Time travel paradoxes, path integrals, and the many worlds interpretation of quantum mechanics

Allen Everett
Department of Physics and Astronomy and Institute of Cosmology, Tufts University, Medford, Massachusetts 02155, USA

(Received 13 September 2002; published 25 June 2004)

We consider two approaches to evading paradoxes in quantum mechanics with closed timelike curves. In a model similar to Politzer's, assuming pure states and using path integrals, we show that the problems of paradoxes and of unitarity violation are related; preserving unitarity avoids paradoxes by modifying the time evolution so that improbable events become certain. Deutsch has argued, using the density matrix, that paradoxes do not occur in the "many worlds interpretation." We find that in this approach account must be taken of the resolution time of the device that detects objects emerging from a wormhole or other time machine. When this is done one finds that this approach is viable only if macroscopic objects traversing a wormhole interact with it so strongly that they are broken into microscopic fragments.

DOI: 10.1103/PhysRevD.69.124023 PACS number(s): 04.20.Gz

I. INTRODUCTION

There has recently been a good deal of interest in possible spacetimes containing closed timelike curves (CTC's) arising either from the presence of traversable wormholes [1] or from the warping of spacetime in such a way as to allow superluminal travel [2], with the possibility of CTC's as a consequence [3–5]. A variety of theoretical considerations (e.g., Refs. [6–8]), either general or addressed to specific models, have been advanced which suggest that the formation of CTC's is not possible. However, while some of these considerations are very persuasive, none appear conclusive [9].

In addition to the problems discussed in the references already cited, CTC's lead to the well-known problems with paradoxes arising from the apparent possibility of inconsistent causal loops. This phenomenon is illustrated by the "grandfather paradox" occurring frequently, in various guises, in science fiction, in which one travels back in time and murders one's own grandfather, thus preventing one's self from being born and traveling back in time in the first place.

Satisfactory physical theories must avoid giving rise to such self-contradictory predictions. One approach to achieving this is to impose consistency constraints on the allowable initial conditions on spacelike surfaces prior to the formation of the CTC's, thus abandoning the principle that initial conditions on such surfaces can be chosen at will. For example, in the case of the grandfather paradox we might insist that the initial conditions just before the prospective murder include the presence of a strategically placed banana peel on which the prospective murderer slips as he pulls the trigger, thus spoiling his aim. One might refer to this approach as the "banana peel mechanism;" it leads to a theory free of logical contradictions, but requires occurrences that would seem, *a priori*, to be highly improbable. This violates strong intuitive feelings. These feelings may simply reflect our lack of experience with phenomena involving CTC's. Nevertheless, a need to invoke constraints on the choice of initial conditions would be quite disturbing for many physicists and contribute to an expectation that CTC's are forbidden.

The first suggestion in the literature that it might be possible to avoid such paradoxes was due to Echeverria, Klinkhammer, and Thorne [10] (henceforth EKT). For simplicity, these authors formulate the problem in terms of billiard balls, thereby avoiding questions of free will. They consider a situation in which, at $t=-\varepsilon$, where $\varepsilon>0$ but may be taken arbitrarily small, there is a billiard ball (henceforth generally denoted by BB), which we take to be at the spatial origin; its trajectory is such that at $t=T$ its leading edge reaches the point $\mathbf{r}_0+\Delta\mathbf{r}$ and enters a wormhole which connects the spacetime point $(\mathbf{r}_0+\Delta\mathbf{r},\ t+T)$ to $(\mathbf{r}_0,t)$. (A procedure for creating such a wormhole is discussed in Ref. [10]. We take Δr, the spatial distance between the wormhole mouths, to be small compared with r_0, and will in general ignore it; however, Δr cannot vanish if the wormhole persists over a time interval $T_0>T$, as one needs to introduce some separation between the two wormhole mouths if they overlap in *t*. In general we will assume $T_0 \ll T$. We take the internal length of the wormhole to be small compared to r_0, and will often work in the approximation in which the two mouths of the wormhole are simply identified with one another.) Upon emerging from the wormhole mouth at $t=0$, the BB may interact with its "younger self" which has not yet entered the wormhole. ("Younger" here means younger in terms of the ball's "personal" time, i.e., the proper time τ measured on a clock attached to the ball.) An inconsistent causal loop, analogous to the "grandfather paradox," can then occur as the result of a BB trajectory such that, on emerging from the wormhole, the ball undergoes a head on collision with its younger self, deflecting the latter so that it does not enter the wormhole in the first place. However, as EKT point out, in the presence of CTC's the trajectory is not unique, and there are also solutions, with the same initial conditions, which give rise to consistent causal loops; e.g., a glancing collision may occur which deflects the ball's younger self so that its trajectory through the wormhole results in the required glancing collision. EKT then suggest adopting a consistency principle according to which only self-consistent solutions are to be considered physical. The EKT consistency principle places constraints on the allowable initial conditions within the region containing CTC's, but does not constrain the initial conditions that may be imposed outside that region. This idea seems physically attractive, and the discussion in Ref. [10] had much to do with stimulating interest in time travel as a subject possibly deserving of serious study.

It is, however, far from clear that the consistency principle always allows one to avoid paradoxes. Because of the variety of types of collision, ranging from glancing to head on, which can occur between two spheres, EKT were able to find self-consistent solutions, in fact an infinite number of them, for a wide variety of initial conditions. However, it is difficult to see how this can be true in general. For example, suppose we place at the early time ($t=0$) mouth of the wormhole a device to detect the BB if it emerges. (One might, e.g., have a spherical grid of current carrying wires enclosing the wormhole mouth at $t=0$ thin enough to be broken by the BB with its given speed and spaced closely enough that a BB cannot emerge from the wormhole without breaking at least one of the wires.) Suppose further that we connect the detector in such a way that, if a BB is detected, a signal is sent at light speed activating a mechanically operated shutter, which deflects the incident ball at some later point on its path so that it does not enter the late-time wormhole mouth. One can include a requirement that the signal is sent only if a BB emerges from the wormhole before the incident ball reaches the shutter. This eliminates the possibility of a second, consistent, solution, in which the shutter starts to close just as the ball passes through it, resulting in a self-consistent time delay in which the ball emerges from the wormhole and causes the shuter to close just as the incident ball reaches it. This arrangement is a modification of somewhat similar ones discussed by Novikov [11] in which consistent solutions exist. In the case discussed here, however, it is difficult to see how there can be any self-consistent, and hence physically acceptable, solution. Thus we seem to be back to the grandfather paradox in the form of a BB which enters the wormhole if and only if it does not enter the wormhole.

We will illustrate these ideas below, making use of a quantum mechanical model due to Politzer [12] (henceforth HDP) which is simple enough to be calculable but has many of the physical features of the BB-wormhole system just discussed. In this model, systems are treated as being in a pure quantum state, as in standard quantum mechanics, even in a spacetime region containing CTC's, and the path integral formalism is used; the treatment in HDP is limited to calculating amplitudes for the case of initial and final states at times, respectively, before and after the era containing CTC's. The Hamiltonian H can be chosen so that there are self-consistent solutions; in accordance with the EKT principle, these solutions can be taken to be the only ones which are physically relevant. We also find, at least in the HDP model, that when unique consistent solutions exist, their time evolution is governed by a unitary operator, so that the probability interpretation of quantum mechanics can be preserved. Hence, if consistent solutions exist, one has a quantum mechanical theory which, in the presence of CTC's, differs from standard quantum mechanics only in the imposition of the EKT criterion for physically relevant solutions, and in the fact that the uniqueness problem remains if there is more than one self-consistent solution.

However, it is also possible, as we will see below, to choose a Hamiltonian in the region of CTC's in the HDP model for which no self-consistent solutions exist, in contrast with the examples in Refs. [10] and [11]. One can attempt to restore consistency by projecting out only that part of the path integral expression for the wave function at $t>T$ which comes from paths satisfying the consistency condition. However, as seen in HDP, this results, because of the consistency requirement, in a time evolution of the wave function which is controlled by a nonunitary operator $X\neq\exp(-iHt)$. One may preserve the probability interpretation of the final state wave function by renormalizing the operator X by a factor which depends on the initial state and which thus introduces nonlinearity into the time evolution. Unexpectedly, this has consequences for the time evolution of the system for $t<0$, i.e., for times *before* CTC's occur, as first observed by Hartle [13]; a more intuitive argument based on the requirement for a consistent probability interpretation is given in HDP. An alternative procedure for renormalizing X by matrix multiplication, proposed by Anderson [14], avoids violations of causality at $t<0$, though perhaps at the cost of discarding essential physics. Both procedures, in effect, lead to the ''banana peel'' mechanism, since one finds that the presence of potential paradoxes insures the occurrence of *a priori* improbable events either before or during the era of CTC's.

Hence, in the general case, the HDP approach avoids the problems associated with inconsistent causal loops only if, in the presence of CTC's, fundamental axioms of quantum mechanics are abandoned. In particular, the time evolution operator which transforms the wave function at t_1 into that at t_2 is no longer unitary and is not given by $U(t_1,t_2)=\exp(-iHt)$, with $t=t_1-t_2$. Moreover, the preservation of a consistent probability interpretation requires the introduction of rather *ad hoc* procedures, possibly involving violations of causality in the era before CTC's are formed.

It would thus be interesting to find a model-independent approach in which the existence of CTC's does not lead to inconsistent causal loops; the existence of such a theory would remove one of the theoretical (or perhaps psychological) objections to CTC's. Hopefully this would avoid the nonunitary time evolution operators, and the consequent difficulties with conservation of probability, which arise in HDP when inconsistent causal loops are present.

Science fiction writers often avoid causal paradoxes in stories involving time travel by invoking the idea of ''alternate universes.'' At first sight this idea seems devoid of any physical foundation. However, the many-worlds interpretation (MWI) of quantum mechanics due to Hugh Everett, III [15] does introduce ideas which have some resemblance to the alternate universes of science fiction; it also provides an interpretation of quantum mechanics which seems difficult or impossible to distinguish experimentally from the more conventional one, and which some might argue is intellectually more satisfying.

It thus seems natural to ask whether the MWI might provide a way out of the problems of logical consistency raised by CTC's. Deutsch [16] (hereafter referred to as DD) has discussed this question. He argues that inconsistent causal loops do not occur in the MWI because, loosely speaking, the pairs of seemingly inconsistent events (e.g., one's birth and one's murdering one's grandfather) occur in different "universes" and hence are not logically contradictory. In Deutsch's approach the MWI becomes more than a mere interpretation of quantum mechanics; in the presence of CTC's it has experimental consequences.

The approach in DD actually involves assumptions that go beyond simply adopting the MWI. The cost of preserving unitarity, or more precisely, conservation of probability, is that, in the presence of CTC's, a system must in general be described by a density matrix, not a wave function. As we will discuss, in the absence of self-consistent solutions, pure states necessarily evolve into mixed states in the region containing CTC's if the violations of unitarity in the HDP approach are to be avoided. Thus, in the same situations in which unitarity fails in the model in HDP, the approach in DD requires one to formulate the theory in terms of the density matrix. The time evolution equation of the density matrix in DD is given, as usual, by

$$\varrho(t_2)=U^{-1}(t_2,t_1)\varrho(t_1)U(t_2,t_1), \tag{1}$$

where (in units with $\hbar=1$) $U(t)=\exp(-iHt)$ and H is the Hamiltonian; this ensures the preservation of the probabilistic interpretation of ϱ. Moreover, Eq. (1) is taken to be valid at all values of t so that the theory determines ϱ during the era in which CTC's exist, as well as before and after. However, the concepts of "mixed state" and "density matrix" in DD are different than in conventional quantum mechanics, where mixed state refers to an ensemble of identically prepared systems whose statistical properties are given by the density matrix. In DD the term mixed state refers to a single system not in a definite quantum state and described not by a wave function but a density matrix. The diagonal elements of ϱ in, say, the R representation, where R is an observable, give the probabilities of observing the possible outcomes of a measurement of R on that single system, and ϱ will not, in general, satisfy the condition $\varrho^2=\varrho$ characteristic of a pure state.

Working only with density matrices and mixed states of the type just discussed goes beyond, at least in principle, simply adopting the MWI as presented in Ref. [14] which deals with systems in pure states described by wave functions. In the MWI, suppose we begin with an object whose initial wave function $\psi=c_1u_1(R_j)+c_2u_2(R_j)$ where R_j are the eigenvalues of an observable R describing the object, and the u_i are eigenstates of R with eigenvalues R_1 and R_2. Let the value of R be measured by a macroscopic measuring apparatus which is left in a state with wave function $\phi_i(q_k)$ when the measurement yields the result R_i, where the ϕ_i are eigenstates of an observable Q with eigenvalues q_k giving the internal state of the measuring apparatus. According to the MWI, the system of object plus apparatus will be described after the measurement by a wave function $f(R_j,q_k)$, where

$$f(R_j,q_k)=c_1'\phi_1(q_k)u_1(R_j)+c_2'\phi_2(q_k)u_2(R_j) \tag{2}$$

and $|c_i'|=|c_i|$. Hence the object-apparatus system remains in a pure state. However, because of the complexity of the internal structure of the measuring apparatus, the eigen-values q_i are highly degenerate. Hence the two terms on the right side of Eq. (2) actually represent effectively infinite sums of terms with varying phases. Thus, once the measurement interaction is over, the two terms on the right side of Eq. (2) become decoherent and matrix elements of operators between states with different ϕ_i effectively vanish. This is the reason the "worlds" in which Q has different well-defined values are unaware of one another so that the MWI, at least in the absence of CTC's, is without observable consequences.

From the foregoing discussion we see that the approach in DD to resolving the paradoxes associated with time travel involves modifying fundamental principles of quantum mechanics; it certainly goes beyond simply adopting the MWI. We will refer to this approach from now on as the "mixed state MWI" (MSMWI) to distinguish it from the original many worlds interpretation of Ref. [14]. However, despite the differences in principle, in practice, when dealing with macroscopic systems, the mixed states which occur in Deutsch's approach are very similar to the nearly decoherent states which occur in the MWI following a measurement, so one might feel that the departure from standard quantum mechanics is relatively minor, and perhaps plausible.

However, as we argue below, once an observation has been made as to whether a BB has or has not emerged from the wormhole the states corresponding to these two possibilities become decoupled, just as in the case of the different "worlds" of the MWI when no CTC's are present. As a result, in situations where, classically, there would be an inconsistent causal loop, while the front part of an object traveling backward in time emerges from the wormhole in a different "world"; another part emerges in the same world which contains its younger self, contrary to the proposal in DD. As a result, in the case of macroscopic objects, when proper account is taken of the finite time required for the object to emerge from the wormhole and be detected, one finds that no self-consistent solutions in which the object passes intact through the wormhole exist in the MSMWI. The object is sliced into two, or more generally into many, pieces in passing through the wormhole, with different pieces winding up in different worlds, i.e., in states of the system labeled by different readings of a macroscopic measuring device. Thus, in the MSMWI, wormholes (or other time machines) which can be traversed intact by macroscopic objects cannot exist. If the MSMWI is correct, such objects must necessarily undergo violent interactions with the time machine which cause the object to disintegrate.

The organization of the remainder of the paper is as follows. In Sec. II we discuss the quantum-mechanical formulation of the consistency condition in the presence of CTC's in terms of its implications for the time evolution operator of the wave function. In Sec. III, we consider the model in HDP in cases where the operator U does, and does not, give rise to the existence of consistent solutions, and observe the connection in the model between the existence and uniqueness of consistent solutions and unitarity. In Sec. IV we review in detail the density matrix approach in DD, and its connection to the MWI, and discuss the relation between the absence of consistent solutions and the transformation of an initial pure quantum state into a mixed state in the region containing CTC's. In Sec. V we examine, following DD, how the MSMWI might resolve the analog of the "grandfather paradox" in the case of a microscopic object, such as an electron, traveling backward in time. In Sec. VI we analyze in detail the difficulties which arise when one attempts to extend the MSMWI to macroscopic objects. We conclude briefly in Sec. VII.

II. CONSISTENCY CONDITION FOR WAVE FUNCTIONS

Here we assume that the rules of quantum mechanics are unchanged in the presence of CTC's except for the imposition of a consistency requirement, whose formulation we wish to examine. Suppose that at $t=-\varepsilon$, where ε is infinitesimal, we have an incident BB at the origin whose trajectory is such that its leading edge reaches the wormhole at $t=T$. We take the BB's proper time τ to be the position of the hand of a clock attached to the ball, which we can treat as a dynamical observable. In contrast, t, the evolution parameter for wave functions, may be thought of as the common reading of a network of synchronized clocks remaining at rest relative to one another. We suppress the (many) other internal variables in addition to τ associated with the internal structure of the BB. For $t<T$, $\tau=t$; however, neglecting the travel time through the wormhole, $\tau=t+T$ for $\tau>T$, i.e., for the ball which emerges from the wormhole. Although a classical object, we assume the BB is, in quantum mechanics, described at the fundamental level by a wave function $\psi_1(\mathbf{r},\tau,t)$ whose dependence on the dynamical variables $\mathbf{r}$ and τ at $t=-\epsilon$ is peaked about their classical values ($r=0$ and $\tau=-\epsilon$) with negligible spread; by continuity this should also be true of $\psi_1(\mathbf{r},\tau,+\epsilon)$. However, there may now be what appears to be a second ball emerging from the wormhole. Since we expect the wave function of the ball near the origin to be determined by continuity, we take the most general form of the wave function for the system to be

$$\psi(\mathbf{r},\tau,\mathbf{r}',\tau',n',t=\epsilon)=\psi_1(\mathbf{r},\tau,\epsilon)\,\psi_2(\mathbf{r}',\tau',n',\epsilon), \quad (3)$$

where the variable n' is an occupation number with two possible values, 1 and 0, denoting, respectively, the presence or absence of a BB emerging from the wormhole. [Thus, e.g., if the incident ball always goes through the wormhole $\psi_2(n'=0)=0$. Excluding the possibility $n'>1$ corresponds to the assumption that the incident BB, if it emerges from the wormhole mouth at $t=0$, is directed in such a way that it does not reenter the wormhole mouth at $t=T$.] For $n'=1$, $\mathbf{r}'$ and τ' are position variables for the emerging BB and the hand of its clock, so that $\psi_2(n'=1,\, t=\epsilon)$ is peaked around the values $\mathbf{r}'=\mathbf{r}_0$ and $\tau'=T+t_0+\epsilon$, where t_0 is the transit time of the BB through the wormhole; since ϵ is arbitrarily small, and the internal length of the wormhole is taken to be such that $t_0\ll T$, $\tau'\approx T$. Note that the product form of the wave function, Eq. (3), which we obtained by continuity in time from the initial condition at $t=-\epsilon$, also follows from the reasonable assumption that the two balls (actually the younger and older versions of the same ball) will not yet have interacted at $t=+\epsilon$.

While traversing the wormhole, we take the subsystem in the vicinity of $\mathbf{r}_o$ to be isolated. For this subsystem, τ, though it may be regarded as a dynamical observable by outside observers, plays the role of the time evolution parameter. The BB evolves through the wormhole in the direction of increasing τ or decreasing t with the evolution governed by a Hamiltonian H', the Hamiltonian of the isolated subsystem. (Instead of a billiard ball, one can picture this in terms of an isolated spaceship inside a superluminal Alcubierre warp bubble [5]. Passengers on the ship would see their world governed by a quantum mechanics in which the reading of clocks on the spaceship would play the role of the time evolution parameter even though the hands of the clocks appear to run backwards to outside observers in some Lorentz frames.)

From the foregoing, we conclude that the wave function at $t=T$ can be obtained from that at $t=\epsilon$ by an operator of the form

$$U(T)=u(T)\otimes\exp[iH't_o] \quad (4)$$

where $u(T)$ is an operator which acts on $\psi_1(\mathbf{r},\tau,\epsilon,)$ and leaves $\psi_2(\mathbf{r}',\tau',n',\epsilon)$ invariant since its evolution is governed solely by $\exp(iH't_0)$. The sign of the exponent is due to the fact that τ decreases in going from the $t=0$ to the $t=T$ mouth of the wormhole. In the approximation that the wormhole mouths are identified so that $t_o=0$,

$$U(T)=u(T)\otimes I \quad (4')$$

and the wave function of the system at $t=T$ has the form

$$\psi(\mathbf{r},\tau,n,\mathbf{r}',\tau',n',T)=\psi_1(\mathbf{r},n,T)\,\psi_2(\mathbf{r}',\tau',n',\epsilon), \quad (5)$$

where, at $t=T$, we include an occupation number variable n for the part of the system separated from the wormhole. (At $t=0$, $n=1$ from the initial conditions.) The direct product structure of Eq. (4) or Eq. $(4')$ and the product form of Eq. (5) are consequences of the fact that H' depends only on the coordinates of the subsystem traversing the wormhole. Thus H' cannot generate any correlations at $t=T$ between the BB's with coordinates $\mathbf{r}$ and $\mathbf{r}'$. If $t_0\neq 0$, a factor of $\exp(-iE_{n'}t_0)$ must be included on the right of Eq. (5), where $E_0=0$ and $E_1=E_{\mathrm{BB}}$, the energy associated with the presence of a BB; in general, this factor, even if present, does not affect the subsequent discussion.

Note, from Eqs. (3) and (5), that the wave function itself is not continuous across the wormhole; only the factor ψ_2 is continuous as a consequence of Eq. $(4')$. If one begins with a system in a pure state (and thus describable by a wave function), then since it is being evolved by a unitary operator the overall system remains in a pure state. The wave function at $t=\epsilon$ has the product structure of Eq. (3) so that each subsystem is separately in a pure state. The continuity across the wormhole, expressed by Eq. (4), then guarantees that structure is preserved in Eq. (5) even though the subsystems outside the wormhole interact with one another between

$t=0$ and $t=T$; the subsystem at the wormhole mouth thus remains separately in a pure state. Thus if $\psi_1(T)$ and/or $\psi_2(T)$ are superpositions of states with different occupation numbers, the values of n and n' must be uncorrelated.

III. CTC's AND THE QUANTUM MECHANICS OF PURE STATES

We begin this section by summarizing Politzer's model referred to above. The model drastically truncates spacetime to a space with two fixed points with coordinates $z=z_1$ and $z=z_2$. For $z=z_1$, $-\infty<t<\infty$. However, at $z=z_2$ there is a time machine in the form of a wormhole connecting $t=0$ and $t=T$; we neglect the transit time through the wormhole t_0, so that the wormhole simply identifies the spacetime points $(z_2,0)$ and (z_2,T). A particle at $z=z_2$, $t<0$ is taken to enter the wormhole at $t=0$ and emerge at $t=T$ to move on into the future, while a particle at z_2 in the range $0<t<T$ enters the $t=T$ mouth of the wormhole and emerges at $t=0$, following a worldline which is an endless CTC at constant $z=z_2$. The physical system is taken to be a single fermion field. Hence the occupation numbers n and n' are restricted to 0 or 1. This models the situation discussed in Sec. II, where $n=1$ is an initial condition at $t=0$, and $n'\leqslant 1$ as a result of the assumed trajectory of a BB emerging from the wormhole. The states of the system lie in a Hilbert space a basis for which is provided by the four states $\uparrow_1\uparrow_2$, $\uparrow_1\downarrow_2$, $\uparrow_2\downarrow_1$, and $\downarrow_1\downarrow_2$, where $\uparrow_1\uparrow_2$, e.g., corresponds to occupation number 1 for the fermion states at both z_1 and z_{2^-}. Henceforth we will omit the subscripts 1 and 2, unless needed for clarity, and simply adopt the convention that the first and second arrows in a pair denote, respectively, the occupation numbers at z_1 and z_2. The field is taken to have effectively infinite mass so that kinetic energy terms in the energy can be ignored. The notation $\uparrow$ and $\downarrow$ for the occupied and unoccupied states, respectively, is motivated by the fact that the model is mathematically equivalent to the presence of spin-$\frac{1}{2}$ particles, each with possible spin states up and down, at z_1 and z_2. We will always deal with the case where the state at z_2 is unoccupied except for $0<t<T$. During this interval the system is governed by a Hamiltonian H whose general form is that of an arbitrary 4×4 Hermitian matrix whose matrix elements are constants, since the particle positions are taken to be given by the occupation numbers and there is no kinetic energy. The assumed freedom to choose the Hamiltonian governing the time evolution in the interval $0<t<T$ is a natural representation in the model of the assumed freedom to impose arbitrary initial conditions at $t=0$ before CTC's exist.

Let $\Psi(t)$ be the state vector of the system, which, in the absence of CTC's, would obey the equation

$$\Psi(t)=\exp(-iHt)\Psi(+\epsilon). \tag{6}$$

Note that an arbitrary 4×4 unitary matrix $U(T)$ can be written as $\exp(-iHT)$ where the Hermitian operator H is a generator of the group $U(4)$; hence the freedom to choose H arbitrarily translates into the freedom to choose an arbitrary $U(T)$.

We will take the model in HDP to give a calculable qualitative guide to the behavior of our system of the billiard ball that travels back in time and interacts with itself. We let z_1 be the position of the incident BB at $t=-\epsilon$ and z_2 the position of the wormhole. As in the HDP model, the state of the system at $t=+\epsilon$ lies in the same Hilbert space as before, where now, e.g., the state $\Psi(+\epsilon)=\uparrow\uparrow$ corresponds to the state with both an incident BB at z_1 and a ball at z_2 which emerged from the wormhole. The behavior of the BB system is, of course, more complicated than that of the model. The actual trajectory of the balls is not one of constant z; the incident ball may move from z_1 to enter the wormhole at z_2, while a ball emerging from the wormhole at $t=0$ might be directed along a trajectory reaching $z=z_{\rm final}$ at $t=T$, where, for simplicity, we could take $z_{\rm final}=z_1$. Moreover during the interval $0<t<T$ the two BB's may interact with each other in complicated ways. For example: they may collide; or they may hit switches which cause shutters to be closed, diverting or stopping one or both of the initial balls. However, the state of the system at $t=T-\epsilon$ (or at least that part of it in which we are interested), can again be described in terms of the same Hilbert space, where now $\uparrow\uparrow$ is the state in which balls are present at both z_2, on the verge of entering the $t=T$ mouth of the wormhole, and at $z_{\rm final}$. Hence, for a prescribed $z_{\rm final}$, which we will take to be z_1, the range of possible time evolutions of the billiard ball system is given by the range of possible 4×4 unitary matrices $U(t)$, as in HDP. Since the HDP model also enforces consistency between the two mouths of the wormhole, it contains much of the essential physics of the BB-wormhole system. Hence it seems reasonable to hope that HDP provides a correct qualitative description of the behavior of the latter system.

We are interested in the case where the initial state at $t=-\epsilon$ is $\uparrow\downarrow$; i.e., we have an initial particle at z_1 but not at z_2. (We will always impose the initial condition at $z=z_2$ that no BB is present, i.e. that $n'=0$, at $t=-\epsilon$.) We assume, from continuity, that the occupation number at z_1 at $t=+\epsilon$ remains equal to 1, so that the two possible states of the system at that time are $\uparrow\uparrow$ and $\uparrow\downarrow$. Politzer in HDP obtains the amplitude for finding occupation number i ($i=0$ or 1), at z_1 at $t=T+\epsilon$, given occupation number 1 at $t=-\epsilon$, by using the Feynman path integral to sum over paths, subject to the consistency condition that the occupation number at $z=z_2$ be the same at $t=0$ and $t=T$ since these two points are identified. The result, as given in HDP, is that the amplitude is given by X_{i1}, where

$$X_{ij}=\sum_k \langle ik|U(T)|jk\rangle, \tag{7}$$

where $|ik\rangle$ denotes a state with occupation numbers $n=i$ and $n'=k$ at z_1 and z_2, respectively. Note that X_{ij} involves a the trace of the 2×2 matrix $U_{ik;jn}$ in the occupation number space at z_2; it does not therefore depend on the choice of the set of two orthonormal basis states in that space.

We define $|X(j)|^2=\Sigma_{i=0}^1|X_{ij}|^2$, $j=0$, 1. It is pointed out in HDP that the overall normalization of $\mathbf{X}$ may be multiplied by a state independent factor that can be absorbed in the functional measure. However, unitarity requires

$$|X(1)|^2=|X(0)|^2. \tag{8}$$

We consider below several different cases, differing in the form of U in Eq. (7). We will always assume, on physical grounds, that U is such that

$$U(T)\downarrow\downarrow=\downarrow\downarrow, \tag{9}$$

i.e., we assume that if no particle is present either at z_1 or z_2 at $t=+\epsilon$, none will be present at $t=T-\epsilon$.

Case 1

First take the example in which the incident BB is on a trajectory to carry it into the wormhole mouth at $t=T$, and a ball emerging from the wormhole mouth at $t=0$ is directed onto a trajectory taking it to $z=z_1$ at $t=T$. Then

$$U(T)\uparrow\uparrow=\uparrow\uparrow \tag{10}$$

and

$$U(T)\uparrow\downarrow=\downarrow\uparrow. \tag{11}$$

Hence the matrix elements $\langle ik|U|1k\rangle$ appearing on the right side of Eq. (7) are nonvanishing only for $i=k=1$. For the $U(t)$ in Eqs. (10) and (11), the states $\uparrow\uparrow$ and $\uparrow\downarrow$ at $t=+\epsilon$ thus give, respectively, completely consistent and completely inconsistent solutions. For the completely inconsistent solution, $\uparrow\downarrow$, the wave functions at $z=z_2$, at $t=0$, and $t=T$ are orthogonal and have no overlap; the incident BB enters the wormhole at $t=T$, but no BB emerges at $t=0$. For the consistent solution, $\uparrow\uparrow$, the wave function at $t=T-\epsilon$ has the product form given in Eq. (5), as demanded by continuity.

It follows from Eq. (7) that, for U given by Eqs. (10) and (11), $X_{01}=0$ because $\langle 0k|U(T)|1k\rangle=0$. Hence Eqs. (7), (10), and (11) give

$$|X(1)|^2=|X_{11}|^2=1. \tag{12}$$

Equations (9)–(11), coupled with unitarity, imply that

$$U(T)\downarrow\uparrow=\uparrow\downarrow \tag{13}$$

so that one sees, from Eqs. (9) and (12), that there is also one consistent and one inconsistent solution for the case that there is no incident BB at $t=-\epsilon$. As a result the calculation of $|X(0)|^2$ exactly parallels that for $|X(1)|^2$ and we find

$$|X(0)|^2=|X_{00}|^2=1. \tag{14}$$

From Eqs. (8) and (14) it follows that unitarity is satisfied in this case in which there is a single self-consistent solution both when there is, and is not, a BB incident at $z=z_1$ at $t=-\epsilon$,

In case 1 the approach in HDP is equivalent to the imposition of the EKT consistency principle; Eq. (7) has the effect of picking out the self-consistent states $\uparrow\uparrow$ or $\downarrow\downarrow$ at $t=+\epsilon$, as the only states which contribute to the path integral; this is in accordance with the consistency principle, according to which only such states are physical. The contribution of the states $\uparrow\downarrow$ or $\downarrow\uparrow$, which do not satisfy the consistency condition and would be regarded as unphysical by EKT, is suppressed by Eq. (7).

One can see in a simple way the connection in the HDP approach between the preservation of unitarity and the existence of a consistent solution. In case 1, Eq. (7) for the X_{ij} is equivalent to the statement that the operator X can be written as $X=U_\epsilon(T)U(T)U_\epsilon(0)$. Here $U_\epsilon(0)=U(\epsilon,-\epsilon)$ is a unitary operator which takes the initial states $\uparrow\downarrow$ and $\downarrow\downarrow$ at $t=-\epsilon$ into the consistent states $\uparrow\uparrow$ and $\downarrow\downarrow$, respectively at $t=\epsilon$. Note that, for $0<t<T$, we need consider only the two dimensional subspace, spanned by the consistent states, of the full four-dimensional Hilbert space, since the two totally inconsistent states make no contribution to the operator X as given by Eq. (7). Similarly, $U_\epsilon(T)=U(T-\epsilon,T+\epsilon)$ takes $\uparrow\uparrow$ and $\downarrow\downarrow$ into the final states $\uparrow\downarrow$ and $\downarrow\downarrow$, respectively, at $t=T+\epsilon$; the relation $U_\epsilon(T)\uparrow\uparrow=\uparrow\downarrow$ reflects the disappearance of the BB at z_2 into the wormhole at $t=T$. $U(T)$ is the unitary time evolution operator from ϵ to $T-\epsilon$ for the consistent states, given by Eqs. (9) and (10). The appearance of the full operator $U(T)$ in X is a consequence of consistency; since $U(T)$ leaves the wave function of a consistent state unchanged at $z=z_2$, limiting the right side of Eq. (7) to terms diagonal in k imposes no restriction. X is thus unitary since it is the product of three unitary operators. While our argument is specific to the highly simplified HDP approach, it seems likely that the conclusion that the unitaruty condition, Eq. (8), holds will be valid in general when a unique consistent solution exists in both the situation when there is, and when there is not, an incident BB.

Case 2

Contrast case 1 with that in which we modify the operator U by replacing Eq. (10) by $U(T)\uparrow\uparrow=a\uparrow\uparrow+b\uparrow\downarrow$, with $|a|^2+|b|^2=1$; Eq. (11) remains unchanged. We can think of this U as simulating the situation where there is a probability $|b|^2$ that, upon emerging from the $t=0$ mouth of the wormhole, the BB hits a switch and transmits an electromagnetic signal closing a shutter and preventing the incident BB from entering the wormhole at $z=z_2$ and $t=T$. Equation (7) now yields the result that $|X(1)|^2=|a|^2$. From unitarity, Eq. (13) is replaced by $U(T)\downarrow\uparrow=a\uparrow\downarrow-b\uparrow\uparrow$, which yields $|X(0)|^2=1+|b|^2$. Hence, unless $b=0$, which is the self-consistent case 1, Eq. (8) does not hold and X is not unitary. Note that, for $U(T)$ obeying Eq. (11), if $b\neq 0$ there is no linear combination of $\uparrow\uparrow$ and $\uparrow\downarrow$ on which the action of $U(t)$ is given by that of an operator with the direct product structure of Eq. (4). The violation of unitarity is thus directly connected to the lack of a completely self-consistent solution within the region containing CTC's, combined with the consistency requirement that only matrix elements diagonal in k appear on the right side of Eq. (7).

The billiard ball version of the grandfather paradox corresponds to the case $|b|^2=1-|a|^2=1$, in which Eq. (10) is replaced by

$$U(T)\uparrow\uparrow=\uparrow\downarrow \tag{10$'$}$$

and we shall primarily concern ourselves with this case, in which the BB emerges from the wormhole if and only if it does not enter it. We describe this case as maximally inconsistent, since the overlap between $U(T)\Psi(0)$ and $\Psi(0)$ at $z=z_2$ vanishes; this also maximizes the violation of unitarity, since it gives $|\Psi(T+\epsilon)|^2=0$. Note that this provides an explicit, albeit highly simplified, example of a model in which no self-consistent solution exists, in contrast to the situations discussed in Refs. [10] and [11].

We can interpret $|b|^2$ as giving the probability that the apparatus producing the interaction between the older and younger versions of the BB is functioning correctly. A small nonzero value of $|a|^2$ could arise, e.g., from a nonzero probability that the transmitter which is to send the message closing the shutter and diverting the incident ball from its original trajectory fails to send the message when the transmitter is activated by a ball emerging from the wormhole at $t=0$. Only in the case of this, or similar, events, whose *a priori* probability one would expect in general to be very small, can the incident ball reach the $t=T$ mouth of the wormhole, thereby allowing the solution in which $\Psi(+\epsilon)=\uparrow\uparrow$ to be self-consistent.

One can seek to preserve the probability interpretation of the wave function by renormalizing the final state Ψ_f at $t>T$ by changing the prescription from $\Psi_f=X\Psi(-\epsilon)$ to $\Psi_f=NX\Psi(-\epsilon)$, where $N=[\Psi^\dagger(-\epsilon)X^\dagger X\Psi(-\epsilon)]^{-1/2}$; the normalizing constant N thus depends on the initial state so that the time evolution is nonlinear. As discussed in HDP, this implies that if the initial state is a superposition of the states $\uparrow_1$ and $\downarrow_1$, the existence of the era of CTC's at $t>0$ affects the time evolution of the system for $t<0$ in such a way that the probability of having $n=1$ at $t=0$ is proportional to $|a|^2$ and thus vanishes in the maximally inconsistent limit $a\rightarrow 0$. Thus in the presence of a time machine we cannot impose initial conditions at $t=0$ so as to have the BB arriving at $z=z_1$ and a properly functioning transmitter leading to Eq. $(10')$ and the resulting potential grandfather paradox.

Anderson [14] suggests the alternative renormalization procedure of replacing the evolution operator X by the unitary operator $U_X=(X^\dagger X)^{-1/2}X$ which is independent of the initial state. The two procedures have the same effect when $n=1$ at $t=0$; in that case either procedure results in multiplying the final state vector at $t>T$ by $1/a$. However, if one begins with a superposition of states with $n=1$ and $n=0$ the effects of the two renormalization procedures differ. In the Anderson procedure the time evolution for $t<0$ is unaffected by the presence of the time machine in the future. However, in our simple model the evolution operator U_X within the region of CTC's, $0<t<T$ is simply the identity operator. Thus all dependence on the physical parameter a is eliminated by this procedure, and the actual evolution is the same for an arbitrary value of a as for the case $|a|=1$, in which case the transmitter is certain to fail and no inconsistency arises. This procedure seems to lack a compelling physical motivation, and to discard much of the essential physics associated with the CTC's. Both of the renormalization procedures have the effect of rendering, essentially by fiat, *a priori* unlikely events certain. If one renormalizes the final state, this happens at $t<0$, before the era of CTC's. In the procedure of Ref. [14], it occurs during that era.

Both the renormalization prescriptions becomes undefined when $a=0$, i.e. when the grandfather paradox is complete. This is, however, presumably an unphysical limit; one expects that, in any actual case, there will always be some nonzero probability, however small, for events to occur which would allow the paradox to be evaded, so this objection is not conclusive. If there are consistent classical solutions, these will dominate the path integral and only the occupation numbers need to be treated quantum mechanically. When this is true, one can expect that the path integral in the HDP model provides a reasonable description of the actual behavior of the system. In the limit $a\twoheadrightarrow 0$ where there are no consistent classical solutions to dominate the actual path integral, it will presumably be given by an integral over many paths, and the argument that it is well represented by the path integral evaluated in HDP ceases to be persuasive; the many degrees of freedom of the BB-detector system not taken into account in HDP presumably become important. However, it seems very plausible that the qualitative conclusion of the HDP model, namely, that the path integral for the case when there is an incident BB at $t=0$ is very small, will remain valid in this limit. One expects this to be true precisely because of the absence of consistent classical solutions, which would normally make the dominant contribution to the path integral for a macroscopic system. Since on physical grounds one does expect that consistent solution(s) will exist when $n=0$ initially and there is no incident BB, i.e., for the vacuum-to-vacuum process, we expect that the conclusion that $|X(0)|^2>|X(1)|^2$, and that hence, from Eq. (8), unitarity is violated when paradoxes occur, will remain true in the full theory. The simple HDP model clearly cannot be expected to give any detailed information about what actually occurs in the $a\rightarrow 0$ limit; that is, it will give no information about the *relative* probability of various *a priori* improbable events. But one does expect that the conclusion that seemingly improbable events of some kind must occur if unitarity is to be preserved will remain valid.

One thus has, in the quantum mechanics of pure states, two rather unappealing alternatives when there are no consistent classical solutions. First, one can accept a nonunitary time evolution operator and the consequent loss of the probability interpretation of quantum mechanics. Alternatively, one can adopt one of the renormalization procedures, meaning seemingly unlikely events become certain either before or during the era of CTC's. This leads to what we have previously referred to as the banana peel mechanism; while highly counterintuitive, it at least allows the theory to be interpreted.

Case 3

As a final example, we examine briefly the case in which there are two self-consistent solutions. Take $U(T)$ to be such that $U(T)\uparrow\downarrow=\uparrow\downarrow$ and $U(T)\uparrow\uparrow=\downarrow\uparrow$. The unitarity of U plus the physical requirement that $U(T)\downarrow\downarrow=\downarrow\downarrow$ then requires that $U(T)\downarrow\uparrow=\uparrow\uparrow$. The states $\uparrow\uparrow$ and $\uparrow\downarrow$ at $t=+\epsilon$ now each give a self-consistent solution, and it is straightforward to show from Eqs. (7) and (8) that unitarity is obeyed. However, a nontrivial linear combination of the states $\uparrow\uparrow$ and $\uparrow\downarrow$ at $t=\epsilon$ does not result in a final state at $T-\epsilon$ with the product structure of Eq. (5). Instead the state of the system at $t=T-\epsilon$ is of the correlated form $c_1\uparrow\downarrow+c_2\downarrow\uparrow$ which, from the discussion in Sec. II, cannot represent a self-consistent pure state. The procedure in HDP, in which the two consistent states $\uparrow\uparrow$ and $\uparrow\downarrow$ are included with equal weight in the path integral, thus implies that the system is in a mixed state in the region containing CTC's. However, the two solutions lead to orthogonal final states at $t>T$, and hence do not interfere with one another, so their undetermined relative phase is irrelevant in Eq. (7).

Thus, in this example, Eq. (7) provides a unique and well behaved solution for $t>T$, but within the region $0<t<T$ which contains CTC's one has a mixed state with two different self-consistent solutions which are present with equal probability. These two solutions are physically quite different. This can be seen most clearly for the initial condition in which there is no BB present at z_1 for $t<0$. Then for one of the two consistent solutions the system is in the state $\downarrow\downarrow$ throughout the interval $0<t<T$; this is precisely the solution one would naively expect. However, there is also a consistent solution in which the system goes from the state $\downarrow\uparrow$ to $\uparrow\uparrow$ in this interval. Here this second solution might be dismissed on grounds of conservation of energy (i.e., of the BB number), but one does not expect this to be true in more realistic models. Intuitively, the first of these solutions seems more likely to be physically relevant. Similar considerations apply to the case where $n=1$ at $t=-\epsilon$. Given the physical differences between the consistent solutions, one might hope that, if CTC's are possible, a complete theory would settle the uniqueness question, perhaps on the basis of entropy considerations [12,16,17], and would select only one of the possible consistent solutions in the interval $0<t<T$. Then if one included only the physical solution in the path integral, case 3 would become mathematically identical to case 1; the system would be in a pure state for all t, with a unique wave function obeying unitarity and having the required continuity across the wormhole.

The form of $U(t)$ for case 3 which we have been discussing appears somewhat unphysical in that involves nonconservation of the BB number. This can be avoided, while having two consistent solutions, only if $U(T)$ is diagonal. (This is an artifact of the simplicity of our model in which U is 4×4 matrix, and there are only two possible states of the system for $t>T$.) The case in which U is diagonal has an interesting property, in that it can lead to the violation of unitarity even though consistent classical solutions exist. This occurs because either in the case $n=1$ or $n=0$ at $t<0$, there are now two consistent solutions which contribute to the same final state and, from Eq. (7), they will interfere with one another. If one chooses $U(t)$ such that the relative phase between the two possible states at $t=T-\epsilon$ is different for the cases $n=0$ and $n=1$, the unitarity condition, Eq. (8), will be violated. (The unitarity violation in the example discussed in HDP in fact occurs for this reason, rather than because of the nonexistence of classical solutions.) Once again, in our example this phenomenon appears as an artifact of the overly simplified model. In general, one would expect the states with $n=0$ and 1 at $t=T-\epsilon$ to lead to different final states at $t>T$ so they would not give rise to quantum interference. The interference occurs in the HDP model since there is only a single state containing (and a single state not containing) a BB at $t>T$. In contrast, the type of unitarity violation found in case 1 would appear to be generic in such situations, since it occurs because the magnitudes of the individual terms on the right side of Eq. (7) become small compared to 1, and does not depend on quantum interference.

IV. CONSISTENCY, THE DENSITY MATRIX, AND THE MIXED STATE MWI

As discussed in the Introduction, the approach in DD requires, in general, describing systems in the presence of CTC's in terms of their density matrix ϱ rather than a wave function. We begin this section by reviewing briefly some simple ideas concerning the density matrix in the present context. Next we show the connection that follows, in Deutsch's approach, between the existence of potential grandfather paradoxes and the necessity for adopting a density matrix description in regions containing CTC's. We then review the argument given in DD that, in quantum mechanics with the MWI, CTC's do not lead to logical contradictions.

In the present case, in which there are four possible states of the system at a given time which can be labeled by pairs of indices ij, where $i,j=0,1$ give the occupation numbers at $z=z_1,\ z_2$, the density matrix ϱ is a 4×4 matrix whose matrix elements can be labeled $\varrho_{ij;mk}$. The diagonal elements $\varrho_{ij;ij}$ give the probability that the system is in the state with occupation numbers i and j; thus $\mathrm{tr}\varrho=1$. For a pure state, described by a wave function, ϱ satisfies the condition $\varrho^2=\varrho$. The matrix elements $\hat{\varrho}_{2jk}$ of the effective density matrix $\hat{\varrho}_2$ for the system at $z=z_2$ are

$$\hat{\varrho}_{2jk}=\varrho_{ij;ik}, \tag{15}$$

where the repeated index i is summed over. The fact that the full system is in a pure state does not imply that the density matrices $\hat{\varrho}_i$ for the two subsystems satisfy the pure state condition. However, when the overall state vector has the structure of a direct product, as in Eq. (16) below, the full density matrix ϱ also has a direct product structure $\varrho=\hat{\varrho}_1\otimes\hat{\varrho}_2$; the pure state condition on ϱ then implies that the separate systems at $z=z_1$ and z_2 are also pure states.

Let us return to our example of the BB which travels backward in time and interacts with its younger self. Suppose we have a pure state at $t=+\epsilon$. Since the occupation number at $z=z_1$ is 1 from the initial conditions, the most general form of the state vector at is

$$\Psi(+\epsilon)=\Psi_1(+\epsilon)\otimes\Psi_2(+\epsilon)=\uparrow\otimes(c_1\uparrow+c_2\downarrow) \tag{16}$$

with $|c_1|^2+|c_2|^2=1$. The most general state of the system is one in which the system at z_2, emerging from the wormhole, is in the pure state $\Psi_2=(c_1\uparrow+c_2\downarrow)$ with a definite phase relation between the occupied and unoccupied components of the state vector. Continuity across the wormhole then means that the state vector at $t=T$ must have the form

$$\Psi(T)=U(T)\Psi(+\epsilon)=\Psi_1(T)\otimes\Psi_2(+\epsilon)$$
$$=(d\uparrow+d_2\downarrow)\otimes\Psi_2(+\epsilon). \qquad (16')$$

In particular, there must be no correlation between the occupation numbers at z_1 and z_2. Thus for a given $U(T)$, the consistency constraint can be satisfied only if it is possible to choose the constants c_1 and c_2 so that $\Psi(T)$ has the form given in Eq. (16′). In case 1 of Sec. III, in which $U(T)$ is given by Eqs. (10) and (11), Eq. (16′) is obeyed, with $d_2=0$, if we take the self-consistent solution with $c_2=0$. Of course we already know this is a self-consistent pure state solution, since we found that the time evolution operator X given by Eq. (7), which has the consistency constraint built in, is unitary.

Now consider case 2 in which $U(T)$ is given by Eqs. (10′) and (11). There is then no consistent solution and the operator X exhibits maximal violation of unitary. Suppose we choose $c_1=c_2=1/\sqrt{2}$, with both taken to be real for simplicity; then, at $t=+\epsilon$, we have the product state,

$$\Psi(+\epsilon)=\uparrow_1(\uparrow_2+\downarrow_2)/\sqrt{2}, \qquad (17)$$

in which the subsystems at z_1 and z_2, and thus the full system, are all in pure states.

From Eqs. (17), (10′), and (11) one obtains

$$\Psi(T)=(\uparrow_1\downarrow_2+\downarrow_1\uparrow_2)/\sqrt{2}. \qquad (18)$$

This is not of the form of Eq. (16′) because of the correlation between the states at z_1 and z_2. Equation (18) describes a situation in which the subsystem at $z=z_2$, considered in isolation, is in a mixed state, with no definite phase relation between the occupied and unoccupied states at $z=z_2$ since the coefficients of $\uparrow_2$ and $\downarrow_2$ depend on the coordinates of the system at $z=z_1$. (Recall that the symbol $\uparrow_1$ is really shorthand in our case for the full wave function of a BB, including the dependence on the internal coordinates.) We can also see that the z_2 subsystem in Eq. (18) is not in a pure state by constructing the density matrix ϱ for the complete system and using Eq. (15) to obtain

$$\hat{\varrho}_2(T)=I_2/2, \qquad (19)$$

where $\hat{\varrho}_2(T)$ is the density matrix for the subsystem at $z=z_2$ at $t=T$ and I_2 is the 2×2 identity matrix. Equation (19) describes a mixed state with equal probabilities of $\frac{1}{2}$ for finding a BB entering, or not entering, the wormhole at $t=T$. Since the subsystem at $z=z_2$ is in a mixed state at $t=T$, this subsystem, and therefore, from Eq. (17), the system as a whole, must be in a mixed state at $t=0$ if there is to be continuity across the wormhole. Thus, in the presence of potential inconsistent causal loops, there are only two possibilities: the first is that the continuity condition across the wormhole may not be exactly satisfied, and when we project out the (possibly nonexistent) part of the wave function satisfying the consistency condition the operator X becomes nonunitary, as in case 2 in Sec. III; the second possibility is that pure states are transformed into mixed states in the region containing CTC's.

It is demonstrated in DD that for any $U(T)$ one can always choose values of c_1 and c_2 in Eq. (16) that yield a ϱ which, when evolved according to Eq. (1), satisfies a modified consistency requirement of the form

$$\hat{\varrho}_2(T)=\hat{\varrho}_2(0). \qquad (20)$$

Consistency in this sense is possible because one is working with ϱ instead of a wave function. The consistency condition on ϱ is satisfied if the correlation is such that the probability for a BB to enter the wormhole at $t=T$ is the same as for one to emerge at $t=0$. In the MSMWI picture, a measurement of whether or not a BB emerges from the wormhole at $t=0$ causes a branching into two MWI "worlds," both of which remain components of the state of the system at $t>0$, and in one of which a BB enters the wormhole mouth at $t=T$. The consistency condition on ϱ, but not on ψ, will then be satisfied if the probabilities of a BB entering the wormhole at $t=T$, and of its emerging at $t=0$, are equal even if a BB traveling backwards in time does not emerge in the same world which it left. For example, while the wave functions in Eqs. (17) and (18) do not satisfy the continuity condition across the wormhole, they each yield a density matrix $\hat{\varrho}_2$ given by Eq. (19) so that Eq. (20) is satisfied.

V. MSMWI AND TIME TRAVEL PARADOXES INVOLVING MICROSCOPIC OBJECTS

Before considering the case of a macroscopic object such as a billiard ball or space ship, we examine how the MSMWI works in a situation where, instead of a macroscopic object, we have a single electron. This is closely analogous to the situation considered in DD. In the spirit of the MWI we include as part of our system a measuring device, e.g., an array of Cerenkov counters surrounding the wormhole mouth, which can presumably be designed to detect with arbitrarily high certainty whether or not an electron emerges from the wormhole. We again consider the grandfather paradox situation with U given by Eqs. (10′) and (11). If an electron is detected, we can imagine the measuring device causes the incident electron at z_1 to be deflected, e.g., by temporarily turning on an electric field near z_1, so that it never reaches the wormhole.

Let the state of the device be designated by q, which will become one of our dynamical variables along with the occupation numbers n and n' at z_1 and z_2. The matrix elements of the full density matrix ϱ appearing in Eq. (15) will now be labeled by two sets of three indices, plus t. Since we are dealing with mixed states with undefined relative phases, the density matrices are diagonal and we can specify the nontrivial density matrix elements at time t uniquely by a single set of three indices, writing the matrix elements as $\varrho(q,n,n',t)$. (The matrix elements of the effective density matrix $\hat{\varrho}$ for the system at z_2 will still be labeled only by the values n', however; q and n are both degrees of freedom associated with the remainder of the system and are both summed over in finding $\hat{\varrho}$.) The Hamiltonian H and thus U in Eq. (1) will now include H_m, the interaction between the electron and the measuring device. Initially, at $t<0$, we take

$q=q_A$, while $q=q_B$ after the detection of an electron by the device. Thus, if an electron emerges from the wormhole at $t=0$, then at $t=\delta>\epsilon$, q becomes equal to q_B; δ is a property of the detection system, and will be finite (though we assume $\delta\ll T$), both because the counters will have a finite response time and because they will be located at a finite distance from the position of the electron at $t=0$, the earliest time at which it could be observed as it emerges from the wormhole.

According to the picture in DD, for $t>0$ the system will be in a mixture of two states, each with probability $\frac{1}{2}$. We will label these states A and B according to the values, q_A and q_B, respectively, of q at $t=T-\epsilon$. Since $q=q_A$ in state A, in that state no electron was detected at $t=\delta$, and it then follows from Eqs. (10′) and (11) that, in state A, $n'=1$ at $t=T-\epsilon$ and the incident electron enters the wormhole mouth at $t=T$. Similarly, in state B, with $q=q_B$ at $T-\epsilon$, no electron enters the wormhole at $t=T$.

Thus, at $t=\epsilon<\delta$, one will have a system with $n=1$ and $q=q_A$ in a mixed state with equal probabilities for finding $n'=0$ and $n'=1$. At $t=\delta$, q becomes equal q_B in the state with $n'=1$; that is, the electron that was in state A at $t=T-\epsilon$ emerges from the wormhole at $t=0$ and is detected in state B at $t=\delta$.

For $t>\delta$ an observer, as in the conventional MWI, has an equal chance of being in the worlds with $q=q_A$ or q_B. In state B, with $q=q_B$, the observer sees the electron initially at z_1 deflected so that it never reaches the wormhole, while the electron leaving the wormhole arrives at z_1 at $t=T$, in accordance with Eq. (10′), so that $n'=0$ at $t=T$, and the observer will conclude that $n'(0)\neq n'(T)$. A similar analysis holds for observers in the world in which $q=q_A$ for $t>\delta$. The time evolution during the period $0<t<T$ will appear perfectly sensible to observers in both worlds. They will be surprised to see that $n'(T)\neq n'(0)$, but this does not constitute an actual logical contradiction, since $n'(0)$ and $n'(T)$ are physically different observables for outside observers so that the theory does not give contradictory predictions for the value of the same observable as seen by the same observer.

A hypothetical observer riding on the electron will also see nothing unusual. The electron apparently evolves normally, in terms of the local time variable τ and Hamiltonian H' discussed in Sec. II, in passing through the wormhole; an observer moving with the electron would see $n'(\tau=T)=n'(\tau=T+t_0)=1$, where t_0 is the transit time through the wormhole, and will see outside clocks reading $t=0$ as he emerges. However, the world in which he now finds himself will be different than he saw when $\tau=t=0$ and the electron was at $z=z_1$, since now he will see $q=q_B$, and find himself in a world with two electrons.

Thus the communication between different MWI worlds postulated in Deutsch's approach actually occurs, if the continuity condition is given by Eq. (20), as a result of the interaction H_m with the measuring apparatus. The electron enters the wormhole in the $q=q_A$ world and appears at $t=0$, also in a state with $q=q_A$. However, at $t=\delta$, when the measurement process results in branching into two separate worlds with different values of q, q becomes equal to q_B in the state containing the electron at z_2. The electron, which is in state A at $t=T$, is able to appear in state B at a later value of its own time τ by traveling back in time through the wormhole to a time $t<\delta$ before the measurement, and the resulting branching into states A and B, has occurred.

More formally, we can understand this as follows. Let us consider the time translation operator $U(t_2,t_1)\equiv U_{21}$ for the case $t_2=\delta+\epsilon'$, $t_1=T$, with ϵ' arbitrarily small and $\delta\ll T$. We can write $U_{21}=U(\delta+\epsilon',0)U(0,T)=U(\delta)U(-T)$; $U(-T)$ is the analog of the corresponding operator in Eq. (4), but does not have a direct product structure because of the correlations between observables at $t=T$. At $t=T$ the system will be, with equal probability, in states with $q=q_A$, $n'=1$ and $q=q_B$, $n'=0$, so the nonzero diagonal density matrix elements $\varrho(q,n,n',t)$ for $t=T$ will be

$$\varrho(q_A,n(T)_A,1,T)=\frac{1}{2}=\hat{\varrho}_2(1,T) \tag{21a}$$

and

$$\varrho(q_B,n(T)_B,0,T)=\frac{1}{2}=\hat{\varrho}_2(0,T), \tag{21b}$$

where $n(T)_A$, e.g., is the value of the occupation number n at $t=T$ and $q=q_A$. The final equality in Eqs. (21) is a consequence of the fact that only the matrix elements of ϱ appearing in Eqs. (21) are nonzero.

The operator $U(-T)$ transforms $\varrho(T)$ into $\varrho(0)$. By Eq. (20), this must leave $\hat{\varrho}_2$ invariant, while at $t=0$ the only nonzero elements of ϱ are for $n=1$ and $q=q_A$. Thus at $t=\epsilon<\delta$ the nonzero elements of ϱ are

$$\varrho(q_A,1,n',\epsilon)=\hat{\varrho}_2(n',\epsilon)=\frac{1}{2},\quad n'=0,1. \tag{22}$$

Comparing Eqs. (21) and (22), we see that Eq. (20) is indeed satisfied.

Since $\delta\ll T$, the factor $U(\delta)$ in U_{21} differs from unity only because of the interaction H_m with the measurement device. Thus, acting on states $|q,n,n'\rangle$, $U(\delta)|q_A,1,1\rangle=|q_B,1,1\rangle$, while $U(\delta)$ leaves $|q_A,1,0\rangle$ unaffected. Thus

$$U_{21}|q_A,n(T)_A,1\rangle=|q_B,1,1\rangle \tag{23a}$$

and

$$U_{21}|q_B,n(T)_B,0\rangle=|q_A,1,0\rangle. \tag{23b}$$

From Eq. (1), there will be a similar transformation of the diagonal density matrix elements $\varrho(q,n,n',t)$ so that, for $n'=1$, we have from Eqs. (21a), (22), and (23a)

$$\varrho(q_A,n(T)_A,1,T)=\varrho(q_A,1,1,0)=\varrho(q_B,1,1,\delta)=\frac{1}{2} \tag{24a}$$

and, similarly, for $n'=0$

$$\varrho(q_B,n(T)_B,0,T)=\varrho(q_A,1,0,0)=\varrho(q_A,1,0,\delta)=\frac{1}{2} \quad (24b)$$

and one sees that the electron, which entered the wormhole at $t=T$ in state A, is found at $t=\delta$ in state B. The continuity condition (20) on the subdensity matrix 2 is satisfied, since the probabilities for finding $n'=0$ and of finding $n'=1$ are both equal to one-half at each end of the wormhole.

Thus the MSMWI, with an object described by a density matrix satisfying Eq. (20), leads, as asserted in DD, to a quantum theory of a microscopic object passing through a time machine which avoids the grandfather paradox. This occurs, as in the parallel universes of science fiction, because the object emerges from the time machine and ''murders'' its younger self in a different world, i.e., an orthogonal quantum state, when it travels back in time.

VI. MSMWI FOR MACROSCOPIC OBJECTS

As we now show, however, problems arise if one applies the MSMWI in the case of macroscopic objects, such as billiard balls, passing through the wormhole. We first specify the meaning we will attach to macroscopic in this context. Let the object in question have linear dimension d in its direction of motion and be moving with speed v, so that it requires a time interval $\Delta t=d/v$ to emerge from the wormhole. That is, for $0<t<\Delta t$, the front portion of the BB exists on a timelike surface $t=t_1$ while the back portion exists on the timelike surface $t=t_1+T$. We will call the object macroscopic if $\Delta t>\delta$, where δ is the time at which the detector recognizes that the object has emerged, and in consequence sends a signal preventing the object from entering the wormhole at $t=T$; as in Sec. V, δ depends on the resolution time of the detector and its distance from the position of the leading edge of the object as it emerges at $t=0$. Since a fraction $f=\delta/\Delta t$ of the object must emerge from the wormhole before the detector is triggered, for a macroscopic object $f<1$ and a fraction $1-f>0$ of the object will not yet have emerged from the wormhole at $t=\delta$.

The above definition of ''macroscopic'' has the problem of depending on δ, and thus on the particular detection device being used. One can introduce a more fundamental definition to avoid this by taking $\Delta t>\delta_{\min}$, where $\delta_{\min}\approx 1/m$, with m the mass of the object and $c=1$, is the smallest possible resolution time for any detector. Then on the fundamental level we would take an object to be macroscopic if $d>1/m$ or $md>1$.

The HDP model must be extended somewhat to accommodate macroscopic objects, but the generalizations do not change the physics in an essential way. Clearly the wormhole mouth must have a finite radius. Also, the wormhole must persist for a time $T_0>\Delta t$ in order for the object to traverse it. One must then generalize the wormhole to identify times t and $t+T$ for $0<t<T_0$. where $\Delta t<T_0<T$; the upper limit on T_0 avoids the necessity of introducing a spatial separation between ends of the wormhole which overlap in time, as discussed earlier. Equation (20) must be correspondingly generalized to

$$\hat{\varrho}_2(t+T)=\hat{\varrho}_2(t). \quad (20')$$

We will place one additional restriction on the wormhole persistence time T_o. Let T_s be the time at which the incident BB reaches the shutter whose closure prevents it from entering the wormhole. We will strengthen the restriction on T_o by requiring $T_o<T_s$. We thus eliminate the possible consistent solution mentioned in the Introduction, in which the BB squeezes past the shutter just as it closes, being slowed down in the process so that it reaches the wormhole at $t=T+T_s$, and reemerges at $t=T_s$ to trigger the shutter just as its younger self reaches it. According to the EKT consistency principle, this would become the physically observed process, thus evading the paradox. However, this consistent solution does not exist if the early-time mouth of the wormhole closes before the BB reaches the shutter, thus eliminating the possibility of a BB emerging from the wormhole at $t=T_s$ and triggering the shutter just as the incident BB reaches it.

Let us consider first, for simplicity, the case $f=\frac{1}{2}.+\epsilon'$; i.e., we assume that, on the average, just over half of the BB emerges from the wormhole before the detector is triggered. By analogy with our discussion in the previous section, at $t=T-\epsilon$, in state A, with $q=q_A$, the incident BB will be about to enter the wormhole, since in that state the detector was not triggered at $t=\delta$, while in state B there will be no BB entering the wormhole. Then at $t<\delta$ one will have a mixture of two states, both with $q=q_A$, in one of which the front portion of a BB will have emerged from the wormhole; this latter state will be state B, with $q=q_B$ for $t>\delta$, since in this state the detection device will be triggered. The density matrix ar $t=\epsilon<\delta$ will be given by Eq. (22), where $n'=1$ denotes the presence of the front edge of the BB at z_2.

For $t>\delta$, q is a constant of the motion since H_m, the interaction Hamiltonian with the detection device, has no matrix elements between the states with $q=q_{A^-}$ and $q=q_B$, after the irreversible measurement has been completed. This is the exact analog of the independence of different worlds from one another in the conventional MWI without CTC's.

This decoupling of states A and B has far reaching consequences for the predicted behavior of a macroscopic object passing through a wormhole. As with the electron, the front half of the BB, which is in state A at $t=T$, appears in state B at $t=\delta$ and $\tau=T+\delta$. This can occur because the front half travels back in time to the range of times $0<t<\delta$ at which time $q=q_A$ in both states A and B. However, the rear half of the BB reaches the wormhole mouth at $t=t_1\equiv T+\Delta t/2=T+\delta$, and hence it begins to emerge from the wormhole at $t=\delta$, after the measurement has occurred. For $t>\delta$ the evolution operator $U'_{21}(t,t_1)=U(t,\delta,)U(\delta,T+\delta)$, plays the analogous role for the back half of the BB that U_{21} played for the electron in Sec. V; U'_{21} does not connect states A and B, since, for $t>\delta$, $q_B\neq q_A$ and these states are decoherent. For $t>\delta$ Eq. (23a) must be replaced by

$$U'_{21}(t)|q_A,n(T)_A,1\rangle=|q_A,1,1\rangle \quad (25)$$

and hence from Eq. (1), the analog of Eq. (24a) for the nonzero matrix elements of the density matrix for $n'=1$ at times t and $T+t$ at opposite ends of the wormhole, when $t>\delta$, is

$$\varrho(q_A,n(T)_A,1,T+t)=\rho(q_A,1,1,t)=\frac{1}{2},\quad t>\delta \quad (26)$$

with analogous changes occurring in Eqs. (23b) and (24b). Hence there is vanishing probability of finding the back half of the BB at $z=z_2$ in the world with $q=q_B$, and the back half of the BB, *unlike the front half*, will necessarily emerge from the wormhole in state A with $q=q_A$.

The MSMWI thus predicts that the two halves of the BB will emerge from the wormhole in different MSMWI worlds! An external observer will, with probability one half, see nothing emerge from the wormhole during the interval $0<t<\delta$, so that the detection device is not triggered, and will end up in state A with $q=q_A$. This observer will then see the rear half of the BB emerge between $t=\delta$ and $t=2\delta$ and go off to reach $z=z_1$ in accordance with Eq. (10′). Since the detector was not triggered, the "younger" BB initially at z_1 at $t=0$ will not be deflected and will enter the wormhole between $t=T$ and $t=T+2\delta$. The front half of the BB, which entered the wormhole at $T<t<T+\delta$, will seem to this observer to have disappeared, since it emerged in the other world; this is similar to the microscopic case. The rear half of the BB will match the rear half which emerged earlier at $t=\delta$, so that observations at the two wormhole mouths at t and $t+T$ will indicate continuity across the wormhole for $t>\delta$, once the discontinuous measurement process, has been completed.

There will also be probability one-half of observing the front half of a BB emerging from the wormhole between $t=0$ and $t=\delta$, triggering the detection device, putting $q=q_B$, and causing the deflection of the young BB, which therefore never reaches the wormhole. In this q_B world, nothing enters the wormhole mouth at $t>T$ and the front half of the BB will seem to appear for no apparent reason; this is again similar to the electron case. However, for $t>\delta$, in state B nothing enters the wormhole at $t+T$ or emerges at t, so that, as in state A, external observers will see continuity between the two mouths of the wormhole for $t>\delta$, after the two worlds have decoupled.

This surprising result is possible because the continuity condition, Eq. (20′), which is the basic assumption in DD, only constrains the elements of the effective density matrix $\hat{\varrho}_2$. The density matrix elements of the macroscopic BB must now be labeled by separate occupation numbers n_f' and n_b' for the front and back segment of the BB. The continuity of $\hat{\varrho}_2$ ensures that the total probability of finding $n_f'=1$, i.e., of detecting the front segment at z_2, is one-half at each mouth of the wormhole. However, the matrix elements of $\hat{\varrho}_2$ for a given value of n_f' involve the sum over q of the matrix elements of the full density matrix ϱ for that value of n_f', and thus the relation between the values of n_f' and the value of q need not be preserved in going through the wormhole. The same holds true for n_b'. In fact, as we have seen, where, classically, there is a grandfather paradox, the relation between q and n' develops a discontinuity. At $t=T$, just before the BB enters the wormhole, the set of observables (q,n_f',n_b') have, with equal probability, the sets of values $(q_A,1,1)$ and $(q_B,0,0)$. However, for the emerging BB at $t>\delta$, the relation between q and n_f' and between q and n_b' differs from that for $t<\delta$ because of the discontinuous change in the value of q resulting from the measurement, and the possible sets of values become $(q_A,0,1)$ or $(q_B,1,0)$. There are equal probabilities at each end of the wormhole of finding each possible value, 0 or 1, for both n_f' and n_b', as required by Eq. (20′). However, the correlation between the values of n_f' and n_b' for a given value of q is different at the two ends of the wormhole. At $t=T$ an observer in an MWI world with a definite value of q sees nonzero values of the density matrix elements for the same values of n_f' and n_b'; i.e., he sees either the whole object or nothing entering the wormhole. At the other end such an observer sees nonzero probabilities for different values of n_f' and n_b' and thus observes only the front or back half of the object. For an elementary particle this problem does not arise since the concept of different parts of such an object is meaningless; for such an object, the discontinuity due to the measurement is simply that associated with the emergence of the object, which occurs suddenly rather than over time as in the macroscopic case.

We can generalize the above discussion to other values of the fraction f. Suppose, e.g., that $f=\frac{1}{3}+\epsilon'$, meaning the detection device can detect the emergence of one-third of a BB, and $\delta=\Delta t/3$. Let us also assume that the detector, after being triggered, reads q_{B1} or q_{B2}, respectively, depending on whether it was triggered at $t=\delta$ by observing the first third of an emerging BB, or at $t=2\delta$ by the middle third. In both of these worlds, since the detector was triggered, the incident BB will not enter the wormhole at $t=T$. Since q is a constant in these worlds for $t>\delta$ or $t>2\delta$, respectively, they will not be coupled to the third world for $t>2\delta$, and in neither of them will the last third of the BB be observed. In this third world, the detector will not be triggered so that q remains equal to q_A. It will couple only to itself for $t>2\delta$, and hence in the q_A world one will observe the rear third of the BB emerging from the wormhole between $t=2\delta$ and $t=3\delta$.

There will thus be three MSMWI worlds. The solution satisfying the consistency condition (20′) on the density matrix is that each of these occurs with probability $\frac{1}{3}$. There is then a one-third probability of having $q=q_A$ and a BB entering the wormhole at $t+T$. This leads to probability $\frac{1}{3}$ for each segment to emerge in its respective world, so that Eq. (20′) is, indeed, satisfied.

More generally, let $f=1/N$, where N is arbitrary, thus including the case of a detector of arbitrarily high sensitivity. One would then have N MSMWI worlds, in each of which a fraction $1/N$ of the BB would be seen to emerge during a time interval $(i-1)\Delta t/N<t<i\Delta t/N$, $i\leqslant N$. As N becomes arbitrarily large and the detector becomes very sensitive, the probability of observing the BB actually reaching the wormhole at $t=T$ without being deflected thus vanishes as $1/N$; in this limit one will observe, essentially with certainty, a microscopic fragment of the BB, which might be indistinguishable from random background, emerging from the wormhole

at some time between $t=0$ and $t=\Delta t$, triggering the detector, and preventing the incident ball from entering the wormhole; thus in the limit of large N the probability of seeing the incident BB enter the wormhole becomes vanishingly small, but the number of fragments into which it is split becomes very large, so that the probability of some fragment emerging in any one of the essentially infinite number of worlds is unity.

The fact that fractions of the BB emerge from the wormhole in states with different values of q means that, if the MSMWI is correct, the Hamiltonian H' controlling the evolution of the BB in its proper time τ through the wormhole cannot be anything like that of a free BB; it must include violent interactions with the matter and/or gravitational fields of the wormhole which lead to the disintegration of the BB. The effect of these interactions is presumably independent of the sensitivity of the device used to detect the emerging BB. Thus it would appear that the MSMWI implies that a macroscopic object traversing a wormhole (or other time machine) must necessarily be broken up into microscopic constituents, presumably elementary particles, which will appear pointlike to the most sensitive detectors possible. This would, e.g., be true in a theory is which stable wormhole can exist only if their radii are of the order of the Planck length. Such a wormhole would not be "traversable" in the sense of Ref. [1]. Hence the MSMWI does not provide a quantum theory which is free of paradoxes and which describes wormholes, or similar objects involving CTC's, which are traversable by macroscopic objects.

VII. CONCLUSIONS

We have considered two general approaches to resolving the problem of apparent paradoxes in theories with CTC's. The first, illustrated by the simplified model presented in HDP, attempts to preserve the quantum-mechanical notion of pure states and imposes an appropriate continuity condition, Eq. (7), across a wormhole or other time machine on the wave function. When the time evolution operator is such that there is a single self-consistent solution, Eq. (7) is equivalent to the EKT consistency principle, and leads to a theory which is both consistent and unitary. If there are multiple consistent solutions, all of which are physical, problems arise because the solution is not uniquely specified by requiring consistency. Hopefully these would be absent in a more complete theory, which solves the uniqueness problem by providing a procedure for selecting only one of the consistent solutions as physical.

However, it is possible to choose the Hamiltonian, in the HDP model, so that no self-consistent solution exists, thus simulating the existence of initial conditions at $t=0$ leading to the formation of inconsistent causal loops as in the grandfather paradox. It seems likely that this is also true in more realistic models. Then the attempt to enforce consistency by means of the continuity constraint in Eq. (7) leads to a violation of unitarity in the operator X connecting the state vectors before and after the region of CTC's. The probability interpretation of quantum mechanics can be preserved only by renormalizing the final state or the operator X. The renormalization has the effect of forcing the probability of some events, e.g., the failure of a piece of apparatus, which would normally be very small, to become equal to 1; depending on the renormalization procedure, the events in question may occur prior to the construction of the time machine, i.e., prior to the formation of a Cauchy horizon. Thus postulating the renormalization procedure required to conserve probability amounts to postulating the banana peel mechanism, i.e., the certain occurrence of some member of a set of *a priori* improbable events which conspire to prevent paradoxes from occurring. The renormalization process fails if the norm of the final state is strictly zero, i.e., if X is singular, meaning there is no sequence of events, however improbable *a priori*, by which the paradox can be evaded.

The alternative approach in DD involves attempting to implement the idea of parallel universes from science fiction so that apparently contradictory events occur in different worlds; if successful, this would preserve the freedom to impose initial conditions arbitrarily. This approach involves two fundamental assumptions: (i) In the presence of CTC's, the MWI as given in Ref. [13] is correct, and not simply an interpretation of quantum mechanics which one is free to adopt or not according to taste. (ii) In the presence of CTC's, individual systems may not be in pure states but in mixed states characterized by a density matrix but not a wave function. This differs from Ref. [13], in which physical systems, with the measuring apparatus included, are taken to be in pure states; we therefore refer to this as the MSMWI, where MS stands for "mixed state." Assumption (ii) has two corollaries. First, the concept of the density matrix is extended to apply to single systems, in contrast to its usual application to ensembles of systems that have been identically prepared. Secondly, the correct formulation of the continuity condition in the presence of a wormhole is not, in general, as a condition on a wave function in the form of Eq. (7), but rather as the condition (20′) on the density matrix.

For the potentially paradoxical case in which the time evolution operator appears to be such that an object emerges from the wormhole at $t=0$ if and only if it does not enter the wormhole at $t=T$, the mechanism suggested in DD for resolving the paradox can be successful if the object is microscopic. The different worlds of the MSMWI correspond to states in which a macroscopic detector, which records whether the object emerged from the wormhole, has different readings, and are thus effectively decoupled. A microscopic object is able to appear intact at $t=0$ in a different world from that in which it entered the wormhole because it emerges from the wormhole at $t=0$ before the measurement leading to the branching of the worlds, has occurred.

If the object is macroscopic, however, it emerges from the wormhole over a finite period of time. If this is greater than the resolving time of the detector, the measurement, and the branching into two or more decoherent states, will occur before the object has emerged completely from the wormhole. The MSMWI worlds then become decoupled, as in the conventional MWI, and the subsequent segment or segments of the macroscopic object cannot emerge in the same world, i.e., in the quantum state with the same reading of the macroscopic detector, as the leading segment. The object is thus split into a number of pieces in its passage through the wormhole. A given observer, who sees a particular reading of the macroscopic detection device, will see only one of these pieces.

The mechanism for eluding the grandfather paradox, proposed in DD, thus appears to imply that macroscopic objects, when traversing a wormhole, undergo interactions which are sufficiently violent as to break up the object. The number of pieces into which the object is observed to be broken depends on the sensitivity of the detector, but becomes very large if the detector is sensitive enough to detect very small fragments. One expects the interaction between the object and the wormhole should be independent of the sensitivity of the device used to detect the emerging object. Hence one concludes that assumptions (i) and (ii), together with their corollaries, can be valid only if macroscopic objects passing through a time machine interact with it so strongly as to be disintegrated into fragments which appear pointlike to the most sensitive possible detectors. One obvious class of theories in which this would be true is the class in which stable wormhole can exist only if their dimensions are of the order of the Planck length.

The approach in DD therefore does not provide an explanation of how paradoxical results can be evaded in a theory with traversable wormholes, or other kinds of traversable CTC's where ''traversable'' is used in the sense of Ref. [1] as meaning traversable intact by macroscopic objects such as billiard balls, space ships, or human beings. Hence the only satisfactory candidate for a theory of such objects appears to be one in which the necessity of renormalizing the future scattering matrix constrains physics in the present in such a way that conditions whose *a priori* probability seems very small, e.g., the presence of a banana peel, are in fact rendered certain.

ACKNOWLEDGMENTS

I am grateful to Larry Ford, Tom Roman, and Serge Winitski for useful discussions, and for helpful comments and criticisms of the manuscript.

[1] M. Morros, J. Thorne, and Y. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).
[2] M. Alcubierre, Class. Quantum Grav. **11**, 173 (1994); S. Krasnikov, Phys. Rev. D **57**, 4760 (1998).
[3] A. Everett, Phys. Rev. D **53**, 7365 (1996).
[4] A. Everett and T. Roman, Phys. Rev. D **56**, 2100 (1997).
[5] A. Ori and Y. Soen, Phys. Rev. D **49**, 3990 (1994).
[6] S. W. Hawking, Phys. Rev. D **46**, 603 (1992).
[7] L. Ford and T. Roman, Phys. Rev. D **51**, 4277 (1995); **53**, 5496 (1996).
[8] M. Pfenning and L. Ford, Class. Quantum Grav. **14**, 1743 (1997); see also Ref. [4].
[9] M. Cassidy, Class. Quantum Grav. **14**, 523 (1997); J. R. Gott and L.-X. Li, Phys. Rev. D **58**, 023501 (1998); M. Visser, S. Kar, and N. Dadhich, Phys. Rev. Lett. **90**, 201102 (2003).
[10] F. Echeverria, G. Klinkhammer, and K. Thorne, Phys. Rev. D **44**, 1077 (1991).
[11] I. Novikov, Phys. Rev. D **45**, 1989 (1992).
[12] H. D. Politzer, Phys. Rev. D **49**, 3981 (1994).
[13] J. Hartle, Phys. Rev. D **49**, 6543 (1994).
[14] A. Anderson, Phys. Rev. D **51**, 5707 (1995).
[15] H. Everett, III, Rev. Mod. Phys. **29**, 454 (1957).
[16] D. Deutsch, Phys. Rev. D **44**, 3197 (1991); Sci. Am. (Int. Ed.) **270** (March), 68 (1994).
[17] G. Romero and D. Tores, Mod. Phys. Lett. A **16**, 1213 (2001).