M}{4\pi\eta^2 r}+\frac{\delta^2}{r^2}+\frac{\delta^4}{3r^4}\right)\right\}dt^2\right.\\&\quad+\left\{1+\frac{8\pi G\eta^2}{1+\alpha_0^2}\left(\frac{M}{4\pi\eta^2 r}+\frac{\delta^2}{r^2}-\frac{2\delta^4}{3r^4}\right)\right\}dr^2\\&\quad\left.+\left\{1-\frac{8\pi G\eta^2}{1+\alpha_0^2}\right\}r^2d\Omega^2\right],\end{aligned} \tag{19}$$

where the relation $\kappa=8\pi G/(1+\alpha_0^2)$ is used, and a coordinate transformation following Barriola and Vilenkin [1] is performed. The above formula is valid for $r_i<r$, and it gives us corrections in $1/r$, up to $\mathcal{O}(1/r^4)$, compared with the results calculated by Teixeira Filho and Bezerra [7] and others.

III. FLAT RCS IN A GALAXY

In a series of works, Matos, Guzmán, Ureña-López, and Núñez [10] discussed the possibility of determining the geometry of a spacetime where the flat RCs in galaxies could be explained and constraining the type of dark matter that generates such geometry. As a candidate for galactic dark matter, we have considered the GM coupled to gravity in a ST theory. In the weak gravity approximation we obtained large-r solutions (19) to Einstein's equations, which determine the geometry of a galactic spacetime.

To discuss RCs in the galaxy, we consider the circular motions of stars in a spacetime with the following metric coefficients:

$$ds^2=-\mathcal{N}^2(r)dt^2+\mathcal{M}^2(r)dr^2+\mathcal{A}^2(r)r^2d\Omega^2. \tag{20}$$

When we consider the case $\theta=\pi/2$, with the definition

$$\frac{dX^\mu}{d\tau}=\left(\frac{dt}{d\tau},\frac{dr}{d\tau},\frac{d\theta}{d\tau},\frac{d\phi}{d\tau}\right)\equiv(\dot{t},\dot{r},\dot{\theta}=0,\dot{\phi}) \tag{21}$$

and dividing Eq. (20) by the square of the infinitesimal proper time $d\tau^2$, we get the equation for the Lagrangian $\mathcal{L}$:

$$\mathcal{L}(r,\dot{r},\dot{\phi},\dot{t};\tau)\equiv-\mathcal{N}^2(r)\dot{t}^2+\mathcal{M}^2(r)\dot{r}^2+\mathcal{A}^2(r)r^2\dot{\phi}^2=-1, \tag{22}$$

where it is used that $ds^2=-d\tau^2$ in our unit system with $c=1$. Since $\partial\mathcal{L}/\partial\phi=0$ and $\partial\mathcal{L}/\partial t=0$, we have the constants of motion

$$\partial\mathcal{L}/\partial\dot{\phi}\equiv2\mathcal{A}^2r^2\dot{\phi}=2L, \quad \partial\mathcal{L}/\partial\dot{t}\equiv-2\mathcal{N}^2\dot{t}=-2E. \tag{23}$$

Using Eqs. (22) and (23), the geodesic equations read

$$\dot{r}^2+V_{eff}(r)=0, \tag{24}$$

where

$$V_{eff}(r)=\frac{1}{\mathcal{M}^2(r)}\left(1+\frac{L^2}{\mathcal{A}^2(r)r^2}-\frac{E^2}{\mathcal{N}^2(r)}\right). \tag{25}$$

We require the following conditions for stars to have circular motions [10]:

$$\dot{r}=0, \quad \partial V_{eff}/\partial r=0, \quad \partial^2V_{eff}/\partial r^2>0. \tag{26}$$

Following the same procedure as in Refs. [10] and [11], we solve the above equations, express $\dot{\phi}$ and $\dot{t}$ as functions of metric coefficients, and have the formula for the rotation velocity

$$v_{rot}\equiv(\mathcal{A}r/\mathcal{N})(d\phi/dt)=\sqrt{\mathcal{N}'/\mathcal{N}(1/r+\mathcal{A}'/\mathcal{A})}. \tag{27}$$

In stable orbits of stars for $r_i<r\ll r_ie^{10^6}$ where $V''_{eff}(r)>0$ within the weak gravity approximation, around a galaxy (of mass M) at the center of which there is a GM coupled to gravity in the ST theory, we apply Eq. (27) to Eq. (19) and obtain their circular velocity,

$$(v_{rot})^2=\frac{8\pi G\eta^2}{1+\alpha_0^2}\left(\alpha_0^2+\frac{(1+\alpha_0^2)M}{8\pi\eta^2r}+\frac{\delta^2}{r^2}+\frac{2(1-2\alpha_0^2)\delta^4}{3r^4}\right). \tag{28}$$

For a GM with a very small size δ compared to astronomical scales [1], the last two terms in the above equation are negligible. Far away from the galactic core, $r\gg r_i$, the rotation velocity (28) approaches the following constant value:

$$\sqrt{8\pi G\eta^2\alpha_0^2/(1+\alpha_0^2)}\equiv v_{rot}^{(0)}. \tag{29}$$

Measurements of the RCs in spiral galaxies give us the asymptotic value of v_{rot}, 100–300 km/s $[v_{rot}/c\sim(3\times10^{-4})-10^{-3}]$ [2,12]. From this and Eq. (5) we have

$$\eta\sim(3\times10^{16})-10^{17}\ \text{GeV}, \tag{30}$$

which is the natural scale for grand unified theories. Estimations of the scale η were already made in previous works about the GM minimally coupled to gravity [1] and nonminimally coupled to gravity [2], and it is interesting that a similar result of the estimation is given when the astronomical constraint (5) for α_0 in a ST theory of gravity is saturated.

When we consider a typical galaxy of radius $r_o\sim30$ kpc and mass $10^{11}M_\odot$, the first and second terms in Eq. (28) are comparable for $r\sim r_o$. Therefore a velocity formula useful in the whole region where the galactic halo exists, $r_i<r<r_h$ (the radius of the galactic halo $r_h\simeq10r_o\simeq200$–400 kpc [13]), can be given by

$$v_{rot}=\sqrt{(v_{rot}^{(0)})^2+GM_{in}(r)/r}. \tag{31}$$

In the above equation the mass parameter M is substituted by the mass $M_{in}(r)$ of the sphere with a radius r, which will be briefly explained in the following section.

IV. SUMMARY AND DISCUSSION

In a ST theory of gravity we have determined a galactic spacetime at the center of which there is a GM. From the geodesic equation in the spacetime, we obtained the formula (31) for the rotation velocity of stars in the galactic halo, which has an extra constant value $v_{rot}^{(0)}$ in addition to the other ordinary terms [12].

In the weak gravity approximation, the metric component $\mathcal{N}^2$ in Eq. (20) with Eq. (19) can be given by $\mathcal{N}^2=1+2\Psi$, with the gravitational potential

$$\Psi=\Psi_s+\Psi_{out}=(v_{rot}^{(0)})^2\ln(r/r_i)-(GM/r), \tag{32}$$

up to $\mathcal{O}(1/r)$. As we can see in Eq. (16), the first term Ψ_s comes only from φ interaction with the GM, and it is also true when we consider the quasistatic source of the spherical mass distribution $\rho(r)$ and substitute the second term Ψ_{out} by the gravitational potential [14]

$$\Psi_{in}=-G\int d^3r'[\rho(r')/|\vec{r}-\vec{r'}|]. \tag{33}$$

Here the φ and g_{tt} contributions to Ψ_{in} are summed as shown in the expression [5] $G = G_*(1+\alpha_0^2)$. The square of the rotation velocity in Eq. (27) can be written as $(v_{rot})^2 \simeq \vec{r}\cdot\nabla\Psi$. Since $-\nabla\Psi_{in} = -\hat{r}GM_{in}(r)/r^2$ is the gravitational force of a sphere of radius r and mass $M_{in}(r) = 4\pi\int_0^r dr' r'^2 \rho(r')$ on a unit mass object at $\vec{r}$, we have Eq. (31).

It seems more plausible to explain the flatness (or even the rising part [15]) of the RCs, including the constant value $v_{rot}^{(0)}$. This originates from a $\ln(r/r_i)$ term of the massless scalar field φ contribution to $\tilde{g}_{tt}$ in Eq. (19), which is Ψ_s. Some other authors also found gravitational potentials similar to Ψ_s in various theories of gravity [16] and in a modified Newtonian dynamics model [17]. In the Brans-Dicke theory of gravity, the weak equivalence principle can be violated only by quantum correction and its possible violation is much smaller than in string theories [18]. Even if a GM induces only a deficit angle in Einstein's theory of gravity [1,4], its energy density, proportional to $1/r^2$, generates the $\ln(r/r_i)$ term in a metric component $\tilde{g}_{tt}$ in Brans-Dicke theory. The force $-\nabla\Psi_s$ responsible for the flatness of the rotation curves in the galactic halo region is a gravitational force derived from the metric $\tilde{g}_{\mu\nu}$ in the physical frame [7,8]. However if we use the more stringent bound on ST theories given by experiments around 1 AU range [19], then we will have a smallervalue of $v_{rot}^{(0)}$. To explain the flat rotation curves in that case, we need more contributions Ψ_{in} from other dark matter [12] in addition to the GM contributions.

By a numerical analysis of the gravitational field of a GM nonminimally coupled to gravity, Nucamendi *et al.* found that the RCs contain a relatively flat region [2]. When we look at Eqs. (28) and (31), it appears that the flatness of the RCs extends beyond the galactic halo. However, the formula (31) is valid for $r_i < r < r_h$, since the GM field (and φ_1) will vanish at distances larger than r_h due to interactions with the nearest topological defect such as an antimonopole [20], a cosmic string, and so on [17,21]. (For example, GM field lines can be absorbed into an antimonopole core.) These defects could be thought of as seeds of structure formation in the Universe [20]. If we perform numerical studies following Nucamendi *et al.* [2] and Banerjee *et al.* [3], then we can draw a more concrete conclusion about flat RCs beyond the weak gravity approximation.

ACKNOWLEDGMENTS

We thank Dr. P. Oh, Dr. J. Lee, Dr. S. Hong, and Dr. J. Yoon for useful discussions. This work was supported by the Soongsil University Research Fund.

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