Nonlinear electrodynamics and the acceleration of the Universe

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It is shown that the addition of a nonlinear term to the Lagrangian of the electromagnetic field yields a fluid with an asymptotically super-negative equation of state, causing an accelerated expansion of the Universe. Some general properties of nonlinear electromagnetism in cosmology are also discussed.

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Observations of the redshift of supernovae type Ia [1] and the cosmic microwave background [2] show that the Universe is undergoing a phase of accelerated expansion. The various possibilities that have been examined to account for this acceleration can be grouped in two classes. In the models in the first class, matter with unusual properties is added to the right-hand side of Einstein equations. Specifically, this matter (dubbed *dark energy*, although a more appropriate name may be *smooth tension* [3]) must have a large negative pressure and, different from the case of dark matter, it must not cluster with visible matter. Amongst the most popular candidates for dark energy, we can mention two. The first one is the cosmological constant Λ, which yields accelerated expansion due to the fact that $p_\Lambda = -\rho_\Lambda$. However, it is strongly disfavored by the (120 orders of magnitude) disagreement between the observed value and that naively predicted from quantum field theory [4,5], which implies a rather unnatural fine tunning. The second candidate is the so-called quintessence [6], the simplest model of which is composed by a scalar field [7] coupled to matter and under the influence of an *ad-hoc* self-interaction potential [8]. The acceleration occurs in a regime in which the potential energy takes over the kinetic energy. Although quintessence has been successful in describing accelerated expansion, some problems remain: the quintessence field has not been identified, and a derivation of the potential from first principles is still lacking.

The second class of models that try to account for accelerated expansion involve a modification of the dynamics of the gravitational field in the low-curvature regime. This modification can be achieved by considering (in five-dimensional scenarios) the effects of the bulk on the dynamics of gravitation on the brane [9], or by directly adding to the four-dimensional gravitational action terms with negative powers of the curvature scalar. An example of the latter type is given by the action [10] $S=(M_{\rm Pl}^2/2)\int\sqrt{-g}(R-\alpha^4/R)d^4x$, where α is a new fundamental parameter with units of mass. In this article a third class of models will be introduced, which is a hybrid between the two classes already mentioned. We shall assume that the action for the electromagnetic field is that of Maxwell with an extra term, namely,

$$S=\int \sqrt{-g}\left(-\frac{F}{4}+\frac{\gamma}{F}\right)d^4x, \tag{1}$$

where $F\equiv F_{\mu\nu}F^{\mu\nu}$. From the conceptual point of view, this phenomenological action has the advantage that it involves only the electromagnetic field, and does not invoke entities that have not been observed (like scalar fields) and/or speculative ideas (like higher-dimensions and brane worlds). At high values of the field invariant F, the dynamics will be that of Maxwell plus corrections which are regulated by the parameter γ, while at low values of F it is the $1/F$ term that dominates [11]. As shown in the Appendix, this modification of electromagnetism seems to be consistent with the observation for small enough values of γ. Notice also that the Lagrangian in S is gauge invariant. Hence charge conservation is guaranteed in this theory.

The action given in Eq. (1) is actually only an example of a more general class of actions for the electromagnetic field with Lagrangians that can be written as $\mathcal{L}=\Sigma_k c_k F^k$, where the sum may involve both positive and negative powers of F. We shall see that in certain cases the "electromagnetic fluid" described by this Lagrangian can be thought of as composed by several noninteracting fluids, each with an equation of state (EOS) of the type $p_i = w_i \rho_i$, allowing for negative values of some of the w_i.

Taking the electromagnetic (EM) field described by the action given in Eq. (1) as source of Einstein's equations, a toy model for the evolution of the Universe will be studied. It will be shown that this model displays accelerated expansion caused by the dominance of the nonlinear EM term over other forms of matter. Let us recall that under the parametrization $p=w\rho$ for the dark energy, the Universe will accelerate if $w<-1/3$. Values of w beyond $w=-1$, which corresponds to the cosmological constant, yield a super-negative EOS which violates the dominant energy condition. However, observation allows for $w<-1$ [12], so it is worthwhile considering models with this type of EOS parameter. Models with $w<-1$ in the literature are usually constructed with a scalar field with negative kinetic energy term [13], the so-called phantom field. The evolution of a universe dominated by this type of matter is very peculiar, ending in a *Big Rip* [14]: the energy density may grow without limit with time, and there is a singularity in the future. As we shall see, the nonlinear electromagnetic model studied here yields ordinary radiation plus a dark energy component with $w<-1$.

The effects of a nonlinear electromagnetic theory in a cosmological setting have been studied in several articles. The authors of [15] showed that nonlinear corrections coming from an Euler-Heisenberg-like Lagrangian can be important in the very early Universe, leading to the avoidance of the singularity. Some attention has been devoted to the dynamics of a universe governed by matter described by Born-Infeld theory [16]. In particular, it has been shown that under certain assumptions (as for instance the existence of a compactified space) a Born-Infeld field as a source yields accelerated expansion [16].

We shall study first some general properties of nonlinear electrodynamics in cosmology, and then pay attention to the specific case of the action given in Eq. (1). Let us start with the theory defined by the Lagrangian $\mathcal{L}=\Sigma_k c_k F^k$, where the sum can have both positive and negative powers of the field invariant. Due to the isotropy of the spatial sections of the Friedman-Robertson-Walker (FRW) model, an average procedure is needed if electromagnetic fields are to act as a source of gravity [17]. Let us define first the volumetric spatial average of a quantity X at the time t by

$$\overline{X}\equiv \lim_{V\to V_0}\frac{1}{V}\int X\sqrt{-g}d^3x, \tag{2}$$

where $V=\int\sqrt{-g}d^3x$ and V_0 is a sufficiently large time-dependent three-volume. In this notation, the electromagnetic field can act as a source for the FRW model if [22]

$$\overline{E_i}=0, \quad \overline{B_i}=0, \quad \overline{E_iB_j}=0, \tag{3}$$

$$\overline{E_iE_j}=-\tfrac{1}{3}E^2g_{ij}, \quad \overline{B_iB_j}=-\tfrac{1}{3}B^2g_{ij}. \tag{4}$$

With these conditions, the energy-momentum tensor of the EM field associated to Maxwell's Lagrangian can be written as

$$\overline{T}_{\mu\nu}=(\rho+p)v_\mu v_\nu-pg_{\mu\nu}, \tag{5}$$

where

$$\rho=3p=\frac{1}{2}(E^2+B^2).$$

Under the same assumptions, a general nonlinear Lagrangian yields the stress-energy tensor given by Eq. (5) with

$$\rho=-L-4E^2L_F, \quad p=L+\tfrac{4}{3}(E^2-2B^2)L_F, \tag{6}$$

where $L_F\equiv dL/dF$. As we shall see below, the case in which $E^2=0$ is the only relevant case in cosmology. It is straightforward to prove from Eq. (6) that when there is only a magnetic field, the fluid can be thought of as composed of a collection of noninteracting fluids indexed by k, each of which obeys the equation of state

$$p_k=(4k/3-1)\rho_k. \tag{7}$$

In particular, the fluid coming from Eq. (1) is composed of ordinary radiation with $p_{(1)}=\frac{1}{3}\rho_{(1)}$ and of another fluid with EOS $p_{(-1)}=-7/3\rho_{(-1)}$. It is precisely this component with negative pressure that may drive the acceleration of the Universe.

Let us remark that since we are assuming that $\overline{B_i}=0$, the magnetic field induces no directional effects in the sky, in accordance with the symmetries of the standard cosmological model.

From Einstein's equations, the acceleration of the Universe is related to its matter content by

$$3(\ddot{a}/a)=-\tfrac{1}{2}(\rho+3p). \tag{8}$$

In order to have an accelerated universe, the matter must satisfy the constraint $(\rho+3p)<0$. Assuming that the nonlinear electromagnetic field is the dominant source of gravity and using the quantities defined in Eq. (7) we can write $\rho+3p=2(L-4B^2L_F)$. Hence the constraint $(\rho+3p)<0$ translates into

$$L_F>L/4B^2. \tag{9}$$

It follows that any nonlinear electromagnetic theory that satisfies this inequality yields accelerated expansion under the conditions given in Eqs. (3) and (4).

As an aside, note that from Eq. (6),

$$\rho+p=-\tfrac{8}{3}(E^2+B^2)L_F. \tag{10}$$

Consequently, nonlinear electromagnetic theories that have a *state* for which $L_F=0$ may generate an effective cosmological constant.

We shall consider next the action given by Eq. (1) in the cosmological setting given by the FRW metric. The corresponding equations of motion (which reduce to ordinary electromagnetism when $\gamma=0$) are

$$[(1+4\gamma/F^2)F^{\mu\nu}]_{;\nu}=0. \tag{11}$$

From now on we shall assume that $E^2=0$, while there is a residual magnetic field characterized by $\overline{B_i}=0,\overline{B_iB_j}=-\frac{1}{3}B^2g_{ij}$. This assumption is consistent with the fact that the electric field will be screened by the charged primordial plasma, while the magnetic field lines will be frozen [18]. In this case,

$$\rho=(B^2/2)+(\mu^8/2)(1/B^2),$$

where we have set $\gamma\equiv-\mu^8$ [11]. From the conservation law $\dot{\rho}+3(\rho+p)(\dot{a}/a)=0$, and setting $a_0=1$ we get that $B=B_0/a^2$. Hence the evolution of the density with the scale factor is

$$\rho=\frac{B_0^2}{2}(1/a^4)+(\mu^8/2B_0^2)a^4. \tag{12}$$

For small a it is the ordinary radiation term that dominates. The $1/F$ term takes over only after $a=\sqrt{B_0}/\mu$, and grows without bound afterwards. In fact, the curvature scalar is $R=T^\mu_\mu=\rho-3p=(4\mu^8/B_0^2)a^4$, showing that there is a curvature singularity in the future of universes for which $a\to\infty$ (see Sec. II B).

Using the expression for the density in Eq. (8) gives $3(\ddot{a}/a)+(B_0^2/2)\,(1/a^4)-\frac{3}{2}\,(\mu^8/B_0^2)\,a^4=0$. To get a regime of accelerated expansion, we must have $(B_0^2/a^4)-3(\mu^8/B_0^2)\,a^4<0$, which implies that the Universe will accelerate for $a>a_c$, with $a_c=(B_0^4/3\mu^8)^{1/8}$. We have shown that when the matter content of the Universe is dominated by a nonlinear EM field with the EOM (11) the Universe enters a phase of accelerated expansion. The nonlinear theory depends upon the yet undetermined parameter μ. It is possible to get an order-of-magnitude estimation for this parameter as follows. Recent observations [19] suggest that the approximate current values for Ω_{de} (the ratio of the dark energy density to the critical density ρ_c) and the Hubble parameter are $\Omega_{de}\approx 0.70$ and $h\approx 0.70$. Assuming that the dark energy can be described by the nonlinear term in Eq. (1), we get that $\rho_{(-1)}\approx 0.70\rho_c$. In other words,

$$(\hbar^2\mu^8/B_0^2)\approx 1.40\rho_c. \tag{13}$$

From this equation, we can get an upper bound for $\hbar\mu^4$ if we take B_0 as that associated to the CMB radiation. This can be obtained from $\Omega_{rad}=2.47\times 10^{-5}h^{-2}$. It follows that $B_0^2=2.47\times 10^{-5}h^{-2}\rho_c$. Inserting this value in Eq. (13) we get

$$\hbar\mu^4\approx 3.74\times 10^{-28}\ \text{g/cm}^3. \tag{14}$$

This value of $\hbar\mu^4$ together with the results given in the appendix show that, in the case of a point charge, the corrections to Maxwell's electromagnetism coming from the nonlinear term in Eq. (1) are negligible.

The dynamics of the Universe with matter density given by Eq. (12) can be obtained qualitatively from the analysis of Einstein's equations. We shall be interested only in the early regime (where radiation with $p=1/3\rho$ drives the expansion), and in the late evolution (where the $1/F$ term is important). Consequently, dust-like matter will be disregarded. Equation $(\ddot{a}/a)=-(B_0^2/6)\,(1/a^4)+\frac{1}{2}\,(\mu^8/B_0^2)\,a^4$, implies that $\ddot{a}$ has a zero at $a=a_c$, while equation $(k/3)\,\rho=(\dot{a}/a)^2+\epsilon/a^2$where $k=8\pi G$, and $\epsilon=0,\pm 1$ depending on the three-section of the geometry, can be written as

$$\dot{a}^2+V(a)=-\epsilon, \tag{15}$$

where $V(a)=-(k/6)\,[B_0^2/a^2+(\mu^8/B_0^2)\,a^6]$. Equation (15) can be interpreted as describing the one-dimensional motion of a particle with energy $-\epsilon$ under the influence of the potential $V(a)$. The potential is always negative, and has a maximum at $a=a_c$.

The analysis of the flat and open cases shows that this system allows for universes that starting with a big bang at $a=0$, expand with negative acceleration up to $a=a_c$, and then run away to infinity with positive acceleration, the difference between the two being the value of the Hubble parameter at $a=a_c$. Note in particular that for $\epsilon=0$, the dynamics is invariant under the change $a\rightarrow 1/a$.

The closed case is more interesting due to the fact that

$$V(a_c)\equiv V_c=-\left(\tfrac{2}{3}\right)^{7/4}kB_0\mu^2 \tag{16}$$

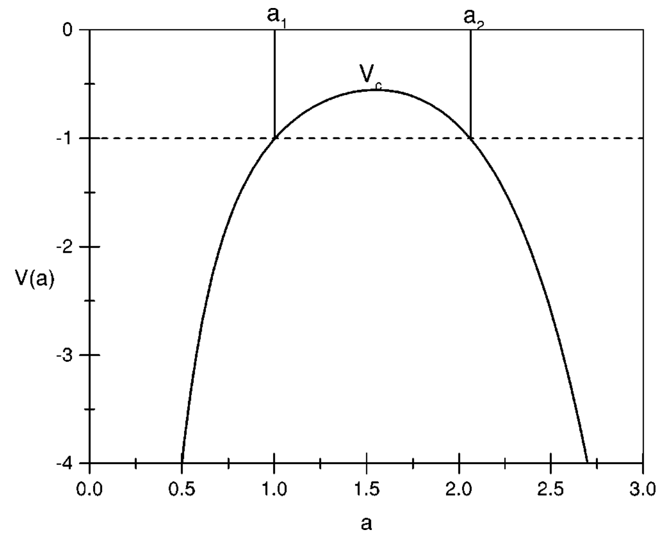

FIG. 1. Qualitative plot of the potential for the case $V_c>-1$. The region between a_1 and a_2 is classically forbidden.

can be $\leqslant -1$ [20]. We shall examine three separate cases.

Case (1): $V_c<-1$. This case displays the same behavior as that of the $\epsilon=1,-1$ cases: the Universe may start from a big bang, then a deaccelerated expansion follows up to $a=a_c$, and afterwards accelerated eternal expansion takes place.

Case (2): $V_c=-1$. The same as the previous case, but this time $H(a_c)=0$, so that $a=a_c$ is an unstable equilibrium point.

Case (3): $V_c>-1$. Two different behaviors are allowed (see Fig. 1). The Universe may start from $a=0$, expand up to a_1 and then re-collapse. Alternatively, it may start from infinity, collapse to $a=a_2$, and then re-expand to infinite.

The accelerated expansion suggests that the EOS parameter w of the dark energy may be smaller than -1, the limit usually imposed by theoretical arguments such as the dominant energy condition. The confirmation of these findings will have a profound impact for fundamental physics. In this article we have given a particular realization of a model that displays a time-dependent EOS, with a super-negative EOS parameter as a limit. Instead of using a negative kinetic energy term for a scalar field, the price to pay was to modify the action for the electromagnetic field by the addition of a nonlinear term that respects gauge invariance. As shown in the appendix, this modification is harmless in the case of a point charge [23]. The result of the modification in FRW cosmology is to accelerate the Universe, ending in a Big Rip, independent of the spatial section of the geometry (except for one particular case in the closed Universe, see Fig. 1).

Attempts are currently under way to reconstruct the EOS from observation [5]. This reconstruction implies the measurement of the *jerk* (third derivative of the scale factor) [21]. Although the toy model we analyzed here yields an EOS with a value ($\omega_{(-1)}=-7/3$) that seems to be rather disfavored by observation [5,14], it would be interesting to see if the addition of other nonlinear terms gives the chance of obtaining more realistic values for the EOS parameter.

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APPENDIX

The least we should require of any modification to Maxwell's equations is consistency with the case of a static electric field generated by a point charged particle. The EOM for a general nonlinear Lagrangian $L=L(F)$ are

$$\partial_\mu(L_F F^{\mu\nu})=J^\nu, \tag{A1}$$

where J^ν is an external current. It follows immediately that $\partial_\nu J^\nu=0$, showing again that charge is conserved in a theory with $L=L(F)$. In the case of the point charge, Eq. (A1) reduces to

$$r^2 L_F E(r)=\text{const.}$$

In the case of the Lagrangian given in Eq. (1) we get

$$E(r)^4+Q/r^2\,E(r)^3-\mu^8=0, \tag{A2}$$

where $Q=4\times$const, and we have set $\hbar=1$. Although this cubic equation could be solved exactly, we shall only need its solution for large and small values of r. For $r^2 \gg Q/\mu^2$ we get

$$E(r)=\mu^2-\tfrac{1}{4}\,(Q/r^2)+O[(Q/r^2)^2], \tag{A3}$$

while in the limit $r^2 \ll Q/\mu^2$,

$$E(r)=-\,(Q/r^2)\,(1-(\mu^2 r^2/Q)^4)+O((\mu^2 r^2/Q)^8). \tag{A4}$$

By taking derivatives of Eq. (A2) it can be shown that the function $E(r)$ has no extrema [24]. Hence, the modulus of the electric field monotonically decreases with increasing r, from an infinite value at the origin to a constant (nonzero but small) value at infinity. This situation is akin to that in the theory defined by the action $S=(M_{\rm P}^2/2)\int\sqrt{-g}(R-\alpha^4/R)d^4x$. It was shown in [10] that the static and spherically symmetric solution of this theory does not approach Minkowski asymptotically; it tends instead to (anti)-de Sitter spacetime. Analogous to the gravitational field in [10], by choosing the parameter μ small enough the electric field of a point particle in the nonlinear theory given by Eq. (1) will pick up only extremely tiny corrections, given by Eqs. (A3) and (A4).

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