Dynamical gauge symmetry breaking and mass generation on the orbifold T^2/Z_2

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(Received 10 March 2004; published 29 June 2004)

Dynamical gauge symmetry breaking on the orbifold T^2/Z_2 is shown to occur through the quantum dynamics of Wilson line phases. Different sets of boundary conditions on T^2/Z_2 can be related to each other by Wilson line phases, forming equivalence classes. The effective potential for Wilson line phases is evaluated at the one-loop level in $SU(2)$ gauge theory. Depending on the fermion content, the $SU(2)$ symmetry can be broken either completely or partially to $U(1)$ without introducing additional Higgs scalar fields. When $SU(2)$ is completely broken, each of three components of the gauge fields may acquire a distinct mass. Masses are generated through the combination of T^2 twists and dynamics of Wilson line phases.

DOI: 10.1103/PhysRevD.69.125014 PACS number(s): 11.15.Ex, 12.10.−g, 12.60.−i

I. INTRODUCTION

Recently, much attention has been paid to gauge theory in space-time with compact extra dimensions. Gauge theory on an orbifold has been studied extensively with hopes of resolving long-standing problems in grand unified theory (GUT) such as the gauge hierarchy problem, the doublet-triplet splitting problem, and the origin of gauge symmetry breaking [1–6]. One intriguing aspect is the gauge-Higgs unification in which Higgs bosons are regarded as a part of extra-dimensional components of gauge fields [7–16].

Extra dimensions are often compactified on topological manifolds. Reflecting the topology of extra dimensions, dynamical gauge symmetry breaking occurs through the Hosotani mechanism [8,9] (gauge symmetry breaking by Wilson lines). Extra-dimensional components of gauge fields (Wilson line phases) become dynamical degrees of freedom and cannot be gauged away. They, in general circumstances, develop nonvanishing vacuum expectation values [17–21]. The extra-dimensional components of gauge fields act as Higgs bosons at low energies. Thus gauge fields and Higgs particles are unified by higher-dimensional gauge invariance. One does not need to introduce extra Higgs fields to break the gauge symmetry.

To construct a realistic GUT, one can choose extra dimensions to be an orbifold, which naturally appears in superstring theory. By having an orbifold in extra dimensions, one can easily accommodate chiral fermions in the four dimensions and also rich patterns of gauge symmetry breaking. In superstring theory, extra six dimensions must be compactified [22–25], and therefore higher-dimensional gauge theory might naturally emerge.

Gauge theory on the simplest orbifold S^1/Z_2 has been studied extensively from various points of view in the literature [26–31]. In this paper, we extend the analysis to six-dimensional space-time, where two of the space coordinates are compactified on the orbifold T^2/Z_2 [32]. In six dimensions there are Weyl fermions which naturally reduce to four-dimensional Weyl fermions after Z_2 orbifolding. Other orbifolds such as T^2/Z_3 and T^2/Z_4 have also been considered to explain the generation structure and violation of discrete symmetry [33]. Our main aim is to study dynamics of gauge symmetry breaking and mass generation on T^2/Z_2. We see that the dynamics of Wilson line phases can reduce or enhance the symmetry of boundary conditions. Such dynamical aspects of gauge symmetry breaking have been studied well in S^1, T^n, and S^1/Z_2. The effects of supersymmetry breaking and finite masses of matter on the dynamics of Wilson line phases have been analyzed [18,11,19]. The dynamics for selecting boundary conditions is also discussed [30,31]. Our analysis given in this paper is expected to provide useful hints for building a realistic unified gauge theory on the orbifold to incorporate electroweak gauge symmetry breaking within the framework of the gauge-Higgs unification [12–15].

In the next section we classify boundary conditions of fields on the orbifold T^2/Z_2 and introduce the notion of equivalence classes of boundary conditions [9,11,30,31]. Those equivalence classes are connected with the existence of Wilson line degrees of freedom. $SU(2)$ gauge theory is investigated in detail. The effective potential for Wilson line phases is evaluated in Secs. III and IV. In Sec. V we examine gauge symmetry breaking in the presence of matter fields in various representations of the gauge group and determine physical symmetry at low energies. It is found that depending on matter content, the $SU(2)$ gauge symmetry is either completely broken or partially broken. It should be emphasized that this makes it plausible to have the electroweak symmetry breaking $SU(2)\times U(1)\to U(1)_{em}$ as a part of the Hosotani mechanism. In Sec. VI we discuss the masses of four-dimensional gauge fields, scalar fields, and fermions. Scalar fields, which are originally the extra-dimensional components of gauge fields, acquire masses by radiative corrections. The final section is devoted to conclusions and a discussion.

II. ORBIFOLD CONDITIONS ON T^2/Z_2

We study gauge theory on $M^4\times T^2/Z_2$, where M^4 is the four-dimensional Minkowski space-time. Let x^μ and y^I be coordinates of M^4 and T^2/Z_2, respectively. The size of the two extra dimensions is denoted by R_I ($I=1,2$). The orbifold T^2/Z_2 is given by identifying a point (x^μ,y^I) with a point $(x^\mu,y^I+2\pi R_I)$ for each $I(=1,2)$ and further identifying $(x^\mu,-y^I)$ and (x^μ,y^I). The resultant extra-dimensional space is the domain $0\leqslant y^1\leqslant \pi R_1$, $0\leqslant y^2\leqslant 2\pi R_2$ with four fixed points, $(y^1,y^2)=(0,0),(\pi R_1,0),(0,\pi R_2),(\pi R_1,\pi R_2)$.

In order for quantum field theory to be defined on spacetime with compactified spaces, boundary conditions of fields in the compactified dimensions must be specified. In our case we need to specify boundary conditions on T^2 and for the Z_2 orbifolding. As a general guiding principle we require that the Lagrangian density be single valued. In gauge theory fields can be twisted up to gauge degrees of freedom when they are parallel transported along noncontractible loops.

A. Gauge field

Let us first consider boundary conditions for the gauge field $A_M(x,y^I)$. The index M runs from 0 to 5. We define boundary conditions of the gauge potential along noncontractible loops on T^2 by

$$T^2:\ A_M(x,\vec{y}+\vec{l}_a)=U_a A_M(x,\vec{y})U_a^\dagger \quad (a=1,2)$$

$$\vec{l}_1=\begin{pmatrix}2\pi R_1\\ 0\end{pmatrix},\quad \vec{l}_2=\begin{pmatrix}0\\ 2\pi R_2\end{pmatrix}, \tag{2.1}$$

where U_I $(I=1,2)$ denote global gauge degrees of freedom associated with the original gauge invariance. Gauge potentials at $A_M(x,y^1+2\pi R_1,y^2+2\pi R_2)$ are related to $A_M(x,y^1,y^2)$ either by a loop translation in the y^1 direction followed by a loop translation in the y^2 direction or by a loop translation in the y^2 direction followed by a loop translation in the y^1 direction. For consistency it follows that

$$[U_1,U_2]=0. \tag{2.2}$$

Let us next consider boundary conditions resulting from the Z_2 orbifolding. To simplify the expressions, we denote four fixed points on T^2/Z_2 by $\vec{z}_i$ $(i=0,1,2,3)$:

$$\vec{z}_0=\begin{pmatrix}0\\ 0\end{pmatrix},\ \vec{z}_1=\begin{pmatrix}\pi R_1\\ 0\end{pmatrix},\ \vec{z}_2=\begin{pmatrix}0\\ \pi R_2\end{pmatrix},\ \vec{z}_3=\begin{pmatrix}\pi R_1\\ \pi R_2\end{pmatrix}. \tag{2.3}$$

Boundary conditions are specified by unitary parity matrices P_i $(i=0,1,2,3)$ at the fixed points:

$$Z_2:\ \begin{pmatrix}A_\mu\\ A_{y^I}\end{pmatrix}(x,\vec{z}_i-\vec{y})=P_i\begin{pmatrix}A_\mu\\ -A_{y^I}\end{pmatrix}(x,\vec{z}_i+\vec{y})P_i^+ \quad (i=0,1,2,3). \tag{2.4}$$

A_{y^I} $(I=1,2)$ must have an opposite sign relative to A_μ under these transformations in order to preserve the gauge invariance. The repeated Z_2 parity operation brings a field configuration back to the original one, so that $P_i^2=1$ $(i=0,1,2,3)$ and, hence, $P_i^\dagger=P_i$.

At this stage, we observe that not all the boundary conditions are independent. The transformation $\pi R_1-y^1\to\pi R_1+y^1$ must be the same as $\pi R_1-y^1\to-\pi R_1+y^1\to y^1+\pi R_1$, from which it follows that $U_1=P_1P_0$. A similar relation holds for U_2. We have

$$U_a=P_aP_0 \quad (a=1,2). \tag{2.5}$$

Finally, as the transformation $(\pi R_1-y^1,\pi R_2-y^2)\to(\pi R_1+y^1,\pi R_2+y^2)$ is the same as a transformation $(\pi R_1-y^1,\pi R_2-y^2)\to(-\pi R_1+y^1,-\pi R_2+y^2)\to(\pi R_1+y^1,-\pi R_2+y^2)\to(\pi R_1+y^1,\pi R_2+y^2)$, the relation $U_2U_1P_0=P_3$ must hold. Taking account of Eqs. (2.4), (2.5), and (2.6), the parity matrix P_3 can be written as

$$P_3=P_2P_0P_1=P_1P_0P_2. \tag{2.6}$$

The boundary conditions for gauge fields are specified with P_i $(i=0,1,2)$ satisfying $P_i=P_i^\dagger=P_i^{-1}$ and $P_iP_jP_k=P_kP_jP_i$.

Discussions can be generalized to the case of T^n/Z_2. The orbifold T^n/Z_2 is defined by identifications

$$T^n:\ \vec{y}+\vec{l}_j\sim\vec{y} \quad (j=1,2,\ldots,n), \tag{2.7}$$

$$Z_2:\ -\vec{y}\sim\vec{y}, \tag{2.8}$$

where $\vec{y}$ is an n-dimensional vector on the n-torus and $\vec{l}_j\equiv(0,\ldots,0,2\pi R_j,0,\ldots,0)^T$ $(j=1,\ldots,n)$. The fixed point satisfies the relation $\vec{y}=-\vec{y}+\Sigma_j m_j\vec{l}_j$ ($m_j=$an integer). In the fundamental domain of T^n, they are given by $\vec{y}=(1/2)\Sigma_j m_j\vec{l}_j$ where $m_j=0$ or 1. In the T^2/Z_2 case, there are four fixed points corresponding to $(m_1,m_2)=(0,0),(0,1),(1,0),(1,1)$. At each fixed point the parity matrix is defined. Repeating the same discussion given above, $n+1$ matrices—for example, $P_0,P_1,\ldots,P_n$—are independent. The consistency of the Z_2 orbifolding and the T^n boundary condition defined by U_j $(j=1,\ldots,n)$ satisfying $[U_j,U_k]=0 (j\neq k)$ yields the relation $U_j=P_jP_0$. The relation $P_iP_jP_k=P_kP_jP_i$ also holds.

B. Matter fields

As for matter fields, it is convenient to first specify Z_2 boundary condition and then derive T^2 conditions. Let us consider a scalar field $H(x,\vec{y})$ which satisfies

$$Z_2:\ H(x,\vec{z}_j-\vec{y})=\eta_j^s T[P_j]H(x,\vec{z}_j+\vec{y}) \quad (j=0,1,2,3). \tag{2.9}$$

Here $T[P_j]$ stands for an appropriate representation matrix under the gauge group associated with P_j. If H belongs to the fundamental or adjoint representation, $T[P_j]H=P_jH$ or $P_jHP_j^\dagger$, respectively. η_j^s is a sign factor taking a value $+1$ or -1. Boundary conditions for the T^2 direction are given by

$$T^2:\ H(x,\vec{y}+\vec{l}_a)=\eta_0^s\eta_a^s T[U_a]H(x,\vec{y}) \quad (a=1,2), \tag{2.10}$$

where U_a is given by Eq. (2.5). Not all of the sign factors are independent: $\eta_3^s=\eta_0^s\eta_1^s\eta_2^s$.

Next we consider a Dirac fermion $\psi(x,y^I)$ in six dimensions. The gauge invariance of the kinetic term of the fermion Lagrangian demands that

$$Z_2:\ \ \psi(x,\vec{z}_j-\vec{y})=\eta_j^f T[P_j](i\Gamma^4\Gamma^5)\psi(x,\vec{z}_j+\vec{y})$$
$$(j=0,1,2,3),$$
$$T^2:\ \ \psi(x,\vec{y}+\vec{l}_a)=\eta_0^f\eta_a^f T[U_a]\psi(x,\vec{y})\ \ (a=1,2). \tag{2.11}$$

The sign factors $\eta_j^f=\pm 1$ satisfy $\eta_3^f=\eta_0^f\eta_1^f\eta_2^f$. Γ^4 and Γ^5 are the fourth and fifth components of six-dimensional 8×8 Dirac's gamma matrices, respectively.

It is instructive to present the explicit form of gamma matrices. We employ the representation

$$\Gamma_\mu=\gamma_\mu\otimes\mathbf{1}_{2\times 2},\quad \Gamma_4=\gamma_5\otimes i\sigma_1,\quad \Gamma_5=\gamma_5\otimes i\sigma_2, \tag{2.12}$$

where γ_μ is the four-dimensional gamma matrix and $\gamma_5\equiv i\gamma_0\gamma_1\gamma_2\gamma_3$ with $(\gamma_5)^2=1_{4\times 4}$. In this representation we have

$$i\Gamma^4\Gamma^5=\mathbf{1}_{4\times 4}\otimes\sigma_3. \tag{2.13}$$

One can define six-dimensional chirality similar to the chirality in four dimensions. It is given by the eigenvalues of Γ^7 defined by

$$\Gamma^7\equiv\Gamma^0\Gamma^1\cdots\Gamma^5(=\gamma_5\otimes\sigma_3). \tag{2.14}$$

Then, we obtain that

$$\Gamma^7\psi_\pm=\pm\psi_\pm,\quad \text{where}\quad \psi_\pm\equiv\frac{1}{2}(1\pm\Gamma^7)\psi. \tag{2.15}$$

If we write

$$\psi_-=\begin{pmatrix}U_L\\ D_R\end{pmatrix},\quad \psi_+=\begin{pmatrix}U_R\\ D_L\end{pmatrix}, \tag{2.16}$$

$\gamma_5 U_L=-U_L$, $\gamma_5 D_L=-D_L$, $\gamma_5 U_R=U_R$, $\gamma_5 D_R=D_R$. In terms of four-dimensional Dirac spinors the boundary conditions (2.11) are recast as

$$Z_2:\ \ U_{L,R}(x,\vec{z}_j-\vec{y})=+\eta_j^f T[P_j]U_{L,R}(x,\vec{z}_j+\vec{y}),$$
$$D_{L,R}(x,\vec{z}_j-\vec{y})=-\eta_j^f T[P_j]D_{L,R}(x,\vec{z}_j+\vec{y}), \tag{2.17}$$

The sign factors $\{\eta_j^s\},\{\eta_j^f\}$ are additional parameters specifying boundary conditions. They play an important role in dynamical gauge symmetry breaking.

C. Equivalence classes and symmetry of boundary conditions

The gauge symmetry is apparently broken by nontrivial parity matrices P_j $(j=0,1,2)$ specifying boundary conditions of the Z_2 orbifolding. Yet the physical symmetry of the theory is not, in general, the same as the symmetry of boundary conditions, once quantum corrections are incorporated.

To elucidate this fact, we first show that different sets of boundary conditions can be related to each other by ''large'' gauge transformations. Under a gauge transformation

$$A'_M=\Omega\left(A_M-\frac{i}{g}\partial_M\right)\Omega^\dagger, \tag{2.18}$$

A'_M obeys a new set of boundary conditions $\{P'_j,U'_a\}$ where

$$P'_j=\Omega(x,\vec{z}_j-\vec{y})P_j\Omega(x,\vec{z}_j+\vec{y})^\dagger,$$
$$U'_a=\Omega(x,\vec{y}+\vec{l}_a)U_a\Omega(x,\vec{y})^\dagger,$$
$$\text{provided}\ \ \partial_M P'_j=\partial_M U'_a=0. \tag{2.19}$$

The relation $U'_a=P'_aP'_0$ follows from Eqs. (2.5) and (2.19). We stress that the set $\{P'_j\}$ can be different from the set $\{P_j\}$. When the relations in Eqs. (2.19) are satisfied, we write

$$\{P'_j\}\sim\{P_j\}. \tag{2.20}$$

This relation is transitive and therefore is an equivalence relation. Sets of boundary conditions form equivalence classes of boundary conditions with respect to the equivalence relation (2.20) [9,11,31].

The residual gauge invariance of the boundary conditions is given by gauge transformations that preserve the original boundary conditions:

$$P_j=\Omega(x,\vec{z}_j-\vec{y})P_j\Omega(x,\vec{z}_j+\vec{y})^\dagger,$$
$$U_a=\Omega(x,\vec{y}+\vec{l}_a)U_a\Omega(x,\vec{y})^\dagger. \tag{2.21}$$

As shown in [11], those residual gauge transformations extend over the entire group space even for nontrivial $\{P_j\}$. All the Kaluza-Klein modes nontrivially mix under those gauge transformations.

The gauge symmetry realized at low energies is given by y^I-independent Ω satisfying

$$[P_j,\Omega(x)]=0\ \ (j=0,1,2). \tag{2.22}$$

We observe that the symmetry is generated by generators of the gauge group which commute with P_j. This is called the symmetry of boundary conditions at low energies.

The gauge symmetry at low energies can also be understood in terms of group generators associated with zero modes of the gauge fields $A_\mu=A_\mu^aT^a$. Let us define

$$\mathcal{H}_{\rm BC}=\{T^a;[T^a,P_j]=0\ \ (j=0,1,2)\},$$
$$\overline{\mathcal{H}}_{\rm BC}=\{T^b;\{T^b,P_j\}=0\ \ (j=0,1,2)\}. \tag{2.23}$$

From the boundary condition (2.4), it follows that zero modes (y^I-independent modes) of A_μ and A_{y^I} can be written as

$$A_\mu(x)=\sum_{T^a\in\mathcal{H}_{\rm BC}}A_\mu^a(x)T^a, \tag{2.24}$$

$$A_{y^I}(x)=\sum_{T^b\in\overline{\mathcal{H}}_{\rm BC}}A_{y^I}^b(x)T^b. \tag{2.25}$$

The residual gauge symmetry at low energies $H_{\rm BC}$ is spanned by those generators belonging to $\mathcal{H}_{\rm BC}$.

The zero modes A_{y^I} in Eq. (2.25), or particularly their x-independent parts, define Wilson line phases and play a critical role in dynamical rearrangement of gauge symmetry at the quantum level, which we elaborate in the following subsection.

D. Wilson line phases and physical symmetry

So far we have discussed the symmetry of the boundary condition $\{P_j\}$ at the tree level. This is not necessarily the same as the physical symmetry of the theory. Once quantum corrections are taken into account, the boundary condition effectively changes as a result of A_{y^I} in Eq. (2.25) developing nonvanishing expectation values. The number of zero modes of four-dimensional gauge fields A_μ^a in the new vacuum also changes. Rearrangement of gauge symmetry takes place. This is called the Hosotani mechanism [8,9].

Constant modes of A_{y^I} satisfying $[A_{y^1},A_{y^2}]=0$ give vanishing field strengths, but become physical degrees of freedom that cannot be gauged away within the given boundary conditions. Indeed the path-ordered integral along a noncontractible loop starting at (x,y)

$$W_I(x,y)=\mathcal{P}\ \exp\left(ig\oint dy^I A_{y^I}\right)\quad (I:\ \text{not summed}) \tag{2.26}$$

transforms, under a gauge transformation, as $W_I(x,\vec{y})\to\Omega(x,\vec{y})W_I(x,\vec{y})\Omega(x,\vec{y}+\vec{l}_I)^\dagger$. Using Eq. (2.21), one finds that

$$W_I(x,\vec{y})U_I\to\Omega(x,\vec{y})W_I(x,\vec{y})U_I\Omega^\dagger(x,\vec{y}). \tag{2.27}$$

The eigenvalues of W_IU_I are invariant under gauge transformations preserving the boundary conditions. The phases of the eigenvalues, called Wilson line phases, cannot be gauge away. They are non-Abelian analogues of Aharonov-Bohm phases.

These Wilson line phases parametrize degenerate classical vacua. At the quantum level the effective potential for Wilson line phases becomes nontrivial. When the effective potential is minimized at nonvanishing Wilson line phases, the physical symmetry of the theory changes from the symmetry of boundary conditions.

The effect of nonvanishing vacuum expectation values of Wilson line phases can be understood as an effective change in boundary conditions. As explained in the previous subsection, there are large gauge transformations which change boundary conditions. The existence of such gauge transformations is in one-to-one correspondence with the existence of physical degrees of freedom of Wilson line phases in a given theory.

Suppose that the effective potential is minimized at nonvanishing $\langle A_{y^I}\rangle\neq 0$, $[\langle A_{y^1}\rangle,\langle A_{y^2}\rangle]=0$. Perform a large gauge transformation

$$\Omega(\vec{y})=\exp\{-ig(\langle A_{y^1}\rangle y^1+\langle A_{y^2}\rangle y^2)\}. \tag{2.28}$$

Then the new gauge potentials satisfy $\langle A'_{y^I}\rangle=0$. Simultaneously the boundary conditions change as in Eqs. (2.19):

$$\begin{aligned}P'_j&=\Omega(\vec{z}_j-\vec{y})P_j\Omega(\vec{z}_j+\vec{y})^\dagger=P_j\Omega(-\vec{z}_j+\vec{y})\Omega(\vec{z}_j+\vec{y})^\dagger\\&=P_j\Omega(-2\vec{z}_j)\equiv P_j^{\rm sym}.\end{aligned} \tag{2.29}$$

In the second equality we made use of the relation $\{\langle A_{y^I}\rangle,P_j\}=0$. Since $\langle A'_{y^I}\rangle=0$, the physical symmetry of the theory $H_{\rm phys}$ is generated by generators belonging to

$$\mathcal{H}_{\rm phys}=\{T^a;[T^a,\ P_j^{\rm sym}]=0\ \ (j=0,1,2)\}. \tag{2.30}$$

The physical symmetry $H_{\rm phys}$ can be either larger or smaller than $H_{\rm BC}$.

As emphasized in Refs. [11] and [31], the physical symmetry $H_{\rm phys}$ is the same in all theories belonging to the same equivalence class of boundary conditions. The dynamics of Wilson line phases guarantees it.

III. ORBIFOLD CONDITIONS AND MODE EXPANSIONS IN $SU(2)$ THEORY

Let us examine $SU(2)$ gauge theory for which complete classification of orbifold boundary conditions can be easily achieved. Boundary condition matrices P_j $(j=0,1,2)$ must satisfy $P_j=P_j^\dagger=P_j^{-1}$ and $P_1P_0P_2=P_2P_0P_1$. A complete classification of boundary conditions in $SU(N)$ gauge theory on the orbifold S^1/Z_2 has been given in Ref. [31].

A. Orbifold conditions

To classify boundary conditions $\{P_j\}$, we first diagonalize P_0, utilizing global $SU(2)$ invariance. Up to a sign factor, $P_0=\mathbf{1}_{2\times2}$ or τ^3. If $P_0=\mathbf{1}_{2\times2}$, P_1 can be diagonalized, and therefore $P_1=\mathbf{1}_{2\times2}$ or τ^3. In the case $P_0=P_1=\mathbf{1}_{2\times2}$, P_2 is diagonalized as well. Even in the case $P_0=\mathbf{1}_{2\times2}$, $P_1=\tau^3$, P_2 must be diagonal to satisfy $P_1P_2=P_2P_1$. In other words, if one of P_j's is $\mathbf{1}_{2\times2}$, all P_j's are diagonal up to a global $SU(2)$ transformation.

In the case $P_0=\tau^3$ and $P_1,P_2\neq\pm\mathbf{1}_{2\times2}$, we recall that the most general form of $P(\neq\pm\mathbf{1}_{2\times2})$ satisfying $P=P^\dagger=P^{-1}$ is given by $P=\tau^3e^{i(\alpha_1\tau^1+\alpha_2\tau^2)}$. Given $P_0=\tau^3$, there still remains $U(1)$ invariance. Utilizing the global $U(1)$ invariance, one can bring P_1 into the form $P_1=\tau^3e^{i\pi a\tau^2}$. Then, to satisfy $P_1\tau^3P_2=P_2\tau^3P_1$, P_2 must be $P_2=\tau^3e^{i\pi b\tau^2}$.

To summarize, boundary conditions $\{P_0,P_1,P_2\}$ are classified as

$$\text{(i)}\quad P_0=\mathbf{1}_{2\times2},\ \ P_1,P_2=\mathbf{1}_{2\times2}\ \text{ or }\ \pm\tau^3,$$

$$\text{(ii)}\quad (P_0,P_1,P_2)=(\tau^3,\tau^3e^{i\pi a\tau^2},\tau^3e^{i\pi b\tau^2}) \tag{3.1}$$

$$P_3=\tau^3 e^{i\pi(a+b)\tau^2},$$

$$U_1=e^{-i\pi a\tau^2}=\begin{pmatrix}\cos\pi a & -\sin\pi a\\ \sin\pi a & \cos\pi a\end{pmatrix},$$

$$U_2=\begin{pmatrix}\cos\pi b & -\sin\pi b\\ \sin\pi b & \cos\pi b\end{pmatrix}. \tag{3.2}$$

Boundary condition (3.2) is periodic in real parameters a,b with a period 2. The symmetry of boundary condition (3.1) is either $SU(2)$ or $U(1)$. The symmetry of boundary condition (3.2) is $U(1)$ if both a and b are integers, and none otherwise.

B. Wilson line phases

There is no degree of freedom of a Wilson line phase with boundary condition (3.1). In the case of boundary condition (3.2) with general values of a and b, there is no zero mode associated with A_μ^a, but there are zero modes for A_{y^I} and may develop expectation values:

$$\langle A_{y^1}\rangle=\frac{\alpha}{2R_1 g}\tau^2,\quad \langle A_{y^2}\rangle=\frac{\beta}{2R_2 g}\tau^2. \tag{3.3}$$

The expectation values α and β are dynamically determined such that the effective potential is minimized. They are related to the Wilson line phases by

$$W_1U_1=\left\langle \exp\left(ig\oint dy^1 A_{y^1}\right)\right\rangle e^{-i\pi a\tau^2}=e^{i\pi(\alpha-a)\tau^2},$$

$$W_2U_2=\left\langle \exp\left(ig\oint dy^2 A_{y^2}\right)\right\rangle e^{-i\pi b\tau^2}=e^{i\pi(\beta-b)\tau^2}. \tag{3.4}$$

C. Equivalence classes of boundary conditions

Consider boundary conditions (3.2). We perform a large gauge transformation with

$$\Omega(c_1,c_2)=\exp\left\{i\left(\frac{c_1}{2R_1}y^1+\frac{c_2}{2R_2}y^2\right)\tau^2\right\}. \tag{3.5}$$

Then the boundary condition matrices change to

$$(P_0',P_1',P_2')=(\tau^3,\tau^3e^{i\pi(a-c_1)\tau^2},\tau^3e^{i\pi(b-c_2)\tau^2}). \tag{3.6}$$

In other words, all sets (P_0,P_1,P_2) of boundary conditions in Eqs. (3.2) are in one equivalence class of boundary conditions. Each set of the boundary conditions (3.1) forms a distinct equivalence class.

Under Eq. (3.5), the zero modes of A_{y^I} in Eqs. (3.3) are transformed as $(\alpha,\beta)\to(\alpha-c_1,\beta-c_2)$. It is recognized that the combination $(\alpha-a,\beta-b)$ is invariant under Eq. (3.5).

Now suppose that the expectation values α,β in Eqs. (3.3) take nontrivial values. With a gauge transformation $\Omega(\alpha,\beta)$, the background field $\langle A_{y^I}\rangle$ can be removed, and in the new gauge we have $\langle A_{y^I}'\rangle=0$. The new boundary conditions are $(P_0^{\rm sym},P_1^{\rm sym},P_2^{\rm sym})=(\tau^3,\tau^3e^{i\pi(a-\alpha)\tau^2},\tau^3e^{i\pi(b-\beta)\tau^2})$. The physical symmetry $H_{\rm phys}$ is generated by the generators of the $SU(2)$ commuting with $P_i^{\rm sym}$ $(i=0,1,2)$.

The physical content of the theory at the quantum level is the same in a given equivalence class. In particular, it does not depend on the parameters (a,b) in Eqs. (3.2). Gauge invariance implies that the effective potential for the Wilson line phases is a function of gauge invariant $\alpha-a$ and $\beta-b$. Depending on the content of matter fields, the effective potential can take the minimum value at nontrivial $(\alpha-a,\beta-b)$ as we will see below.

D. Mode expansions

Given the orbifold boundary conditions, each field is expanded in eigenmodes. On T^2/Z_2 there are two types of mode expansions, Z_2 singlets and Z_2 doublets.

A Z_2 singlet field $\phi(x,y)$ obeys

$$\phi(x,\vec z_j-\vec y)=P_j\phi(x,\vec z_j+\vec y),\quad P_j=+\ \text{or}\ -. \tag{3.7}$$

Each singlet field is specified with (P_0,P_1,P_2). Mode expansions are

$$\phi(x,\vec y)=\frac{1}{\sqrt{2\pi^2R_1R_2}}\phi_{00}(x)\begin{pmatrix}1\\0\end{pmatrix}+\frac{1}{\sqrt{\pi^2R_1R_2}}\times\sum_{(n,m)\in K_+}\phi_{nm}(x)\begin{pmatrix}\cos\\ \sin\end{pmatrix}\left(\frac{ny^1}{R_1}+\frac{my^2}{R_2}\right)$$
$$\text{for}\ (P_0,P_1,P_2)=\begin{cases}(+,+,+),\\(-,-,-),\end{cases} \tag{3.8}$$

$$\frac{1}{\sqrt{\pi^2R_1R_2}}\sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty}\phi_{nm}(x)\begin{pmatrix}\cos\\ \sin\end{pmatrix}\left(\frac{ny^1}{R_1}+\frac{\left(m+\frac{1}{2}\right)y^2}{R_2}\right)$$
$$\text{for}\ (P_0,P_1,P_2)=\begin{cases}(+,+,-),\\(-,-,+),\end{cases} \tag{3.9}$$

$$\frac{1}{\sqrt{\pi^2R_1R_2}}\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}\phi_{nm}(x)\begin{pmatrix}\cos\\ \sin\end{pmatrix}\left(\frac{\left(n+\frac{1}{2}\right)y^1}{R_1}+\frac{my^2}{R_2}\right)$$
$$\text{for}\ (P_0,P_1,P_2)=\begin{cases}(+,-,+),\\(-,+,-),\end{cases} \tag{3.10}$$

$$\frac{1}{\sqrt{\pi^2R_1R_2}}\sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty}\phi_{nm}(x)\begin{pmatrix}\cos\\ \sin\end{pmatrix}\left(\frac{\left(n+\frac{1}{2}\right)y^1}{R_1}+\frac{\left(m+\frac{1}{2}\right)y^2}{R_2}\right)\quad\text{for}\ (P_0,P_1,P_2)=\begin{cases}(+,-,-),\\(-,+,+).\end{cases} \tag{3.11}$$

In Eq. (3.8)

$$\sum_{(n,m)\in K_+} B_{n,m} = \sum_{n=1}^{\infty} B_{n,0} + \sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty} B_{n,m}. \quad (3.12)$$

Zero modes exist only for $(P_0,P_1,P_2)=(+,+,+)$.

A Z_2 doublet field $\phi=\binom{\phi_1}{\phi_2}$ appears when a boundary condition of the type (3.2) is considered. It obeys

$$\phi(x,-y^1,-y^2)=\begin{pmatrix}1 & \\ & -1\end{pmatrix}\phi(x,y^1,y^2),$$

$$\phi(x,y^1+2\pi R_1,y^2)=\begin{pmatrix}\cos\pi a & -\sin\pi a\\ \sin\pi a & \cos\pi a\end{pmatrix}\phi(x,y^1,y^2),$$

$$\phi(x,y^1,y^2+2\pi R_2)=\begin{pmatrix}\cos\pi b & -\sin\pi b\\ \sin\pi b & \cos\pi b\end{pmatrix}\phi(x,y^1,y^2). \quad (3.13)$$

Its mode expansion is given by

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}(x,\vec{y})=\frac{1}{\sqrt{2\pi^2R_1R_2}}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\phi_{nm}(x)\begin{pmatrix}\cos\\ \sin\end{pmatrix}\times\left[\frac{\left(n+\frac{1}{2}a\right)y^1}{R_1}+\frac{\left(m+\frac{1}{2}b\right)y^2}{R_2}\right]. \quad (3.14)$$

Z_2 doublets appear when the Scherk-Schwarz supersymmetry (SUSY) breaking [34,18] is implemented in SUSY theories as well. We see below that twists specified with (a,b) play an important role to give fermions nonvanishing masses in four dimensions.

IV. EFFECTIVE POTENTIAL IN $SU(2)$ GAUGE THEORY

In order to study physical symmetry of the theory, one must take into account quantum corrections. To this end one needs to evaluate the effective potential for Wilson line phases. Wilson line phases are related to zero modes of the component gauge fields in extra dimensions.

We study patterns of gauge symmetry breaking in $SU(2)$ gauge theory on $M^4\times T^2/Z_2$ in order to get insight into the electroweak gauge symmetry breaking and the gauge-Higgs unification in a more realistic framework. We believe that the analysis here provides us many useful and important hints.

Dynamical gauge symmetry breaking or enhancement can take place when the boundary condition in Eqs. (3.2) is adopted. As explained in the previous section, all the boundary conditions in Eqs. (3.2) are in one equivalence class, and therefore the same physical results are obtained independently of the values of the parameters (a,b), provided that the dynamics of Wilson line phases is taken into account. Hence we adopt, without loss of generality,

$$(P_0,P_1,P_2)=(\tau^3,\tau^3,\tau^3), \quad (4.1)$$

which implies that $P_3=\tau^3$ and $U_1=U_2=1_{2\times 2}$. With this boundary condition, zero modes for A_μ^a and $A_{y^I}^a$ $(I=1,2)$ exist for $A_\mu^{a=3}$ and $A_{y^I}^{a=1,2}$ $(I=1,2)$, respectively. The symmetry of the theory at the tree level is $U(1)$.

We employ the standard background filed method to evaluate the effective potential for Wilson line phases. Zero modes for $A_{y^I}^{a=1,2}$ are parametrized as

$$A_{y^1}=\frac{1}{2gR_1}(\alpha_1\tau^1+\alpha_2\tau^2)\equiv\frac{\alpha}{2gR_1}\begin{pmatrix}0 & e^{-i\theta}\\ e^{i\theta} & 0\end{pmatrix},$$

$$A_{y^2}=\frac{1}{2gR_2}(\beta_1\tau^1+\beta_2\tau^2)\equiv\frac{\beta}{2gR_2}\begin{pmatrix}0 & e^{-i\tilde{\theta}}\\ e^{i\tilde{\theta}} & 0\end{pmatrix}. \quad (4.2)$$

Here one should note that, contrary to the case S^1/Z_2, there are two directions of the compactified dimensions, so that the tree-level potential is induced for the background given above,

$$V_{tree}=-\frac{g^2}{2}\mathrm{tr}[A_{y^1},A_{y^2}]^2=\frac{1}{4g^2(R_1R_2)^2}(\alpha_1\beta_2-\alpha_2\beta_1)^2. \quad (4.3)$$

The vanishing tree-level potential is achieved when

$$\alpha_1\beta_2-\alpha_2\beta_1=0, \quad (4.4)$$

which implies the vanishing field strength $\langle F_{y^1y^2}\rangle=0$. Once we restrict ourselves to the case (4.4), the parametrization of background fields is further simplified. The relation (4.4) means $\theta=\tilde{\theta}$, and by using the $U(1)$ gauge degrees of freedom, one can take $\theta=\tilde{\theta}=0$. To summarize, we take, as background fields,

$$A_{y^1}=\frac{1}{2gR_1}\alpha\tau^1,\quad A_{y^2}=\frac{1}{2gR_2}\beta\tau^1. \quad (4.5)$$

The effective potential for (α,β) is obtained by integrating quantum fluctuations of every field.

A. Gauge fields and ghosts

Contributions from the gauge fields and ghosts to the effective potential are given by

$$V^{gauge+ghost}=-\frac{i}{2}\mathrm{tr}\ln D_LD^L(A_{y^I}), \quad (4.6)$$

where $D_LD^L(A_{y^I})=\partial_\mu\partial^\mu-\Sigma_{I=1}^2D_{y^I}^2(A_{y^I})$. One needs to find eigenvalues of the mass operator $D_{y^I}^2(A_{y^I})$ to evaluate the effective potential.

With Eq. (4.1), the parity assignment for A_{y^I} is given by

$$A_{y^I}^{a=1,2}:\ (P_0,P_1,P_2)=(+++), \quad (4.7)$$

$$A_{y^I}^{a=3}:\ (P_0,P_1,P_2)=(---). \quad (4.8)$$

The mass operator for $A^a_{y^I}$ $(a=1,2,3)$ for the background field configuration (4.5) is obtained by inserting the mode expansion (3.8). As a result of the nonvanishing background (α,β), $A^2_{y^I}$ and $A^3_{y^I}$ mix with each other. It is given by

$$\sum_{I=1}^{2} D^2_{y^I} = \frac{1}{R_1^2}\begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2+\alpha^2 & 2n\alpha \\ 0 & 2n\alpha & n^2+\alpha^2 \end{pmatrix} + \frac{1}{R_2^2}\begin{pmatrix} m^2 & 0 & 0 \\ 0 & m^2+\beta^2 & 2m\beta \\ 0 & 2m\beta & m^2+\beta^2 \end{pmatrix} \quad \text{for } (n,m)\in K_+ . \tag{4.9}$$

The eigenvalues of the operator for $(n,m)\neq(0,0)$ are easily obtained as

$$\left(\frac{n}{R_1}\right)^2+\left(\frac{m}{R_2}\right)^2, \quad \left(\frac{n\pm\alpha}{R_1}\right)^2+\left(\frac{m\pm\beta}{R_2}\right)^2 \quad \text{for } (n,m)\in K_+ . \tag{4.10}$$

Zero modes $(n,m)=(0,0)$ exist only for $A^{a=1,2}_{y^I}$. Eigenvalues for the zero modes are given by

$$0, \quad \left(\frac{\alpha}{R_1}\right)^2+\left(\frac{\beta}{R_2}\right)^2. \tag{4.11}$$

In a similar way, we can compute contributions from $A^{a=1,2,3}_\mu$ to the effective potential. In this case the parity assignment is

$$A^{a=1,2}_\mu:\ (P_0,P_1,P_2)=(---), \tag{4.12}$$

$$A^{a=3}_\mu:\ (P_0,P_1,P_2)=(+++). \tag{4.13}$$

The mass matrix has the same structure as before. Only the $a=3$ component of A^a_μ has a zero mode. Hence eigenvalues of the mass operator are

$$\left(\frac{n}{R_1}\right)^2+\left(\frac{m}{R_2}\right)^2, \quad \left(\frac{n\pm\alpha}{R_1}\right)^2+\left(\frac{m\pm\beta}{R_2}\right)^2 \quad \text{for } (n,m)\in K_+ , \quad \left(\frac{\alpha}{R_1}\right)^2+\left(\frac{\beta}{R_2}\right)^2. \tag{4.14}$$

The mass matrix for ghost fields is the same as that for A_μ. Contributions to the effective potential from A_μ and ghosts are, therefore, $4-2=2$ times contributions coming from the spectrum (4.14).

In six dimensions there are two extra-dimensional components A_{y^I}. Therefore, if one adds Eqs. (4.10), (4.11), and (4.14), one obtains two copies of

$$\left(\frac{n}{R_1}\right)^2+\left(\frac{m}{R_2}\right)^2, \quad \left(\frac{n+\alpha}{R_1}\right)^2+\left(\frac{m+\beta}{R_2}\right)^2, \quad \left(\frac{n-\alpha}{R_1}\right)^2+\left(\frac{m-\beta}{R_2}\right)^2 \quad (-\infty<n,m<+\infty), \tag{4.15}$$

for the mass spectrum. Here we used the fact that K_+ covers a half of the integer lattice plane after (0, 0) is removed.

The contributions from the gauge fields and ghost fields are summarized as

$$V_{\rm eff}(\alpha,\beta)^{gauge} = 2\,\frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\,\frac{1}{2\pi^2R_1R_2}\times\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left\{2\ln\left[p_E^2+\left(\frac{n+\alpha}{R_1}\right)^2+\left(\frac{m+\beta}{R_2}\right)^2\right] + \ln\left[p_E^2+\left(\frac{n}{R_1}\right)^2+\left(\frac{m}{R_2}\right)^2\right]\right\}. \tag{4.16}$$

Here the Wick rotation has been made and p_E stands for the Euclidean momenta in four dimensions. As shown in Refs. [20,12],

$$\begin{aligned} I(\alpha,\beta) &\equiv \frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\,\frac{1}{2\pi^2R_1R_2}\times\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\ln\left[p_E^2+\left(\frac{n+\alpha}{R_1}\right)^2+\left(\frac{m+\beta}{R_2}\right)^2\right] \\ &= -\frac{1}{16\pi^9}\left\{\frac{1}{R_1^6}\sum_{n=1}^{\infty}\frac{\cos(2\pi n\alpha)}{n^6} + \frac{1}{R_2^6}\sum_{m=1}^{\infty}\frac{\cos(2\pi m\beta)}{m^6} + 2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos(2\pi n\alpha)\cos(2\pi m\beta)}{(n^2R_1^2+m^2R_2^2)^3}\right\} \\ &\quad + (\alpha,\beta-\text{independent terms}). \end{aligned} \tag{4.17}$$

In terms of $I(\alpha,\beta)$,

$$V_{\rm eff}(\alpha,\beta)^{gauge} = 4I(\alpha,\beta)+2I(0,0), \tag{4.18}$$

which is depicted in Fig. 1. We note that one unit of I represents contributions to the effective potential from two physical degrees of freedom on $M^4\times(T^2/Z_2)$.

B. Scalar fields in the fundamental representation

A scalar field $H(x,y)=(H_1,H_2)^t$ in the fundamental representation satisfies Eq. (2.9) or

$$H(x,\vec{z}_j-\vec{y}) = \eta_j^s\tau^3 H(x,\vec{z}_j+\vec{y}) \quad (j=0,1,2,3). \tag{4.19}$$

Each component of H is a Z_2 singlet. The parity assignment is

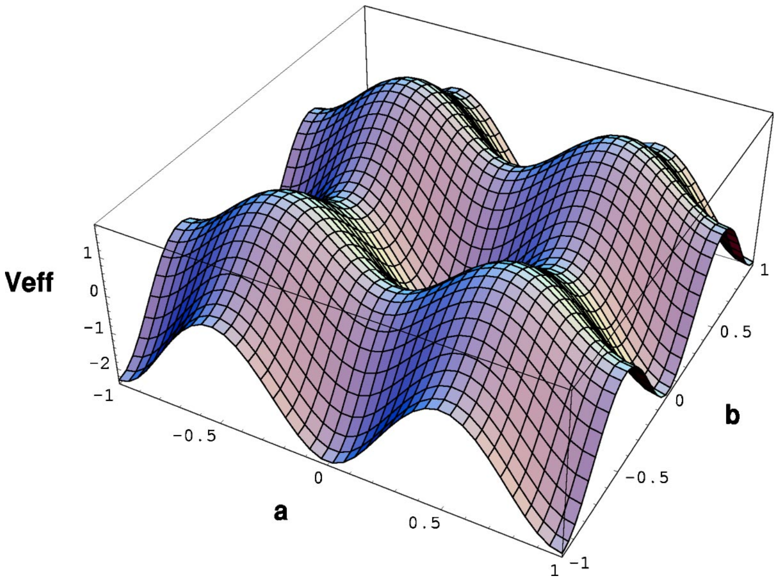

FIG. 1. The effective potential $V_{\rm eff}(\alpha,\beta)$, Eq. (4.18), in the pure gauge theory with $R_1=R_2$. There are four degenerate minima at $(\alpha,\beta)=(0,0)$, (1,0), (0,1), and (1.1). All of them correspond to $U(1)$ symmetric states.

$$H_1:\ (P_0,P_1,P_2)=(+\eta_0,+\eta_1,+\eta_2),$$

$$H_2:\ (P_0,P_1,P_2)=(-\eta_0,-\eta_1,-\eta_2). \tag{4.20}$$

Consequently the mode expansion of the doublet H is given by one of the pairs in Eqs. (3.8)–(3.11).

Let us first examine the case $\eta_0=\eta_1=\eta_2=+1$ or -1. The mode expansion of (H_1,H_2) is given by a pair in Eq. (3.8). When the mass operator

$$\sum_{I=1}^{2} D_{y^I}^2=\begin{pmatrix} \partial_{y^1} & -i\alpha/2R_1 \\ -i\alpha/2R_1 & \partial_{y^1}\end{pmatrix}^2 +\begin{pmatrix} \partial_{y^2} & -i\beta/2R_2 \\ -i\beta/2R_2 & \partial_{y^2}\end{pmatrix}^2$$

acts on (n,m) $(\in K_+)$ components in the mode expansion of H, it yields a matrix

$$\frac{1}{R_1^2}\begin{pmatrix} n^2+\frac{1}{4}\alpha^2 & i\alpha n \\ -i\alpha n & n^2+\frac{1}{4}\alpha^2\end{pmatrix} +\frac{1}{R_2^2}\begin{pmatrix} m^2+\frac{1}{4}\beta^2 & i\beta m \\ -i\beta m & m^2+\frac{1}{4}\beta^2\end{pmatrix}, \tag{4.21}$$

which has eigenvalues

$$\frac{\left(n+\frac{1}{2}\alpha\right)^2}{R_1^2}+\frac{\left(m+\frac{1}{2}\beta\right)^2}{R_2^2},\quad \frac{\left(n-\frac{1}{2}\alpha\right)^2}{R_1^2} +\frac{\left(m-\frac{1}{2}\beta\right)^2}{R_2^2}\quad \text{where}\ \ (n,m)\in K_+ . \tag{4.22}$$

Only one of H_1 or H_2 has a zero mode (y^I-independent mode). Its eigenvalue for $\Sigma_{I=1}^2 D_{y^I}^2$ is

$$\frac{\alpha^2}{4R_1^2}+\frac{\beta^2}{4R_2^2}. \tag{4.23}$$

Combining Eqs. (4.22) and (4.23), one obtains

$$\frac{\left(n+\frac{1}{2}\alpha\right)^2}{R_1^2}+\frac{\left(m+\frac{1}{2}\beta\right)^2}{R_2^2}\quad (-\infty<n,m<+\infty). \tag{4.24}$$

The analysis in other cases of parity assignment (η_0,η_1,η_2) is almost the same. The mode expansion is given by one of the pairs in Eqs. (3.9)–(3.11). There is no zero mode. At this junction it is convenient to introduce δ_j by

$$\delta(\eta)=\begin{cases} 0 & \text{for}\ \ \eta=+1,\\ 1 & \text{for}\ \ \eta=-1,\end{cases}$$

$$\delta_j=\delta(\eta_0\eta_j)\quad (j=1,2). \tag{4.25}$$

The only change arising when the mass operator $\Sigma_{I=1}^2 D_{y^I}^2$ acts on (n,m) components in the mode expansion is that n

and m in the matrix (4.21) are replaced by $n+(1/2)\delta_1$ and $m+(1/2)\delta_2$, respectively. Consequently the eigenvalues of $\Sigma_{I=1}^2 D_{y^I}^2$ are given by

$$\frac{\left[n+\frac{1}{2}(\alpha+\delta_1)\right]^2}{R_1^2}+\frac{\left[m+\frac{1}{2}(\beta+\delta_2)\right]^2}{R_2^2}\quad(-\infty<n,m<+\infty) \tag{4.26}$$

in all cases.

The contributions of one scalar doublet to the effective potential is found, from Eq. (4.26), to be

$$V_{\rm eff}(\alpha,\beta)^{sF}=2I\left[\frac{1}{2}(\alpha+\delta_1),\frac{1}{2}(\beta+\delta_2)\right]. \tag{4.27}$$

Here the factor of 2 accounts for the complex nature of the field H.

C. Weyl fermions in the fundamental representation

Let us next consider contributions to the effective potential from fermions in the fundamental representation. We start with a Weyl fermion satisfying $\Gamma^7\psi=-\psi$ and take all the sign factor $(\eta_0,\eta_1,\eta_2)=(+,+,+)$. Then, the mode expansion with the boundary condition (2.11) or (2.17) is given by

$$\begin{pmatrix}U_{L1}\\ U_{L2}\end{pmatrix}(x,y^I)=\frac{1}{\sqrt{2\pi^2R_1R_2}}\begin{pmatrix}U_{L1(00)}(x)\\ 0\end{pmatrix}+\frac{1}{\sqrt{\pi^2R_1R_2}}\sum_{(n,m)\in K_+}\begin{pmatrix}U_{L1(nm)}(x)\\ U_{L2(nm)}(x)\end{pmatrix}\begin{pmatrix}\cos\\ \sin\end{pmatrix}\times\left(\frac{n}{R_1}y^1+\frac{m}{R_2}y^2\right),$$

$$\begin{pmatrix}D_{R1}\\ D_{R2}\end{pmatrix}(x,y^I)=\frac{1}{\sqrt{2\pi^2R_1R_2}}\begin{pmatrix}0\\ D_{R2(00)}(x)\end{pmatrix}+\frac{1}{\sqrt{\pi^2R_1R_2}}\sum_{(n,m)\in K_+}\begin{pmatrix}D_{R1(nm)}(x)\\ D_{R2(nm)}(x)\end{pmatrix}\begin{pmatrix}\sin\\ \cos\end{pmatrix}\times\left(\frac{n}{R_1}y^1+\frac{m}{R_2}y^2\right). \tag{4.28}$$

Note that each of U_{La} or D_{Ra} is a four-component spinor with definite four-dimensional chirality. The mass operator for U_{La} in this basis is given by

$$\sum_{I=1}^2 D_{y^I}^2\Rightarrow\frac{1}{R_1^2}\begin{pmatrix}n^2+\frac{1}{4}\alpha^2 & i\alpha n\\ -i\alpha n & n^2+\frac{1}{4}\alpha^2\end{pmatrix}+\frac{1}{R_2^2}\begin{pmatrix}m^2+\frac{1}{4}\beta^2 & i\beta m\\ -i\beta m & m^2+\frac{1}{4}\beta^2\end{pmatrix} \tag{4.29}$$

for $(n,m)\in K_+$. Eigenvalues are given by

$$\left(\frac{n+\frac{1}{2}\alpha}{R_1}\right)^2+\left(\frac{m+\frac{1}{2}\beta}{R_2}\right)^2,$$

$$\left(\frac{n-\frac{1}{2}\alpha}{R_1}\right)^2+\left(\frac{m-\frac{1}{2}\beta}{R_2}\right)^2\quad\text{for}\quad(n,m)\in K_+ \tag{4.30}$$

and

$$\left(\frac{\alpha}{2R_1}\right)^2+\left(\frac{\beta}{2R_2}\right)^2 \tag{4.31}$$

for the zero mode $U_{L1(00)}$. Combining Eqs. (4.30) and (4.31), one finds

$$\left(\frac{n+\frac{1}{2}\alpha}{R_1}\right)^2+\left(\frac{m+\frac{1}{2}\beta}{R_2}\right)^2\quad(-\infty<n,m<+\infty). \tag{4.32}$$

The spectrum is the same as for a scalar field in the fundamental representation.

Eigenvalues of the mass operator for D_{Ra} are the same as those for U_{La}. Therefore the contributions to the effective potential from a Weyl fermion in the fundamental representation with $(\eta_0,\eta_1,\eta_2)=(+,+,+)$ is given by

$$V^{fF}=-4I\left(\frac{\alpha}{2},\frac{\beta}{2}\right). \tag{4.33}$$

The minus sign is due to fermi statistics.

Extension to other cases of (η_0,η_1,η_2) is straightforward. The basis for the mode expansion (4.28), which corresponds to Eq. (3.8), is changed to one of Eqs. (3.9)–(3.11). The resultant spectrum of the mass operator $\Sigma_{I=1}^2 D_{y^I}^2$ is the same as in the scalar field case. (n,m) in Eq. (4.32) is replaced by $(n+(1/2)\delta_1,m+(1/2)\delta_2)$ where δ_j is defined in Eqs. (4.25). Consequently the contributions to the effective potential from a Weyl fermion in the fundamental representation is summarized as

$$V^{fF}=-4I\left(\frac{\alpha+\delta_1}{2},\frac{\beta+\delta_2}{2}\right). \tag{4.34}$$

D. Weyl fermions and scalars in the adjoint representation

Contributions of matter fields in the adjoint representation are easily obtained as in the preceding subsections. Consider a Weyl fermion. Note that $D_M\psi=\partial_M\psi+ig[A_M,\psi]$. With the background fields (4.5),

$$\begin{aligned}2\mathrm{Tr}\overline{\psi}i(\Gamma^2 D_5+\Gamma^5 D_6)\psi=\overline{\psi^1}i(\Gamma^5\partial_{y^1}+\Gamma^6\partial_{y^2})\psi^1 \\ +(\overline{\psi^2},\overline{\psi^3})i\left\{\Gamma^5\begin{pmatrix}\partial_{y^1} & \alpha/R_1\\ -\alpha/R_1 & \partial_{y^1}\end{pmatrix}\right. \\ \left.+\Gamma^6\begin{pmatrix}\partial_{y^2} & \beta/R_2\\ -\beta/R_2 & \partial_{y^2}\end{pmatrix}\right\}\begin{pmatrix}\psi^2\\ \psi^3\end{pmatrix}.\end{aligned} \tag{4.35}$$

The parity assignment for a Weyl fermion satisfying $\Gamma^7\psi=-\psi$ is

$$\begin{aligned}\begin{pmatrix}U_L^{a=1}\\ D_R^{a=1}\end{pmatrix},\begin{pmatrix}U_L^{a=2}\\ D_R^{a=2}\end{pmatrix}:\ (P_0,P_1,P_2)=\begin{cases}(-\eta_0,-\eta_1,-\eta_2),\\ (+\eta_0,+\eta_1,+\eta_2),\end{cases}\\ \begin{pmatrix}U_L^{a=3}\\ D_R^{a=3}\end{pmatrix}:\ (P_0,P_1,P_2)=\begin{cases}(+\eta_0,+\eta_1,+\eta_2),\\ (-\eta_0,-\eta_1,-\eta_2).\end{cases}\end{aligned} \tag{4.36}$$

The net consequence in the evaluation of the mass operator $\Sigma_{I=1}^2 D_{y^I}^2$ is that (α,β) in the case of fermions in the fundamental representation is replaced by $(2\alpha,\ 2\beta)$. The contributions to the effective potential are summarized as

$$V^{f,Ad}=-2\left\{I\left(\frac{1}{2}\delta_1,\frac{1}{2}\delta_2\right)+2I\left(\alpha+\frac{1}{2}\delta_1,\beta+\frac{1}{2}\delta_2\right)\right\}. \tag{4.37}$$

Similarly, for a real scalar field in the adjoint representation we have

$$V^{s,Ad}=\frac{1}{2}\left\{I\left(\frac{1}{2}\delta_1,\frac{1}{2}\delta_2\right)+2I\left(\alpha+\frac{1}{2}\delta_1,\beta+\frac{1}{2}\delta_2\right)\right\}. \tag{4.38}$$

Adding contributions from gauge fields and ghosts, we immediately see that $V^{gauge+ghost}+V^{f,Ad}=0$ if $\delta_1=\delta_2=0$. This is because in six dimensions (A_M,ψ_{adj}) forms the vector multiplet of $\mathcal{N}=1$ supersymmetry [35] and their on-shell degrees of freedom are equal to each other. Therefore, the contributions from bosons and fermions are canceled to yield the vanishing effective potential. It is important to observe that the cancellation holds only for the sign assignment $(\eta_0,\eta_1,\eta_2)=(+++)$ or $(---)$. For the other cases, the effective potential does not vanish. The $\mathcal{N}=1$ supersymmetry is broken by the different assignment of the sign factors η_i for bosons and fermions. This is similar to the Scherk-Schwarz breaking of supersymmetry [34], in which different boundary conditions for bosons and fermions are imposed.

E. Z_2 doublets

Twists along noncontractible loops on T^2 can be introduced for each field by doubling the number of degrees of freedom. As we see below, these T^2 twists give fermions additional masses in four dimensions. This may be very important in the phenomenological viewpoint, as these twists can substitute Yukawa interactions. We prepare a pair of Weyl fermions (ψ,ψ') satisfying

$$\begin{aligned}\begin{pmatrix}\psi\\ \psi'\end{pmatrix}(x,-\vec{y})=\eta_0 T[P_0](i\Gamma^4\Gamma^5)\begin{pmatrix}\psi\\ -\psi'\end{pmatrix}(x,\vec{y}),\\ \begin{pmatrix}\psi\\ \psi'\end{pmatrix}(x,y^1+2\pi R_1,y^2)=\begin{pmatrix}\cos\pi a & -\sin\pi a\\ \sin\pi a & \cos\pi a\end{pmatrix}\eta_0\eta_1 T[U_1]\\ \times\begin{pmatrix}\psi\\ \psi'\end{pmatrix}(x,\vec{y}),\\ \begin{pmatrix}\psi\\ \psi'\end{pmatrix}(x,y^1,y^2+2\pi R_2)=\begin{pmatrix}\cos\pi b & -\sin\pi b\\ \sin\pi b & \cos\pi b\end{pmatrix}\eta_0\eta_2 T[U_2]\\ \times\begin{pmatrix}\psi\\ \psi'\end{pmatrix}(x,\vec{y}).\end{aligned} \tag{4.39}$$

Nonvanishing a and b give twists on the pair (ψ,ψ'). Note that each pair can have its own $(a,\ b)$.

Let us illustrate it by considering fermions in the fundamental representation for which $T[P_0]\psi=P_0\psi$, etc. Take $\eta_0=1$, $P_0=P_1=P_2=\tau^3$, $U_1=U_2=1_{2\times 2}$. With the notation in Eqs. (2.16), (U_a,U_a') and (D_a,D_a') $(a=1,2)$ form Z_2 doublets. Their mode expansions are given, as in Eq. (3.14), by

$$\begin{aligned}\begin{pmatrix}U_{R1}\\ U_{R1}'\end{pmatrix}(x,\vec{y})=\frac{1}{\sqrt{2\pi^2 R_1R_2}}\sum_{n,m=-\infty}^{\infty}\hat{U}_{R1,nm}(x)\begin{pmatrix}\cos z_{nm}(\vec{y})\\ \sin z_{nm}(\vec{y})\end{pmatrix},\\ \begin{pmatrix}U_{R2}\\ U_{R2}'\end{pmatrix}(x,\vec{y})=\frac{1}{\sqrt{2\pi^2 R_1R_2}}\sum_{n,m=-\infty}^{\infty}\hat{U}_{R2,nm}(x)\\ \times\begin{pmatrix}-\sin z_{nm}(\vec{y})\\ \cos z_{nm}(\vec{y})\end{pmatrix},\\ z_{nm}(\vec{y})=\left(n+\frac{a+\delta_1}{2}\right)\frac{y^1}{R_1}+\left(m+\frac{b+\delta_2}{2}\right)\frac{y^2}{R_2}.\end{aligned} \tag{4.40}$$

Similar expansions hold for D_{La} and D_{La}' as well.

The nonvanishing Wilson line phases α and β mix $\hat{U}_{1,nm}$ and $\hat{U}_{2,nm}$ as in Sec. IV C. The resultant mass matrix takes the same form as in Eq. (4.29) where n and m are replaced by $n+1/2(a+\delta_1)$ and $m+1/2(b+\delta_2)$, respectively. Hence the eigenvalues are given by

$$\frac{1}{R_1^2}\left(n+\frac{a+\delta_1+\alpha}{2}\right)^2+\frac{1}{R_2^2}\left(m+\frac{b+\delta_2+\beta}{2}\right)^2,$$

$$\frac{1}{R_1^2}\left(n+\frac{a+\delta_1-\alpha}{2}\right)^2+\frac{1}{R_2^2}\left(m+\frac{b+\delta_2-\beta}{2}\right)^2,$$

$$-\infty<n,m<+\infty. \tag{4.41}$$

To summarize, the contributions to the effective potential from each Z_2 doublet of Weyl fermions in the fundamental representation are given by

$$V_{\rm doublet}^{fF}=-4\left\{I\left[\frac{1}{2}(\alpha+a+\delta_1),\frac{1}{2}(\beta+b+\delta_2)\right]+I\left[\frac{1}{2}(\alpha-a+\delta_1),\frac{1}{2}(\beta-b+\delta_2)\right]\right\}. \tag{4.42}$$

Extension to fields in other representation is straightforward.

F. Total effective potential

Adding all the contributions of Z_2 singlet fields, we find that the total effective potential for the Wilson line phases is given by

$$\begin{aligned}V_{\rm eff}(\alpha,\beta)=&4I(\alpha,\beta)+2I(0,0)+\sum_{\delta_1,\delta_2}2\{N^{s,F}_{(\delta_1\delta_2)}-2N^{f,F}_{(\delta_1\delta_2)}\}\\&\times I\left[\frac{1}{2}(\alpha+\delta_1),\frac{1}{2}(\beta+\delta_2)\right]\\&+\sum_{\delta_1,\delta_2}\frac{1}{2}\{N^{s,Ad}_{(\delta_1\delta_2)}-4N^{f,Ad}_{(\delta_1\delta_2)}\}\left\{I\left(\frac{1}{2}\delta_1,\frac{1}{2}\delta_2\right)\right.\\&\left.+2I\left(\alpha+\frac{1}{2}\delta_1,\beta+\frac{1}{2}\delta_2\right)\right\}.\end{aligned} \tag{4.43}$$

Here $N^{f,F}_{(\delta_1\delta_2)}$ and $N^{f,Ad}_{(\delta_1\delta_2)}$ are the numbers of Weyl fermion multiplets in the fundamental and adjoint representations with the parity assignment $(\delta_1\delta_2)$, respectively. $N^{s,F}_{(\delta_1\delta_2)}$ and $N^{s,Ad}_{(\delta_1\delta_2)}$ are defined similarly for scalar fields. ($N^{s,Ad}_{(\delta_1\delta_2)}$ counts the number of real scalar field multiplets.) If there exist fields of Z_2 doublets, their contributions need to be added.

The true vacuum is given by the global minimum of Eq. (4.43). As we see in the following section, the global minimum can be located at nonvanishing (α,β).

V. GAUGE SYMMETRY BREAKING

The true vacuum is determined by the global minimum of the effective potential for the Wilson line phases (4.43). We recall that α and β are phase variables with a period 2. The function $I(\alpha,\beta)$, which is defined in Eq. (4.17), satisfies $I(\alpha+1,\beta)=I(\alpha,\beta+1)=I(\alpha,\beta)$. It has the global minimum at (0, 0), the global maximum at (1/2, 1/2), and saddle points at (1/2, 0) and (0, 1/2) (mod 1), respectively.

A. Pure gauge field theory

The case of the pure $SU(2)$ gauge theory has been already examined in Sec. IV A. The effective potential is given by Eq. (4.18). The configurations that minimize the potential are found to be

$$(\alpha,\beta)=(0,0),(1,1),(1,0),(1,1). \tag{5.1}$$

We have seen that the phases α,β are determined dynamically.

Let us discuss the gauge symmetry at low energies. First of all, the Wilson line for the parametrization is given by

$$W_1=e^{i\pi\alpha\tau^1},\quad W_2=e^{i\pi\beta\tau^1}. \tag{5.2}$$

Let us move to a new gauge, in which $\langle A'_{y^I}\rangle=0$, by a gauge transformation

$$\Omega(\vec{y};\alpha,\beta)=\exp\left\{i\left(\frac{\alpha y^1}{2R_1}+\frac{\beta y^2}{2R_2}\right)\tau^1\right\}. \tag{5.3}$$

Then, new parity matrices in Eqs. (2.19) become

$$P'_0=\tau^3,\quad P'_1=e^{i\pi\alpha\tau^1}\tau^3,\quad P'_2=e^{i\pi\beta\tau^1}\tau^3. \tag{5.4}$$

As we have discussed, generators commuting with the new P'_i $(i=0,1,2)$ form a symmetry algebra at low energies. For $(\alpha,\beta)=(0,0)$, we have $P'_0=P'_1=P'_2=\tau^3$. Here $(1/2)\tau^3$ commutes with all the P'_i, so that the $U(1)$ symmetry survives at low energies. The symmetry of boundary conditions at the tree level is not broken even at the quantum level.

Taking into account the periodicity of the effective potential, the configurations $(\alpha,\beta)=(1,0)$,(0, 1),(1, 1) also give the vacuum configurations. These configurations are physically equivalent with $(\alpha,\beta)=(0,0)$. In order to see that, let us consider $(\alpha,\beta)=(1,0)$, for which we have $P'_0=\tau^3$, $P'_1=-\tau^3$, $P'_2=-\tau^3$. Again, $\tau^3/2$ commutes with these parity matrices, so that there is $U(1)$ gauge symmetry at low energies. One can also confirm that the mass spectrum on each vacuum is the same. Indeed, masses for $A^{a=3}_\mu$ are given by $(n+\alpha)^2R_1^{-2}+(m+\beta)^2R_2^{-2}$. Here $A^{a=3}_{\mu(n,m)=(0,0)}$ becomes a massless mode corresponding to the $U(1)$ gauge symmetry for the configuration $(\alpha,\beta)=(0,0)$, while $A^{a=3}_{\mu(n,m)=(-1,0)}$ is a massless mode for $(\alpha,\beta)=(1,0)$. Likewise, a massless mode for the $U(1)$ gauge symmetry is given by $A^{a=3}_{\mu(n,m)=(0,-1)}$ and $A^{a=3}_{\mu(n,m)=(-1,-1)}$ for $(\alpha,\beta)=(0,1)$ and $(\alpha,\beta)=(1,1)$, respectively. Hence, the vacuum configurations related by the periodicity of the potential are physically equivalent to each other and the mass spectrum on each vacuum is obtained by shifting the Kaluza-Klein (KK) modes by the same amount of periodicity.

B. With fermions in the fundamental representation

When there are additional fermions in the fundamental representation, one of the configurations in Eq. (5.1) becomes the global minimum of the effective potential. Take, as an example, the case $N^{f,F}_{00}\neq 0$. The potential becomes

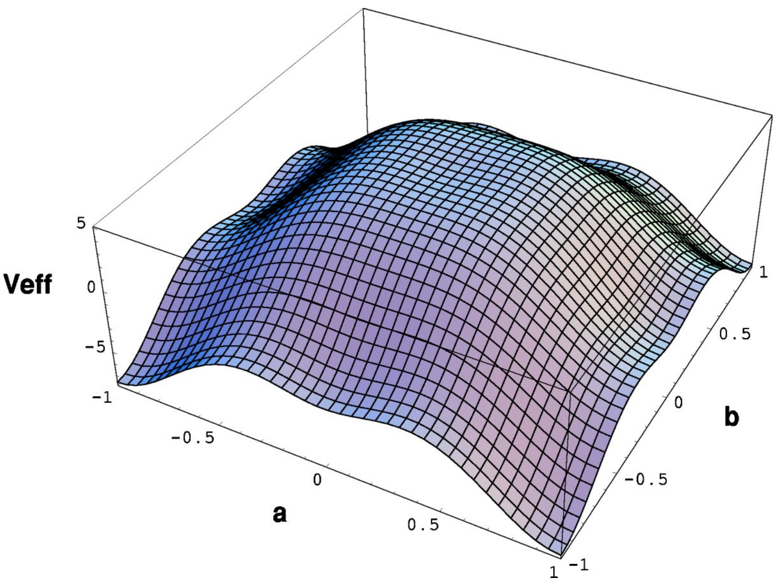

FIG. 2. $V_{\rm eff}(\alpha,\beta)$ for $N^{f,F}_{(00)}=3$ and $R_1=R_2$.

$$V_{\rm eff}(\alpha,\beta)=4I(\alpha,\beta)+2I(0,0)-4N_{00}^{f,F}I\left[\frac{1}{2}\alpha,\frac{1}{2}\beta\right]. \tag{5.5}$$

As $-I[(1/2)\alpha,(1/2)\beta]$ takes the minimum value at $(\alpha,\beta)=(1,1)$ (mod 2), the global minimum is located at $(\alpha,\beta)=(1,1)$. The physical symmetry is $U(1)$. The effective potential for $N^{f,F}_{(00)}=3$ is depicted in Fig. 2.

If $N_{00}^{f,F}=0$ and $N_{11}^{f,F}\neq 0$, the effective potential becomes

$$V_{\rm eff}(\alpha,\beta)=4I(\alpha,\beta)+2I(0,0)-4N_{11}^{f,F}I\left[\frac{1}{2}\alpha+\frac{1}{2},\frac{1}{2}\beta+\frac{1}{2}\right]. \tag{5.6}$$

In this case the global minimum is located at $(\alpha,\beta)=(0,0)$ (mod 2).

C. With fermions in the adjoint representation

Let us consider the cases with fermions in the adjoint representation. The effective potential is given by

$$V_{\rm eff}(\alpha,\beta)=4I(\alpha,\beta)+2I(0,0)-\sum_{\delta_1,\delta_2}2N^{f,Ad}_{(\delta_1\delta_2)}\times\left\{I\left(\frac{1}{2}\delta_1,\frac{1}{2}\delta_2\right)+2I\left(\alpha+\frac{1}{2}\delta_1,\beta+\frac{1}{2}\delta_2\right)\right\}. \tag{5.7}$$

If only fermions with $(\delta_1\delta_2)=(00)$ exist, then

$$V_{\rm eff}(\alpha,\beta)=2(1-N_{00}^{f,Ad})\{2I(\alpha,\beta)+I(0,0)\}. \tag{5.8}$$

For $N_{00}^{f,Ad}\geqslant 2$, the global minimum of the effective potential is given by the global maximum of $I(\alpha,\beta)$. There are four degenerate minima located at $(\alpha,\beta)=(1/2,1/2)$ (mod 1).

For the vacuum configuration $(\alpha,\beta)=(1/2,1/2)$, for instance, the new parity matrices in Eqs. (5.4) are given by

$$P_0'=\tau^3,\quad P_1'=\tau^2,\quad P_2'=\tau^2. \tag{5.9}$$

There is no $SU(2)$ generator that commutes with all the P_i', so that the $U(1)$ gauge symmetry is broken. As a result, there is no massless gauge boson. In fact, remembering that the mass spectrum for $A^{a=3}_{\mu(n,m)}$ is given by $(n+\alpha)^2R_1^{-2}+(m+\beta)^2R_2^{-2}$, for the vacuum configuration $(\alpha,\beta)=(1/2,1/2)$, we immediately see that none of $A^{a=3}_{\mu(n,m)}$ can be massless. There is no massless mode in $A^{a=3}_{\mu(n,m)}$ for noninteger values of α, β in general.

D. With $N^{f,F}$, $N^{f,Ad}\neq 0$

In the examples described above, the configuration corresponding to the global minimum of the effective potential is located at the special points $(\alpha,\beta)=(0,0)$ where α and β are integers or half-odd integers. More generic configurations can be chosen if fermions in the fundamental representation and fermions in the adjoint representation coexist.

As an example let us examine the case with $N_{00}^{f,F}, N_{01}^{f,Ad}\neq 0$. The effective potential is given by

$$V_{\rm eff}(\alpha,\beta)=4I(\alpha,\beta)-4N_{00}^{f,F}I\left[\frac{1}{2}\alpha,\frac{1}{2}\beta\right]-4N_{01}^{f,Ad}I\left(\alpha,\beta+\frac{1}{2}\right). \tag{5.10}$$

In the case $N_{00}^{f,F}=0$ the global minimum is located at $(\alpha,\beta)=(0,0)$ (mod 1) for $N_{01}^{f,Ad}\leqslant 1$, while at $(\alpha,\beta)=(1/2,0)$ (mod 1) for $N_{01}^{f,Ad}\geqslant 2$.

Now add fermions in the fundamental representation with $N_{00}^{f,F}\neq 0$. In the vicinity of $(\alpha,\beta)=(1/2,0)$ (mod 1), $I[1/2\alpha,1/2\beta]$ has a nonvanishing slope in the α direction. Hence the location of the global minimum is shifted in the α direction. Furthermore, the fourfold degeneracy existing in the case of $N_{00}^{f,F}=0$ is partially lifted. For instance, the two

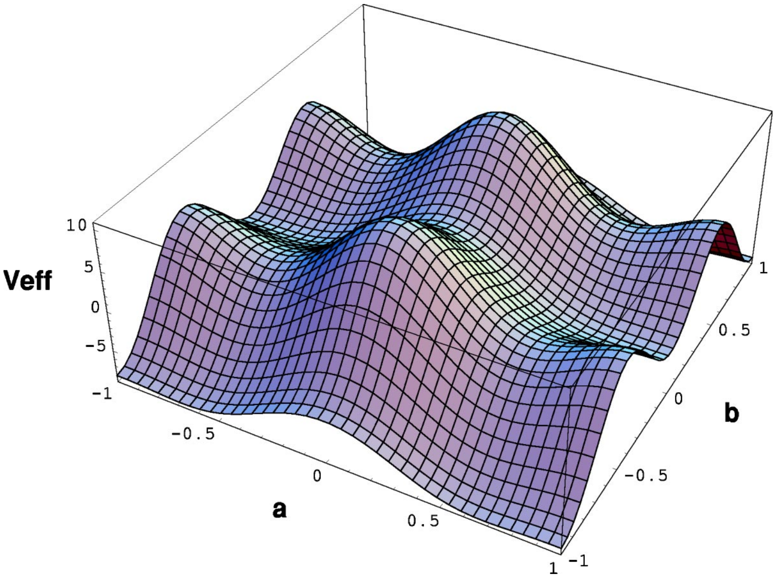

FIG. 3. $V_{\text{eff}}(\alpha,\beta)$ for $N_{(00)}^{f,F}=N_{(01)}^{f,Ad}=3$ with $R_1=R_2$. The global minima are located at $(\alpha,\beta)=(\pm 0.678,1)$ (mod 2).

degenerate global minima are located at $(\alpha,\beta)=(\pm 0.555,1)$ (mod 2) for $(N_{00}^{f,F},N_{01}^{f,Ad})=(1,3)$ with $R_1=R_2$. For $(N_{00}^{f,F},N_{01}^{f,Ad})=(3,3)$, the global minima are located at $(\alpha,\beta)=(\pm 0.678,1)$ (mod 2) for $R_1=R_2$ and $(\alpha,\beta)=(\pm 0.636,1)$ (mod 2) for $R_2/R_1=1.3$. See Figs. 3 and 4. The minima are shifted to $(\alpha,\beta)=(\pm 0.600,1)$ (mod 2) for $(N_{00}^{f,F},N_{01}^{f,Ad})=(3,4)$.

E. With fermions in Z_2 doublets

It is of great interest from the phenomenological viewpoint to incorporate fermions in Z_2 doublets. Intriguing models are obtained if there are fermions in the adjoint representation $(N_{00}^{f,Ad},N_{01}^{f,Ad}\neq 0)$ and fermions in Z_2 doublets in the fundamental representation $(N_{00}^{f,F}\neq 0)$ with the twist parameters $(a,b)\sim(0.5,-0.5)$. The effective potential becomes

$$V_{\text{eff}}(\alpha,\beta)=4I(\alpha,\beta)-4N_{00}^{f,Ad}I(\alpha,\beta)-4N_{01}^{f,Ad}I\left(\alpha,\beta+\frac{1}{2}\right)-4N_{00,\text{doublet}}^{f,F}\left\{I\left[\frac{1}{2}(\alpha+a),\frac{1}{2}(\beta+b)\right]+I\left[\frac{1}{2}(\alpha-a),\frac{1}{2}(\beta-b)\right]\right\}. \tag{5.11}$$

First take $(N_{00}^{f,Ad},N_{01}^{f,Ad})=(2,0)$. When $N_{00,\text{doublet}}^{f,F}=0$, there are four degenerate global minima at $(\alpha,\beta)=(\pm 1/2,\pm 1/2)$ and $(\pm 1/2,\mp 1/2)$. We add three generations of fermions in the fundamental representation, $N_{00,\text{doublet}}^{f,F}=3$. For $(a,b)=(1/2,-1/2)$, the degeneracy is partly lifted. The effective potential has the global minima at $(\alpha,\beta)=(\pm 1/2,\pm 1/2)$. Now we vary the values of a and b. For $(a,b)=(0.51,-0.51)$, the global mimima move to (α,β)

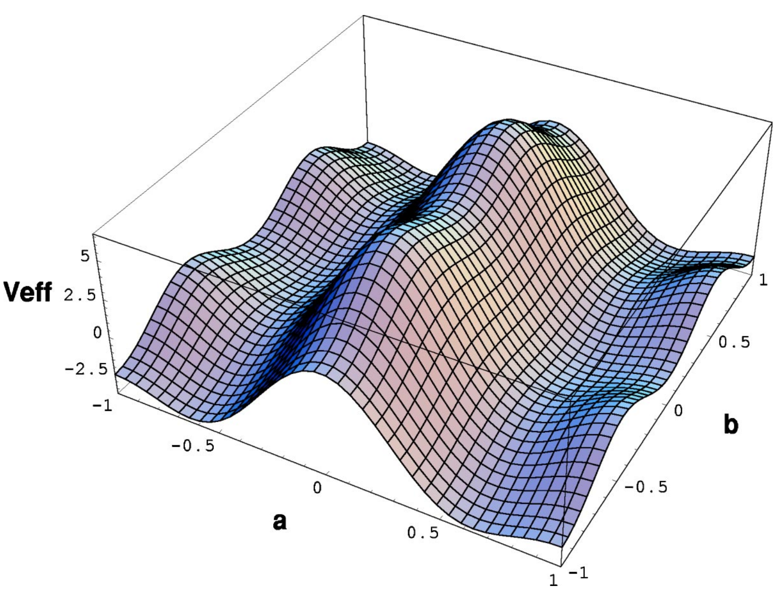

FIG. 4. $V_{\text{eff}}(\alpha,\beta)$ for $N_{(00)}^{f,F}=N_{(01)}^{f,Ad}=3$ with $1.3R_1=R_2$. The global minima are located at $(\alpha,\beta)=(\pm 0.636,1)$ (mod 2).

$=(\pm 0.486,\pm 0.486)$. For $(a,b)=(0.52,-0.52)$, the global mimima move to $(\alpha,\beta)=(\pm 0.472,\pm 0.472)$.

As a second example, take $(N_{00}^{f,Ad},N_{01}^{f,Ad})=(0,3)$. When $N_{00,\mathrm{doublet}}^{f,F}=0$, there are four degenerate global minima at $(\alpha,\beta)=(\pm 1/2,0)$ and $(\pm 1/2,1)$. Again we add three generations of fermions in the fundamental representation, $N_{00,\mathrm{doublet}}^{f,F}=3$. For $(a,b)=(0.5,0)$, the degeneracy is partly lifted. The effective potential has the global minima at $(\alpha,\beta)=(\pm 0.5,1)$. For $(a,b)=(0.5,1)$, the global minima are located at $(\alpha,\beta)=(\pm 0.5,0)$. For $(a,b)=(0.52,0)$, the global minima are located at $(\alpha,\beta)=(\pm 0.479,1)$.

In all these cases the $SU(2)$ symmetry is completely broken.

VI. MASS GENERATION

As the Wilson line phases develop nonvanishing expectation values $(\alpha,\beta\neq 0)$, the mass spectrum changes from that at the tree level. We are particularly interested in the mass spectrum in four dimensions.

A. Four-dimensional gauge fields and scalars

Extra-dimensional components of gauge potentials A_{yI}^a play the role of four-dimensional Higgs scalar fields. With the boundary condition (4.1), the components $a=1,2$ of A_{yI}^a have zero modes which serve as lower-dimensional scalars. They are massless at the tree level, but acquire nonvanishing masses at the quantum level.

The fields A_{yI}^a acquire masses in two steps. When the global minimum of the effective potential $V_{\mathrm{eff}}(\alpha,\beta)$ is located at $(\alpha_{\mathrm{min}},\beta_{\mathrm{min}})\neq(0,0)$ (mod 2), the fields are expanded around this configuration. Through the gauge coupling all fields in the four dimensions acquire masses of $O(\alpha_{\mathrm{min}}/R_1)$ and of $O(\beta_{\mathrm{min}}/R_2)$. Some of A_{yI}^a may not be affected by this correction, but they acquire nonvanishing masses from one loop corrections. It is a part of the Hosotani mechanism [9,11]. The mechanism is similar to that of pseudo-Nambu-Goldstone bosons and that of the little Higgs boson [36].

The best way to understand this is to go to a new gauge in which expectation values of Wilson line phases vanish. Perform a large gauge transformation $\Omega(\vec{y};\alpha_{\mathrm{min}},\beta_{\mathrm{min}})$ defined in Eq. (5.3). In the new gauge $\langle A_{yI}\rangle=0$. The boundary conditions change to $(P_0,P_1,P_2)=(\tau^3,e^{i\pi\alpha_{\mathrm{min}}\tau^1}\tau^3,e^{i\pi\beta_{\mathrm{min}}\tau^1}\tau^3)$ and $(U_1,U_2)=(e^{i\pi\alpha_{\mathrm{min}}\tau^1},e^{i\pi\beta_{\mathrm{min}}\tau^1})$.

Let us look at the mass spectrum of four-dimensional gauge fields. In this gauge $A_\mu^1(x,\vec{y})$ has a mode expansion of a Z_2 singlet field with $(P_0,P_1,P_2)=(-,-,-)$ in Eq. (3.8), while $(A_\mu^3(x,\vec{y}),A_\mu^2(x,\vec{y}))$ forms a Z_2 doublet with $(a,b)=(2\alpha_{\mathrm{min}},2\beta_{\mathrm{min}})$ in Eq. (3.14). The spectrum is, therefore,

$$\begin{aligned} A_\mu^1&:\ \left(\frac{n}{R_1}\right)^2+\left(\frac{m}{R_2}\right)^2 \qquad \text{where}\ \ (n,m)\in K_+,\\ \begin{pmatrix}A_\mu^3\\ A_\mu^2\end{pmatrix}&:\ \left(\frac{n+\alpha_{\mathrm{min}}}{R_1}\right)^2+\left(\frac{m+\beta_{\mathrm{min}}}{R_2}\right)^2\\ &\text{where}\ \ -\infty<n,m<+\infty. \end{aligned} \tag{6.1}$$

When $(\alpha_{\mathrm{min}},\beta_{\mathrm{min}})=(0,0)$, A_μ^1 and A_μ^2 have the same spectrum and only A_μ^3 has zero modes. When $(\alpha_{\mathrm{min}},\beta_{\mathrm{min}})\neq(0,0)$, A_μ^2 and A_μ^3 mix to form mass eigenstates. With this mixing in mind, it can be said that all three $SU(2)$ components of the gauge fields have distinct masses.

Similarly the spectrum of the extra-dimensional components A_{yI}^a is found. $A_{yI}^1(x,\vec{y})$ has a mode expansion of a Z_2 singlet field with $(P_0,P_1,P_2)=(+,+,+)$ in Eq. (3.8), while $(A_{yI}^2(x,\vec{y}),A_{yI}^3(x,\vec{y}))$ forms a Z_2 doublet with $(a,b)=(-2\alpha_{\mathrm{min}},-2\beta_{\mathrm{min}})$ in Eq. (3.14). The mass spectrum at the tree level is

$$\begin{aligned} A_{yI}^1&:\ 0,\left(\frac{n}{R_1}\right)^2+\left(\frac{m}{R_2}\right)^2 \qquad \text{where}\ \ (n,m)\in K_+,\\ \begin{pmatrix}A_{yI}^2\\ A_{yI}^3\end{pmatrix}&:\ \left(\frac{n-\alpha_{\mathrm{min}}}{R_1}\right)^2+\left(\frac{m-\beta_{\mathrm{min}}}{R_2}\right)^2\\ &\text{where}\ \ -\infty<n,m<+\infty. \end{aligned} \tag{6.2}$$

There are four zero modes associated with A_{yI}^a for $(\alpha_{\mathrm{min}},\beta_{\mathrm{min}})=(0,0)$ (mod 1), while only two otherwise. These zero modes become massive at the quantum level.

Case 1: $(\alpha_{\mathrm{min}},\beta_{\mathrm{min}})=(0,0)$ (mod 2). In this case there remains $U(1)$ symmetry. There are four zero modes associated with $A_{y^1}^1$, $A_{y^2}^1$, $A_{y^1}^2$, $A_{y^2}^2$. The effective potential is given by

$$\begin{aligned} \hat{V}_{\mathrm{eff}}[A_{y^1}^1,A_{y^2}^1,A_{y^1}^2,A_{y^2}^2]&=\hat{V}_{\mathrm{eff}}^{\text{1-loop}}\\ &+g^2\{(A_{y^1}^1)^2(A_{y^2}^2)^2+(A_{y^1}^2)^2(A_{y^2}^1)^2-2A_{y^1}^1A_{y^1}^2A_{y^2}^1A_{y^2}^2\}, \end{aligned} \tag{6.3}$$

where the second term comes from $1/2\mathrm{Tr}(F_{y^1y^2})^2$ at the tree level. The evaluation of $\hat{V}_{\mathrm{eff}}^{\text{1-loop}}$ for general configurations with $F_{y^1y^2}\neq 0$ is difficult. We observe that the mass spectrum is $U(1)$ symmetric and expect that fluctuations with vanishing $F_{y^1y^2}$ form a normal basis for the zero modes. We therefore make an approximation

$$\hat{V}_{\mathrm{eff}}^{\text{1-loop}}\sim V_{\mathrm{eff}}[\alpha,\beta], \tag{6.4}$$

where $V_{\mathrm{eff}}[\alpha,\beta]$ is the effective potential obtained in the preceding sections with $\alpha=2gR_1\sqrt{(A_{y^1}^1)^2+(A_{y^1}^2)^2}$ and $\beta=2gR_2\sqrt{(A_{y^2}^1)^2+(A_{y^2}^2)^2}$.

As an example, take the pure gauge theory. The effective potential is given by $V_{\mathrm{eff}}[\alpha,\beta]=4I(\alpha,\beta)$. [See Eq. (4.18).] The mass matrix is given by the second derivatives of $\hat{V}_{\mathrm{eff}}$ with respect to A_{yI}^a evaluated at vanishing A_{yI}^a. One finds that

$$(\mathrm{mass})^2=\begin{cases} 8\pi^2g_4^2R_1^3R_2\left.\dfrac{\partial^2V_{\mathrm{eff}}}{\partial\alpha^2}\right|_{(\alpha,\beta)=(0,0)} & \text{for}\ \ A_{y^1}^1,\ \ A_{y^1}^2,\\ 8\pi^2g_4^2R_1R_2^3\left.\dfrac{\partial^2V_{\mathrm{eff}}}{\partial\beta^2}\right|_{(\alpha,\beta)=(0,0)} & \text{for}\ \ A_{y^2}^1,\ \ A_{y^2}^2. \end{cases} \tag{6.5}$$

Here the four-dimensional gauge coupling is given by $g_4^2 = g^2/2\pi^2 R_1 R_2$. We used the fact $\partial^2 V_{\rm eff}/\partial\alpha\partial\beta|_{(\alpha,\beta)=(0,0)}=0$. When $R_1=R_2$, the masses are given by

$$(\text{mass})^2 = \frac{8C_1 g_4^2}{\pi^5 R^2},$$

$$C_1 = \sum_{n=1}^{\infty}\frac{1}{n^4} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{(n^2+m^2)^2} \approx 1.507 \tag{6.6}$$

for all zero modes.

Case 2: $(\alpha_{\rm min},\beta_{\rm min})=(1,1)$ (mod 2). In the example discussed in Sec. V B, the global minimum of the $V_{\rm eff}(\alpha,\beta)$ is located at $(\alpha_{\rm min},\beta_{\rm min})=(1,1)$. In the new gauge $(P_0,P_1,P_2)=(\tau^3,-\tau^3,-\tau^3)$. There are no zero modes for the fermions in the fundamental representation with $(\delta_1,\delta_2)=(0,0)$.

There still remains the $U(1)$ symmetry. The masses of the four zero modes associated with $A_{y_I}^a$ are given by Eq. (6.5) with $V_{\rm eff}$ in Eq. (5.5). For $R_1=R_2=R$ they are given by

$$(\text{mass})^2 = \frac{2(4C_1 + N_{00}^{f,F} C_2) g_4^2}{\pi^5 R^2},$$

$$C_2 = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^4} - \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}}{(n^2+m^2)^2} \approx 0.753. \tag{6.7}$$

Case 3: $(\alpha_{\rm min},\beta_{\rm min})\neq(0,0)$ (mod 1). The examples discussed in Secs. V C and V D belong to this category. There are only two zero modes associated with $A_{y^1}^1$ and $A_{y^2}^1$. The lightest modes of Z_2 doublet $(A_{y_I}^2, A_{y_I}^3)$ has $(\text{mass})^2 = (\overline{\alpha}_{\rm min}/R_1)^2 + (\overline{\beta}_{\rm min}/R_2)^2$ where $\overline{\alpha}_{\rm min}$ and $\overline{\beta}_{\rm min}$ are the distances to the nearest integers of $\alpha_{\rm min}$ and $\beta_{\rm min}$, respectively.

The masses of the two zero modes of $A_{y_I}^1$ are evaluated from $\hat{V}_{\rm eff}(A_{y^1}^1, A_{y^2}^1) = V_{\rm eff}(\alpha_{\rm min}+2gR_1 A_{y^1}^1, \beta_{\rm min}+2gR_2 A_{y^2}^1)$. Take the example in Sec. V D with $N_{00}^{f,F}=0$ and $N_{01}^{f,Ad}\geqslant 2$. The global minimum is located at $(\alpha_{\rm min},\beta_{\rm min})=(1/2,0)$ (mod 1). It follows from Eq. (5.10) that, for $R_1=R_2=R$,

$$(\text{mass})^2 = \begin{cases} (-C_3 + N_{01}^{f,Ad} C_2)\,\dfrac{8g_4^2}{\pi^2 R^2} & \text{for } A_{y^1}^1, \\[2ex] (+C_4 + N_{01}^{f,Ad} C_2)\,\dfrac{8g_4^2}{\pi^2 R^2} & \text{for } A_{y^2}^1, \end{cases}$$

$$C_3 = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^4} + 2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n-1} n^2}{(n^2+m^2)^3} \approx 1.152,$$

$$C_4 = \sum_{m=1}^{\infty}\frac{1}{m^4} + 2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n} m^2}{(n^2+m^2)^3} \approx 0.776. \tag{6.8}$$

B. Four-dimensional fermions

From the phenomenological viewpoint it is necessary to accommodate fermions with small, but nonvanishing masses. In the four-dimensional standard model of electroweak interactions, Yukawa interactions provide such small masses. In higher dimensional gauge theory, however, Yukawa interactions are sometimes absent, or a part of gauge interactions so that it becomes difficult to allow small, but nonvanishing fermion masses.

We would like to point out that such small masses might be accommodated in the framework of gauge theory on orbifolds through the combination of T^2 twists and dynamics of Wilson line phases. At the moment such a scenario is realized only if special combinations of matter fields are arranged. It might occur naturally in supersymmetric theories. We reserve discussions of supersymmetric theories for the future publication.

The models discussed in Sec. V E give nice examples. In the model described by Eq. (5.11) with $(N_{00}^{f,Ad}, N_{01}^{f,Ad}, N_{00,\rm doublet}^{f,F})=(2,0,3)$, one of the global minima of the effective potential is located at $(\alpha,\beta)=(0.5,0.5)$ and (0.486, 0.486) for $(a,b)=(0.5,-0.5)$ and (0.51, -0.51), respectively. Fermions in the fundamental representation have the mass spectrum given by Eq. (4.41) with $\delta_j=0$. The relevant parameters are $(a+\alpha, b+\beta)$ and $(a-\alpha, b-\beta)$. Unless one of these two pairs has elements equal or close to even integers, fermions acquire masses of $O(R_1^{-1})$ or $O(R_2^{-1})$. We see that none of four-dimensional fermions in the model are light. In the model with $(N_{00}^{f,Ad}, N_{01}^{f,Ad}, N_{00,\rm doublet}^{f,F})=(0,3,3)$, the situation does not change. There are no light fermions in four dimensions. For $(a,b)=(0.5,0)$, the global minima of $V_{\rm eff}$ are located at $(\alpha,\beta)=(\pm 0.5,1)$, whereas for $(a,b)=(0.5,1)$ they are located at $(\alpha,\beta)=(\pm 0.5,0)$.

This is a general feature. Fermions either in Z_2 singlets or in Z_2 doublets give contributions to the effective potential for Wilson line phases such that the effective potential is minimized by four-dimensional massive fermions, as can be inferred from Eq. (5.11). The tendency is reversed by contributions from bosons. In supersymmetric theories contributions from bosons and fermions cancel if supersymmetry remains unbroken. When supersymmetry is softly broken as in the Scherk-Schwarz breaking, a nontrivial dependence of the effective potential on twist parameters and Wilson line phases appears [18,11]. Then fermions in four dimensions may have small nonvanishing masses.

VII. CONCLUSIONS AND DISCUSSION

We have studied gauge theory with matter on $M^4 \times T^2/Z_2$. We have classified general boundary conditions for fields on the orbifold T^2/Z_2. The equivalence relation among various sets of boundary conditions holds as a result of the existence of boundary-condition-changing gauge transformations. By incorporating Wilson line degrees of freedom correctly, one can establish the same physics in each equivalence class of boundary conditions.

The Z_2-orbifolding boundary conditions, which are specified by parity matrices P_i $(i=0,1,2)$, break the gauge symmetry at the tree level. In order to find physical symmetry of the theory at low energies, which, in general, is different from the symmetry of boundary conditions, one must take into account dynamics of Wilson line phases by the Hosotani mechanism, through which further gauge symmetry breaking can be induced at the quantum level.

We have studied the $SU(2)$ gauge theory in detail to clarify physical symmetry at low energies. We have chosen boundary conditions of the Z_2 orbifolding that break the $SU(2)$ gauge symmetry down to $U(1)$. Depending on the matter content, the residual $U(1)$ gauge symmetry is further broken through the Hosotani mechanism and the original $SU(2)$ gauge symmetry is completely broken. This indicates that the electroweak gauge symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$ can be realized by the Hosotani mechanism, once a larger gauge group is chosen to start with. Indeed, such implementation of symmetry breaking has been attempted in the literature under the name of the gauge-Higgs unification. The $SU(6)$ model on $M^4 \times (S^1/Z_2)$ realizes such a scenario [14].

Regarding gauge symmetry breaking, the study in the present paper has been limited mostly to the case where the ratio of the size of the two extra dimensions are equal $r \equiv R_2/R_1 = 1$. Varying r modifies the shape of the effective potential to give different gauge symmetry breaking patterns. This study may be important in the model building. One can introduce two distinct scales: the GUT scale and electroweak scale.

We have also studied the particle spectrum in four dimensions. Some of the extra-dimensional components of gauge fields, four-dimensional ‘‘Higgs’’ scalar fields, are massless at the tree level, but become massive by radiative corrections. Their typical mass is given by g_4/R_1 or g_4/R_2, where g_4 is the four-dimensional gauge coupling constant.

It is interesting to extend our work to higher-rank gauge groups and to study more realistic models of gauge symmetry breaking and gauge-Higgs unification. It is particularly important to consider supersymmetric gauge theory in this framework. A realistic fermion mass spectrum in four dimensions might be achieved in supersymmetric theories as a result of dynamics of Wilson line phases, additional T^2 twists on matter fields, and supersymmetry breaking. We hope to come back to this point in the near future.

ACKNOWLEDGMENTS

This work was supported in part by Scientific Grants from the Ministry of Education and Science, Grant No. 13135215, Grant No. 13640284, and Grant No. 15340078 (Y.H.) and by the 21st Century COE Program at Osaka University (K.T.).

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