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Moduli potentials in string compactifications with fluxes: Mapping the discretuum

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We find de Sitter and flat space solutions with all closed string moduli stabilized in four-dimensional supergravity theories derived from heterotic and type II string theories, and explain how all the previously known obstacles to finding such solutions can be removed. Further, we argue that if the compact manifold allows a large enough space of discrete topological choices then it is possible to tune the parameters of four-dimensional supergravity such that a hierarchy is created, and the solutions lie in the outer region of moduli space in which the compact volume is large in string units, the string coupling is weak, and string perturbation theory is valid. We show that at least two light chiral superfields are required for this scenario to work; however, one field is sufficient to obtain a minimum with an acceptably small and negative cosmological constant. We discuss the cosmological issues of the scenario and the possible role of anthropic considerations in choosing the vacuum of the theory. We conclude that the most likely stable vacua are in or near the central region of moduli space where string perturbation theory is not strictly valid, and that anthropic considerations cannot help much in choosing a vacuum.

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I. INTRODUCTION

Time was when string theory was expected to produce a unique four-dimensional theory. However, even though in ten or eleven dimensions superstring theory is perhaps unique in that the five perturbative formulations are expected to be different limits of one underlying theory, it was already realized in the mid 1980s that there are many different possibilities for supersymmetric vacua when the theory is compactified down to four dimensions. At that time it was thought that the only phenomenologically viable theory was the heterotic theory, so it was possible to argue that the gauge group had to be a subgroup of $E_8 \times E_8$ or $SO(32)$. Work done in the 1990s with F theory removed even this constraint, so that now it appears that one can find supersymmetric vacua with almost any gauge group up to a rank of $O(1000)$, as well as many different numbers of generations.

In order to get some perspective on the current state of the theory it is useful to recall the steps that have led us to the models in the so-called discretuum. We start with the basic theoretical conjecture (T) and then add the different experimental and observational inputs (E) that need to be used in order to get a model of the real world.

A. The saga of weakly coupled strings

T: *The assumption of* (*weakly interacting*) *quantized superstrings in a Lorentz invariant background.* This yields a startling outcome—the graviton (coupling precisely as expected from general relativity at low energies) as well as the quanta of gauge fields, thus giving us a viable candidate for a unified theory. However, space-time is ten dimensional. Of course, since only two of these dimensions need to be geometrical, the rest being contributions to the central charge of a superconformal field theory, this leaves open the possibility of a four-dimensional space-time.

$E1$: *Four dimensions.* Since the observed world is four dimensional and this fact does not emerge automatically from the basic conjecture, it has to be put in as an extra assumption. Thus, the topological criterion that the ten-dimensional space is of the form $R^4 \times M_6$, M_6 being some compact manifold or some abstract conformal field theory, was imposed, relegating all other solutions of the theory including the simplest one R^{10} to a theoretical limbo.

$E2$: $\mathcal{N} \leqslant 1$. Simple, for instance toroidal, compactifications yield 16 (or 32) supersymmetries in four dimensions which certainly cannot yield the chiral structure of the observed world. Thus an additional input, that only four supersymmetries survive, was added. So only internal manifolds such as Calabi-Yau (CY) manifolds (including their orbifold limits) were to be considered. The choice of such a solution is characterized by a number of parameters—the h_{12} complex structures and h_{11} Kahler structures of the manifold. There are arguments that the space of such manifolds is connected, although there may also be isolated points in this so-called super moduli space that correspond to nongeometrical compactifications such as asymmetric orbifolds.

$E3$: *No massless moduli.* The (super) moduli appear as massless four-dimensional (chiral super) fields in the low energy four-dimensional action that is supposed to describe the real world at scales below the string scale. However, they couple with gravitational strength to other fields including the standard model ones, and such fields are definitely ruled out by experiment since they affect Newtonian gravity at large distances. The same is true of the dilaton superfield whose ground state value sets the coupling strength. Thus,

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additional input is needed which would generate a potential for these moduli fields.

$E4$: *Supersymmetry is broken.* A moduli potential can be generated in many different ways. The earliest solution was to consider gaugino condensation in a gauge group [1–3] (see [4] for a recent review). Typically this yields a runaway potential for the moduli but if there is a direct product of gauge groups (obtained, say, by turning on discrete Wilson lines in the internal manifold) then one has the possibility of developing a critical point in the so-called "racetrack" models [5,6]. Similar effects can be obtained by considering brane instanton effects [7]. In addition, contributions to the potential can be generated by turning on fluxes in internal compact directions [2]. Often the minima are supersymmetric with a string scale negative cosmological constant (CC). However, the world is not supersymmetric and at best has broken supersymmetry (SUSY) with mass splitting at a scale $10^{-15}M_P$.

$E5$: *The cosmological constant is small and positive.* The world appears to have a positive (or perhaps zero) CC at a scale $10^{-120}M_P^4$. Recent work [8] has indicated that even though the natural scale of string theory $M\sim M_P$ it may still be possible to find a small positive CC.

$E6$ and $E7$: *Three generations and* $SU(3)\times SU(2)\times U(1)$. It may turn out that a string theory that satisfies all of the above criteria will be unique and result in three generations of chiral fermions with just the standard model gauge group. However, given the enormous number of possibilities, this seems unlikely and so we may need these two experimental inputs as well.

B. The discretuum

What then is left for string theory to predict? It is likely that the Yukawa couplings can be calculated once a model satisfying all the other criteria is found. Here, again, although the superpotential terms can be calculated exactly and will not be renormalized, the Kahler terms will acquire corrections, but perhaps these can be calculated perturbatively to a reasonable accuracy. However, even if this succeeds it is clear that we are far from a fundamental theory involving just some basic theoretical input(s).

Now the question arises as to what to make of the huge number of solutions that do not satisfy the observational inputs $E1$–$E7$. Do they exist as different universes? For instance, is there a ten-dimensional (noncompact) supersymmetric universe? If one uses only $E1$ then there is a continuous infinity to the power of the dimension of moduli space of supersymmetric compactifications corresponding to values of the moduli. These are perfectly valid solutions of perturbative string theory. Are they all to be included as Universes that actually exist? $E2$ selects a subset of the above but $E4$ and $E5$ give a new set that comes from giving a potential to the moduli.[1] It is this discrete set that is referred to in the literature as the "discretuum."

[1]Except when the minimum of the potential, the CC, is exactly zero in which case what is selected is a discrete subset of the set of vacua obtained after $E2$.

In supergravity (SUGRA) phenomenology after picking the field content and the gauge group one is left with three arbitrary functions. A realistic phenomenology with a zero or a tiny CC could be obtained by fine-tuning. In string theory there was no obvious mechanism that would allow for this fine-tuning, let alone finding a solution to the problem of generating a constant that is 120 orders of magnitude smaller than the string scale. Our interpretation of the work of Bousso and Polchinski [8] is that such fine-tuning is actually possible in string theory in spite of the quantization of the parameters (fluxes) in terms of the string scale.

In this paper we will consider both heterotic and type IIB weakly coupled string theories compactified on large volume 6D manifolds, and examine under what conditions one can get a small positive CC with hierarchically larger, but still parametrically small in string units, SUSY breaking. We will attempt to find such minima from the F terms of the $\mathcal{N}=1$ potential in racetrack type models. We feel that finding solutions for which SUSY is spontaneously broken through F terms is more reliable than invoking either explicit breaking [9] [Kachru-Kallosh-Linde-Trivedi (KKLT)] or D terms [10]. If there is only one light modulus as in the KKLT case we find that it is not possible to find a SUSY breaking minimum with zero or positive CC. However, with at least two light moduli such minima do exist. In general, we find that each of the vacua in the discretuum develops its own discretuum with several vacua, including some that have negative CC and unbroken SUSY. With one light modulus, however, it is possible to find a minimum in which SUSY is broken and the CC is negative although acceptably small.

We have not yet produced a concrete string model which realizes all the constraints. However, we do show that all known obstacles that were previously found in racetrack models (see, for example, [11]) can be removed in this framework by a choice of topological or geometrical properties of the compactification manifold. We explain along the way what the obstacles are and why previous attempts failed.

We will argue that all solutions in the discretuum that are in the outer region of moduli space, including ours, are not cosmologically viable—being subject to the overshoot problem first discussed in [12] and recently called appropriately the "bat from hell" problem [13]. We also discuss the possible application of the anthropic principle to choose among the variety of vacuua and find that it is not very useful. We expect that solutions in the central region of moduli space will not suffer from the cosmological overshoot problem.

II. MODULI POTENTIAL IN THE HETEROTIC STRING THEORY

A. The potential of the complex structure moduli and the dilaton

The first attempt at using fluxes and gaugino condensation to stabilize the moduli was that of Dine *et al.* [2] (for a recent discussion of this model, see [14]). The main argument for rejecting this model as a model of moduli stabilization had been the observation that flux is quantized in string units [15], and hence the dilaton is stabilized in an unrealistic strong coupling region. Here we reconsider the argument and show that it should be modified, and that it is possible to stabilize the dilaton at weak (or intermediate) coupling. This, as far as we know, could have been observed at the time that the original paper was written.

The ten-dimensional low energy effective action is reduced to a four-dimensional action using the ansatz

$$ds_{10}^2 = e^{-6u(x)} ds_4^2 + e^{2u(x)} g_{mn}^0 dy^m dy^n,$$

where m,n go over the dimensions of the internal space which is taken to be a CY threefold X. For simplicity we will assume that X has only one Kahler modulus but may have an arbitrary number of complex structure moduli. The four-dimensional dilaton φ and the four-dimensional volume scalar ρ are related to the ten-dimensional dilaton ϕ and the modulus u by $\varphi = \phi/2 - 6u$ and $\rho = \phi/2 + 2u$. The chiral superfields S,T are then defined by $S = e^{-\varphi} + ia$ and $T = e^{\rho} + ib$ where a,b are the corresponding axions. The argument proceeds from the observation that the low energy ten-dimensional effective action for the heterotic string contains the following contribution that can be interpreted as an effective potential in four dimensions for the moduli:[2]

$$V_{action} = \frac{1}{4\alpha'^4} \frac{1}{S_R T_R^3} \int_X \left(H_3 - \frac{\alpha'}{16} T_R^{3/2} S_R^{3/2} T_3 \right) \wedge *_6 \left(H_3 - \frac{\alpha'}{16} T_R^{3/2} S_R^{3/2} T_3 \right). \tag{1}$$

In the above, H_3 is the Neveu-Schwarz–Neveu-Schwarz (NSNS) three-form flux which is taken to be nonzero only on X, and T_3 is a fermionic bilinear three-form (not to be confused with the chiral superfield T) which is assumed to be represented upon gaugino condensation by

$$T_3 = 2U\Omega + \text{c.c.}, \tag{2}$$

where $U = \langle \mathrm{tr}\, \lambda\lambda \rangle$ is an effective low energy scalar field representing the gaugino condensate and Ω is the holomorphic (3,0)-form on X [2]. S_R, T_R stand for the real parts of S,T, respectively. The dynamics of $U$is governed by the Veneziano-Yankielowicz (VY) superpotential [16]

$$W_{np} = \frac{U}{4} \left[f + \frac{C(G)}{8\pi^2} \ln(\alpha'^{3/2} U) \right], \tag{3}$$

where f is the gauge coupling function and $C(G)$ is the dual Coxeter number of the gauge group, which we have assumed here to be simple. We have also assumed that the model does not contain matter that is charged under the gauge group. Classically $f = S$, so extremizing the VY effective superpotential one finds $U = \alpha'^{-3/2} e^{-[8\pi^2/C(G)]S - 1}$.[3] We also note for future reference that the nonperturbative superpotential for the modulus S that is generated is

$$W_{np} = -C(G)\mu^3 e^{-[8\pi^2/C(G)]S - 1}, \tag{4}$$

where $\mu^3 = 1/32\pi^2 \alpha'^{3/2}$.

Apart from Chern-Simons terms which are $O(\alpha')$ corrections, H_3 is closed, and the *classical* equation of motion implies that it is co-closed as well. Thus it may be expanded in terms of a basis of harmonic three forms on X,

$$H = a\Omega + b^\alpha \chi_\alpha + \overline{a\Omega} + \overline{b^{\bar\beta}} \overline{\chi_{\bar\beta}}, \tag{5}$$

where the sums over $\alpha, \bar\beta$ go over $1, \ldots, h_{12}$, the dimension of the complex structure moduli space of X. The volume of the CY manifold can be expressed in terms of Ω by $v \equiv i \int \Omega \wedge \bar\Omega$, and the metric on the moduli space is given by $G_{\alpha\bar\beta} = -(i/v) \int \chi_\alpha \wedge \chi_{\bar\beta}$. Using the expansion (5) and the expression for T_3 in Eq. (1), we get

$$V_{action} = \frac{1}{2\alpha'^3} \frac{v}{S_R T_R^3} \left[\left| a - \frac{\alpha'}{8} \frac{3\pi^2}{C_G} T_R^3 S_R^{1/2} W_{np} \right|^2 + G_{\alpha\bar\beta} b^\alpha b^{\bar\beta} \right]. \tag{6}$$

We now wish to express the action (6) in the $N=1$ SUGRA form

$$V_{SUGRA} = e^K (K^{i\bar j} F_i F_{\bar j} - 3|W|^2). \tag{7}$$

From the classical action and the properties of the manifold X the Kahler potential is found to be

$$K = -\ln(S+\bar S) - 3\ln(T+\bar T) - \ln\left(\frac{v}{4\alpha'^3} \right). \tag{8}$$

The superpotential is the sum of two contributions. One contribution comes from the flux superpotential of Gukov, Vafa, and Witten [17], which is given by

$$W_{flux} = \frac{4}{\alpha'^4} \int H \wedge \Omega = -\frac{4i}{\alpha'^4} av. \tag{9}$$

To obtain the second equality we have used the expansion (5). In the case that W_{np} vanishes,W_{flux} and the Kahler potential (8) result in the potential coming from the *classical* action (1) with W_{np} set to zero. Now, if gaugino condensation does occur and W_{np} does not vanish, the total superpotential is the sum of the flux superpotential (9) and the gaugino condensate superpotential given by (4),

$$W_{tot} = W_{flux} + W_{np}.$$

Computing Eq. (7) with W_{tot} gives

[2]In this section we use the same normalization conventions for the classical action as in [14].

[3]Note that the precise normalization depends on the cutoff scale chosen in Eq. (3). Here we have chosen it to be the string scale.

$$V_{SUGRA}=\frac{\alpha'^4}{32T_R^3S_R v}\left\{\left|\frac{4i}{\alpha'^4}\overline{av}+\left(1+\frac{16\pi^2}{C(G)}S_R\right)W_{np}\right|^2 + G^{\alpha\bar\beta}\left(\frac{4i}{\alpha'}b^{\bar\gamma}G_{\alpha\bar\gamma}\bar v+\partial_\alpha KW_{np}\right)\times\left(\frac{4i}{\alpha'}b^{\bar\beta}G_{\delta\bar\beta}\bar v+\partial_{\bar\beta}KW_{np}\right)\right\}. \tag{10}$$

Here the differentiation ∂_α is with respect to the complex structure moduli.

Comparing Eq. (6) to Eq. (10) we see that there is agreement only when we set $W_{np}=0$, i.e., only at the classical level.[4] This should not be surprising. One should not expect to obtain the correct nonperturbative four-dimensional action from the *classical* ten-dimensional action. The difference between Eqs. (1) and (10) is significant. If Eq. (1) had been the correct formula for the potential then it would be impossible to find an $O(1)$ solution for the four-dimensional dilaton S and hence a weak 4D gauge coupling. This follows from integrating the relation between H and T_3 that is obtained at the minimum of V_{action} in Eq. (1) over a three-cycle on X and using Eq. (2) for T_3 and the quantization of the three-form field $H=dB$ that was first observed in [15] (for a recent discussion see [14]). However, this is not the correct relation at the minimum since the $\mathcal{N}=1$ SUGRA potential is Eq. (10), and the correct equation is the vanishing of the F term for the dilaton,

$$\frac{4i}{\alpha'^4}\overline{av}+\left(1+\frac{16\pi^2}{C(G)}S_R\right)W_{np}=0. \tag{11}$$

Substituting the explicit expression for W_{np} from Eq. (4) we get

$$\frac{4i}{\alpha'^4}\overline{av}-\frac{C(G)}{32\pi^2\alpha'^{3/2}}\left(1+\frac{16\pi^2}{C(G)}S_R\right)e^{-[(8\pi^2/C(G))S+1]}=0. \tag{12}$$

Thus, getting a weak coupling solution with S_R of order a few amounts to finding a small value in the "discretuum" for the flux superpotential, specifically for the product av. This is similar to the corresponding type IIB case discussed by KKLT, and, as in that case, one expects such values to exist in CY manifolds with large numbers of complex structures. This mechanism would be an alternative to the proposal of [14] where the Chern-Simons contributions to H [3] were included and integrated over spaces where the corresponding invariants are fractional.[5]

In addition to Eq. (11), at the potential minimum the F term of the complex structure moduli needs to vanish,

$$\frac{4i}{\alpha'}b^{\bar\gamma}G_{\alpha\bar\gamma}\bar v+\partial_\alpha KW_{np}=0. \tag{13}$$

Using Eq. (11) and the relation $b^{\bar\gamma}G_{\alpha\bar\gamma}=b_\alpha$, we get,

$$\partial_\alpha K=\alpha'^3\{1+([16\pi^2/C(G)])S_R\}\frac{b_\alpha}{\bar a}. \tag{14}$$

Recall that the derivation here is with respect to the complex structure moduli. Thus, with generic fluxes the potential (6) fixes all the complex structure moduli in addition to the dilaton S. Note that, unlike in the case of type IIB, here in order to get a solution we need the nonperturbative term W_{np}. From Eq. (14) it is clear that the relation $b_\alpha=0$ imposed in [14] [and which would have been obtained if we had used Eq. (6) rather than Eq. (10)] is valid only if $a=0$ or $S_R\to\infty$.

B. The potential of the Kahler moduli

The Kahler moduli, and in particular the volume modulus T which is present in any compactification, are clearly not fixed in the models that we have discussed so far. Additionally, SUSY is generically broken since

$$F_T=K_TW=-\frac{3}{2T_R}\left(\frac{4i}{\alpha'^4}\overline{av}+W_{np}\right) =\frac{6i}{T_R\alpha'^4}\frac{2+[16\pi^2/C(G)]S_R}{1+[16\pi^2/C(G)]S_R}\overline{av} \tag{15}$$

is nonzero for a finite value of T_R and generic fluxes. In fact, unbroken SUSY ($F_T=0$) occurs only in the decompactification limit $T_R\to\infty$, as long as the flux superpotential (in effect a) is nonzero. The situation here is somewhat different from that in type IIB, where S and the complex structure moduli were fixed classically, i.e., without any nonperturbative superpotentials [18], and where even though generically SUSY was broken there were flux configurations that preserved it.

We would like to consider possible modifications of the models so that they will stabilize all moduli, including the Kahler moduli. For simplicity we discuss now the case of only one Kahler modulus, the volume.

A dependence on the T modulus can arise from threshold effects which have been calculated for various compactifications. We will first consider the T dependent contribution to the gauge function coming from anomaly considerations. Then we will make some remarks about the case when the compact manifold X is an orbifold where the complete one-loop string theory correction has been worked out.

There are now two possibilities for analyzing the theory with all moduli stabilized.

The first possibility is to integrate out the complex structure moduli and the dilaton by arguing that they are fixed at the minimum of the potential (10) at a high scale. This would

[4]Aspects of this difference have been noticed already in [2].

[5]In this reference it was argued that one is forced to do this—based on the constraint mentioned above from using the form of the potential coming from the ten-dimensional action. Here we have seen that this constraint is not the appropriate one.

generically be the case. In fact without threshold corrections (i.e., just using the classical relation $f=S$) the T modulus would have zero mass, and the other moduli would have string scale masses.[6] In this respect the situation is similar to the type IIB case. Then by incorporating threshold corrections, we get a theory for the single light field T with a constant in the superpotential (W_{flux} evaluated at the minimum of the potential). As we have shown elsewhere [19] the modulus in the SUSY breaking direction needs to be light but there is no such requirement on the other moduli.

The second possibility is to integrate out the complex structure moduli at a high scale but arrange by a choice of a point in the discretuum to have the dilaton light so it is not integrated out at this stage.

It is plausible that within the discretuum there are choices of CY manifolds and fluxes where this is justified. So we solve the equations $\partial_\alpha V=0$ to express the complex structure moduli in terms of S and T. It is not obvious that this can be done holomorphically but since SUSY cannot be broken by this procedure it must be the case that the result is an $\mathcal{N}=1$ SUGRA with just S and T moduli. Then after including threshold corrections (that would introduce T dependence in the superpotential) we would be left with a two-moduli minimization problem. Since now we do not have a positive definite potential, finding actual solutions is complicated but can be done. However, in this case one would in general expect the Kahler potential to be different from the naive classical form obtained by just suppressing the complex structure dependent part of K.

The complete minimization problem (if none of the moduli are integrated out at a high scale) is prohibitively complicated. But as an alternative to the previous possibility we can consider the case where $h_{12}=0$ as in the original paper [2]. In this case obviously we cannot use cancellations among different three-cycles to get a small value for W_{flux} and we would have to resort to the mechanism of [14], which uses the Chern-Simons terms with the classical Kahler potential to get realistic examples with two light moduli.

We will show in Sec. IV that within models with only one light modulus it is impossible to get a true minimum of the potential with a zero or positive CC from the F terms. With two light moduli we have found examples where such minima exist. Thus, only the last two cases will lead to models with all moduli stabilized with a positive or zero CC.

1. Threshold corrections from Green-Schwarz terms

Regardless of the compactification manifold there are some one-loop corrections that can be computed due to the existence of the Green-Schwarz anomaly cancellation mechanism. As pointed out by Banks and Dine [20] (see also [14] for a recent discussion) from reduction of the $B\wedge X_8$ term in the ten-dimensional action (where X_8 is a certain polynomial in the gauge and curvature two-forms), it is possible to see that the gauge coupling function(s) takes the form $f_i=S+\beta_i T$ where the β_i are numbers determined by the topology of the gauge bundle and the tangent bundle of X. Let us also assume that X has a nontrivial fundamental group so that we can turn on discrete Wilson lines to break the original ten-dimensional group $E_8\times E_8$ or $SO(32)$ to a product of several simple groups. This adds another layer of discrete choices in the discretuum. Then the superpotential arising from gaugino condensation (4) takes the form

$$W_{np}=-\sum_i C(G_i)\mu^3 e^{-[8\pi^2/C(G_i)](S+\beta_i T)-1}. \tag{16}$$

The potential is then given by Eq. (10) with the above expression for W_{np} and a term

$$\Delta V_{SUGRA}=\frac{\alpha'^4}{32T_R^3 S_R v}\frac{4T_R^2}{3}\Bigg(|\partial_T W_{np}|^2 - \frac{3}{T_R}\mathrm{Re}[\partial_T W_{np}(\overline{W}_{flux}+\overline{W}_{np})]\Bigg). \tag{17}$$

If the fluxes, gauge group parameters, and various topological numbers are such that the dilaton and the complex structure are heavy, then they can be integrated out by setting the expression (10) to zero, and then effectively we have a potential for the modulus T given by Eq. (17) with S and the complex structure moduli fixed. As we have already mentioned, in this case with only one remaining light modulus it is not possible to find de Sitter (dS) or Poincaré minima. The general situation is of course prohibitively complicated. Thus we will focus on a situation where only the complex structure moduli (assumed heavy) are integrated out leaving two light moduli S,T. Alternatively, we could have considered a manifold X with two Kahler moduli with S and the complex structure moduli integrated out at a high scale.

2. Modular invariance for orbifolds

Another source of T dependence (at least in orbifold compactifications) comes from requiring modular invariance under $M:T\rightarrow(aT-ib)/(icT+d)$, $a,\ldots,d\in\mathcal{Z}$ [21] as might be expected from T duality (of course, in this case there would be three Kahler moduli, but for the sake of simplicity we will identify them).

Then (assuming that the complex structure moduli have been integrated out at a high scale) we take

$$K=-3\ln(T+\overline{T})-\ln(S+\overline{S}), \tag{18}$$

$$W=\left(c+\sum d_i e^{-8\pi^2 S/C(G_i)}\right)\Big/\eta(T)^6 \equiv \omega(S)/\eta(T)^6, \tag{19}$$

where $\eta(T)=e^{-\pi T/12}\prod_n(1-e^{-2\pi n T})$ is the Dedekind eta function. We have assumed here that the Kahler potential is equal to its classical form. The constant in the superpotential arises from the H flux as in Eq. (9). The Kahler invariant combination of K,W and hence the potential is M invariant.

The potential resulting from K and W of Eqs. (18), (19) is

[6]A caveat noticed by Michael Dine is that the mass matrix of moduli is a large matrix, and therefore it can have some particularly small or large eigenvalues.

$$V=\frac{|\eta(T)|^{-12}}{2S_R(2T_R)^3}\left\{|2S_R\omega_S-\omega|^2+\left(\frac{3T_R^2}{\pi^2}|\hat{G}_2|^2-3\right)|\omega|^2\right\}, \tag{20}$$

where $\hat{G}_2=-(\pi/T_R+4\pi\eta^{-1}\partial\eta/\partial T)$ is a modular function of weight 2. The potential has SUSY extrema at

$$2S_RW_S-W(S)=0, \tag{21}$$

$$\hat{G}_2=0. \tag{22}$$

With an appropriate choice of value of c in the superpotential (19) there would be a solution for S perhaps even at weak coupling, and the corresponding Hessian in the S direction is positive definite as discussed in Sec. IV. However, the zeros of G_2 (i.e., $T=1,\ e^{i\pi/6},0$) are saddle points ($T=1$) or maxima in the T direction. In addition, there is a true minimum (again, a result of a numerical calculation), at $T=1.2$ independently of the value of S at the minimum. At this point the volume of the compact manifold is not large in string units, and hence we may expect large α' corrections to this, and the solution is not under complete control.

3. Threshold corrections for orbifolds

It is not clear that in the presence of fluxes the theory is modular invariant. In fact, if one strictly follows the logic as in the type IIB case (discussed by KKLT) what one gets is a superpotential

$$W=c+\sum d_a e^{-3kS/2\beta_a}/\eta(T)^6\equiv c+\omega(S)/\eta(T)^6,$$

where $c=\int H\wedge\Omega$ evaluated at the minimum of the classical flux potential at which generically all complex structure moduli will be fixed. In computing the gaugino condensate from Eq. (3) we have used the (Wilsonian—hence holomorphic) gauge coupling function

$$f=kS+\frac{1}{4\pi^2}\left(\frac{1}{2}b'-k\delta^{GS}\right)\ln\eta(T)^2,$$

which comes from calculation of the threshold effects in orbifolds [22]. In contrast to this, the T dependence of the first term in W in Eq. (19) comes from the *requirement* of M invariance in the potential.

However, now (with the same K as before) there is no modular invariance. The potential is

$$V=\frac{|\eta(T)|^{-12}}{2S_R(2T_R)^3}\Bigg\{|2S_R\omega_S-\omega-c\,\eta^6(T)|^2+3\left|\frac{T_R}{\pi}\hat{G}_2\omega+c\,\eta^6(T)\right|^2-3|\omega+c\,\eta^6(T)|^2\Bigg\}.$$

This potential for two moduli is we believe the correct replacement of the formula (20). It is a potential for two (possibly light) moduli and we will discuss the minima of such potentials in Sec. IV.

4. Non-Kahler manifolds

It is well known that in the presence of H flux the heterotic string does not admit supersymmetric compactifications on Kahler manifolds [23].[7] Such compactifications are possible, however, on non-Kahler manifolds, and recently there have been a number of papers on this subject (see, for example, [24,25] and references therein). The non-Kahler manifolds are not Ricci flat and so there are in general two contributions to the classical potential—one from the fluxes and one from the curvature. Actually, one might think that this is the case even if the internal space is taken to be conformally CY. But in that case, as we will see below, there is no solution unless the conformal factor is trivial and the flux is zero.

The metric of the ten-dimensional space is parametrized as

$$ds^2=e^{2\omega(y)-6u(x)}\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu+e^{-2\omega(y)+2u(x)}\tilde{g}_{mn}(x,y)dy^m dy^n. \tag{23}$$

Then from the ten-dimensional heterotic action we have the *classical* potential

$$V=-\frac{1}{S_R}\int d^6y\sqrt{\tilde{g}^{(6)}(y)}\left[\frac{1}{T_R^2}[\tilde{R}^{(6)}-8(\tilde{\partial}_m\omega)^2]-\frac{e^{4\omega(y)}}{12T_R^3}\tilde{H}_3^2\right].$$

This potential is a runaway potential in S and some quantum (or stringy) effect such as the gaugino condensate term discussed earlier is needed to stabilize it. On the other hand for T the situation is different. If $\int d^6y\sqrt{\tilde{g}^{(6)}(y)}\tilde{R}^{(6)}>8\int d^6y\sqrt{\tilde{g}^{(6)}(y)}(\tilde{\partial}_m\omega)^2$, then it seems that it is possible to classically stabilize the T modulus. Of course, if $\tilde{g}$ metric is CY as assumed in the previous subsections, then $\tilde{R}=0$ and there is no extremum point for T either.

It has been suggested that the potential (in the non-Kahler case) can be expressed in terms of a superpotential $W=\int(H-idJ)\wedge\Omega$ where J is the Kahler form ([24,25] and references therein). However it is not clear what the Kahler potential is, and it is not known how to express the potential coming from the ten-dimensional action in the $\mathcal{N}=1$ SUGRA form. Once this is done then we will have a situation which is dual to the type IIB case of Giddings, Kachru, and Polchinski (GKP) with the S modulus being exchanged with the T modulus and the former being stabilized by invoking nonperturbative effects.[8] We leave further discussion of these issues to future work.

[7] This can be seen from the discussion after Eq. (15).

[8] We mention in passing that in the above mentioned references it is argued that the T modulus stabilization is a stringy effect needing the incorporation of α' corrections. We are somewhat puzzled by this since the above argument does not require any such corrections.

III. MODULI POTENTIAL IN TYPE IIB STRING THEORY

In type IIB string theory it was shown (GKP) [18] that all the complex structure moduli and the dilaton can be stabilized by an appropriate choice of fluxes. The resulting 4D models are of the no scale type. An important question is whether the Kahler moduli can be stabilized with SUSY broken, in a Poincaré or dS background.

A. Review of the proposals to stabilize Kahler moduli

Let us first briefly review the proposals of KKLT and Burgess, Kallosh, and Quevedo (BKQ) [10]. Both of these constructions start with the potential for the complex structure moduli and the dilaton given by GKP.[9] This is a classical $N=1$ SUGRA potential which can be obtained by considering ten-dimensional low energy type IIB theory compactified on a CY orientifold with D3-branes and D7-branes—essentially a limit of an *F*-theory construction.

The metric is taken to be Eq. (23) where $e^{4u}=T_R$ is the real part of the Kahler modulus (volume modulus) which sets the overall size of the internal space and e^{ω} is a warp factor which effectively changes the scale of four-dimensional physics at different points on the internal manifold. Additionally, we impose the constraint $\partial_\mu \det \tilde{g}_{mn}=0$ on the $\tilde{g}_{mn}$ metric which can in turn be parametrized in terms of the other Kahler moduli as well as the complex structure moduli. GKP considered the case where ten-dimensional space is compactified on a CY manifold with only one Kahler modulus but an arbitrary number of complex structure moduli. They derived a potential for these moduli and the dilaton[10]

$$V=\int_X d^6y\sqrt{\tilde{g}^{(6)}}\,\frac{e^{4\omega(y)-12u(x)}}{24\tau_I}|iG_3\widetilde{-*}_6G_3|^2. \tag{24}$$

Here $G_3=F_3-\tau H_3$, H_3 is the NS three-form flux of type IIB, F_3 is the Ramond-Ramond three-form flux, and $\tau=C_0+ie^{-\phi}$ is the complex axion dilaton field. The integration is over the CY manifold *X*, and we have set $2\kappa_{10}^2=1$. The tilde over the absolute value means that the Hodge dual and the tensor contractions are evaluated with the metric $\tilde{g}_{mn}$. The potential (24) can be derived using the standard SUGRA form from a Kahler potential *K* and a superpotential *W* given by

$$K=-\ln[-i(\tau-\bar{\tau})]-3\ln(T+\bar{T})-\ln\left(i\int_X\Omega\wedge\bar{\Omega}\right), \tag{25}$$

$$W=8\int_X G_3\wedge\Omega, \tag{26}$$

[9] We use the same conventions as GKP in this section.

[10] Strictly speaking, this derivation is valid only when the warp factor is trivial [26]—the solutions though are valid even for nontrivial ω.

where Ω is the holomorphic three form on the CY manifold *X*. The potential is clearly positive definite and is of the no-scale form: at the minimum of the complex structure moduli and the dilaton the potential vanishes so that the scale *T* is undermined. At this point the fluxes must satisfy the imaginary self-duality condition $iG_3=*_6G_3$ and SUSY is broken if $W=c\neq 0$ at this point.

To stabilize the volume modulus one may introduce nonperturbative contributions to the superpotential, coming for instance from gaugino condensation in the gauge theory on the stack of D7-branes wrapping a four-cycle (with betti number $b_1=0$) in the internal manifold, as suggested by KKLT. In this case it is easily seen that the corresponding gauge coupling function of the super-Yang-Mills theory is given by $f=T$ so that by standard arguments (reviewed in the previous section) a superpotential for this modulus is generated—thus giving a total superpotential

$$W=c-C(G)\mu^3e^{-8\pi^2T/C(G)}. \tag{27}$$

With the (classical) Kahler potential (25) the resulting potential has a single negative minimum at which SUSY is preserved, rather than being broken as before.

The KKLT proposal is to add the contribution of a $\bar{\mathrm{D}}_3$-brane to this four-dimensional effective action. The anti-D-brane gives a positive contribution to the potential,

$$\delta V=\frac{D}{T_R^3},$$

where *D* is positive and proportional to the $\bar{D}_3$ tension. KKLT add the $\bar{\mathrm{D}}$-brane to the four-dimensional effective action; however, the $\bar{\mathrm{D}}$-branes, like the D-branes, are string theoretic objects and should be added to the classical ten-dimensional theory. There does not seem to be a reason to add the D-branes to the ten-dimensional action, as GKP do, and not the antibranes. However, if both the branes and antibranes are treated in the same manner, the classical potential becomes

$$V=\int d^6y\sqrt{\tilde{g}^{(6)}}\,\frac{e^{4\omega(y)-12u(x)}}{24\tau_I}|iG_3\widetilde{-*}_6G_3|^2 +2e^{-12u(x)}\sum_{\bar{D}} T_3e^{4\omega(y^{\bar{D}})}. \tag{28}$$

Note that T_3 is the brane tension, not to be confused with the gaugino condensation field discussed in the previous section or with the *T* modulus. The contribution of the anti-D-branes is local and therefore their contribution is determined by the warp factor at their positions $y^{\bar{D}}$.

If we follow KKLT and integrate out (classically) the complex structure moduli and the dilaton, we are left with an effective four-dimensional theory with a potential for the volume modulus

$$V=\frac{2}{T_R^3}\sum_{\bar{D}} T_3 e^{4\omega(y^{\bar{D}})}. \quad (29)$$

However, this is not a four-dimensional SUGRA theory any more. From the four-dimensional standpoint SUSY is explicitly broken by the anti-D-branes, as is evident from the term (29) in the potential. Moreover, it is a runaway potential which pushes the theory toward the decompactification limit $T_R \to \infty$. In this limit ten-dimensional SUSY will be restored. This behavior is reminiscent of what happens with the Scherk-Schwarz mechanism where a runaway potential is generated for a modulus, although in that case the sign is opposite to that in the above.

Since in the resulting four-dimensional theory SUSY is explicitly broken, it is no longer possible to derive the moduli potential from a superpotential. Hence it is unclear how the addition of a nonperturbative contribution (coming, say, from gaugino condensation) to the Gukov-Vafa-Witten (GVW) superpotential evaluated at the minimum of the complex structure moduli and the dilaton potential, can be justified. A possible consistent derivation would be possible if the term (29) can be interpreted as a D term. However, it is unclear how this can be done in this case. In particular, the D-term breaking discussed by BKQ [10] has a very different structure.

In BKQ, SUSY is broken by turning on an electric flux E on D7-branes which can be interpreted in $\mathcal{N}=1$ SUGRA context as a $U(1)$ D term. This then gives (after integrating out the complex structure moduli and the dilaton) a potential of the standard D-term form

$$V=g_{YM}^2\frac{D^2}{2}=\frac{2\pi}{T_R}\left(\frac{E}{T_R}+\sum q_I|Q_I|^2\right)^2,$$

where the Q_I are any additional massless matter fields which are charged under the gauge field on the D7-branes with charges q_I.

There are some uncertainties in this scenario. Generically, charged massless matter exists and acquires vacuum expectation values so as to set the D term to zero. This is exactly what was realized in the context of the heterotic string (see, for example, the discussion in Sec. 18.7 of [27]). However, in [10] it was argued that there are special situations in which such massless matter is absent, and the D-term contribution is not canceled. This seems to require a certain open string modulus to be fixed at a nonzero value but it is unclear how this is done. In addition, it should be noted that this scenario implies the existence of an anomaly in the $U(1)$ group since the D-term interpretation of the E^2/T_R^3 term depends on gauging the PQ symmetry associated with T_I, the axionic partner of T_R. This anomaly would need to be canceled by some chiral fermions having $\mathrm{tr}\, Q \neq 0$. The models of [10] do not have such fermions, though it is possible that such models can be constructed.

B. Racetrack models for Kahler moduli

In view of the uncertainties associated with the proposals that we have just described, we will examine another possibility, which in some sense is a more conservative one, for finding a positive or vanishing minimum as well as stabilizing the volume modulus. We will consider field theoretic nonperturbative effects in the superpotential incorporating multiple gaugino condensates in the spirit of the old "racetrack" models, thus generalizing the analysis of KKLT.

If we use only the ingredients of GKP without the antibranes and without turning on fluxes on the branes, the resulting effective four-dimensional theory is an $\mathcal{N}=1$ SUGRA. In this case it is meaningful to add nonperturbative contributions to the superpotential. Multiple gaugino condensates arise when the gauge group is broken by turning on discrete Wilson lines on the four-cycle of the CY manifold which is wrapped by the D7-branes. Note that we have added another discrete choice to the discretuum. The additional layer increases the number of vacua in a way that depends on the gauge group and the desired pattern of breaking.

It is likely that there will be some corrections to the Kahler potential which we will ignore for the moment, since as long as they are small their exact form is not particularly important to us. The SUGRA potential obtained with all the ingredients mentioned above is not necessarily a nonnegative potential, and it may have both positive and negative minima. There does not seem to be a general argument which says that positive or vanishing minima are somehow excluded. To rule out the positive minima on the basis of a generalization of the classical no-go theorem would require one to show that there is a ten-dimensional action which incorporates the nonperturbative terms *and* satisfies the strong energy condition. As far as we know such an action does not exist, and *a priori* there is no reason to dismiss the possibility of a dS or Minkowski minimum for the $\mathcal{N}=1$ SUGRA potential in the case that we have discussed.

As in the heterotic case, there are two possibilities which are feasible to analyze.

The one-light-modulus case. Here we integrate out (classically) the complex structure moduli and the dilaton, both of which generically will have string scale masses. Then by adding the gaugino condensate terms to the constant superpotential coming from the flux we are left with a potential for T. With one condensate it is easy to see that the only minimum is an anti–de Sitter (AdS) one, and that SUSY is preserved. On the other hand, with more than one condensate one might have expected to find dS or flat space minima. We will show that this is impossible.[11]

Two light moduli. With nongeneric fluxes it should be possible to keep the dilaton light while the complex structure moduli will still have string scale masses. Alternatively, we could consider compactification on a CY with $h_{11}=2$ and proceed as in the previous case after integrating out the dilaton and the complex structure moduli at a high scale and getting a superpotential for the Kahler moduli from gaugino condensation. In this case we will have an $\mathcal{N}=1$ theory with two light moduli and then, as we will show, it is possible to have dS or Poincaré minima.

[11]A particular case of this general result was noticed in [28].

IV. POINCARÉ AND DE SITTER MINIMA OF ONE- AND TWO-MODULI POTENTIALS

We will be considering the minima of the SUGRA action (7) where all but one or two moduli have been integrated out at a high scale. The origin of the SUGRA action can be either type IIB string theory or heterotic string theory.

In type IIB string theory compactified on a CY manifold with only one Kahler modulus the situation analyzed in the literature (for instance by KKLT) would fall into the category of the one-modulus case since the dilaton and the complex structure moduli have been integrated out classically. The classical Kahler potential of the SUGRA is

$$K=-3\ln(T+\overline{T}), \tag{30}$$

and the superpotential is of the form

$$W=c+\sum d_i e^{-8\pi^2 T/C(G_i)}. \tag{31}$$

The constant c in the superpotential is the value of the GVW superpotential [17] evaluated at the point that minimizes the classical superpotential, and $d_i=-\mu^3 C(G_i)$. In the type IIB case the sum originates from multiple gaugino condensates that may occur if the original gauge group living on the D7-branes is broken by discrete Wilson lines on the four-cycle which is wrapped by the branes. The sum of exponential terms in the superpotential would be over the simple gauge group factors.

In the heterotic string theory case a similar effective four-dimensional SUGRA action for a single field can arise if (as is generically the case) the dilaton and the complex structure moduli would get string scale masses from Eq. (10), and a potential for the volume modulus arises from the mechanisms discussed in Sec. II.

The two-light-moduli case can arise in both string theories. One possibility is that the overall volume modulus and the dilaton are light. This can happen by tuning the parameters of the 10D action by a choice in the discretuum. Other possibilities can arise as well. For example, as an alternative to keeping the dilaton and the overall volume modulus light, we may consider compactification on a CY manifold with two Kahler moduli, T_1,T_2 say. Now we will have two four-cycles (each assumed to have $b_1=0$ so that there are no open string moduli) and it is possible have a stack of seven branes wrapping each cycle. We do not know how to parameterize the corresponding metric in the CY case, but we might proceed in analogy with the torus (or orbifold) case where there will be three Kahler moduli. After fixing the complex structure moduli the metric may be written as

$$ds^2=\exp-\left(2\sum_{i=1}^{3}u_i(x)\right)\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu+e^{2u_1(x)}dz^1d\overline{z^1}+e^{2u_2(x)}dz^2d\overline{z^2}+e^{2u_3(x)}dz^3d\overline{z^3}.$$

We need to identify the axionic partners of the Kahler moduli in order to complete the chiral scalars in the corresponding supermultiplets. They can be identified by writing the four-form gauge field as $C_4=\Sigma_{i=1}^3 a^i(x)\wedge J^i$ where $J^i=dz^i\wedge d\overline{z^i}$ and the a^i are two-forms in four dimensions. The axions are then the pseudoscalars b^i defined by writing $da_2^i=\exp-(2\Sigma_j u^j-2u^i)\tilde{*}_4 db^i$ and the chiral scalars in the Kahler moduli superfields take the form $T^i=b_i/\sqrt{2}+ie^{4u_i}$ with the Kahler potential being

$$K=-\sum_{i=1}^{3}\ln(T^i+\overline{T^i}).$$

In this case we can have three stacks of D7-branes each wrapping a different four-cycle and then with a certain choice of normalization of the integral over the three-cycles, we get for the gauge coupling functions

$$f^1=\sqrt{T^2T^3},\quad f^2=\sqrt{T^3T^1},\quad f^3=\sqrt{T^1T^2}.$$

The corresponding VY superpotential would then be

$$W=c+d_i e^{-8\pi^2 f^i/C(G_i)}.$$

Again, if one introduces discrete Wilson lines on each four-cycle then each exponential term would be replaced by a sum of exponentials as before. The point is that now we have a theory of three light moduli. As long as the extrema of the potential are away from zero,[12] we can expand around any of them as before to determine whether they are minima. However, the three-moduli case is technically quite complicated to analyze since the number of terms in the potential is large. We wish to consider a simpler case with only two light moduli, in other words we need the analogous theory when the compactification manifold is a CY with $h_{11}=2$. It is not clear to us how to compute the gauge coupling functions in this case. However, all that we really need is that the gauge coupling functions f^1,f^2 coming from branes wrapping different four-cycles have different dependencies on the two moduli. By analogy with the case of the torus (with, say, $T^2=T^3$) this would appear to be the case. If so, this case can be analyzed in exactly the same way as the one with the dilaton and the overall volume.

In [19] we have considered the constraints on a four-dimensional SUGRA with stable moduli if hierarchically small SUSY breaking is desired, with an acceptably small CC. More precisely, defining the ratio of the gravitino mass to the Planck mass $\varepsilon=m_{3/2}/M_{Pl}$ the SUSY breaking is $O(\varepsilon)$ and the CC is $\ll O(\varepsilon^2)$. We showed that for general Kahler potentials, the mass of the modulus in the SUSY breaking direction had to be $O(\varepsilon)$; however, the masses of the other moduli could be of the string scale. Assuming a canonical form of the Kahler potential we found concrete examples of a one-modulus potential with a stable minimum. Our result does not preclude the existence of more than one light modulus—it just requires at least one.

[12]In any case the theory breaks down for values of T^i smaller than unity, corresponding to compactification scales smaller than the string scale.

In earlier work (see [29] and references therein), it was found that single field steep superpotentials do not allow extrema with broken SUSY and a non-negative CC that is a true minimum in the resulting SUGRA potentials. Steep superpotentials are defined by the condition that their derivatives are large,

$$\frac{|(T+\bar{T})\partial_T^{(n+1)}W|}{|\partial_T^n W|} \gg 1, \quad n=0,1,2,3. \tag{32}$$

The reason that $n \leqslant 3$ appears will be explained shortly.

The steepness property holds for all the gaugino-condensation superpotentials that were previously discussed, and it is generic to all models of moduli stabilization near the boundaries of moduli space. The typical example of a superpotential satisfying Eq. (32) is a sum of exponentials $W(T) = \Sigma_i d_i e^{-\beta_i T}$, as in Eq. (31), in the region $|T\beta_i| \gg 1$. In general, the precise definition of the region in which the potential is steep will depend on its detailed properties. Our result did not depend on the particular form of the Kahler potential, provided that it was regular at the extremum. Thus, to obtain a true minimum with broken SUSY and a vanishing or positive CC requires that the superpotential and its first three derivatives can be tuned so that the conditions in Eq. (32) are avoided in a certain region of field space.

We have further defined a criterion for what constitutes an acceptably small CC: that the value of the CC be much smaller than $\varepsilon^2 M_{Pl}^4$. The reason for choosing such a criterion, and not requiring that the cosmological constant vanishes or is of the order of the critical energy density as suggested by recent observations, is that we expect corrections to the CC coming from low energy field theoretic effects. Generically, loop contributions to the CC can be as large as $\varepsilon^2 M_{Pl}^4$, and in addition we expect contributions of order $\varepsilon^4 M_{Pl}^4$ from electroweak scale physics. There are models in which the leading order loop corrections can be canceled (see, for example, [30]) but there would be corrections that are at least as large as $\varepsilon^4 M_{Pl}^4$. Therefore, at energies just below the string scale, a model with a negative CC that is as large as, say, $\varepsilon^4 M_{Pl}^4$, is at the level of accuracy that we (and others) are working at, as good a model as one with a positive CC whose magnitude is $(10^{-3}\ \mathrm{eV})^4$.

Here we extend and complete our previous results. We will prove the following results.

If the Kahler potential for a single modulus Φ is of the form that comes from classical string theory $K = -A \ln(\Phi + \bar{\Phi})$, for $1 \leqslant A \leqslant 3$, for example, $K = -3 \ln(T+\bar{T})$ or $K = -\ln(S+\bar{S})$, then there does not exist a minimum with a positive or zero CC for an F-term potential for any superpotential. A minimum with a negative CC and broken SUSY in a region in which string perturbation theory is under control can be found for such Kahler potentials, and with enough tuning of the parameters of the superpotential it can be made acceptably small.

In the case of two moduli (say, S and T) with a classical string theory Kahler potential, for example $K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T})$, it is possible to find a superpotential that yields a stable minimum with a positive or vanishing CC. It is also possible to find minima with a negative CC and broken SUSY which, with enough tuning, can be made acceptably small.

The first result means that if all but one modulus is integrated out at a high scale, leaving us with an effective four-dimensional SUGRA as in the KKLT case, then such a potential (without introducing D terms) cannot have a minimum with a zero or positive CC. As we have argued above, the situation with D terms is unclear so in our view the existence of such a dS or even Poincaré minimum is still not established. Our result also explains why the racetrack models with one modulus that have been discussed in the literature over the past 15 years have failed to produce a model for stabilizing moduli with zero (or positive) CC.

The second result establishes the possible existence of a minimum in the region of moduli space where the Kahler potential is approximately of its classical form if all but two moduli are integrated out at a high scale. In this case the effective low energy theory is a four-dimensional SUGRA with two light moduli. We expect that the tuning that is available in the discretuum will be sufficient to yield such a potential.

To establish our results we will begin with some general considerations. We investigate a SUGRA potential given by Eq. (7) with Kahler potential $K = -A \ln(S+\bar{S}) - B\ln(T+\bar{T})$. Here S and T are generic names for two moduli, for instance they could be the dilaton and a Kahler modulus, or two Kahler moduli. Let us assume that there is an extremum of V_{SUGRA} at $S=S_0$, $T=T_0$. Then near the extremum we may expand the superpotential in a power series of the form

$$W(S,T) = \sum_{ij} a_{ij}(S-S_0)^i (T-T_0)^j. \tag{33}$$

We may simplify the superpotential. If we make a translation $S \to S + i\,\mathrm{Im}\,S_0$, $T \to T + i\,\mathrm{Im}\,T_0$, the Kahler potential is unchanged, so without loss of generality we can take S_0, T_0 to be real. Since S,T are non-negative fields and in fact S_{0R} and T_{0R} have the meaning of a coupling or a geometric object such as a radius of a cycle of the compact manifold, both of them are positive. We may now rescale S and T by S_0 and T_0, respectively, to get

$$W(\tilde{S},\tilde{T}) = \sum_{ij} a_{ij} S_0^i T_0^j (\tilde{S}-1)^i (\tilde{T}-1)^j. \tag{34}$$

The Kahler potential is changed under this rescaling,

$$K \to K(\tilde{S},\tilde{T}) - A \ln(2S_{0R}) - B \ln(2T_{0R}), \tag{35}$$

which means that the potential is rescaled as $V(S,T) \to [1/(2S_{0R})^A (2T_{0R})^B] V(\tilde{S},\tilde{T})$. If the extremum is a minimum, this rescaling does not change its nature: A dS minimum remains a dS minimum and a Poincaré minimum remains a Poincaré minimum. Further simplification occurs because the superpotential appears as an absolute value squared in the potential, and therefore if we multiply it by a constant phase, the potential is unchanged. We may use this freedom to set a_{00} to be real. To summarize: We may take the minimum to be at $S_0=1$, $T_0=1$, and the first (constant) term in the superpotential to be real without loss of generality.

To establish the existence of a Poincaré or dS minimum we need to establish the following:

$$V|_0\geq 0,\ \partial_S V|_0=\partial_T V|_0=0,\ \text{eigenvalues}\ [\partial^2_{ij}V|_0]\geq 0,$$

where $|_0$ means that the expressions are to be evaluated at $S=S_0$, $T=T_0$, and the last expression is the Hessian matrix of second derivatives of V. Now from the holomorphicity of the superpotential, we see from the form (33) that in calculating these quantities we need to keep only terms up to the third order in S and T in W. This is because in the expression for the potential only W and its first derivatives appear. So for analyzing the existence of a minimum we may limit ourselves to a superpotential of the form

$$W=\sum_{i=0,j=0}^{i+j=3} a_{ij}(S-1)^i(T-1)^j,$$

with a_{00} real. We will see later that to prove that a minimum cannot exist it is sometimes possible to restrict this to a quadratic superpotential.

A. The one-modulus case

1. de Sitter and Poincaré minima are not possible

Here we would like to prove that, if the Kahler potential for a single modulus Φ is $K=-A\ln(\Phi+\bar{\Phi})$, for $1\leq A\leq 3$, for example, then there does not exist a minimum with a positive or zero CC for an F-term potential for any superpotential. We will prove our result by showing that under the conditions stated above there is at least one direction in which the extremum is a maximum rather than a minimum. Thus, we will show that an extremum can be a saddle point or a maximum but not a true minimum.

Rather than computing the Hessian directly and showing that under the conditions mentioned above it has at least one negative eigenvalue, namely, that it is not a positive definite matrix, we will show that $\partial^2V/\partial\Phi_R\partial\Phi_R+\partial^2V/\partial\Phi_I\partial\Phi_I<0$. If this quantity is negative then the Hessian matrix cannot be positive definite. This is because the condition that it is positive definite is that its determinant and all of its principal minors be positive. If $\partial^2V/\partial\Phi_R\partial\Phi_R+\partial^2V/\partial\Phi_I\partial\Phi_I<0$ then $\partial^2V/\partial\Phi_R\partial\Phi_R<0$ and/or $\partial^2V/\partial\Phi_I\partial\Phi_I<0$, so at least one of the principal minors is negative. There are two advantages to choosing this quantity as a diagnostic. One is that it is linear in the potential rather than quadratic like the determinant. The second advantage is due to the fact that $\partial^2V/\partial\Phi_R\partial\Phi_R+\partial^2V/\partial\Phi_I\partial\Phi_I=4\partial^2V/\partial\Phi\partial\bar{\Phi}$. Because the superpotential is a holomorphic function, $\partial^2V/\partial\Phi\partial\bar{\Phi}$ at the extremum depends only on the superpotential and its first and second derivatives at the extremum. Therefore we may use a quadratic superpotential to analyze this quantity. This greatly simplifies the analysis.

We consider a Kahler potential

$$K=-A\ln(\Phi+\bar{\Phi}),\ \ 1\leq A\leq 3, \tag{36}$$

and a quadratic superpotential

$$W=a0+(a1_R+ia1_I)(\Phi_R+i\Phi_I-1)+(a2_R+ia2_I)(\Phi_R+i\Phi_I-1)^2, \tag{37}$$

which depends on five real parameters $\{a0,a1_R,a1_I,a2_R,a2_I\}$. The expression for the resulting potential has many terms and it is not practical to present it here. We use MATHEMATICA to compute it symbolically and manipulate it.

We then impose the condition that $V(1)=\epsilon\geq 0$, and that $\Phi=1$ is an extremum $\partial_{\Phi_R}V|_1=0$, $\partial_{\Phi_I}V|_1=0$. This results in three equations for the five parameters, leaving two of them free. We then compute $[\partial^2V/\partial\Phi_R\partial\Phi_R+\partial^2V/\partial\Phi_I\partial\Phi_I]|_{\Phi=1}$ in terms of the remaining two parameters and check whether there is a region of parameter space for which it is positive. If we choose the two free parameters to be $a1_R,a1_I$ we find that

$$\left[\frac{\partial^2V}{\partial\Phi\partial\bar{\Phi}}\right]_{|_{\Phi=1}}=-\frac{2^{3-A}}{A(3-A)}\Big[(3-A)a1_I^2+(3+A)a1_R^2+\sqrt{A}a1_R\sqrt{12a1_R^2-(3-A)(2^AA\epsilon-4a1_I^2)}\Big],\quad A\neq 3$$

$$\left[\frac{\partial^2V}{\partial\Phi\partial\bar{\Phi}}\right]_{|_{\Phi=1}}=-2\epsilon,\quad A=3. \tag{38}$$

We then check whether this expression can be positive for any value of $a1_R$ and $a1_I$ and we find that it is always negative. This is of course obvious for the $A=3$ case.

If we look specifically for a Poincaré vacuum, for which $\epsilon=0$, the analysis simplifies, and the results remain the same; the Hessian has always at least one negative eigenvalue. For the case $A=3$ it is less obvious, but it is nevertheless correct since in this case we have from the above $[\partial^2V/\partial\Phi_R\partial\Phi_R]_{|_{\Phi=1}}=-[\partial^2V/\partial\Phi_I\partial\Phi_I]_{|_{\Phi=1}}$.

2. AdS minima with SUSY breaking

We would like to show that it is possible to find AdS minima with an acceptably small CC. As can be seen from Eq. (38), if the value of potential at the minimum is negative $\epsilon<0$, it is no longer possible to deduce that one of the eigenvalues is negative. In fact, it is rather easy to find examples when both eigenvalues are positive and therefore the candidate extremum is indeed a minimum.

For example, in the case of a quadratic superpotential with real coefficients $W(\tilde{T})=a_0+a_1(\tilde{T}-1)+a_2(\tilde{T}-1)^2$, and a Kahler potential $K=-3\ln(T+\bar{T})$, the conditions for a true minimum are that $a_1=a_2$, $a_0=a_2/3-2\epsilon/a_2$, and $0<a_1<\sqrt{-6\epsilon}$. SUSY is generically broken at the minimum since $F=-\frac{3}{2}a_0+a_1=a_2/2+3\epsilon/a_2$ generically does not vanish. For the case $K=-\ln(T+\bar{T})$, the conditions are a_0

$=4a_2$, $a_1=2a_2+\sqrt{12a_2^2+\epsilon/2}$, and at the minimum SUSY is generically broken $F=-a_0/2+a_1=\sqrt{12a_2^2+\epsilon/2}$.

To discuss the issue of scaling let us explicitly present the relationship between exponential and polynomial potentials. Comparing $W=c+\Sigma d_i e^{-8\pi^2 T/C(G_i)}$ to $W(\tilde{T})=\Sigma a_i(\tilde{T}-1)^i$, we find that $a_0=c+\Sigma d_i e^{-8\pi^2 T_0/C(G_i)}$, $a_1=\Sigma -d_i\beta_i T_0 e^{-8\pi^2 T_0/C(G_i)}$, $a_2=\Sigma d_i[(\beta_i T_0)^2/2]e^{-8\pi^2 T_0/C(G_i)}$, and so on. One needs to tune the coefficients such that the superpotential and its first three derivatives are of the same order in a region in the vicinity of the minimum. To make the CC acceptably small it needs to be less than $O(\varepsilon^2)$; recall that $\varepsilon=m_{3/2}/M_{Pl}$. Therefore the parameters in the superpotential need to be tunable to an accuracy which is roughly ε [recall that the true potential is rescaled by a factor $(2T_{0R})^3$].

The constant c can be tuned by choosing parameters in the discretuum, while the amount by which the other coefficients are tunable is determined by the values of T_0 and $C(G_i)$. In the type IIB case there seem to be enough possibilities to tune the coefficients to the desired accuracy, while in the heterotic case compactified on a Kahler manifold the amount of tuning seems to be insufficient as long as the constraint that the rank of the group be less than 22 exists.

B. The two-moduli case: Examples with minima

The two-moduli case can be analyzed using the same methods as in the single modulus case. Because of the complexity of the analysis we cannot give the results for the general case, rather we give some specific examples where it is possible to find a true minimum with a positive or vanishing CC and broken SUSY.

The examples that we were able to find are $K=-\ln(S+\bar{S})-3\ln(T+\bar{T})$, and a general cubic superpotential with ten real coefficients,

$$\begin{aligned}W=&a0+a1(S_R+iS_I-1)+a2_R(S_R+iS_I-1)^2\\&+a3(S_R+iS_I-1)^3+b1(T_R+iT_I-1)\\&+b2(T_R+iT_I-1)^2+b3(T_R+iT_I-1)^3+ab1\\&\times(S_R+iS_I-1)(T_R+iT_I-1)+ab2(S_R+iS_I-1)\\&\times(T_R+iT_I-1)^2+ba2(S_R+iS_I-1)^2(T_R+iT_I-1).\end{aligned} \tag{39}$$

The conditions that $T=1$, $S=1$ is an extremum and that the value of CC is ϵ constitute three equations leaving seven free parameters. We were able to find true minima for a range of these parameters such that the CC is positive or vanishing. For example, $a1=1$, $b1=1$, $ab1=1$, $a3=0.1$, $b3=-0.1$, $ab2=0.1$, $ba2=0.1$, $\epsilon=0.6$, or $a1=0.8$, $b1=0.8$, $ab1=0.5$, $a3=0.2$, $b3=-0.15$, $ab2=0.2$, $ba2=-0.15$, $\epsilon=0.3$. The range of parameters is finite. It is also possible to find examples of true minima with vanishing or positive CC for simpler superpotentials, for example a general quadratic superpotential with real coefficients.

For other forms of the Kahler potential we were unable to find solutions with a positive or vanishing CC; however, the analysis seems more complicated and our inability to find such solutions does not necessarily mean that they do not exist. We do believe that the existence of such solutions is quite sensitive to the form of the Kahler potential. From the previous discussion it is also clear that in addition to the solutions that we have found for the particular form of the Kahler potential it is also possible to find minima with a negative CC and broken SUSY, and that with enough tuning their CC can be made small enough so that they are acceptable according to the criteria that we have defined previously. The amount of fine-tuning required can be estimated by comparing the polynomial form of the superpotential to its original form as a sum of a constant and exponential terms as was done in the single field case.

V. COSMOLOGICAL ISSUES AND CONCLUSIONS

A. The discretuum and the anthropic principle

In the last few years it has become fashionable to apply the anthropic principle (AP) to the discretuum. Imagine applying it to the larger set of vacua that we have discussed. In order to discuss the applicability of AP in string theory we first need a precise statement of it. A scientifically acceptable statement would be the following weak form of the AP.

AP: Given a theory that predicts a range of values for some fundamental parameters, observers will measure values for these parameters that are typical of those universes which are consistent with the existence of the observers.

To formulate this more precisely would require one to know exactly what parameters of the standard model are relevant for our existence. Weinberg [31,32] has argued that if the CC is not within a factor of a few of the currently observed value then galaxies would not have formed (and hence we would not have come into being). This argument may constitute an explanation of the coincidence problem (why the CC is of the same order of magnitude as the matter density); however, it is not an explanation of its actual value. It is possible to imagine universes where both the CC and the matter density are much higher but galaxies are formed.[13]

As observed by many authors, the anthropic principle makes sense only in the context of a theory that allows a wide range of values for the parameters in question. The example of Newtonian dynamics, which allows for the existence of planets at various distances from the sun, is often cited. There this is just a matter of a set of initial conditions, chosen presumably at random, with one of them putting a planet just at such a distance that its surface temperature is between the freezing point and boiling point of water, so that life as we know it can form. In this analogy the point is that there is no fundamental explanation of why the planet Earth is at a certain distance from the sun. If it was at a different distance (outside some small range) then we could not inhabit it. So in the same way we should not ask why it is that we live in a universe with a particular value of the CC since if it were different (again, outside some range) then we would not be here to ask the question.

Notice that the argument assumes that all possible distances from the sun are allowed and in principle can be

[13]A detailed analysis along these lines is in [33].

achieved. If, for instance, in spite of the theory, observation showed that there was only one planet in the universe and it was at such a distance that liquid water existed on its surface, then the anthropic explanation would not be tenable. Thus, the reality of planets at other distances is a necessary condition for this explanation to make sense. Similarly, an anthropic explanation of the cosmological constant (or any other parameter) requires the reality of other solutions to string or M theory that have different values of this parameter. We can see that there are other planets at a variety of different distances, but so far we have not detected any other universe, and it is not clear that we ever would [34]. The detection of other universes might even be impossible in principle.

Thus the anthropic explanation actually entails a prediction—that other universes exist, and that there is a correlation between the values of their CC's and the existence of galaxies capable of supporting intelligent life in them. The latter does not make any sense unless the former is true. On the other hand, for most of the history of fundamental physics, theories and models that do not satisfy observational criteria have been discarded as unphysical. Of course, in the past physicists have not attempted anything as ambitious as the construction of a theory of all fundamental phenomena. But an analogy from general relativity may illustrate the point. As is well known, without additional criteria this theory can lead to bizarre solutions, including naked singularities, universes with closed timelike geodesics, etc.[14] Even solutions which are much more acceptable, such as, say, a Bianchi cosmology, are usually rejected because they are not in accord with the observed homogeneity and isotropy of the universe.

Application of the anthropic principle in the discretuum would make sense only if one were to treat all possible solutions of perturbative string theory as having a real existence. Since it is highly unlikely that any of them other than our own (assuming it is a member of the discretuum) is ever going to be observed it does not appear to be a meaningful principle to use. In this paper we have argued that there are points in the discretuum where all the moduli are stabilized. Perhaps it is possible to find such a point with the observed CC and small supersymmetry breaking. If so, rather than appeal to the anthropic principle, we would argue that what has been done is good old fashioned fine-tuning.

B. The overshoot problem

A generic problem with the moduli stabilization and SUSY breaking scenarios discussed above is that cosmologically they are all subject to the overshoot problem first pointed out in [12]. To see this let us estimate the height of the barrier separating the SUSY breaking minimum from the runaway decompactifying region of the potential. In both the KKLT and the BKQ proposals this is clearly set by the size of the extra term that is added. In either case at the minimum this number sets the scale of SUSY breaking. To have low energy SUSY the D term in the BKQ proposal should satisfy $|D| < O(10^{-14})$ in Planck (or string) units. In the anti-D-brane case the SUSY breaking is explicit, but if one needs this breaking to be at a low scale then we will have a similar result. As one moves away to the right of the minimum, this term gets reduced (since it is proportional to a negative power of T_R) and hence the barrier height is $\leqslant |D|^2 \sim 10^{-28}$. In the case of F-term breaking (with multiple gaugino condensates) discussed here the argument is slightly more involved. The point is that at the SUSY breaking minimum, in order to get a nearly zero CC, the value of $|\sqrt{3}W|$ should be equal to $|F|$, i.e., the value of the SUSY breaking order parameter. At this point then the value of the exponential terms is of the same order as the constant in the superpotential. Beyond this minimum the exponential terms are smaller (in absolute value) and an extremum at a positive value of V may arise but the barrier is expected to be of the same order as $|F|^2$ at the SUSY breaking scale.

In addition to the SUSY breaking minima that we have worked so hard to establish, in generic situations there are also nearby SUSY preserving minima with a large negative CC. The reason is that for superpotentials that can be approximated by polynomials (and this can always be done near the SUSY breaking minimum) there are also solutions to the equations $F=0$. Whenever the F term vanishes the potential is negative (or in some special situations vanishes), and in general one of these solutions will be the global minimum in the region where the approximation holds.

Now the problem is that generically one would expect the initial conditions on the T modulus to be set by the string or Planck era of the universe when one expects string scale energy densities and temperatures. Clearly, if the modulus starts with energy density $\leqslant O(1)$ in string units then it is not going to remain in this extremely shallow minimum and will roll right over into the decompactifying region or into the SUSY preserving AdS minimum. This classical rolling rather than quantum tunneling through the barrier is the real problem with any cosmology based on such outer region compactifications. Of course, it is possible (though unlikely) that with enough tuning of the parameters such that the height and width of the barrier are much larger, some of these problems may be avoided, but in the absence of a concrete example one has to regard this issue as a serious problem.

C. Toward a resolution

The main focus of this paper has been the possibility of obtaining models with all moduli stabilized. We have established the following for $\mathcal{N}=1$ SUGRA potentials with the classical string theory form for the Kahler potential, and a positive or zero CC.

If all but one modulus is stabilized at a high scale then it is not possible to have the remaining light modulus stabilized by F terms.

If there are two light moduli then there are examples where stabilization can be achieved in regions where string perturbation theory is under control.

[14]Recent work seems to indicate that Goedel universes are valid solutions of string theory too.

Our results depend on the form of Kahler potential, as we have emphasized on several occasions along the way. If the corrections to the classical form of the Kahler potential are small, as expected in regions of moduli space in which string perturbation theory is a good approximation, then our results should be valid. If for the scales at which the CC is measured (or even at the standard model scale) it turns out that there are significant corrections to the Kahler potential and the classical form is drastically modified, then we expect our results to be significantly modified. For example, if the Kahler potential is modified in such a way that it can be approximated by the canonical form, then one can find a good minimum even in the one-modulus case [19]. Clearly, for two moduli there is a wider range of possibilities.

The minima that we have discussed, as well as all others discussed in the literature, fall into the category of outer region solutions in the terminology of [29]. They are separated from the runaway regions of the moduli potentials by tiny barriers and are thus subject to the cosmological overshoot problem discussed in the previous subsection. This means that even if one finds a model of this sort which contains the (supersymmetric extension of the) standard model, such a theory—though it would serve as an existence proof that an ultraviolet completion of the standard model coupled to gravity exists—would not give a viable cosmology.

If one includes the requirement of a viable cosmology it appears unlikely that one could get a satisfactory theory in the outer region of moduli space. Thus, as has been the recurrent theme of our previous work, we need to focus on building models in the central region of moduli space—i.e., the region that is not related by any dualities to weak coupling large volume compactifications. Obviously, it is technically hard to compute in this region at this stage of development of string theory, and perhaps one has to await the successful formulation of some nonperturbative description of string theory to be able to calculate meaningful quantities in this region. However, as explained in [29] and in [19], by combining information from the different theories in the outer region and using information from a bottom up approach, one may gain some insight into the physics of this region.

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