PHYSICAL REVIEW D **69**, 124031 (2004)

Reflection, radiation, and interference near the black hole horizon

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(Received 8 October 2003; published 29 June 2004)

The event horizon of black holes is capable of reflection: there is a finite probability for any particle that approaches the horizon to bounce back. The albedo of the horizon depends on the black hole temperature and the energy of the incoming particle. The reflection shares its physical origins with the Hawking process of radiation; both of them arise as consequences of the mixing of the incoming and outgoing waves that takes place due to quantum processes on the event horizon.

DOI: 10.1103/PhysRevD.69.124031 PACS number(s): 04.70.Dy, 04.20.Gz

I. INTRODUCTION

Classically the event horizon of black holes is presumed to be unable to emit anything into the outside world and is supposed to possess perfect absorption ability, i.e., to be able to take in everything that comes close to the horizon. There is, however, a known limitation to this simple intuitive picture that stems from thermodynamics, which attributes temperature and entropy to black holes. The first indication that gravitational fields could have entropy came when the investigation of Christodoulou [1] of the Penrose process [2] for extracting energy from a Kerr black hole showed that there is a quantity which could not decrease. Hawking found [3] that it is proportional to the area of the horizon. Further research of Bardeen *et al.* [4] demonstrated that black holes should obey laws similar to the laws of thermodynamics. An important step made by Bekenstein [5–7] revealed that the area was actually the physical entropy. This suggestion was supported and enriched by the discovery of the Hawking radiation phenomenon [8,9]. These works provided the foundation for the thermodynamics approach to black holes; for a recent review, see Wald [10] and references therein and also the books by Frolov and Novikov [11], Thorne [12], and Chandrasekhar [13] for a comprehensive discussion of other black hole properties.

The thermodynamics properties of black holes reveal that the black hole horizon has a finite temperature T and, correspondingly, is capable of radiation through the Hawking mechanism, in contradiction to the naive expectations. In this work I address another, new property of the horizon, its ability for reflection. The classical description of motion in the vicinity of the black hole horizon includes two types of trajectories. There are the ingoing trajectories, describing the motion toward the black hole center. There are also the outgoing trajectories that lead out of the black hole center. Classically, these two types of motion are quite different. If a particle following the ingoing trajectory approaches the event horizon, then it inevitably crosses it into the inside region. After that it stays inside; there is no classically allowed way for it to switch to any outgoing trajectory that leads into the outside region, in full accord with intuitive feelings. Discussing this point later on, we will use Fig. 1 below as an illustration of this statement. Thus classically the horizon cannot reflect approaching particles.

The quantum description reveals a new unexpected feature of the problem. The wave function of the incoming particle necessarily includes both the incoming and outgoing waves. The presence of the outgoing wave in the wave function of the incoming particle has important physical implications. One of them is the effect of *reflection from the horizon* (RH) which is discussed in some detail in this work. The RH means that there is a finite probability for an incoming particle to be reflected off the event horizon, back to the outside region. Another effect is the well-known Hawking mechanism of radiation. We show that the radiation can be considered as a consequence of the mentioned interference. This new point of view provides an attractive physical picture that sheds new light on the radiation process.

An important, intrinsic property of the RH is that it is due to those events that take place *strictly* on the horizon. This feature distinguishes it from a number of known phenomena that take place outside the horizon. One of them is related to the well-known graybody factors that arise from energy-dependent potential barriers outside the horizon. These factors filter the incoming and outgoing waves, producing a strong impact on the scattering process. In particular, they make the absorption cross sections finite, proportional to the event horizon area in the infrared region [14]. The graybody factors also manifest themselves in the Hawking radiation process [8,9], filtering the initially blackbody spectrum emanating from the horizon (see Ref. [15] devoted to a number of different aspects relevant to the graybody effect). The distinction between the RH effect and the graybody factors becomes particularly prominent in the infrared region, when the distances related to the graybody factors are much larger than the radius of the horizon, while the RH remains localized on the horizon. There are a number of other effects related to the potential barriers outside the horizon, for example, gravitational lensing (for theory and references, see the book [16]). Lensing, in particular, can be caused by strong bending of light which, for the Schwarzschild black hole, happens in the vicinity of $r=(3/2)r_g>r_g$, where r_g is the Schwarzschild radius (see Ref. [17] and references therein).

In contrast with other phenomena mentioned above, the RH is localized precisely on the horizon. This means that, in principle, this phenomenon can be studied both theoretically

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and experimentally separately from other phenomena, which are localized outside the horizon. The corresponding experimental study would require that an experimental facility is brought into the close vicinity of the horizon (the obvious difficulty of such an experimental setup is irrelevant here; it is important only that this is possible in principle). However, if we consider experimental conditions in which a particle propagates from large distances toward the horizon, then the events in the outside region, in particular those described by the graybody factors, and the effect of the RH should be considered simultaneously.

A situation of this type takes place in scattering. The discovery of the Penrose process [2] and the works of Zel'dovich [18] and Misner [19] devoted to the energy extraction from the Kerr black hole greatly stimulated interest in the scattering problem, which for the Kerr black hole can be formulated in terms of the superradiation process. The corresponding amplification factor was calculated numerically by Press and Teukolsky [20,21] and analytically by Starobinsky [22] for the scalar field and by Starobinsky and Churilov [23] for electromagnetic and gravitational waves. Independently, in parallel with this line of research, Unruh [24] found the absorption cross section for scalar and fermion particles scattered off a Schwarzschild black hole. The results of these, as well as the following works (see, e.g., [25]), take into account phenomena associated with the graybody factors; see the books [11,13,26] which summarize the results of the mentioned studies and provide further references on the subject. However, Ref. [27], which follows in the footsteps of the present work and Refs. [28–30], shows that the RH effect has a significant, qualitative influence on scattering, which has not been considered previously. This fact allows the RH effect to be measured by an observer located far away from the black hole. However, this topic will remain outside the scope of the present work, which is focused on those events that take place in the close vicinity of the horizon. This formulation makes the discussion more transparent (and permits one to neglect complications induced by the graybody factors).

To describe the main result of this work, consider a particle in the outside region that approaches the black hole horizon. It is shown that there is a finite probability $\mathcal{P}$ for the particle to be reflected off the horizon,

$$\mathcal{P}=\exp\left(-\frac{\varepsilon-Q\Phi-J\omega}{kT}\right), \qquad (1.1)$$

in other words, the horizon possesses albedo.[1] The probability of RH depends on the energy of the incoming particle ε, its charge Q, and its projection of the orbital momentum J on the axis of rotation of the black hole. The essential parameters of the black hole that govern the process are the temperature T, the electric potential on the horizon Φ, and the angular velocity of the horizon ω.

[1]This should not be confused with the well-known albedo of the black hole related to the graybody factors.

Notably, the probability of RH (1.1) coincides with the temperature factor that governs the Hawking radiation process, although the physical manifestation of the RH differs from the radiation since the flux of the reflected particles is proportional to the magnitude of the incoming flux. Nevertheless, the similarity between the probability of RH (1.1) and the temperature factor is not accidental. As was mentioned above, the RH and radiation share the same physical origin, namely, the interference of the incoming and outgoing waves due to effects that take place on the horizon. We develop a convenient way to prove the existence of this interference and to establish its magnitude by deriving Eq. (1.1) for the RH.

This paper is based on the eternal approach to black holes. For *practical* applications one needs to verify that the results obtained are applicable to the collapsing black holes as well. There are reasons indicating that this is probably the case. First, the result is very robust. This paper employs two different (though related) approaches to verify it. The recent reference [29] presents another two different ways that lead to the same conclusion. Considered by itself, this fact (however positive it is) is probably not decisive, because all the above mentioned methods of derivation are based on the eternal approach. However, there is a second reason supporting the validity of the presented results for collapsing black holes. This paper shows that the reflection on the horizon and the Hawking radiation process share similar physical origins. This claim is discussed from different perspectives in [27–30], making this conclusion reliable. Therefore, since it is firmly established that the radiation phenomenon is relevant to the collapsing black hole, one should expect the effect of reflection to possess this property also.

Relativistic units $\hbar=c=1$ are used, supplemented by the condition $2Gm=1$ imposed on the gravitational constant G and the black hole mass m, if not stated otherwise. The Schwarzschild radius in these units reads simply $r_g=2Gm/c^2\equiv 1$.

II. SINGULARITY OF THE WAVE FUNCTION ON THE HORIZON

Consider a static black hole described by the conventional Schwarzschild metric

$$ds^2=-\left(1-\frac{1}{r}\right)dt^2+\frac{dr^2}{1-1/r}+r^2d\Omega^2, \qquad (2.1)$$

where $d\Omega^2=d\theta^2+\sin^2\theta d\varphi^2$. The Hamilton-Jacobi classical equations of motion $g^{\kappa\lambda}\partial_\kappa S\partial_\lambda S=-\mu^2$ for a particle with the mass μ in the metric (2.1) take the form

$$\frac{\dot S^2}{1-1/r}=\left(1-\frac{1}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial S}{\partial\varphi}\right)^2+\mu^2. \qquad (2.2)$$

Separating the variables $S(\mathbf{r},t)=-\varepsilon t+L\varphi+S(r)$, where ε and L are the energy and the momentum of the particle, and φ is its azimuthal angle, one finds the radial action

$$S(r) = \mp \int^r \left[\varepsilon^2 - \left(\mu^2 + \frac{L^2}{r} \right) \left(1 - \frac{1}{r} \right) \right]^{1/2} \frac{dr}{1 - 1/r}. \quad (2.3)$$

In the vicinity of the black hole horizon $r \to 1$, which plays an important role in the following discussion, the action (2.3) simplifies to

$$S(r) = \mp \varepsilon \ln(r-1), \quad (2.4)$$

which gives $S(\mathbf{r},t) = -\varepsilon t \pm \varepsilon \ln(r-1) + L\varphi$. The corresponding equation of motion $\partial_\varepsilon S(\mathbf{r},t) = 0$ yields the radial trajectories

$$r = 1 + \exp(\mp t). \quad (2.5)$$

The minus and plus signs in Eqs. (2.3),(2.4),(2.5) correspond to the incoming and outgoing trajectories, respectively. These equations are conveniently written for the outside region $r > 1$ (the inside region is discussed in Sec. V). It is important that the classical action for a probing particle has the logarithmic singularity (2.4) on the horizon. The coefficient in front of the logarithm function is equal to the energy of the particle ε ($\varepsilon r_g / c$ in absolute units), which plays an important role in what follows, eventually finding its way into the exponential function in Eq. (1.1). Importantly, the logarithmic singularity is an invariant property of the action; it persists even in those coordinates that eliminate the singularity of the metric on the horizon, as is discussed in Sec. V.

The classical action allows one to find the semiclassical wave function $\Phi(\mathbf{r},t)$ that describes the coordinate motion of the particle (leaving aside possible spin variables). Separating the variables, $\Phi(\mathbf{r},t) = \exp(-i\varepsilon t) Y_{LM}(\theta,\varphi)\phi(r)$, where $Y_{LM}(\theta,\varphi)$ is the conventional spherical function describing the motion with orbital momentum L and its projection M, one presents the semiclassical radial wave function $\phi(r)$ as

$$\phi(r) \propto \exp[iS(r)] \simeq \exp[\mp i\varepsilon \ln(r-1)]. \quad (2.6)$$

We will verify below [see after Eq. (2.7)] that the preexponential factor in Eq. (2.6) is a constant, which we chose to be unity. Thus the singularity of the action at $r = 1$ results in the corresponding singularity of the wave function.

In order to scrutinize this result one needs to assess the validity of the semiclassical description in the vicinity of the horizon. To this end, consider the wave function $\Phi(\mathbf{r},t)$ as a solution of the Klein-Gordon equation for the scalar field. From Eq. (2.1) one finds that the radial wave function $\phi(r)$ satisfies the equation

$$\phi'' + \left(\frac{1}{r} + \frac{1}{r-1} \right) \phi' + \frac{1}{1-1/r} \left(\frac{\varepsilon^2}{1-1/r} - \mu^2 - \frac{L(L+1)}{r^2} \right) \phi = 0. \quad (2.7)$$

In the vicinity of the horizon $r = 1$ the solution can be approximated by $\phi(r) \simeq (r-1)^\eta$ where Eq. (2.7) yields $\eta = \pm i\varepsilon$. The agreement with the semiclassical result (2.6) supports its validity and verifies that the preexponential factor in Eq. (2.6) is, indeed, a constant. It is instructive also to look at the singularity of the wave function (2.6) from the point of view of the conventional Schrödinger-type equation. Making the substitution $\phi(r) \to \psi(r) = [r(r-1)]^{1/2}\phi(r)$, one rewrites Eq. (2.7) as

$$p^2 \psi(r) = -\psi''(r) + U(r)\psi(r), \quad (2.8)$$

where

$$U(r) = -\frac{1}{(r-1)^2} \left(\varepsilon^2 + \frac{1}{4r^2} \right) - \frac{1}{r-1} \left(\varepsilon^2 + p^2 - \frac{L(L+1)}{r} \right). \quad (2.9)$$

Equation (2.9) has the form of a Schrödinger-type equation if we consider $U(r)$ as an effective, energy-dependent potential and accept the momentum p^2 on the left-hand side as the eigenvalue. For $r \to 1$ the potential exhibits the notable feature

$$U(r) \to -\frac{\varepsilon^2 + 1/4}{(r-1)^2}. \quad (2.10)$$

It is well known in nonrelativistic quantum mechanics [33] that in the potential $U(z) = -U_0 / z^2$ for $U_0 > 1/4$ the wave function collapses to the point $z = 0$. Since the necessary inequality is obviously satisfied in Eq. (2.10), $\varepsilon^2 + 1/4 > 1/4$, one concludes that Eq. (2.8) indicates the collapse of the wave function on the event horizon $r = 1$. This fact could be interpreted as the absorption of the particle by the black hole. Thus at first sight the quantum description seems to agree with classical arguments based on the incoming trajectory in Eq. (2.5) which converges to the event horizon, supporting also the intuitive perception of the black hole horizon as an ideal absorber. However, a more careful discussion below exposes the limitations of this point of view.

Summarizing, we demonstrated that the wave function $\phi(r)$ has a singularity (2.6) on the event horizon.

III. REFLECTION BY THE HORIZON

Consider a particle that approaches the event horizon of the black hole. Let us describe its radial motion with the help of the wave function $\phi(r)$. According to Eq. (2.6), the wave function in the vicinity of the horizon can be written as

$$\phi(r) = \exp[-i\varepsilon \ln(r-1)] + \mathcal{R} \exp[i\varepsilon \ln(r-1)]. \quad (3.1)$$

The first term here describes the proper incoming wave, while the second one, which presents the outgoing wave, is written in order to allow for the opportunity of possible interference of the incoming and outgoing waves in the wave function. If this interference takes place, i.e., if $\mathcal{R} \neq 0$, then the outgoing wave in Eq. (3.1) clearly indicates that there is a probability for the incoming particle to be reflected on the horizon. The unitarity condition implies that $|\mathcal{R}| \leqslant 1$. Moreover, intuitively one would expect the reflection coefficient in Eq. (3.1) to be zero, $\mathcal{R} = 0$. This assumption would agree with a naive perception of the black hole horizon as a perfect absorber. However, in order to verify, approve, or reject this intuitive claim (we will reject it, in fact) one needs to examine carefully what happens to the wave function on the horizon.

A straightforward discussion of the events that happen strictly at $r=1$ faces an obstacle produced by the singular nature of the wave function (3.1) at this point. Fortunately, one can avoid discussion of the events that take place strictly on the horizon $r=1$ by using the analytical continuation of the wave function in the vicinity of this point. Consider the distance from the horizon $z=r-1$, treating z as a complex variable. The wave function (3.1) is explicitly analytical in z, except for the power-type singularity at $z=0$ which induces a cut emerging from this point on the complex plane z. Let us take r in the outside region of the black hole in the close vicinity of the event horizon, which means that $0<z\ll 1$, and examine what happens to the wave function when one rotates z in the complex z plane over an angle of 2π clockwise [the counterclockwise rotation is forbidden; see the discussion after Eq. (3.9)]. We can keep $|z|$ small, $|z|\ll 1$, during this rotation, thus justifying the validity of the semiclassical wave function (3.1). This analytical continuation necessarily incorporates a crossing of the cut on the complex plane. Therefore, after finishing this rotation and returning to a real, physical value $z>0$, the wave function acquires a new value on its Riemann surface; let us call it $\phi^{(2\pi)}(r)$. A procedure of this type is usually referred to as a monodromy. In our case the monodromy can be read from Eq. (3.1):

$$\phi^{(2\pi)}(r)=\varrho\,\exp[-i\varepsilon\,\ln(r-1)]+\frac{\mathcal{R}}{\varrho}\exp[i\varepsilon\,\ln(r-1)], \tag{3.2}$$

where $\varrho=\exp(-2\pi\varepsilon)$. The analytically continued function $\phi^{(2\pi)}(r)$ satisfies the same real differential equation as the initial function $\phi(r)$. Moreover, one has to expect that the wave function $\phi^{(2\pi)}(r)$ satisfies the same normalization conditions as the initial wave function $\phi(r)$. This implies that one of the coefficients in Eq. (3.2), either ϱ or $\mathcal{R}/\varrho$, should have an absolute value equal to unity. Since $\varrho<1$, we deduce that $|\mathcal{R}|/\varrho=1$, thus concluding that

$$|\mathcal{R}|=\exp\left(-\frac{2\pi r_g\varepsilon}{\hbar c}\right), \tag{3.3}$$

where the conventional units are used to make the result more transparent. We see that the reflection coefficient is nonzero. In other words, the black hole horizon is capable of reflection, i.e., the RH takes place.

There is a more general and rigorous way to prove this statement [28] that uses a symmetry of the black hole space-time. It is convenient to present this argument in the Kruskal coordinates U,V [31], which in the outside region $r>1$ are defined by

$$U=-\sqrt{r-1}\,\exp[(r-t)/2], \tag{3.4}$$

$$V=\sqrt{r-1}\,\exp[(r+t)/2]. \tag{3.5}$$

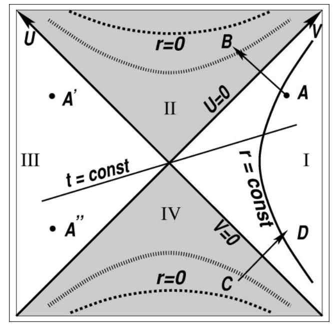

FIG. 1. Kruskal coordinates. Areas I and III represent two identical copies of the outside region; II and IV show two inside regions. Hyperbolic curves $UV=\text{const}$ describe the condition $r=\text{const}$, the dotted curve shows the location of $r=0$, and the inclined straight line presents the condition $t=\text{const}$. The direction of time flow in I and III is opposite. The incoming particle follows *AB*, crossing the horizon $U=0$ and residing in II. The outgoing particle *CD* escapes from IV, crossing the horizon $V=0$ and coming to I. Areas II and IV are not connected, which ensures classical confinement in II. The wave function (3.1) or (5.3) describes mixing of events that correspond to incoming and outgoing classical trajectories (*AB* and *CD*), resulting in the phenomena of RH and radiation. The symmetrically located points A, A', A'' are used to reveal the symmetry (3.8) of the space-time. The wave function (3.1) describes mixing of events corresponding to incoming and outgoing trajectories (*AB* and *CD*), which results in the phenomena of RH and radiation.

An appropriate analytical continuation allows one to define them in the inside region $r<1$ as well. Overall, the U,V plane shown in Fig. 1 represents the complete space-time; for a comprehensive discussion of the Kruskal coordinates, see Ref. [32].

The areas I and III in the Kruskal plane give two *identical* representation of the outside world [31,32]. This means that a transformation that brings an arbitrary point A of the region I into the symmetrically located point A' in the region III (see Fig. 1) is a space-time symmetry. Therefore the wave function transforms according to some representation of the symmetry group related to this transformation. The wave function (3.1) is a scalar; therefore this symmetry transformation can manifest itself only as $\phi\to\phi'=\lambda\phi$, where λ is a constant. Applied twice, the considered transformation brings the point A back to its initial value, being accompanied by the transformation of the wave function $\phi\to\lambda^2\phi$, which can differ from its initial value only by a phase factor. This shows that $\lambda=\exp i\alpha$, where α is a phase that is not determined by the symmetry conditions. Thus the transformation $A\to A'$ results in

$$\phi\to\phi'=\exp(i\alpha)\phi. \tag{3.6}$$

There is a convenient way to make use of this symmetry condition. Let us first fulfill the complex rotation $z\to\exp(-2\pi i)z$. Equations (3.4),(3.5) show that it results in the transformation $U\to -U, V\to -V$ which brings the point A to A'' in Fig. 1. After that we can use the operation of time inversion $\hat{T}$. According to Eqs. (3.4),(3.5) the time inversion

$t\rightarrow -t$ is equal to the transformation $U\rightarrow -V, V\rightarrow -U$, which transforms A'' to A' in Fig. 1. The appearance of the time inversion is related to the fact that the arrows of time flow in areas I and III are opposite; see the inclined straight line of constant time in Fig. 1. As usual, the inversion of time $t\rightarrow -t$ in the argument of the wave function should be accompanied by the complex conjugation of the function, i.e., the operator of the time inversion is defined as $\hat{T}[\phi(r,t)] \equiv \phi^*(r,-t)$. For the stationary wave function $\phi \propto \exp(-i\varepsilon t)$, this definition reads

$$\hat{T}[\phi(r)] \equiv \phi^*(r). \tag{3.7}$$

Combining the 2π rotation on the complex z plane with the time inversion Eq. (3.7), we fulfill the transformation $A \rightarrow A'$. The symmetry of the space-time under this transformation Eq. (3.6) gives that

$$[\phi^{(2\pi)}]^* = \exp(i\alpha)\phi. \tag{3.8}$$

Using the wave functions (3.1) and (3.2), we find that the symmetry Eq. (3.8) requires that $\mathcal{R} = \exp(-2\pi\varepsilon - i\alpha)$, in accord with Eq. (3.3). An alternative derivation of this result [29], which relies more heavily on dynamical properties of the problem, supports the validity of Eq. (3.3) as well; it also provides a way to determine the phase α [27], which vanishes for low energies, i.e., $\alpha = 0$ when $\varepsilon \ll 1$.

From Eq. (3.3) we see that the incoming and outgoing waves interfere in the wave function (3.1). Correspondingly, there is RH. The probability of RH can be found as $\mathcal{P} = |\mathcal{R}|^2$, which in view of Eq. (3.3) gives $\mathcal{P} = \exp(-4\pi\varepsilon)$ in full accord with Eq. (1.1) stated in Sec. I. The parameter T that appears in Eq. (1.1) arises from the coefficient in front of the logarithmic function in Eq. (2.4):

$$kT = \frac{\hbar c}{4\pi r_g} \tag{3.9}$$

(absolute units). Notably, it proves equal to the Hawking temperature of the black hole. In applying Eq. (1.1), one should remember, of course, that the electric potential and rotational frequency for the Schwarzschild case are absent, $\Phi = \omega = 0$.

Let us go back to examine why it was necessary to use specifically the clockwise rotation when the analytical continuation of the wave function (1.1) in the complex z plane was obtained. A simplified answer to this question is that an attempt to use the counterclockwise rotation leads to a self-contradiction. Trying it, i.e., making the counterclockwise rotation, one arrives at a result similar to (3.2), but with a different coefficient ϱ' instead of ϱ, $\varrho \rightarrow \varrho' = 1/\varrho = \exp(2\pi\varepsilon)$. Proceeding further, one would be forced to conclude that the reflection coefficient is $|\mathcal{R}'| = \exp(2\pi\varepsilon)$, which comes into obvious contradiction with the unitarity condition for the reflection which specifies $|\mathcal{R}'| \leqslant 1$. It is a known, common feature of the semiclassical wave function that different ways of analytical continuation lead to different results, and one needs to choose carefully an appropriate method of continuation (3.1). To outline the deeper roots of this problem, it is convenient to use the Kruskal coordinates (3.4),(3.5). It is known from the analysis of Hartle and Hawking [34] that the propagator of the scalar particle in the Schwarzschild metric is an analytical function of U and V in the upper half plane of the complex U plane and in the lower half plane of the complex V plane. In terms of the variable z, this means that the propagator remains an analytical function when it is continued from the real semiaxis $z > 0$ in the clockwise direction over the angle 2π. There is a slight distinction in our case. Our analysis relies on the wave function, while the work [34] refers to the properties of the propagator. However, the analytical properties of the wave function are similar to those of the propagator. We conclude that the analytical continuation used in the derivation of Eq. (3.2) is justified. In contrast, an attempt to use the analytical continuation rotating z from the region $z > 0$ in the counterclockwise direction meets a difficulty.

Equations (3.1),(3.3) indicate that the horizon is capable of reflecting the incoming particle with a probability given in Eq. (1.1). By the same token, it means that the probability for the incoming particle to cross the horizon $\mathcal{P}_{\rm cr}$, penetrating into the inside region, is less than unity,

$$\mathcal{P}_{\rm cr} = 1 - \mathcal{P}. \tag{3.10}$$

We will use Eq. (3.10) in Sec. V when discussing the Hawking radiation process.

Let us summarize the ideas used in the derivation of Eqs. (3.1) and (3.3). We employed two important facts, the discrete symmetry of the space-time presented by Eqs. (3.6),(3.7),(3.8), and the logarithmic singularity in the wave function Eq. (3.1) in the vicinity of the event horizon. The symmetry condition given by Eqs. (3.6),(3.7), (3.8) implies that the wave function $\phi^{(2\pi)}(r)$ satisfies the same normalization conditions as the initial wave function $\phi(r)$. This statement can be considered as a shortcut way to express the symmetry of the space-time. Its simplicity makes it convenient for applications to the more sophisticated black holes discussed below.

IV. REFLECTION BY HORIZONS OF DIFFERENT BLACK HOLES

This section extends the results derived above for the Schwarzschild black hole to other, more complex types of black holes.

A. Charged black holes

Consider the Reissner-Nordström black hole with mass m and charge q. Its metric is given by

$$ds^2 = -\left(1 - \frac{1}{r} + \frac{\mathfrak{q}^2}{r^2}\right)dt^2 + \frac{dr^2}{1 - 1/r + \mathfrak{q}^2/r^2} + r^2 d\Omega^2. \tag{4.1}$$

We use conventional gravitational units for the charge, which means that $\mathfrak{q}^2 \equiv 4\pi G q^2$. The Hamilton-Jacobi equation for the particle with mass μ, charge Q, and orbital momentum L for the metric (4.1) reads

$$\frac{(\dot{S}-Q\Phi)^2}{1-1/r+\mathfrak{q}^2/r^2}=\left(1-\frac{1}{r}+\frac{\mathfrak{q}^2}{r^2}\right)\left(\frac{\partial S}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2+\mu^2, \tag{4.2}$$

where $\Phi(r)=q/r$ is the black hole electric potential. Separating the variables $S(\mathbf{r},t)=-\varepsilon t+L\varepsilon+S(r)$, one derives

$$S(r)=\mp\int\left[(\varepsilon-Q\Phi(r))^2-\left(\mu^2+\frac{L^2}{r}\right)\left(1-\frac{1}{r}+\frac{\mathfrak{q}^2}{r^2}\right)\right]^{1/2}\times\frac{dr^2}{1-1/r+\mathfrak{q}^2/r^2}. \tag{4.3}$$

The poles of $g_{rr}=1-1/r+\mathfrak{q}^2/r^2$ are located on two spherical surfaces with radii

$$r_\pm=\frac{1}{2}\pm\sqrt{\frac{1}{4}-\mathfrak{q}^2}. \tag{4.4}$$

The larger of them, with the radius r_+, represents the black hole horizon. In the vicinity of the horizon $r\to r_+$ one finds from Eq. (4.3)

$$S(r)\simeq\mp\zeta\ln(r-r_+), \tag{4.5}$$

where

$$\zeta=[\varepsilon-Q\Phi(r_+)]\frac{r_+^2}{r_+-r_-}. \tag{4.6}$$

In analogy with Eq. (2.4) the action (4.5) possesses a logarithmic singularity. We can therefore follow the way paved by Eqs. (3.1), (3.2), and (3.3). First we construct the wave function

$$\phi(r)=\exp[-i\zeta\ln(r-1)]+\mathcal{R}\exp[i\zeta\ln(r-1)], \tag{4.7}$$

which describes the radial motion of the particle in the vicinity of the event horizon. Then, introducing the variable $z=r-r_+$ and assuming that $z>0,|z|\ll r_+-r_-$, i.e., taking r in the external region in a close vicinity of the event horizon, we make the analytical continuation of rotating z in the complex plane $z\to\exp(-i\gamma)z,\gamma\geqslant 0$, eventually taking $\gamma=2\pi$. The discrete symmetry of the space-time requires that the analytical continuation does not change the normalization conditions of the wave function; see the discussion at the end of Sec. III.

The described procedure gives the coefficient of reflection $\mathcal{R}=\exp(-2\pi\zeta)$ and the probability of RH $\mathcal{P}=|\mathcal{R}|^2=\exp(-4\pi\zeta)$. The latter result agrees with Eq. (1.1), where the value for the parameter T follows from Eqs. (4.5),(4.6),

$$kT=\frac{\hbar c}{4\pi}\frac{r_+-r_-}{r_+^2} \tag{4.8}$$

(absolute units). It proves equal to the Hawking temperature of the charged black hole.

B. Rotating black holes

Consider the Kerr black hole, which possesses the mass m and the spin j, which is conveniently parametrized by $a=j/m$. The Kerr metric reads

$$ds^2=-\frac{\Delta}{\rho}(dt-a\sin^2\theta d\varphi)^2+\frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\varphi-a dt]^2+\frac{\rho^2}{\Delta}dr^2+\rho^2 d\theta^2. \tag{4.9}$$

Here $\Delta=r^2-r+a^2$ and $\rho^2=r^2+a^2\cos^2\theta$. The poles of $g_{rr}=\rho^2/\Delta$, i.e., the nodes of Δ, are located on two spheres with radii

$$r_\pm=\frac{1}{2}\pm\sqrt{\frac{1}{4}-a^2}, \tag{4.10}$$

the larger of which represents the black hole horizon. The Hamilton-Jacobi equations of motion for the metric (4.9),

$$\frac{1}{\Delta}\left(r^2+a^2+\frac{ra^2}{\rho^2}\sin^2\theta\right)^2\dot{S}^2-\frac{\Delta}{\rho^2}\left(\frac{\partial S}{\partial r}\right)^2-\frac{1}{\rho^2}\left(\frac{\partial S}{\partial\theta}\right)^2-\frac{1}{\Delta\sin^2\theta}\left(1-\frac{r}{\rho^2}\right)\left(\frac{\partial S}{\partial\varphi}\right)^2+2\left(\frac{ra}{\rho^2\Delta}\right)\frac{\partial S}{\partial\varphi}\dot{S}=\mu^2, \tag{4.11}$$

allow the full separation of variables $S(\mathbf{r},t)=-\varepsilon t+J\varphi+\Sigma(\theta)+S(r)$, which results in the following equation for the radial action $S(r)$:

$$S(r)=\int\Delta^{-1}\sqrt{R}dt, \tag{4.12}$$

$$R=P^2-\Delta[\mu^2r^2+K], \tag{4.13}$$

$$P=\varepsilon(r^2+a^2)-aJ. \tag{4.14}$$

Here J is the conserved projection of the orbital momentum of the particle on the axis of rotation of the black hole, and K is an additional ("accidental") integral of motion. In the vicinity of the horizon $r\to r_+,\Delta\to 0$, one finds from Eq. (4.12) that $S(r)$ has a logarithmic singularity that satisfies Eq. (4.5) in which the parameter ζ equals

$$\zeta=(\varepsilon-J\omega)\frac{r_+^2+a^2}{r_+-r_-}. \tag{4.15}$$

Here $\omega=a/(r_+^2+a^2)$ is the frequency of rotation of the black hole horizon. Using the method well discussed above, we derive from the logarithmic singularity that the rotating black hole is capable of RH; the probability of reflection is given by Eq. (1), in which Eq. (4.15) predicts for the parameter T

$$kT=\frac{\hbar c}{4\pi}\frac{r_+-r_-}{r_+^2+a^2} \tag{4.16}$$

(absolute units), which coincides with the temperature of the rotating black hole.

C. Charged rotating black holes

Consider the general case of the Kerr-Neumann black hole, which possesses both the charge q and the spin j. The Kerr-Newman metric in the Boyer-Lindquist coordinates is described by Eq. (4.9), in which the parameter Δ reads

$$\Delta=r^2-r+a^2+q^2. \tag{4.17}$$

The nodes of Δ are located on spheres with radii

$$r_\pm=\frac{1}{2}\pm\sqrt{\frac{1}{4}-a^2-q^2}, \tag{4.18}$$

the larger of which represents the black hole horizon. The electromagnetic field of the black hole is described by the vector potential $A_\mu dx^\mu=-(qr/\rho^2)(dt-a\sin^2\theta d\varphi)$. The Hamilton-Jacobi equations of motion for a charged particle moving in the gravitational and electromagnetic fields created by a black hole allow the full separation of variables; see, e.g., Ref. [32], p. 901. One promptly finds that the radial action is described by Eqs. (4.12),(4.13), in which the parameter P equals

$$P=\varepsilon(r^2+a^2)-aJ-qQr. \tag{4.19}$$

From Eqs. (4.12),(4.13),(4.19) we find that on the horizon $r\to r_+$ the action has the logarithmic singularity

$$S(r)\simeq\mp\zeta\ln(r-r_+), \tag{4.20}$$

where

$$\zeta=[\varepsilon-Q\Phi(r_+)-J\omega]\frac{r_+^2+a^2}{r_+-r_-}. \tag{4.21}$$

Here $\Phi(r_+)=qQr_+/(r_+^2+a^2)$ is the potential describing interaction of the particle with the electromagnetic field of the black hole on the horizon. Using Eq. (4.21) and applying the method well described above, one proves that the reflection probability for the Kerr-Newman black hole is given by Eq. (1.1). The parameter T that appears in Eq. (1.1) satisfies Eq. (4.16) with $r_\pm$ from Eq. (4.18); this T coincides with the temperature of the Kerr-Newman black hole. Setting in Eqs. (4.16),(4.18) either q or j or both of them to zero, one returns to the cases of the Kerr black hole, the Reissner-Nordström black hole, and the Schwarzschild black hole, respectively.

We relied above on the semiclassical approach. Equation (4.18) can be improved to account more accurately for the quantum properties of the momentum j by substituting $j^2\to j(j+1)$ in a^2 in Eq. (4.18). This issue becomes important when the quantum properties of the black hole itself are considered (see recent work of Bekenstein devoted to this subject [35,36]). However, for the purpose of this work this subtlety is not essential.

We discussed in this section several types of black holes that possess either charge or momentum or both, verifying that in each and every case the black hole is capable of reflection. Our most general result, which is presented for the Kerr-Newman solution, is described in Eqs. (1.1),(4.16),(4.18). A number of more sophisticated solutions for black holes with hair are known (see the review [37]) but we leave them outside the scope of the present work.

V. REFLECTION AND RADIATION

Let us show that the reflection ability of the black hole horizon and the phenomenon of Hawking radiation have the same physical origin. Consider the Schwarzschild case for simplicity. It is convenient to rewrite our results in the Kruskal coordinates. As a first step let us return to the time-dependent description. The time-dependent action equals $S(r,t)=S(r)-\varepsilon t$, in which the radial component in the vicinity of the horizon reads $S(r)=\mp\varepsilon\ln(r-1)$; here the signs "minus" and "plus" correspond to the incoming and outgoing trajectories, respectively. Using the Kruskal coordinates Eqs. (3.4),(3.5), the action can be conveniently rewritten as

$$S(r)=-\varepsilon\ln(r-1)-\varepsilon t\simeq-\varepsilon\ln V^2 \tag{5.1}$$

for the incoming trajectory, and

$$S(r)=\varepsilon\ln(r-1)-\varepsilon t\simeq\varepsilon\ln U^2 \tag{5.2}$$

for the outgoing trajectory. Let us rewrite correspondingly the radial wave function (3.1). Multiplying it by the time-dependent factor $\exp(-i\varepsilon t)$, one can present it in a convenient abstract notation

$$|\phi\rangle=|\,\mathrm{in}\rangle+\mathcal{R}|\,\mathrm{out}\rangle, \tag{5.3}$$

where $|\phi\rangle=\exp(-i\varepsilon t)\phi(r)$, and $\phi(r)$ is given in Eq. (3.1). The two terms on the right-hand side of Eq. (3.1) are

$$|\mathrm{in}\rangle=\exp[-i\varepsilon\ln(V^2)], \tag{5.4}$$

$$|\mathrm{out}\rangle=\exp[i\varepsilon\ln(U^2)]. \tag{5.5}$$

We restrict our discussion to the events that take place in the vicinity of the horizon, where the semiclassical description holds, justifying Eqs. (5.4),(5.5). The classical trajectory that corresponds to the incoming wave $|\,\mathrm{in}\rangle$ follows from the equation of motion $\partial_\varepsilon S=0$, where the action is given in Eq. (5.2). Therefore the ingoing trajectory is described by the equation $V=\mathrm{const}$. Similarly, the outgoing wave $|\mathrm{out}\rangle$ in the vicinity of the horizon corresponds to the classical trajectory $U=\mathrm{const}$. In r,t variables these two trajectories are presented in Eq. (2.5) for the outside region.

Figure 1 shows classical trajectories in Kruskal coordinates. This graphical presentation emphasizes the unexpected, nontrivial nature of the interference between the incoming and outgoing waves in Eq. (5.3). A particle that follows the incoming trajectory has no classically allowed chance to switch to the outgoing trajectory in the classical approximation. Figure 1 visualizes this argument, showing that inside the event horizon the incoming and outgoing trajectories belong to different regions of the U-V plane. Thus the incoming and outgoing trajectories seem to be completely unrelated. However, Eq. (5.3) indicates that on the quantum level there arises a connection between the incoming and outgoing waves. It manifests itself as the interference of these waves in the wave function. We verified this statement above for the outside region $r>1$, but it holds for the inside region as well. Indeed, the Kruskal coordinates in Eqs. (5.4),(5.5) show that the logarithmic singularity of the wave function does not depend on the sign of U and V, i.e., it exists on both sides of the horizon. Therefore inside the horizon one can use same method that we used above for the outside region, which leads to the same result, which remains valid on both sides of the horizon: the incoming and outgoing waves do interfere in the wave function (5.3) (see also the discussion in Ref. [28]).

We discussed in Secs. III and IV the physical manifestation of this interference for the outside region, claiming that it leads to the reflection of the probing incoming particle from the event horizon. Let us now consider the physical manifestation of this interference for the region inside the horizon. The classical ingoing trajectory $V=\mathrm{const}$ describes here the motion toward the black hole center; the outgoing $U=\mathrm{const}$ trajectory describes the motion that eventually brings the particle from the inside region, over the horizon, into the outside region $r>1$. If a particle follows the ingoing classical trajectory then, as mentioned above, there is no classical way for it to switch to the outgoing trajectory and escape into the outside region. However, Eq. (5.3) shows that the perception based on the classical picture is not completely correct. In the quantum wave function the proper ingoing wave $|\mathrm{in}\rangle$ is mixed with the proper outgoing wave $|\mathrm{out}\rangle$. This mixing indicates that the particle that moves toward the black hole center in the inside region has a finite chance to simultaneously populate the outgoing wave that brings it to the outside region. Thus there is a finite probability for the particle to escape from the region inside the horizon into the outside region.

Let us calculate this probability. Suppose that there is a particle confined in the inside region. Assume that this particle occupies a state with the quantum numbers ε,L,M moving from the horizon deeper inside the black hole, eventually aiming at the singularity at the origin.[2] According to the above discussion one should describe this particle by the wave function (5.3), which shows that there is an admixture of the outgoing wave. The probability of populating this wave is $\mathcal{P}=|\mathcal{R}|^2$. Following the classical outgoing trajectory, which corresponds to this wave, the particle can reach the event horizon and therefore can escape into the outside world.

Thus there exists the probability that the particle escapes, $\mathcal{P}_{\mathrm{esc}}\propto|\mathcal{R}|^2=\mathcal{P}$. We can be more specific. We know that the wave that reaches the event horizon is partially reflected. According to Eq. (3.10) the probability of RH equals $\mathcal{P}_{\mathrm{cr}}=1-\mathcal{P}$. We proved this result when we considered the scattering process that takes place in the outside region. One can verify that this result holds when we consider the scattering that takes place for the wave that comes to the horizon on its way from inside out as well. Combining the two factors, the probability of populating the outgoing wave and the probability of crossing the event horizon we conclude that the probability for the particle to escape into the outside world equals $\mathcal{P}_{\mathrm{esc}}=\mathcal{P}(1-\mathcal{P})$. It is instructive to compare this result with the probability of the particle to be absorbed. Suppose we have an incoming particle in the outside region in a state described by the wave function (5.3) with the same quantum numbers ε,L,M. The probability for this particle to populate the ingoing wave in Eq. (5.3) is unity; therefore the probability to be absorbed $\mathcal{P}_{\mathrm{abs}}$ into the inside region equals the probability to cross the event horizon (3.10), which gives $\mathcal{P}_{\mathrm{abs}}=1-\mathcal{P}$. We can consider now the ratio of the probability for a particle to escape from the inside region to the probability to be absorbed:

$$\frac{\mathcal{P}_{\mathrm{esc}}}{\mathcal{P}_{\mathrm{abs}}}=\mathcal{P}=\exp\left(-\frac{\varepsilon}{kT}\right). \tag{5.6}$$

Discussing the probabilities above, we considered only those factors that originate directly from the wave function (5.3). The physical probabilities include also additional normalization factors related to the flux of particles and the surface area of the event horizon. However, these additional factors are canceled out in the ratio (5.6), which presents therefore the result for the ratio of the two physical rates, emittance and absorption. It states that the ratio of the emittance and absorption rates coincides with the conventional temperature factor that describes the ratio of these rates for a blackbody with temperature T. This means that if the black hole is put inside a thermostat with temperature T, it remains in equilibrium with it. One concludes therefore that Eq. (5.6) indicates that the black hole possesses temperature T, radiating as a blackbody with this temperature, as was first discovered by Hawking [8,9] using different arguments.

There is a conventional physical explanation for the Hawking process that refers to the creation of pairs. The gravitational field in the vicinity of the horizon creates a pair; then a particle goes into the outside world, while its antipartner is absorbed by the black hole. This explanation of the process needs effort to prove the fact that the antiparticle brings into the black hole the negative amount of energy that compensates the energy of the created particle. Equation (5.3) suggests an alternative simple explanation. The radiation happens because the particle confined inside the horizon can escape into the outside world. This point of view automatically accounts for the reduction of the mass of the black

[2]Generally speaking, to be certain that the particle is located in the inside region, one needs to describe its motion with the help of the wave packet that propagates from the horizon into the deeper region. However, for our purposes it suffices to take into account only one wave with the given quantum numbers ε,L,M. In proving below that each wave of the packet has a chance to escape from the inside region, we prove simultaneously that the wave packet can escape as well.

hole; when the particle escapes from the black hole it no longer contributes to the mass of the black hole.

Summarizing, we verified that both the RH and the Hawking radiation stem from the interference of the incoming and outgoing waves in the wave function (5.3).

VI. DISCUSSION AND CONCLUSION

The existence of the event horizon that separates the outside and inside regions is the main property of black holes. It is well known that one can always choose a coordinate frame that makes the metric smooth on the horizon. Correspondingly, the classical equations of motion for a probing particle in these coordinates are also smooth on the horizon. From this fact follows a known conclusion: a probing particle that follows the classical trajectory on its way to the black hole crosses the horizon quite smoothly, but after that will be forced to stay inside forever. However, quantum corrections influence the fate of this particle. The arguments presented indicate that the horizon makes a strong impact on the wave function of a probing particle. It manifests itself in the form of interference, mixing of the incoming and outgoing waves in the wave function (5.3). Without this mixing the incoming wave crosses the event horizon quite uneventfully, in accord with the similar smooth transition through the horizon of the classical trajectory. The mentioned mixing indicates that the incoming wave inevitably incorporates some admixture of the outgoing wave.

This result was derived from two facts: the discrete symmetry of the space-time and the semiclassical nature of the wave function in the vicinity of the horizon. The mixing coefficient $\mathcal{R}$ obtained possesses a typically semiclassical nature for a classically forbidden quantity,

$$|\mathcal{R}| = \exp\left(-\frac{\mathcal{A}}{\hbar}\right), \tag{6.1}$$

where A has the meaning of some effective classical action. For example, for the Schwarzschild geometry of the black hole $\mathcal{A} = \varepsilon\tau$, where τ has the dimension of time with the typical value $\tau = 2\pi r_g / c$. In the classical limit $\hbar \to 0$ the mixing (6.1) disappears. Thus, from the point of view of the classical approximation the physical manifestations of quantum interference look unusual. Generally speaking, there is nothing unusual about interference between the incoming and outgoing waves; on the contrary, it is quite normal (it happens, for example, due to the graybody factors). The point is that the black hole horizon is very special. It is supposed to absorb very well everything incoming; therefore naively the particle that approaches the horizon is described by a purely incoming wave. From this perspective the existence of the interference and, consequently, the existence of the reflected outgoing wave is surprising.

We discussed two effects that originate from the interference between the incoming and outgoing waves. One of them is a novel effect, RH. For any particle that approaches the event horizon there is a finite probability to bounce back, into the outside world. The probability of RH depends on the energy ε of the incoming particle and the temperature T of the black hole. For $\varepsilon < T$ the black hole horizon behaves as a reflector, which is unusual.

Another effect that follows from the interference of the incoming and outgoing waves is the well-known phenomenon of Hawking radiation. The suggested new explanation for this effect is simple and appealing. The radiation happens because, when the incoming particle is confined in the inside region, it still maintains an opportunity to escape back into the outside world. This fact changes the perception of the event horizon. Conventional arguments claim that, when the incoming particle comes into the inside region, it stays there forever; the horizon is impassable for the backward transition. This argument, however, holds only in the classical approximation. Quantum corrections make the horizon partially transparent; the particles can cross it and go away, creating the Hawking radiation spectrum of the black hole.

Both the radiative and reflective abilities of the black hole horizon arise from quantum corrections; both these processes are governed by the Hawking temperature of the black hole, but experimentally they are easily distinguishable. The reflected flux depends on the nature, flux, and spectrum of the incoming particles, as well as on the black hole properties, while the radiation is governed entirely by the black hole. The radiation phenomenon provides support for important thermodynamic properties of black holes. The suggested new approach to the origins of the radiation may help one to look anew at the thermodynamics properties of black holes as well, but this topic lies ahead.

In conclusion, we showed that the black hole horizon is capable of reflection and found a general common physical origin of this effect and the Hawking radiation.

ACKNOWLEDGMENTS

This work was supported by the Australian Research Council. Discussions with V. V. Flambaum are appreciated.

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