The end of unified dark matter?

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Despite the interest in dark matter and dark energy, it has never been shown that they are in fact two separate substances. We provide the first strong evidence that they are separate by ruling out a broad class of so-called unified dark matter models that have attracted much recent interest. We find that they produce oscillations or exponential blowup of the dark matter power spectrum inconsistent with observation. For the particular case of generalized Chaplygin gas models, 99.999% of the previously allowed parameter space is excluded, leaving essentially only the standard ΛCDM limit allowed.

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I. INTRODUCTION

Despite the broad interest in dark matter and dark energy, their physical properties are still poorly understood. Indeed, it has never even been shown that the two are in fact two separate substances. The goal of this paper is to show that they are.

There is strong evidence from a multitude of observations that there is about six times more cold dark matter (CDM) than baryons in the cosmic matter budget, making up of the order of 25% of the critical density [1–4]. In addition to this clustering dark component, observations of supernovae, the cosmic microwave background fluctuations and galaxy clustering provide mounting evidence of a uniformly distributed dark energy with a negative pressure which has come to dominate the Universe recently (at redshifts $z \lesssim 1$) and caused its expansion to accelerate. It currently constitutes about two-thirds of the critical density [1,2,5].

Although the dark energy can be explained by introducing the cosmological constant (Λ) into general relativity [lending the standard model the name the cold dark matter model with a cosmological constant (ΛCDM)], this "solution" has two severe problems, frequently triggering anthropic explanations and general unhappiness. The first problem is explaining its magnitude, since theoretical predictions for Λ lie many orders of magnitude above the observed value. The second problem is the so-called cosmic coincidence problem: explaining why the three components of the Universe (matter, radiation and Λ) presently are of similar magnitudes although they all scale differently with the Universe's expansion.

As a response to these problems, much interest has been devoted to models with dynamical vacuum energy, the so-called quintessence [6]. These models typically involve scalar fields with a particular class of potentials, allowing the vacuum energy to become dominant only recently. Although quintessence is the most studied candidate for the dark energy, it generally does not avoid fine-tuning in explaining the cosmic coincidence problem. Recently several alternative models have also been proposed such as [7–9].

An alternative to quintessence which has attracted great interest lately is the so-called generalized Chaplygin gas (GCG) [10–25] (see also the related earlier work of [26]). Rather than fine-tuning some potential, the model explains the acceleration of the Universe via an exotic equation of state causing it to act like dark matter at high density and like dark energy at low density. The model is interesting for phenomenological reasons but can be motivated by a braneworld interpretation [11,12]. An attractive feature of the model is that it can explain both dark energy and dark matter in terms of a single component, and has therefore been referred to as unified dark matter (UDM) or "quartessence" [27]. (For a tachyonic scalar field UDM model, see also [28].)

This approach has been thoroughly investigated for its impact on the 0th order cosmology, i.e., the cosmic expansion history (quantified by the Hubble parameter $H[z]$) and corresponding spacetime-geometric observables. An interesting range of models was found to be consistent with SN Ia data [27] and CMB peak locations [29].

Some work has also studied constraints from 1st order cosmology (the growth of linear perturbations), finding an interesting range of models to be consistent with cosmic microwave background (CMB) measurements [30]. There is, however, a fatal flaw in UDM models that manifests itself only at recent times and on smaller (galactic) scales and has therefore not been revealed by these studies.[1] As we will see, this flaw rules out all GCG models except those that are for all practical purposes identical to the usual ΛCDM model.

The rest of this paper is organized as follows. In the next section, we review the fundamentals of the GCG model. We then consider in Sec. III the evolution of density inhomogeneities in the model and use the predicted matter power spectrum to constrain it with observational data. We close a baryon-related loophole in Sec. IV and conclude by describing how the basic flaw that rules out the GCG model is

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[1]The work by [30] did in fact evolve *one* k-mode up to the present time for both GCG and baryons, but focused the analysis on the effects on the CMB.

indeed a generic feature of a broad class of unified dark matter models.

II. THE CHAPLYGIN GAS

A standard assumption in cosmology is that the pressure of a single substance is, at least in linear perturbation theory, uniquely determined by its density. A generalized Chaplygin gas [10,12,13] is simply a substance where this relation $p(\rho)$ is a power law

$$p=-A\rho^{-\alpha} \tag{1}$$

with A a positive constant. The original Chaplygin gas had $\alpha=1$. The standard ΛCDM model has two separate dark components, both with $\alpha=-1$, giving a constant equation of state $w\equiv p/\rho$ that equals 0 for dark matter and -1 for dark energy.

By inserting Eq. (1) into the energy conservation law, one finds that the GCG density evolves as [27]

$$\rho(t)=\left[A+\frac{B}{a^{3(1+\alpha)}}\right]^{1/(1+\alpha)}, \tag{2}$$

where $a(t)$ is the cosmic scale factor normalized to unity today, i.e., $a=(1+z)^{-1}$ where z denotes redshift. Here B is an integration constant. The striking feature here is that although the GCG has $\rho\propto a^{-3}$ when sufficiently compressed, its density will never drop below the value $A^{1/1+\alpha}$ regardless of how much it is expanded. Defining

$$\Omega_m^*\equiv\frac{B}{A+B},\quad \rho_*\equiv(A+B)^{1/(1+\alpha)}, \tag{3}$$

Eq. (2) takes the form

$$\rho(a)=\rho_*[(1-\Omega_m^*)+\Omega_m^* a^{-3(1+\alpha)}]^{1/(1+\alpha)}. \tag{4}$$

For comparison, a standard flat model with current CDM density parameter Ω_m as well as dark energy density $(1-\Omega_m)$ whose equation of state w_* is constant gives

$$\rho(a)=\rho_*[(1-\Omega_m)a^{-3(1+w_*)}+\Omega_m a^{-3}]. \tag{5}$$

We see that the last two equations bear a striking similarity even though the former involves a single substance and the latter involves two. Both have two free parameters. Both have the current density $\rho(1)=\rho_*$. Making the identification $\Omega_m^*=\Omega_m$, both have $\rho(a)\rightarrow\Omega_m\rho_* a^{-3}$ at early times as $a\rightarrow 0$ (for $w_*<0$), showing that Ω_m^* can be interpreted as an effective matter density in the GCG model. Indeed, for the special case $\alpha=0$ and $w_*=-1$, we see that both models coincide with standard ΛCDM. For $\alpha=0$ the GCG model becomes equivalent to ΛCDM not only to 0th order in perturbation theory as above but to all orders, even in the nonlinear clustering regime.

The 0th order cosmology determined by Eq. (4) together with the Friedman equation

$$H\equiv\frac{\dot{a}}{a}=\left[\frac{8\pi G}{3}\rho\right]^{1/2} \tag{6}$$

[which determines $a(t)$ and the spacetime metric to 0th order] has been thoroughly investigated in previous work [10,12], and by studying constraints from supernovae observations, Makler *et al.* [27] have placed interesting constraints on the (α,Ω_m^*) parameter space.

III. GROWTH OF INHOMOGENEITIES

Let us now consider the evolution of density perturbations in this UDM model. Following the standard calculations of [31], we obtain for the relativistic analog of the Newtonian 1st order perturbation equation in Fourier space that a density fluctuation δ_k with wave vector $\mathbf{k}$ evolves as

$$\ddot{\delta}_k+H\dot{\delta}_k[2-3(2w-c_s^2)]-\frac{3}{2}H^2\delta_k[1-6c_s^2-3w^2+8w]$$
$$=-\left(\frac{kc_s}{a}\right)^2\delta_k, \tag{7}$$

where the equation of state $w\equiv p/\rho$ and the squared sound speed $c_s^2\equiv\partial p/\partial\rho$ are evaluated to 0th order and hence depend only on time, not on position. (We use units where the speed of light $c=1$ throughout.) This equation is valid on subhorizon scales $|\mathbf{k}|\gg H/c$. In other words, the growth of density fluctuations is completely determined by the two functions $w(a)$ and $c_s^2(a)$. Combining Eq. (1) and Eq. (4), these two functions are [27]

$$w=-\left[1+\frac{\Omega_m^*}{1-\Omega_m^*}a^{-3(1+\alpha)}\right]^{-1}, \tag{8}$$

$$c_s^2=-\alpha w=\alpha\left[1+\frac{\Omega_m^*}{1-\Omega_m^*}a^{-3(1+\alpha)}\right]^{-1}. \tag{9}$$

This shows a second reason why the GCG has been considered promising for cosmology it starts out behaving like pressureless CDM ($w\approx 0$, $c_s\approx 0$) early on (for $a\ll 1$) and gradually approaches cosmological constant behavior ($w\approx -1$) at late times. There is also an intermediate state where the effective equation of state is $p=\alpha\rho$ [10]. (Going beyond 1st order perturbation theory, the GCG that gets gravitationally bound in galactic halos maintains its density high enough to keep acting like CDM forever. Once formed, the contribution of such halos to the total density is diluted towards zero by cosmic expansion as a^{-3}, since the halo volume is constant in physical rather than comoving coordinates.)

To solve Eq. (7) numerically, we change the independent variable from t to $\ln a$. Using the properties

$$\frac{d}{dt}=H\frac{d}{d\ln a},\quad \ddot{\delta}_k=H^2\delta''+\frac{1}{2}(H^2)'\delta', \tag{10}$$

where $' \equiv d/d \ln a$, and defining

$$\xi \equiv \frac{(H^2)'}{2H^2} = -\frac{3}{2}(1+(1/\Omega_m^* - 1)a^{3(1+\alpha)})^{-1}, \tag{11}$$

Eq. (7) takes the form

$$\delta_k'' + [2+\xi - 3(2w - c_s^2)]\delta_k' = \left[\frac{3}{2}(1-6c_s^2+8w-3w^2) - \left(\frac{kc_s}{aH}\right)^2\right]\delta_k. \tag{12}$$

Even before solving this, it is obvious that a nonzero sound speed, if present for a sufficiently long time span, is going to have a dramatic effect on the k dependence of the perturbation growth. If $c_s^2 > 0$, then fluctuations with wavelength below the Jeans scale $\lambda_J = \sqrt{\pi |c_s^2| / G\rho}$ will be pressure-supported and oscillate rather than grow. This oscillation is confirmed by the numerical solutions, and is analogous to the acoustic oscillations in the photon-baryon fluid in the predecoupling epoch. If $c_s^2 < 0$, corresponding to negative α, fluctuations below this wavelength will be violently unstable and grow exponentially [32].

A key point which has apparently been overlooked in prior work is that whereas all the other terms in Eq. (12) are of order unity or smaller, the sound speed term $(kc_s/aH)^2$ can be much larger even if the sound speed is tiny, $|c_s| \ll 1$. This is because c_s is multiplied by the prefactor k/aH which can be enormous, since it the Horizon scale divided by the perturbation scale. Defining a critical wavelength λ_c by

$$\lambda_c^2 \equiv \frac{c_s^2}{(aH)^2} = -\frac{\alpha w}{(aH)^2}, \tag{13}$$

the pressure term in Eq. (12) becomes simply $(\lambda_c k)^2$, so we expect oscillations or exponential blowup in the power spectrum on scales $k \gtrsim \lambda_c^{-1}$. These are created mainly during the recent transition period when both a and $-w$ are of order unity (growing from 0 to 1), and since neither effect is seen in observed data, we therefore expect to obtain constraints of order $|\alpha| \lesssim (H/k)^2$, the squared ratio of the perturbation scale to the horizon scale. This heuristic argument thus suggests that Galaxy clustering constraints on scales down to $10h^{-1}$Mpc would give the constraint $|\alpha| \lesssim (10h^{-1}\text{Mpc}/3000h^{-1}\text{Mpc})^2 \approx 10^{-5}$—we will see that this approximation is in fact fairly accurate.

For our numerical calculations, we evolved a scale invariant Harrison-Zeldovich spectrum forward in time to redshift $z=100$ (before which the GCG is indistinguishable from ΛCDM) with CMBfast [34] to correctly include all the relevant effects (early super-horizon evolution, prerecombination acoustic oscillations, Silk-damping, etc.), with cosmological parameters given by the concordance model of [2]. We then used Eq. (12) to evolve the fluctuations from $z=100$ until today. Results for a sample of α values are plotted in Fig. 1, and show how tiny nonzero values of α result in large changes on small scales as expected.

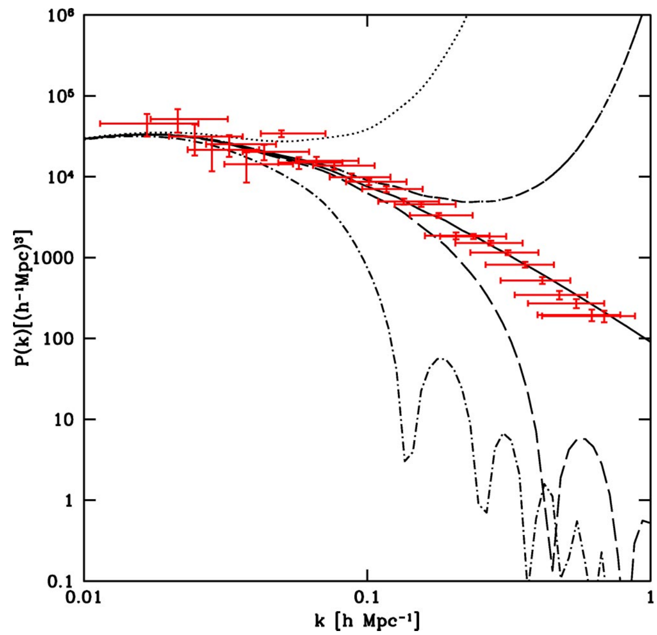

FIG. 1. UDM solution for perturbations as a function of wave number, k. From top to bottom, the curves are GCG models with $\alpha = -10^{-4}$, -10^{-5}, 0 (ΛCDM), 10^{-5} and 10^{-4}, respectively. The data points are the power spectrum of the 2df galaxy redshift survey.

We constrain α by making a χ^2 fit of the theoretically predicted power spectrum against that observed with the 2dF 100k Galaxy Redshift Survey [35] as analyzed by [36]. For each α, we use the best fitting power normalization to ensure that our constraints come only from the shape of the power spectrum, not from the overall amplitude which involves mass-to-light bias. To be conservative and stay in the linear regime, we discard data with $k > 0.3h/\text{Mpc}$. We run our code for a fine grid of models with $-1 < \alpha < 1$ to find the corresponding χ^2 values. The likelihood function $e^{-\Delta\chi^2/2}$ is plotted in Fig. 2. It predictably peaks around $\alpha \approx 0$, and the observed skewness is simply due to the fact that the oscillating solution ($\alpha > 0$) is easier to fit than the exponentially unstable solution ($\alpha < 0$). Using a $\Delta\chi^2 = 1$ cutoff as in a crude Bayesian analysis gives the constraints $-0.00000081 < \alpha < 0.0000079$.

To place this result in context, Fig. 3 shows the 0th order constraints from Makler *et al.* [27] with our new strict constraints superimposed.

IV. CLOSING THE BARYON LOOPHOLE

After the posting of the first version of this paper, Ref. [23] suggested a possible loophole related to the effect baryons. This was explored by [17] who by repeating the above calculations with an added baryonic component showed that the galaxy power spectrum is only marginally affected by the disappearance of structure in the unified dark matter component. In essence, the late-time gravitational effects of dark matter can add new fluctuations to the baryons, but cannot erase the fluctuations that are already there. In the following, we confirm this behavior and demonstrate how this loophole

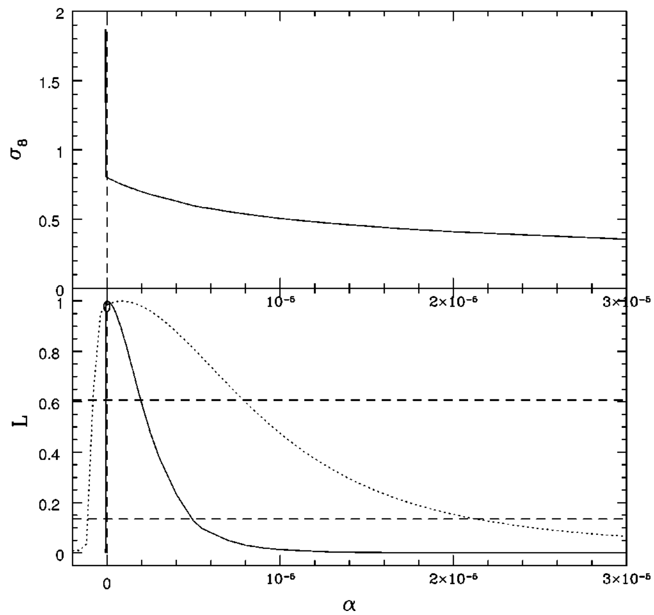

FIG. 2. Effects of varying the GCG parameter α. The top panel shows that σ_8 blows up exponentially for $\alpha<0$ and plummets for $\alpha>0$ as the dark matter fluctuations get erased. The bottom panel shows that the likelihood function $e^{-\Delta\chi/2}$ is sharply peaked around $\alpha=0$, which is equivalent to the ΛCDM model. The solid curve corresponds to the σ_8 constraint from weak gravitational lensing and the dashed curve corresponds to the shape of the galaxy power spectrum (the latter curve ignores baryon effects, the former does not). From top to bottom, the horizontal dashed lines correspond to $\Delta\chi^2=1$ and 4, respectively.

is closed by current gravitational lensing constraints, which render the UDM scenario virtually indistinguishable from the ΛCDM model at about the same quantitative level as obtained in the previous section.

In order to consistently include the baryon component, we introduce definitions analogous to Eq. (3):

$$\Omega_d^* \equiv (1-\Omega_b)\frac{B}{A+B} \tag{14}$$

and

$$\rho^*(1-\Omega_b)=(A+B)^{1/1+\alpha}, \tag{15}$$

which means the total energy density in the Chaplygin gas may be written

$$\rho_d(a)=\rho^*(1-\Omega_b)^{\alpha/(1+\alpha)}[(1-\Omega_d^*-\Omega_b + \Omega_d^* a^{-3(1+\alpha)}]^{1/(1+\alpha)}. \tag{16}$$

By going through a tedious derivation similar to the previous section but complicated by the presence of two fluids (see again [31]) we arrive at the analog to Eq. (12),

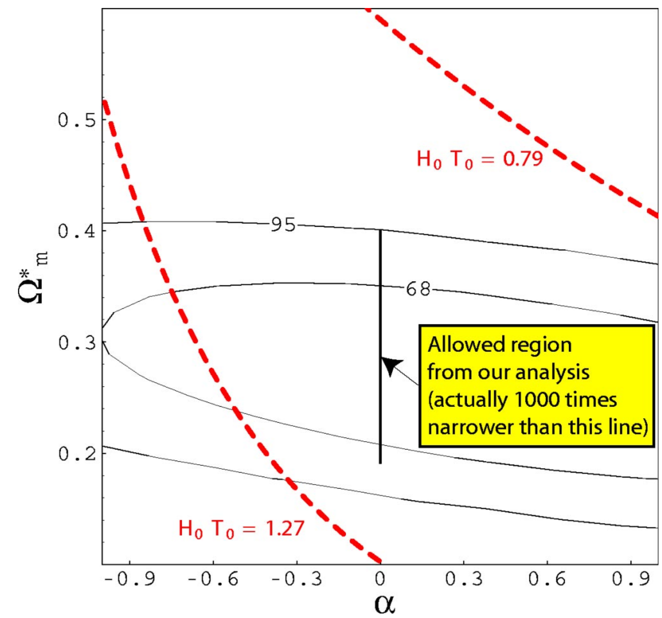

FIG. 3. The graph is showing constraints from previous work by Makler *et al.* Our new constraints from first order perturbation theory are superimposed on the plot as shown.

$$\delta_N''+\delta_N'(2+\xi-3(2w_N-c_{sN}^2)) + \frac{3}{2}\delta_N(-7w_N+3w_N w+6c_{sN}^2) = \frac{3}{2}(1+w_N)\frac{\Sigma_M\rho_M\delta_M}{\Sigma_M\rho_M}-\left(\frac{c_{sN}^2 k}{aH}\right)\delta_N, \tag{17}$$

where $N,M\in\{b,d\}$. We take $w=c_s=0$ for the baryon component. Figure 4 shows both the UDM and baryon power spectra for $\alpha=10^{-4}$ and clearly demonstrates the results of [17]. The only notable effect of a nonzero, positive value[2] of α on the total power spectrum is a vertical shift on scales $k\gtrsim\lambda_c^{-1}$ due to fact that the UDM becomes more uniform and therefore stops sourcing further baryon fluctuation growth at late times. Given the current uncertainties in the value of the galaxy bias parameter $b\equiv[P_{\rm galaxy}(k)/P(k)]^{1/2}$, the constraint from this slight suppression in the small-scale baryon power is relatively weak. To be conservative, we will not use it for our constraints on $\alpha>0$ models.

Whereas galaxies are made of baryons and thus teach us about the baryon power spectrum, weak gravitational lensing measurements probe the power spectrum of the *total* matter distribution, baryonic plus dark. Figure 2 (top panel) shows the power spectrum normalization σ_8 for the total matter distribution as a function of α, and is easy to understand. For $\alpha<0$, σ_8 blows up exponentially since with the UDM fluctuations do. For $\alpha\gg10^{-5}$, the UDM fluctuations drop to

[2]In contrast, the behavior of models with negative values of α is essentially unaltered by the inclusion of baryons, since the exponentially growing dark matter oscillations cause exponentially large baryon fluctuations as well.

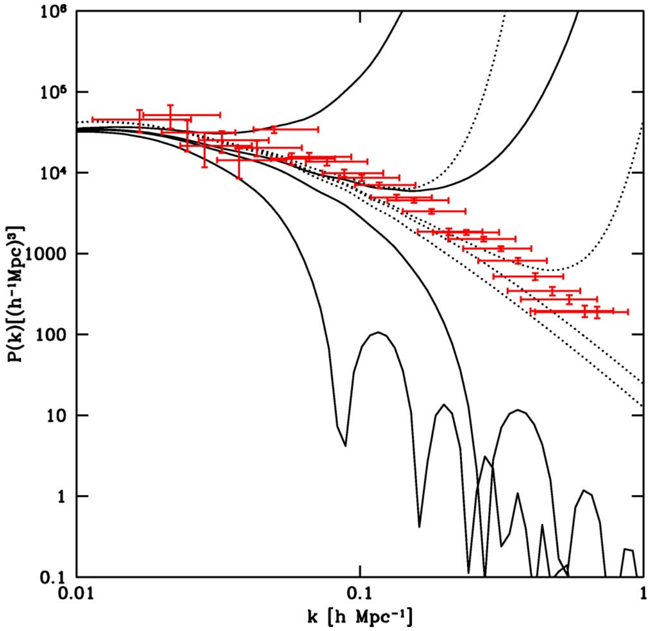

FIG. 4. The graph shows UDM (solid) and baryon (dotted) power spectra for $\alpha=-10^{-4}$, -10^{-5}, 10^{-5} and 10^{-4} from top to bottom. For negative values of α both the baryon and UDM power spectra still exhibits blow-ups strongly inconsistent with observation. For positive values of α the baryon power spectra are still marginally consistent with the 2dFGRS power spectrum despite oscillations in the UDM power spectrum. However, the respective normalizations, σ_8, are inconsistent with weak lensing constraints.

negligible levels whereas the baryon fluctuations become only marginally suppressed. Thus for a baryon fraction $f_b \approx 1/6$, we would naively assume $\delta\approx\frac{1}{6}\delta_b+\frac{5}{6}\delta_d\rightarrow\frac{1}{6}\delta_b$, and so σ_8 would drop by about a factor of six. However as also noticed by [17], the changes in the UDM component also slows down the growth in the baryons so that σ_8 is further suppressed for $\alpha\gg 10^{-5}$. Current lensing measurements like [37] (see Table V in [4] for a recent summary) conservatively suggest $\sigma_8\approx 0.8\pm 0.1$ (1σ). Quantitatively, $\alpha=\{10^{-5},10^{-4},10^{-3}\}$ gives $\sigma_8(\alpha)\approx\{0.50,0.22,0.095\}$ all of which are well below the range allowed by recent lensing measurements.

V. CONCLUSIONS

Above we showed that GCG models with $|\alpha|\gg 10^{-5}$ are ruled out, since they cause fluctuations or blowup in the unified dark matter power spectrum that are inconsistent with observation. For positive and negative α values, this follows from weak gravitational lensing measurements of the total power spectrum normalization. For negative values, the same conclusion follows from the observed shape of the galaxy power spectrum. Let us now examine the assumptions that went into our analysis and the broader implications.

First of all, our extremely tight constraints imply that the narrow range of allowed GCG models will be completely indistinguishable from ΛCDM both to 0th order and at the early times when primary CMB anisotropies are produced. This means that the corresponding standard constraints on cosmological parameters from CMB, SN Ia etc. apply also to the GCG models when making the identification $\Omega_m^*=\Omega_m$, so that there are no interesting degeneracies between α and other parameters that can significantly widen the allowed α-range. We have therefore used standard constraints $0.2<\Omega_m<0.4$ for the allowed region in Fig. 3.

Second, limiting our constraints to the nearly linear scales probed by galaxy clustering and weak lensing σ_8 constraints was probably overly conservative. As reviewed in [38,39], there are quite strong constraints on the power spectrum on much smaller scales from weak lensing, from the Lyman α forest and perhaps even from lensing of halo substructure [40] which if used would tighten our upper limit on α to 10^{-6}, 10^{-7} and 10^{-10}, respectively.

Third, we saw that all that really mattered in Eq. (12) as far as the constraints were concerned was the pressure term $(\lambda_c k)^2$. This means that our results apply more generally than merely to the GCG case: *any* unified dark matter model where p is a unique function of ρ is ruled out if the effective sound speed is non-negligible, i.e., if the function $p(\rho)$ departs substantially from a constant over the range where pressure has an effect—quantitatively, if $|d\ln p/d\ln\rho|\gtrsim 10^{-5}$, again rendering it indistinguishable from standard ΛCDM. In other words, a viable UDM model must have negligible pressure gradients, i.e., pressure that is essentially spatially constant like a Λ term.

In contrast, standard quintessence models have no such problems. Although they typically have high sound speeds causing oscillations as above, this does not prevent the dark matter from clustering since it is a separate dynamic component. Quintessence models would fail as above if there the two components were tightly coupled, and this is effectively what happens with UDM since the two are one and the same substance.

To salvage UDM idea in some form pressure must not be uniquely determined by its density, or perturbations should not be adiabatic. As worked out in detail by Hu [32], if perturbations are not adiabatic, the effective sound speed will differ from the adiabatic sound speed. If the former vanishes our constraints do not apply. This possibility was investigated recently by one of us [33], by considering nonadiabatic perturbations with a specific initial condition $(\delta p=0)$. Another way to try to avoid these problems could be by introducing some sort of scale dependence into the equations. The tachyonic scalar field model of [28] is a UDM for which the dark energy equation of state, w, becomes scale dependent. This may also alleviate some of the problems discussed in our work and should be investigated. Although the above examples demonstrate that it is possible to construct models which escape the conclusion of this paper, we feel that this means giving up much of the simplicity that gave the unified dark matter idea its appeal.

In conclusion, precision data is gradually allowing us to test rather than assume the physics underlying modern cosmology. We have taken a step in this direction by ruling out a broad class of so-called unified dark matter models. With the above mentioned caveats, our results indicate that dark energy is either indistinguishable from a pure cosmological constant or a separate component from the dark matter with a life of its own.

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APPENDIX: EQUIVALENCE BETWEEN THE GCG $\alpha=0$ LIMIT AND ΛCDM

To demonstrate the equivalence between ΛCDM and the $\alpha=0$ limit of the GCG, we here write down the governing fluid mechanical equations plus the Poisson equation (in the Newtonian limit).

In the limit of $|\alpha|\ll 1$ the equation of state is given by

$$p_u \approx -\rho^*(1-\alpha \ln \rho_u). \tag{A1}$$

We can then evaluate temporal and spatial derivatives of p

$$\dot{p}_u = \alpha\rho^* \frac{d}{dt}\ln \rho_u = \alpha\rho^* \frac{\dot{\rho}_u}{\rho_u} \tag{A2}$$

and

$$\boldsymbol{\nabla} p_u = \alpha\rho^* \boldsymbol{\nabla} \ln \rho_u = \alpha\rho^* \frac{\boldsymbol{\nabla}\rho_u}{\rho_u}. \tag{A3}$$

This enables us to write down the full Euler, continuity and Poisson equations

$$\frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla}\Phi + \alpha\frac{\rho^*}{\rho_u}(\boldsymbol{\nabla}\rho_u + \mathbf{v}\dot{\rho}_u)/(\rho_u + p_u) \tag{A4}$$

$$\left(1+\alpha\frac{\rho^*}{\rho_u}\right)\frac{D}{Dt}\rho_u = \alpha\frac{\rho^*}{\rho_u}\frac{\partial}{\partial t}\rho_u - (\rho_u + p_u)\nabla\cdot\mathbf{v} \tag{A5}$$

$$\nabla^2\Phi = 4\pi G(\rho_u + 3p_u). \tag{A6}$$

For $\alpha=0$ and the identifications $\rho_u=\rho_d+\rho_\Lambda$ $p_u=-\rho_\Lambda$ these equations reduce to the equations for ΛCDM, thus in this limit the two scenarios are identical.

It is clear that all extra terms come in factors of α, and we expect the equations to be also *asymptotically* identical to ΛCDM for α close to zero. Note that there is a further suppression from the factor ρ^*/ρ_u demonstrating that the difference from ΛCDM is further suppressed in dense regions, as well as up to the very recent cosmic epoch.

This simple argument demonstrates not only that the linear calculations are equivalent to ΛCDM in the $\alpha=0$ limit but that the behavior must be identical to *any* order. Only if the model deviates significantly from $\alpha=0$, may we expect new nonlinear effects to manifest themselves.

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