Time evolution of tunneling in a thermal medium: Environment-driven excited tunneling

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Time evolution of tunneling phenomena proceeding in a thermal medium is studied using a standard model of environmental interaction. A semiclassical probability formula for the particle motion in a metastable state of a one-dimensional system put in a thermal medium is combined with the formula of the quantum penetration factor through a potential barrier to derive the tunneling rate in the medium. The effect of environment, its influence on time evolution in particular, is clarified in our real-time formalism. A nonlinear resonance effect is shown to enhance the tunneling rate at finite times of order $2/\eta$, with η the friction coefficient unless η is too small. In the linear approximation this effect has relevance to the parametric resonance. This effect enhances the possibility of early termination of the cosmological phase transition much prior to the typical Hubble time.

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I. INTRODUCTION

Tunneling phenomena, when they occur in isolation, are genuinely a quantum effect. But when they occur in some surrounding medium, the important question arises as to whether the tunneling rate is enhanced or suppressed by the environmental effect. There are already many works on this subject [1], and most past works deal with a system in equilibrium as a whole. A Euclidean technique, such as the bounce solution [2], is often used in this context [3–8]. We would like to examine this problem by working in the real-time formalism [9]. We find it more illuminating to use the real-time description instead of the Euclidean method much employed in the literature. An advantage of the real-time description is the possibility of identifying an enhanced tunneling rate at a finite time of dynamical evolution of the tunneling [10]. Our preliminary application to tunneling phenomena are given in Refs. [10–12].

Our approach is based on the separation of a subsystem from the thermal environmental, and integrating out the environmental degrees of freedom. This method is best suited to a (by itself) nonequilibrium system which is immersed in a larger thermal equilibrium state. The detailed properties of the environment and its interaction form with the subsystem are expected to be insensitive to the behavior of the subsystem in question. We use the standard model of environment consisting of an infinite number of harmonic oscillators [3,9]. In this picture the dissipation seen in the behavior of the subsystem is due to our ignorance of the huge environmental degrees of freedom. Although the subsystem itself is an open system having an interaction with the environment, the entire system including the environment obeys quantum mechanical laws in the closed system.

We work out consequences of the formalism for a simple subsystem in one dimension; a tunneling potential system with one local minimum. The most interesting result that comes out in this study is the resonance enhanced tunneling [10]. Our result demonstrates that the enhancement occurs at a time scale of order $1/\eta$, with η the friction coefficient. The enhanced tunneling probability goes with η as $\eta^{-1.5}$, at least in the range of friction $0.001\omega_p < \eta < 0.02\omega_p$ according to our numerical analysis in the present work. Here ω_p is the curvature at the potential well. We offer an interpretation of the resonance enhancement due to an energy flow into the system from the environment. Furthermore, the enhanced tunneling rate is related to the parametric resonance. We call this enhanced tunneling ''environment-driven excited tunneling'' (EET). The popular Euclidean approach cannot deal with the finite time behavior of tunneling, hence it misses this resonance enhancement at finite times.

Our method, with a certain extension, should be applicable to field theory models. In the future we hope to investigate applications to cosmology, in which one may deal with the electroweak first order phase transition relevant to baryogenesis [13]. An important question here is the time scale of tunneling. If the usual Hubble time scale that gives rise to the potential (or more properly the free energy) change (hence regarded in the conventional picture as the termination of the first order phase transition), is replaced by a shorter tunneling time scale of order $1/\eta$, the conventional picture of the first order phase transition may drastically change. For instance, bubbles of the true vacuum may be formed by the resonance enhanced tunneling prior to the potential change. Since this process is stochastic, it may enhance the out-of-equilibrium condition necessary for baryogenesis [14,15]. The bubble formation via EET may also resurrect the old scenario of grand unified theory (GUT) phase transition [16] which was once discarded due to the graceful exit problem [17].

The rest of this paper is organized as follows. In Sec. II our setup is shown and the environmental model and the tunneling rate formula are explained. For simplicity, we examine the environmental effect for a simple tunneling system described by a one-dimensional potential. In Sec. III we derive equations needed for numerical calculations, and some analytical results are given. The semiclassical approximation is then introduced and the crucial simplification, the linear approximation of the environment variable, is further worked out. This approximation, used while the particle in the one-dimensional potential is in the metastable region, determines how the particle is excited in the potential well due to the environment interaction. In Sec. IV a detailed numerical

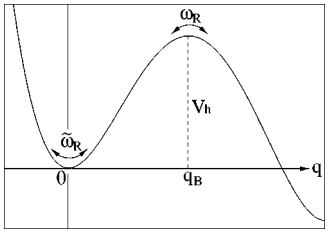

FIG. 1. A typical potential for tunneling in a one-dimensional system. Curvature parameters $\tilde{\omega}_R$ at the bottom of the potential, ω_R at the top of the potential, along with the barrier height V_h and the barrier width, characterize a global structure of the potential.

analysis in the metastable region is presented, taking a concrete example of the potential. It is shown that excitation to higher levels proceeds via the parametric resonance effect in the linear approximation. A numerical analysis is then presented, quantitatively demonstrating the effect of EET. In the final section an application to cosmological baryogenesis is briefly discussed. In the Appendix we explain the renormalization effect along with the Langevin equation and the friction coefficient along with the relevant formulas in the standard model of particle physics, which may be applicable to discussions of the electroweak baryogenesis in the standard model.

II. ENVIRONMENT MODEL AND TUNNELING RATE FORMULA

We consider as the first step of our investigation the simplest, yet most basic problem of this kind, a one-dimensional system described by the potential $V(q)$. The potential $V(q)$, as illustrated in Fig. 1, is assumed to have some local minimum at $q=0$ with $V(0)=0$, which is separated at the barrier top $q=q_B$ (>0) from a global minimum, which we take in the present work to be at $q=\infty$ such that there is no reflection from the wall at the far right.

This system is put in a thermal medium which acts as an environment. The environment has infinitely many, continuously distributed harmonic oscillators given by their coordinates $Q(\omega)$. Its coupling to the tunneling system is given by a bilinear form [3,9]. The total Hamiltonian is

$$H_{\rm tot}=\frac{\dot{q}^2}{2}+V(q)+q\int_{\omega_c}^{\infty}d\omega c(\omega)Q(\omega) + \int_{\omega_c}^{\infty}d\omega\left(\frac{\dot{Q}(\omega)^2}{2}+\frac{\omega^2}{2}Q(\omega)^2\right). \tag{1}$$

The frequency-dependent coupling strength is $c(\omega)$ and ω_c is some threshold frequency. One may imagine a generalized case in which the subsystem variable q is the order parameter for the first order phase transition in cosmology, the homogeneous Higgs field, and the environment oscillator $Q(\omega)$ is a collection of various forms of matter fields coupled to the Higgs field.

In considering our problem of tunneling in a thermal medium we assume that the effects of thermal environment are most important in the time period when the particle in a one-dimensional system remains in the metastable potential well. Thus, we anticipate that at tunneling the simple quantum mechanical formula may directly be used. Our basic formula for the tunneling rate is

$$\Gamma(t)=\frac{\omega_p}{2\pi}\sum_{n=0}^{\infty}\mathcal{W}_n(t)|T(E_n)|^2. \tag{2}$$

Here ω_p is the physical frequency of the potential well, after taking into account the renormalization of the potential curvature at the bottom $(q=0)$ due to the interaction with the environment. A detailed explanation of the renormalization is given in Appendix Sec. 1. The factor $\omega_p/2\pi$ in the right-hand side is a 1/period for the periodic motion in the left potential well. The complete set of states $|n\rangle$ that appear in Eq. (2) is defined by introducing harmonic oscillator states with respect to the reference frequency ω_p. The quantity $\mathcal{W}_n(t)$ is the occupation probability of the energy (E_n) eigenstate at the potential region left to the barrier top, and $|T(E_n)|^2$ is the time-independent penetration factor for the potential $V(q)$.

Energy eigenstates are special among pure quantum states since they time evolve only with phase factors. Thus, if one takes for the projection $|n\rangle\langle n|$, the energy eigenstate $|n\rangle$ of the total Hamiltonian, the density matrix does not change with time. On the other hand, if one takes the energy eigenstate of the subsystem Hamiltonian, then the density matrix changes solely due to the environment interaction. It is thus best to use a pure eigenstate of, or its superposition of, the subsystem Hamiltonian when one wishes to determine how the tunneling rate is modified in the thermal medium.

The formula (2) is intuitively understandable and is also used in many papers and textbooks [4,18,19]. The time dependence of $\mathcal{W}_n(t)$ comes from the interaction with the environment. In a sufficient time, an arbitrary initial tunneling state is expected to be thermalized due to the interaction with the environment. For instance, the initial ground state at the local potential bottom goes as

$$\mathcal{W}_n(0)=\delta_{n0}\rightarrow\mathcal{W}_n(\infty)=\frac{e^{-\beta E_n}}{\sum_n e^{-\beta E_n}}. \tag{3}$$

Here $T=\beta^{-1}$ is the temperature of the environment. We shall derive this limiting formula at the end of Sec. III. A fundamental paper by Affleck [4] uses the form (2) of the tunneling probability, using the equilibrium formula for $\mathcal{W}_n(\infty)$

$$\Gamma_{\rm th}=\frac{\omega_p}{2\pi}\sum_{n=0}^{\infty}\frac{e^{-\beta E_n}}{\sum_n e^{-\beta E_n}}|T(E_n)|^2. \tag{4}$$

Reference [4] also compares this method with an imaginary-time calculation, obtaining the same result (4).

Our formula (2) is a generalization of the equilibrium formula (4) to finite times. Ideally, the formula (2), under ap-

propriate circumstances, should be derived from the real-time formalism of Feynman and Vernon [9], which has been unsuccessful so far.

A comment on the potential $V(q)$ might now be useful. We do not include temperature-dependent corrections in this potential due to the environmental interaction. They automatically arise in our formalism as coupling-$[c(\omega)]$ dependent terms, when one solves the system of Eq. (1) completely. In approximation (2) the temperature-dependent correction to the potential is ignored, which is justified for a sufficiently low temperature. Alternatively, as a phenomenological approach one could include temperature-dependent terms in the potential such as Eq. (A8) of Appendix Sec. 2 valid in the standard particle physics model, if one carefully avoids the double counting problem which might arise from the interaction term.

We now show the basic strategy for calculating the time dependence of the occupation probability $\mathcal{W}_n(t)$. This part of the calculation is an ideal problem to apply the Feynman-Vernon real-time formalism to. By using this formalism we can calculate the density matrix or the Wigner function of a small system immersed in a huge thermal environment.

We start with the density matrix for the total (tunneling system+environment) system. The density matrix that describes the entire quantum-mechanical system obeys the equation of motion

$$i\hbar\frac{\partial\rho}{\partial t}=[H_{\rm tot},\rho(t)]. \tag{5}$$

For the time being, we explicitly write the Planck constant $\hbar$. In the configuration space this density matrix is given by its matrix elements

$$\rho(q,q';Q(\omega),Q'(\omega);t)=\langle q,Q(\omega)|\rho(t)|q',Q'(\omega)\rangle. \tag{6}$$

Its Fourier transform with respect to relative coordinates $q-q',Q(\omega)-Q'(\omega)$ is called the Wigner function and is denoted by f_W:

$$\begin{aligned} f_W[q,p;Q(\omega),P(\omega);t] &\equiv \int_{-\infty}^{\infty}\frac{d\xi}{2\pi}\prod_\omega\frac{dX(\omega)}{2\pi}\exp\left[-i\xi p-i\int d\omega X(\omega)P(\omega)\right] \\ &\times\rho[q+\xi/2,q-\xi/2;Q(\omega)+X(\omega)/2,Q(\omega) \\ &-X(\omega)/2;t]. \end{aligned} \tag{7}$$

Reduction to the subsystem by integrating out environment variables $Q(\omega)$ is an essential part of the Feynman-Vernon formalism. We thus define the reduced density matrix of the tunneling system by

$$\rho^{(R)}(q,q';t)\equiv\int\prod_\omega dQ(\omega)dQ'(\omega)\delta[Q(\omega)-Q'(\omega)]\cdot\rho[q,q';Q(\omega),Q'(\omega);t]. \tag{8}$$

The fact that there is a delta function $\delta[Q(\omega)-Q'(\omega)]$ for the environment variable reflects our understanding that one does not observe the environment. A similar reduction is made to the Wigner function, to get the reduced Wigner function

$$\begin{aligned} f_W^{(R)}(q,p;t)&\equiv\int\prod_\omega dP(\omega)dQ(\omega)f_W[q,p;Q(\omega),P(\omega);t] \\ &=\int_{-\infty}^{\infty}\frac{d\xi}{2\pi}e^{-i\xi p}\rho^{(R)}(q+\xi/2,q-\xi/2;t). \end{aligned} \tag{9}$$

A detailed calculation of the reduced Wigner function is given in Sec. III.

The reduced density matrix and reduced Wigner function all have quantum and statistical information about the small system immersed in a huge environment. Thus the occupation probability can be calculated by performing projection onto the energy eigenstate of the tunneling system on the left side of the potential barrier

$$\begin{aligned} \mathcal{W}_n(t)&=\langle E_n|\rho^{(R)}(t)|E_n\rangle \\ &=\int dqdq'\varphi_{E_n}(q)\varphi^*_{E_n}(q')\rho^{(R)}(q,q';t) \\ &=\int dqdq'dp\,\varphi_{E_n}(q)\varphi^*_{E_n}(q')e^{ip(q'-q)} \\ &\times f_W^{(R)}\left(\frac{q+q'}{2},p;t\right), \end{aligned} \tag{10}$$

where $\varphi_{E_n}(q)$ is the wave function of eigenenergy E_n. Our basic idea is to limit application of this formula to the left region of the metastable state.

The reduction to the subsystem here is in spirit opposite to that common in the effective field theory approach; integration over heavy degrees of freedom is performed. What is integrated out in the present case is the majority of constituents which are taken invariant under frequent thermalizing interaction.

III. SEMICLASSICAL REDUCED WIGNER FUNCTION AND OCCUPATION PROBABILITY

A. Semiclassical reduced Wigner function

It is easy to show from the master equation (5) that the Wigner function of the total system defined in Eq. (7) obeys

$$\begin{aligned} \frac{\partial f_W}{\partial t}&=-p\frac{\partial f_W}{\partial q}+\frac{1}{i\hbar}\left\{V\left(q+\frac{i\hbar}{2}\frac{\partial}{\partial p}\right)-V\left(q-\frac{i\hbar}{2}\frac{\partial}{\partial p}\right)\right\}f_W \\ &-\int d\omega\left\{P(\omega)\frac{\partial f_W}{\partial Q(\omega)}+\omega^2Q(\omega)\frac{\partial f_W}{\partial P(\omega)}\right. \\ &\left.+c(\omega)\left(q\frac{\partial}{\partial P(\omega)}+Q(\omega)\frac{\partial}{\partial p}\right)f_W\right\}. \end{aligned} \tag{11}$$

We concentrate on the equation at the potential region left of the barrier top, in which $\hbar \to 0$ may be used, resulting in a limit. Then we have in Eq. (11)

$$\frac{1}{i\hbar}\left\{ V\left(q+\frac{i\hbar}{2}\frac{\partial}{\partial p}\right) - V\left(q-\frac{i\hbar}{2}\frac{\partial}{\partial p}\right)\right\} \to \frac{dV}{dq}\frac{\partial}{\partial p}. \quad (12)$$

The resulting equation for the Wigner function is identical to the classical Liouville equation familiar in classical statistical mechanics. It has an obvious formal solution

$$\begin{aligned} f_W[q,p;Q(\omega),P(\omega);t] &= \int dq_i dp_i \int \prod_\omega dQ_i(\omega) dP_i(\omega) \\ &\times f_W^{(i)}[q_i,p_i;Q_i(\omega),P_i(\omega)]\,\delta(q-q_{\rm cl})\,\delta(p-p_{\rm cl}) \\ &\times \prod_\omega \delta\,[Q(\omega)-Q_{\rm cl}(\omega)]\,\delta\,[P(\omega)-P_{\rm cl}(\omega)], \end{aligned} \quad (13)$$

where $q_{\rm cl}, Q_{\rm cl}(\omega)$, etc., are the solution of equation of motions for the Hamiltonian (1) taken as the classical equation with the specified initial condition written in terms of $q_i, p_i, Q_i(\omega), P_i(\omega)$:

$$\frac{d^2q}{dt^2}+\frac{dV}{dq} = -\int_{\omega_c}^{\infty} d\omega c(\omega) Q(\omega),$$

$$\frac{d^2Q(\omega)}{dt^2}+\omega^2 Q(\omega) = -c(\omega) q. \quad (14)$$

We consider the circumstance under which the tunneling system is initially in a state uncorrelated to the rest of the environment and the environment is taken to be a thermal state of the temperature $T=\beta^{-1}$. We do not specify the detailed form of the initial Wigner function of the tunneling system for the time being. Thus we take the form of the initial Wigner function of a factorized form

$$\begin{aligned} f_W^{(i)}[q_i,p_i,Q_i(\omega),P_i(\omega)] &= f_{W,q}^{(i)}(q_i,p_i)\prod_\omega f_{W,Q}^{(i)}[Q_i(\omega),P_i(\omega)], \end{aligned} \quad (15)$$

$$f_{W,Q}^{(i)}[Q_i(\omega),P_i(\omega)] = \frac{1}{\pi}\tanh\frac{\beta\omega}{2}\exp\left[-\tanh\frac{\beta\omega}{2}\left(\omega Q_i^2(\omega)+\frac{P_i^2(\omega)}{\omega}\right)\right]$$

to get the reduced Wigner function after the environment $Q(\omega), P(\omega)$ integration,

$$f_W^{(R)}(q,p;t) = \int dq_i dp_i f_{W,q}^{(i)}(q_i,p_i) K(q,p,q_i,p_i;t), \quad (16)$$

$$K(q,p,q_i,p_i;t) = \int \prod_\omega dQ_i(\omega) dP_i(\omega) f_{W,Q}^{(i)}(Q_i,P_i) \times \delta(q-q_{\rm cl})\,\delta(p-p_{\rm cl}). \quad (17)$$

The quantity $q_{\rm cl}$ and $p_{\rm cl}$ are now solutions of the Langevin equation

$$\frac{d^2q}{dt^2}+\frac{dV}{dq}+2\int_0^t ds\,\alpha_I(t-s)q(s) = F_Q(t), \quad (18)$$

$$\alpha_I(t) \equiv -\int_{\omega_c}^{\infty} d\omega r(\omega)\sin(\omega t), \quad \text{with} \quad r(\omega)=\frac{c^2(\omega)}{2\omega},$$

$$F_Q(t) = -\int_{\omega_c}^{\infty} d\omega c(\omega)\left(Q_i(\omega)\cos(\omega t)+P_i(\omega)\frac{\sin(\omega t)}{\omega}\right), \quad (19)$$

which is obtained from the equation of motion (14) by eliminating the environment variable $Q(\omega)$ from the second equation [20].

An often used model is the local, Ohmic model [22,23] taking

$$r(\omega) = \frac{\eta\omega}{\pi} \quad (20)$$

with $\omega_c = 0$. Then the Langevin equation takes the well-known local form

$$\ddot{q}+V'(q)+\eta\dot{q} = F_Q(t). \quad (21)$$

Here the overdot means the time derivative and the prime indicates the derivative with respect to its argument. The expected friction term $\eta(dq/dt)$ and the fluctuation term $F_Q(t)$ are realized in the above equation through the interaction between the tunneling system and the environment. We use this local Ohmic model in the rest of our paper.

The solution of the Langevin equation is regarded as a function of initial values $q_{\rm cl}(q_i,p_i,Q_i,P_i;t)$. The problem of great interest is how further one can practically simplify the kernel function K introduced in Eq. (16).

In many situations one is interested in the tunneling probability when the environment temperature is low enough. At low temperatures of $T \ll$, a typical frequency or curvature scale ω_s of the potential V, one has

$$\omega_s \sqrt{\overline{Q_i^2(\omega)}}, \sqrt{\overline{P_i^2(\omega)}} = O[\sqrt{T}] \ll \sqrt{\omega_s}. \quad (22)$$

Here an overbar denotes the thermal average. In the electroweak and GUT phase transitions this corresponds to $T \ll m_H$ (Higgs mass). Expansion of $q_{\rm cl}$ in terms of $Q_i(\omega), P_i(\omega)$ and its truncation to linear terms is then justified. Thus, we use

$$q_{\rm cl} \simeq q_{\rm cl}^{(0)} + \int d\omega \{ Q_i(\omega) q_{\rm cl}^{(Q)}(\omega) + P_i(\omega) q_{\rm cl}^{(P)}(\omega)\}, \quad (23)$$

valid to the first order of $Q_i(\omega), P_i(\omega)$. The zeroth order term is $q_{\rm cl}^{(0)}$ obeying

$$\ddot{q}_{\rm cl}^{(0)} + V'(q_{\rm cl}^{(0)}) + \eta \dot{q}_{\rm cl}^{(0)} = 0, \tag{24}$$

with the initial condition $q_{\rm cl}^{(0)}(t=0)=q_i$, $\dot{q}_{\rm cl}^{(0)}(t=0)=p_i$. On the other hand, the coefficient functions of the first order terms $q_{\rm cl}^{(Q)}(\omega), q_{\rm cl}^{(P)}(\omega)$ can be written by the complex function $z(\omega,t)$ as $z(\omega,t)=q_{\rm cl}^{(Q)}(\omega,t)+i\omega q_{\rm cl}^{(P)}(\omega,t)$ and $\dot{z}(\omega,t) = p_{\rm cl}^{(Q)}(\omega,t)+i\omega p_{\rm cl}^{(P)}(\omega,t)$. Then $z(\omega,t)$ obeys the inhomogeneous linear equation

$$\ddot{z}(\omega,t) + V''(q_{\rm cl}^{(0)}) z(\omega,t) + \eta \dot{z}(\omega,s) = -c(\omega) e^{i\omega t}, \tag{25}$$

with the initial condition $z(\omega,t=0)=\dot{z}(\omega,t=0)=0$. A similar expansion for $p_{\rm cl}$ using $p_{\rm cl}^{(0)}, p_{\rm cl}^{(Q)}(\omega), p_{\rm cl}^{(P)}(\omega)$ also holds.

With Eq. (23) and the Gaussian initial Wigner function (15) one can perform integrations for kernel K in Eq. (16) analytically. The result of this integral leads to an integral transform [12] of the Wigner function $f_W^{(i)}$ (initial) $\rightarrow$ $f_W^{(R)}$ (final) using the following form of the kernel function:

$$K(q,p,q_i,p_i;t) = \frac{\sqrt{\det \mathcal{J}}}{2\pi} \exp\left[-\frac{1}{2}(q-q_{\rm cl}^{(0)}, p-p_{\rm cl}^{(0)}) \mathcal{J} \begin{pmatrix} q-q_{\rm cl}^{(0)} \\ p-p_{\rm cl}^{(0)} \end{pmatrix} \right], \tag{26}$$

where the matrix elements of $\mathcal{J}_{ij}^{-1} = I_{ij}$ are given by

$$I_{11}(t) = \frac{1}{2}\int_0^\infty d\omega \coth\frac{\beta\omega}{2} \frac{1}{\omega} |z(\omega,t)|^2, \tag{27}$$

$$I_{22}(t) = \frac{1}{2}\int_0^\infty d\omega \coth\frac{\beta\omega}{2} \frac{1}{\omega} |\dot{z}(\omega,t)|^2, \tag{28}$$

$$I_{12}(t) = \frac{1}{2}\int_0^\infty d\omega \coth\frac{\beta\omega}{2} \frac{1}{\omega} \mathrm{Re}[z(\omega,t)\dot{z}^*(\omega,t)] = \frac{\dot{I}_{11}}{2}. \tag{29}$$

The physical picture underlying the formula for the integral transform (16) along with Eq. (26), should be evident; the probability at a phase space point (q,p) is dominated by the semiclassical trajectory $q_{\rm cl}^{(0)}$ [where the environmental effect of dissipation is included in its determination by the third term of Eq. (24)] reaching (q,p) from an initial point (q_i,p_i) whose contribution is weighed by the probability $f_W^{(i)}$ given initially. The contributing trajectory is broadened by the environmental interaction with the width factor $\sqrt{I_{ij}}$. The quantity I_{11} given by Eq. (27), for instance, is equal to $\overline{(q-q_{\rm cl}^{(0)})^2}$, an environmental driven fluctuation under the stochastic force $F_Q(t)$.

We note that in the exactly solvable model of the inverted harmonic oscillator potential, the identical form of the integral transform (26) was derived [11] without resort to the semiclassical approximation. The explicit form of the classical and fluctuation functions ($q_{\rm cl}^{(0)}$), and $z(\omega,t)$ is given there, which is valid beyond the Ohmic approximation.

A convenient form of the fluctuation formula is derived by separating the zero-temperature result along with $\coth\beta\omega/2 = 1 + 2/(e^{\beta\omega}-1)$ in Eqs. (27)–(29). We denote the temperature-dependent part of the fluctuation by $I_{ij}^{(T)}$. The zero-temperature part $I_{ij}^{(0)}$ must be renormalized. After performing renormalization the zero-temperature part is approximated by the result of the harmonic approximation explained in the next paragraph. The renormalized fluctuation is then well approximated using the physical curvature ω_p. We shall use the same notation I_{ij} for the renormalized fluctuation in the rest of our paper.

B. Harmonic approximation

It would be instructive to discuss the harmonic approximation taking the potential $V(q)$ in Eqs. (23) and (25) as

$$V(q) \simeq \frac{\omega_p^2}{2} q^2, \tag{30}$$

where the physical curvature is given by $\omega_p = \sqrt{\tilde{\omega}_R^2 - \eta^2/4}$ in the local Ohmic model. (The derivation of the relation between the physical curvature ω_p and the renormalized curvature ω_R is shown in Appendix Sec. 1.) Then the zeroth order term $q_{\rm cl}^{(0)}(t)$ and $z(\omega,t)$ are obtained analytically, and given by

$$q_{\rm cl}^{(0)} = u(t) q_i + v(t) p_i, \tag{31}$$

$$u(t) = \left(\cos\omega_p t + \frac{\eta}{2\omega_p} \sin\omega_p t \right) e^{-\eta t/2},$$

$$v(t) = \frac{\sin\omega_p t}{\omega_p} e^{-\eta t/2}, \tag{32}$$

$$z(\omega,t) = \frac{c(\omega)}{\omega^2 - \tilde{\omega}_R^2 - i\omega\eta} [e^{i\omega t} - u(t) - i\omega v(t)]. \tag{33}$$

The function $z(\omega,t)$ has a Breit-Wigner form for a small η. In particular, the infinite time ($t\rightarrow\infty$) limit of the fluctuation has

$$I_{11}(\infty) = \int_0^\infty d\omega \coth\frac{\beta\omega}{2} \frac{r(\omega)}{(\omega^2 - \tilde{\omega}_R^2)^2 + \eta^2\omega^2}. \tag{34}$$

The friction η appears as the resonance width in this formula. The real part of the pole position in a frequency integral such as Eq. (34), $\omega_p \equiv$ Re (pole position), is $\omega_p = \sqrt{\tilde{\omega}_R^2 - \eta^2/4}$ as expected.

The harmonic approximation is excellent at late times, as will be demonstrated explicitly later, but at early and intermediate times nonlinear resonance effects are important as discussed in Ref. [10]. The nonlinear effect will be taken into account in the discussion of the finite time behavior of the tunneling rate.

C. Occupation probability

We now discuss how to obtain the occupation probability $\mathcal{W}_n(t)$ from the reduced Wigner function by using Eq. (10). From Eqs. (16) and (26), explicit p integration of Eq. (10) gives

$$\mathcal{W}_E(t)=\int dqdq'\frac{dq_i dp_i}{2\pi}\varphi_E(q)\varphi_E^*(q')\frac{f_W^{(i)}(q_i,p_i)}{\sqrt{2\pi I_{11}}}\times\exp\left\{-\frac{1}{2I_{11}}\left(\frac{q+q'}{2}-q_{\rm cl}^{(0)}(t)\right)^2-\frac{\det I}{2I_{11}}(q-q')^2+i(q'-q)\left[p_{\rm cl}^{(0)}(t)+\frac{I_{12}}{I_{11}}\left(\frac{q+q'}{2}-q_{\rm cl}^{(0)}(t)\right)\right]\right\}. \tag{35}$$

Unless the energy eigenstate near the barrier top is important for the computation of the tunneling rate, one may approximate the wave function $\varphi_E(q)$ in the left region to $q=q_B$ by that of the harmonic oscillator at the potential bottom $q=0$. The wave function $\varphi_E(q)$ is thus approximated by using the Hermite polynomial H_n as

$$\varphi_E(q)=\left(\frac{\sqrt{\omega_p}}{\sqrt{\pi}2^n n!}\right)^{1/2}H_n(\sqrt{\omega_p}q)e^{-\omega_p q^2/2}, \tag{36}$$

where $E=n\omega_p$. It is convenient to use the generating function

$$\mathcal{W}(t,z)\equiv\sum_{n=0}^{\infty}\mathcal{W}_n(t)z^n \tag{37}$$

for computing the projected probability $\mathcal{W}_n(t)$. With the help of the Mehler formula [26] for the Hermite polynomial

$$\sum_{n=0}^{\infty}\frac{(z/2)^n}{n!}H_n(x)H_n(y)=(1-z^2)^{-1/2}\exp\left[\frac{2xyz-(x^2+y^2)z^2}{1-z^2}\right], \tag{38}$$

one has Gaussian integrals for q and q' in Eq. (10), resulting in

$$\mathcal{W}(t,z)=\int dq_i dp_i\frac{f_W^{(i)}(q_i,p_i)}{\sqrt{\det\tilde{A}}}\times\exp\left[-\frac{1-z}{2\det\tilde{A}}(q_{\rm cl}^{(0)},p_{\rm cl}^{(0)})\tilde{A}\begin{pmatrix}q_{\rm cl}^{(0)}\\ p_{\rm cl}^{(0)}\end{pmatrix}\right],$$

$$\tilde{A}=\begin{pmatrix}(1-z)(I_{22}-\omega_p/2)+\omega_p & -(1-z)I_{12}\\ -(1-z)I_{12} & (1-z)[I_{11}-1/(2\omega_p)]+1/\omega_p\end{pmatrix}. \tag{39}$$

Quantities I_{ij} are to be computed at finite time t. The important constraint of the probability conservation $\Sigma_{n=0}^{\infty}\mathcal{W}_n(t)=1$ is automatically satisfied due to $\mathcal{W}(t,1)=1$. For a precise computation, one expands this generating function to powers of z, and identifies the projected probability $\mathcal{W}_n(t)$ by Eq. (37).

Let us discuss the infinite time limit of the occupation probability $\mathcal{W}_n(\infty)$. The harmonic approximation for the kernel function K is excellent in this limit, as stated in the previous section. Then the fluctuations I_{ij} are independent of the initial conditions of q_i, p_i, as exemplified by Eq. (34). On the other hand, solutions of the homogeneous equation (24), $q_{\rm cl}^{(0)}$ and $p_{\rm cl}^{(0)}$, are identically zero due to the friction term. Thus the integration of initial variables q_i, p_i in Eq. (39) becomes trivial and can be solved analytically. The resultant occupation probability is

$$\mathcal{W}_n(\infty)=(1-e^{-\beta\omega})e^{-n\beta\omega}+O(\eta), \tag{40}$$

the thermal one up to the order η. We note that the subsystem thermalizes at the infinite time despite that it was initially an arbitrary initial state, hence a nonequilibrium state. This proves that the infinite time limit coincides with the thermal limit $\Gamma(\infty)=\Gamma_{\rm th}$.

IV. NUMERICAL ANALYSIS AND RELEVANCE OF PARAMETRIC RESONANCE

We first specify a particular model of the potential for our numerical calculation. The potential we take (illustrated in Fig. 2) is given by

$$\begin{aligned}V(x)&=\frac{\tilde{\omega}_R^2}{2}x^2+g\tilde{\omega}_R^3x^4 \quad (x<x_b),\\ &=V_h \quad (x_b<x<x_f),\\ &=-V(\infty) \quad (x_f<x).\end{aligned} \tag{41}$$

The two parameters V_h and x_b are related by $V(x_b^-)=V_h$ for our choice of a particular quartic potential in the metastable well. We consider for definiteness, $g=0.01$, $V_h=5\omega_p(\omega_p=\sqrt{\tilde{\omega}_R^2-\eta^2/4})$ for the parameters of Eq. (41). We use the friction η in the range of $0.001\omega_p<\eta<0.02\omega_p$ and the temperature of the environment in the range of $0.1\omega_p<T<\omega_p$. We set $x_f-x_b=0.5\omega_p$ in this calculation.

The initial Wigner function of the system $f_W^{(i)}(q_i,p_i)$ is taken to be that of the ground state of the harmonic oscillator at the potential bottom. It is given by

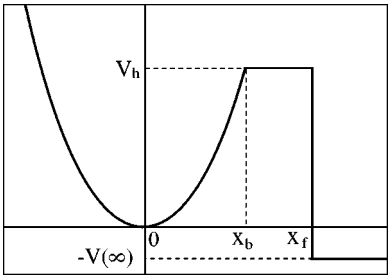

FIG. 2. The tunneling potential used in the numerical calculation.

$$f_W^{(i)}(q_i,p_i)=\frac{1}{\pi}\exp\left[-\omega_p q_i^2-\frac{p_i^2}{\omega_p}\right]. \tag{42}$$

The effect of the nonlinear term, the quadratic term $g\tilde{\omega}_R^3\,x^4$, is clarified by a comparison to the harmonic approximation, because the term is absent in the approximation.

A. Relevance of the parametric resonance

We point out that a relation of the semiclassical Wigner function to the parametric resonance, as briefly mentioned in Ref. [10]. The presence of anharmonic terms is inevitable in any realistic tunneling potential. This introduces a nontrivial oscillating term in the coefficient function $V''(q_{\rm cl}^{(0)})$ of the fluctuation equation (25), thus giving a differential equation akin to the Mathiew equation. Depending on whether or not the relevant parameters fall into the instability band of this Mathiew-type equation, the fluctuation may indefinitely increase in the zero friction limit [24]. What happens is more subtle; the parameters fall right on the boundary of stability and instability bands. We shall show this by taking an example of the tunneling potential (42).

We now consider Eq. (25) of the fluctuation $z(\omega,t)$, using the potential of Eq. (41),

$$V''(q_{\rm cl}^{(0)})=\tilde{\omega}_R^2+12g\tilde{\omega}_R^3(q_{\rm cl}^{(0)})^2. \tag{43}$$

We first take the zero friction limit $\eta=0$ and consider the homogeneous equation of the fluctuation $z_h(t)$,

$$\ddot{z}_h(t)+[h+2\theta(q_{\rm cl}^{(0)})^2]z_h(t)=0,\quad \ddot{q}_{\rm cl}^{(0)}+V'(q_{\rm cl}^{(0)})=0, \tag{44}$$

where

$$h=\tilde{\omega}_R^2,\quad \theta=6g\tilde{\omega}_R^3\quad (g=0.01). \tag{45}$$

The second equation in Eq. (44) gives a periodic solution for $q_{\rm cl}^{(0)}$. A notable feature of the homogeneous equation (44) in the $\eta=0$ limit is that $q_{\rm cl}^{(0)}$ appearing in the coefficient function is periodic, hence it gives rise to solutions $z_h(t)$ either of the Bloch wave type or of the parametric resonance type [24]. Despite the definite values of h and θ in Eq. (45), we regard (h,θ) as free parameters to thoroughly investigate the stable-unstable band structures, because that would more definitely clarify the relation between the band structure and the relevant parameter of this model.

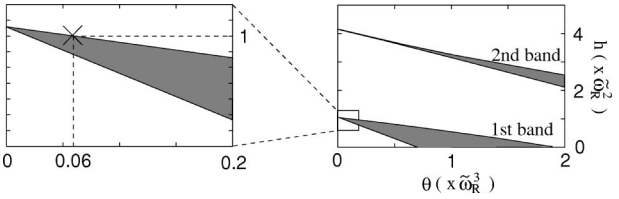

FIG. 3. Band structure for the relevant fluctuation. The shaded areas correspond to the instability band. The first band is enlarged separately (the left figure), where the crossed point corresponds to our model relation (46).

In a relevant region of $0<h<5\tilde{\omega}_R^2$, $0<\theta<2\tilde{\omega}_R^3$, which includes the model parameter (46), we numerically checked the time evolution of the homogeneous solution of Eq. (44). We observed the power-law increase of the fluctuation in the model parameter (46). Indefinite increase of the fluctuation $z(\omega,t)$ as time increases implies that the assumed parameter lies either within the unstable band or on the boundary between the bands. The linear rise $z(\omega,t)\propto t$, as indicated by our numerical analysis, suggests that the parameter is on the boundary.

The result of the band structure is given in Fig. 3, and the relevant parameter Eq. (45) (corresponding to the crossed point) is just on the boundary between the stable and first unstable band. This is consistent with the linear rise, as is expected in the case of the exact Mathiew function on the boundary [25]. For this numerical calculation we had to choose an initial condition for $q_{\rm cl}^{(0)}$. We took several conditions for investigating band structures, and we always obtained the same result of the linear power-law increase.

We next consider effects of the friction η. The inclusion of the friction gives a modification of Eq. (44) to

$$\ddot{z}(t)+[h+2\theta(q_{\rm cl}^{(0)})^2]z(t)+\eta\dot{z}(t)=0,$$

$$\ddot{q}_{\rm cl}^{(0)}+V'(q_{\rm cl}^{(0)})+\eta\dot{q}_{\rm cl}^{(0)}=0. \tag{46}$$

The effect of the friction term $\eta\dot{z}(t)$ is studied by changing the variable $z(t)$ to $y(t)\equiv z(t)e^{\eta t/2}$. Then the equation for $y(t)$ becomes

$$\ddot{y}+[(h-\eta^2/4)+2\theta(q_{\rm cl}^{(0)})^2]y=0. \tag{47}$$

When the effect of friction for the behavior of the zeroth order $q_{\rm cl}^{(0)}$ is small, $q_{\rm cl}^{(0)}$ is nearly periodic and the equation for y is nearly of the parametric resonance type. In this case then, the behavior of $z(t)=y(t)e^{-\eta t/2}$ is rather simple; it is a product of the linearly rising function $[y(t)\sim t]$ and the exponentially decreasing function $(e^{-\eta t/2})$. The rate of the exponential decrease is $\eta/2$, hence this product function has a maximum at a time around $2/\eta$. This seems essentially what we observe in the numerical computation for $z(\omega,t)$.

With the presence of friction, the function y approaches the behavior of simple harmonic oscillator as time increases since $q_{\rm cl}^{(0)}\to 0$ by friction. This means, as seen in Fig. 4, that the linear increase of y is saturated at the time of order $2/\eta$.

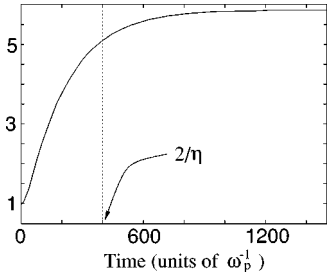

FIG. 4. Time evolution of the function $y(t)$, a solution of Eq. (47). The amplitude of the function $y(t)$ with the friction $\eta = 0.005\omega_p$ is depicted.

The net effect of the friction in the second equation of Eq. (46) is simply the minor change of a coefficient of A/η, where A is close to 1.7.

The effect of the presence of the inhomogeneous term in Eq. (25) is as follows. This term acts as an external force, and gives a resonance effect when the frequency ω is close to the inherent frequency of the system. We illustrate this point. In Fig. 5 the ω dependence of $\omega_p|z(\omega,t)|^2 + |\dot{z}(\omega,t)|^2/\omega_p$ at a particular time is shown, and one observes, in addition to the simple resonance, higher modes due to the nonlinear effect. Without a strong resonance effect such as this one, the fluctuation $z(\omega,t)$ is very small, since we have the initial condition $z(\omega,0)=0=\dot{z}(\omega,0)$. Thus, the environment acts here as a force of the resonant kick from zero amplitude, which then makes it possible for the parametric resonance effect to work, typically very important for large amplitude oscillation. In Fig. 5(a), the growth of the first $(\omega\sim\omega_p)$ and second $(\omega\sim 3\omega_p)$ modes is shown. Other higher modes (third, fourth, ...) are also amplified in the same manner as in the second mode case at $t\sim 360\omega_p^{-1}$. On the other hand, these higher modes disappear at late times, and only the first mode remains, as seen in Fig. 5(b).

B. Numerical results of physical quantities

We now discuss the behavior of the frequency integrated fluctuation $I(t)$ in Eqs. (27)–(29). The increase of the fluctuation due to the parametric resonance gives an interesting time evolution of I. In Fig. 6(a), the time evolution of the quantity $\omega_p I_{11}^{(T)} + I_{22}^{(T)}/\omega_p$ is shown for a few values of the

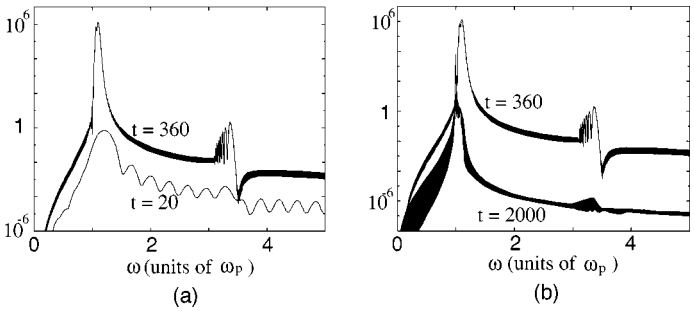

FIG. 5. Examples of the ω dependence of $z(\omega,t)$. The quantity $\omega_p|z(\omega,t)|^2+|\dot{z}(\omega,t)|^2/\omega_p$ at both (a) early and (b) late times are shown. The unit of time is ω_p^{-1}.

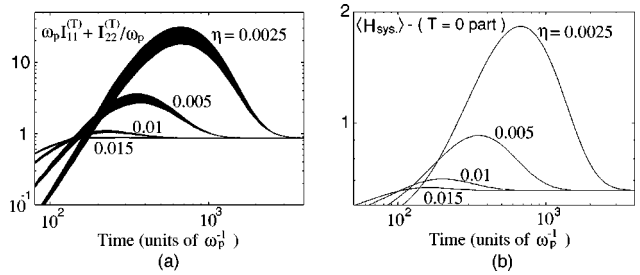

FIG. 6. Time evolution of (a) the fluctuation $\omega_p I_{11}^{(T)} + I_{22}^{(T)}/\omega_p$ and (b) the subsystem energy $\langle H_{\rm sys}\rangle$ ($T=0$ part). A temperature $T=0.5\omega_p$ is taken with four cases of the friction η (in the unit of ω_p).

friction η. This function depends on the initial value q_i, p_i, for which we took a typical value in the ground state at the potential minimum.

What is the physical picture behind the resonance enhancement? We would like to present a suggestive interpretation. We first derive a relation between the fluctuation here and the subsystem energy defined by

$$\langle H_{\rm sys}\rangle \equiv \left\langle \frac{p^2}{2} + \frac{\omega_p^2}{2} q^2 \right\rangle, \tag{48}$$

where $\langle A(q,p)\rangle$ means $\int dqdp f^{(R)}(q,p)A(q,p)$. By making the approximation on $q_{\rm cl}$ and $p_{\rm cl}$, namely, taking up to linear terms of the initial environment value as used in Eq. (23), this equation is reduced to

$$\langle H_{\rm sys}\rangle = (T=0 \ \ \text{part}) + \frac{1}{2}\int_0^\infty \frac{dp_i dq_i}{2\pi} f_W^{(i)}(q_i,p_i) \times \{I_{22}^{(T)}(q_i,p_i;t) + \omega_p^2 I_{11}^{(T)}(q_i,p_i;t)\}. \tag{49}$$

Thus, the temperature-dependent part of the subsystem energy is given by the fluctuation averaged over initial values. In short, the subsystem energy is driven by the fluctuation. If initial values for $\langle q_{\rm cl}^2(0)\rangle$ and $\langle p_{\rm cl}^2(0)\rangle$ are small (in the $\omega_p = 1$ unit), the zero-temperature part ($T=0$) part is close to $0.5\omega_p$ (zero-point fluctuation of the system). In Fig. 6(b) the quantity $H_{\rm sys} - (T=0$ part) is shown, taking as the initial state the ground state of the harmonic oscillator at the potential bottom.

The time at which the maximum of $H_{\rm sys}$ occurs is $1.7/\eta$, which is the same as that of the fluctuation, thus indicating a close correlation between the fluctuation and our subsystem energy $\langle H_{\rm sys}\rangle$. The implication to the occupation probability and the tunneling rate seems obvious; the parametric resonance we observe here enhances excitation to higher energy states at the potential region left of the barrier top in the quantum-mechanical terminology. This would then enhance the tunneling rate from the metastable state of the potential well.

In Fig. 7, a few examples of the projected probability are shown. It is clearly observed that excitation to higher energy levels is correlated to an evacuation of the ground state. The infinite time limit of $\mathcal{W}_n(\infty)$ coincides with the result of the harmonic approximation as is expected. We note that the

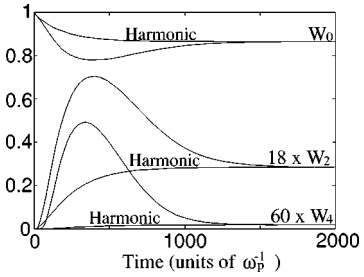

FIG. 7. Time evolution of the projected probability for ground, second excited, and forth excited states. The case of $T=0.5\omega_p$ and $\eta=0.005\omega_p$ is depicted. For comparison the result in the harmonic approximation is shown.

subsystem thermalizes at infinite time despite the fact that it was initially in the ground state of a harmonic oscillator, hence it is in a nonequilibrium state.

We now clarify the relation between the subsystem energy $H_{\rm sys}$ and the occupation probability $\mathcal{W}_n(t)$

$$\sum_{n=0}^{\infty} E_n \mathcal{W}_n(t) = \langle H_{\rm sys} \rangle, \tag{50}$$

where $\langle H_{\rm sys} \rangle$ is calculated by Eq. (49). This relation was numerically checked, as might be expected. We have no analytic proof of this relation, although there is no doubt on the correctness of this relation.

In computing the tunneling rate $\Gamma(t)$ by using Eq. (2) we have to calculate the penetration coefficient $|T(E_n)|^2$, and we used the simple quantum-mechanical formula

$$|T(E)|^2 \approx \exp\left[-2\int_{q_1(E)}^{q_2(E)} dx \sqrt{2(V(x)-E)}\right]\theta(V_h-E) + \theta(E-V_h), \tag{51}$$

where $q_i(E)$ are turning points separating the subbarrier region.

In Fig. 8(a) the tunneling rate $\Gamma(t)$ is shown for a few values of the friction. The tunneling rate is observed to become maximal at the time $t\sim 1.7/\eta$.

A salient feature, and the most important result of the present work, is that the tunneling rate is enhanced around a time of order $2/\eta$, which is caused by nonlinear resonance effects. The enhanced tunneling rate at $t\approx O(2/\eta)$ is clearly related to the power-law rise of the fluctuations I_{ii}, as observed in our previous paper [10] and as already explained. The maximal enhancement factor for the tunneling rate is plotted in Fig. 8(b), relative to the thermal tunneling rate [thus the rate is given by $\Gamma(\infty)$]. A large enhancement is obtained for small η and small T values. The maximal tunneling rate goes as $\eta^{-1.5}$ at fixed T unless η is too small.

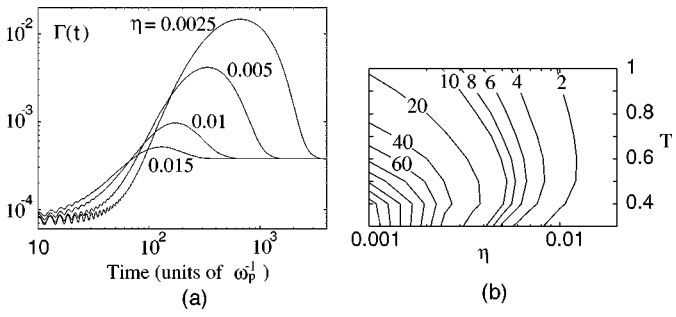

FIG. 8. (a) Time evolution of the tunneling rate $\Gamma(t)$. A temperature $T=0.5\omega_p$ is taken with four cases of the friction η. (b) Ratio of the maximal tunneling rate at $t\sim O(2/\eta)$ to $\Gamma(\infty)$ (a traditional one) is depicted as a contour map in the η-T plane.

At first sight one might think that the maximal tunneling rate goes to infinity if we take the $\eta\to 0$ limit. Our result is valid for a range of $(0.001<\eta)$ in our numerical calculation. Our approximation used in Eq. (23) breaks down when we take a very small η. In this approximation we expand the solution of the Langevin equation by the environment variables $Q_i(\omega)$, $P_i(\omega)$. The subleading term proportional to $Q_i(\omega)$ or $P_i(\omega)$ exhibits the parametric resonance. The effect of the parametric resonance is, however, suppressed by the friction term. If the friction η is very small, this suppression is not enough, giving a bad convergence of the expansion in Eq. (23).

We confirmed that the expansion has good convergence in the range of our numerical calculation $0.002<\eta<0.02$. Therefore, our final form of the tunneling rate is not to be applied to a very small η. Fortunately, many cases of practical interest are in the range of our numerical calculation, for example, the case of electroweak symmetry breaking in the standard model shown in the Appendix. If we are forced to take the formal $\eta\to 0$ limit, a decoupling of the tunneling system and environment occurs, because η is nothing but the coupling strength between the system and the environment. In this limit the tunneling rate becomes time independent and the simple QM tunneling rate is obtained. Thus, there is no peculiarity of the $\eta\to 0$ limit.

We may summarize what happens, in the following way. The nonlinear resonance inherent to the tunneling potential excites higher modes that have larger tunneling probabilities. This effect is, however, terminated by the friction caused by an environmental interaction, which is the origin of the final decrease of the tunneling rate at large times.

We finally study the correlation between the tunneling rate $\Gamma(t)$ and the tunneling system energy $H_{\rm sys}(t)$ at a quantitative level. In order to evaluate the correlation quantitatively, we define the correlation coefficient r used in statistics:

$$r=\frac{\langle \Gamma H_{\rm sys}\rangle - \langle \Gamma\rangle\langle H_{\rm sys}\rangle}{\langle \Gamma\rangle\langle H_{\rm sys}\rangle}, \quad \langle A\rangle \equiv \frac{1}{\rm time}\int_0^{\rm time} dt A(t). \tag{52}$$

The correlation coefficient r was numerically calculated and we obtained results that can be summarized by $r=0.99\sim 1$ in the parameter region of temperature and friction $0.2\leqslant T/\omega_p \leqslant 1.0$ and $0.001\leqslant \eta/\omega_p \leqslant 0.02$. This proves the correlation between the two quantities.

We next turn to the discussion of the survival probability, the probability that the metastable state remains in the same initial state. This quantity $P(t)$ is defined by

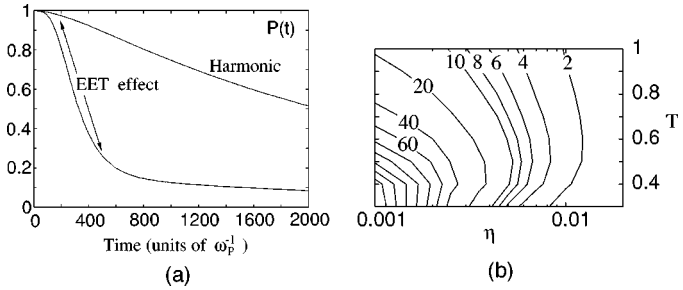

FIG. 9. (a) Time evolution of the survival probability $P(t)$. The case of $T=0.5\omega_p$ and $\eta=0.005\omega_p$ is depicted. For comparison the result in the harmonic approximation is shown. (b) Ratio of the lifetime as defined in Eq. (54) to the traditional one is depicted as a contour map in the η-T plane.

$$\dot{P}(t)=-\Gamma(t)P(t), \quad P(0)=1,$$
$$\Rightarrow \quad P(t)=\exp\left[-\int_0^t dt'\Gamma(t')\right]. \tag{53}$$

In Fig. 9(a) the survival probability is depicted along with the result of the harmonic approximation of the potential bottom. The effect of EET is unquestionable.

A traditional definition of the lifetime $t_{\rm trad}$ which many authors use is $t_{\rm trad}=\Gamma^{-1}(\infty)$, while in the case of time dependent tunneling one may effectively use $t_{\rm LT}$ given by the e-holding time

$$P(t_{\rm LT})=e^{-1}. \tag{54}$$

In Fig. 9(b) a contour plot of $t_{\rm LT}/t_{\rm trad}$ is shown in the parameter (η,T) space. For small values of η and T a substantial reduction of the lifetime $t_{\rm LT}$ is observed, which means an early termination of the tunneling. We hope that we have convinced the reader of the power and the usefulness of EET, even though our idea is supported by a detailed numerical analysis which is necessarily limited to special cases.

V. DISCUSSION

There may be many applications of the environment-driven excited tunneling (EET) presented in this paper. We shall only mention two possible applications to cosmology, an enhancement mechanism of electroweak baryogenesis and a possible resurrection of the old inflationary scenario of the GUT phase transition. Furthermore in the present discussion we shall focus on one aspect of the baryogenesis condition; the out-of-equilibrium condition [14,15]. Needless to say, there will be many problems to solve in an actual application of these ideas to realistic models.

Electroweak baryogenesis is expected to occur if the electroweak phase transition is strong first order. The first order phase transition usually proceeds via the bubble formation. Inside the bubble the true electroweak broken phase is realized in which the baryon number is effectively conserved. On the other hand, outside the bubble the universe remains in the high-temperature symmetric phase in which the baryon number is not conserved via sphaleron related processes [27]. This mismatch inside and outside the bubble gives rise to the possibility of electroweak baryogenesis, although details of a promising mechanism are yet to be worked out. Thus, the strong first order of the phase transition has been considered to be a requisite for the out-of equilibrium condition necessary for baryogenesis.

In an ordinary circumstance, namely, in the standard model, the strong first order phase transition requires a small Higgs mass, which is inconsistent with observation. Along with the small *CP* violation effect of Kobayashi-Maskawa theory, this excludes [28] the possibility [29] of the standard model as a viable model for baryogenesis. Thus, a more complicated model such as two Higgs doublet model needs to be invoked for electroweak baryogenesis. Regardless of whether an extended Higgs model is needed, we would like to point out that the environment-driven excited tunneling (EET) found in this paper has a chance of enhancing the out-of equilibrium condition necessary for baryogenesis.

Although not much discussed in the literature, there is an important factor to consider for the out-of equilibrium condition. This is the factor Γt_H, where Γ is the nucleation rate of the bubble and $t_H=1/H$ is the Hubble time. In the usual scenario of the first order phase transition the effectiveness of electroweak baryogenesis is in proportion to this quantity. The nucleation rate Γ in the usual estimate without EET is exponentially suppressed by the potential barrier, and the factor Γt_H is much less than 1, which means that the first order phase transition is never completed by a merger of nucleated bubbles. Under this circumstance it is expected that the phase transition is terminated by proceeding to disappearance of the local minimum of the symmetric phase so that the condition for the first order phase transition is not met. The effectiveness of the out-of equilibrium condition is then reduced by a factor proportional to the quantity Γt_H.

On the other hand, with the aid of EET, the factor Γt may become of order unity at some finite time, and the phase transition may be completed much prior to the Hubble time, as discussed in previous sections. One should check the new time scale $2/\eta$ which is roughly the time at the maximum for the tunneling rate in comparison to the Hubble time. The factor Γt thus may become of order unity at $t\sim 2/\eta$. In this case there is no reduction factor similar to Γt_H.

We shall check the numerical relation between two time scales t_H and $2/\eta$. The size of the friction η in the standard model is of the order of the Higgs mass m_H times some power of coupling factors, while the Hubble rate is given by

$$H=1/t_H=\sqrt{\frac{4\pi^2}{45}N}\,\frac{T^2}{m_{pl}}, \quad N=106.75. \tag{55}$$

The calculation, as summarized in the Appendix, indicates that

$$\eta\simeq\frac{y_t^2}{32\pi}\omega_p\tanh\left(\frac{\omega_p}{4T}\right),$$
$$\omega_p^2=V''(0)\simeq 0.2\times(T^2-m_H^2-0.04v^2), \tag{56}$$

where y_t is the top Yukawa coupling and $V(\phi)$ is the effective potential of the standard model at finite temperature, with v the vacuum expectation value of the Higgs field. Thus, the Hubble rate is much smaller than η by the inverse Planck factor since $m_H/m_{\rm Pl} \ll y_t^2$.

One may expect that even in the case of weak first order phase transition the quantum tunneling enhanced by EET may give rise to a sufficient condition for the out-of equilibrium in the early phase of the phase transition. Much remains to be seen in this regard.

Another interesting effect of EET concerns the resurrection of the old inflationary scenario of the GUT phase transition [16]. The old inflationary scenario was once rejected because the bubble formation proceeds too slowly, hence leaving a very inhomogeneous universe [17]. The essential reason for this problem, called the graceful exit problem, is that the exponentially suppressed tunneling rate is too small so the bubbles of the true vacuum do not merge sufficiently to give rise to a homogeneous universe which the inflationary scenario intends to derive. Our mechanism of enhanced tunneling may help this situation. The problem must, however, be addressed quantitatively; one has to obtain an early termination of the first order phase transition to promote merger of bubbles, yet one also has to obtain sufficient inflation to solve the intended problems. We leave this quantitative study for the future.

ACKNOWLEDGMENTS

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APPENDIX

1. Renormalization effect

We explain the renormalization effect to the potential $V(q)$ along with the Langevin equation (18). We rewrite the equation with the counter term adopted in Ref. [3], explicitly,

$$\frac{d^2q}{dt^2}+\frac{dV}{dq}+2\int_0^t ds\,\alpha_I(t-s)q(s)-q(t)\int d\omega c(\omega)^2 = F_Q(t). \tag{A1}$$

The third term on the left-hand side is the counter term arising from the interaction with environment. In the local Ohmic model, the kernel function α_I becomes [22,23]

$$\alpha_I(\tau)=\eta\delta'(\tau)+2\,\delta(\tau)\int d\omega\frac{\eta}{\pi}. \tag{A2}$$

The second term on the right-hand side is interpreted as a potential renormalization or a mass renormalization in the field theory analogy, since by changing the bare frequency parameter to the renormalized ω_R^2, the term $\delta\omega^2 q$ is cancelled by the counterterm in the potential.

We now discuss the relation between the renormalized curvature $\tilde{\omega}_R(\omega_R)$ at the potential bottom (barrier top) and the physical curvature $\omega_p(\omega_B)$ here. We first illustrate this for the curvature at the barrier top. In the vicinity of the barrier top, the renormalized potential is given by $V(q)\simeq -\omega_R^2(q-q_B)^2/2+V_h+$counterterms ($\omega_R>0$ by definition). The physical curvature ω_B (the tachyon mass in the field theory analogy, which is defined by a positive value) shifted by the environment is determined as a pole position of the Green's function $\langle 0|T[q(t)q(0)]|0\rangle$ at the barrier top, thus as a solution for the isolated pole of the following equation:

$$\omega_B^2-\omega_R^2+\int_{\omega_c}^{\infty}d\omega\frac{2\omega_B^2 r(\omega)}{\omega(\omega_B^2+\omega^2)}=0. \tag{A3}$$

In many past works, the renormalized curvature ω_R was used as a reference curvature to evaluate the environmental effect. Due to the positivity of $r(\omega)$, one has $\omega_B^2<\omega_R^2$ in general. A consequence of this is that the potential barrier is further suppressed by using ω_B instead of taking the renormalized curvature ω_R as the reference point. It is instructive to give an example. In the Ohmic model of Eq. (20) the explicit form of the physical curvature ω_B is given by

$$\omega_B=\sqrt{\omega_R^2+\frac{\eta^2}{4}}-\frac{\eta}{2}. \tag{A4}$$

The relation between the renormalized and the physical curvature at the local minimum of the potential is obtained in the same way, to give in the Ohmic model

$$\omega_p=\sqrt{\tilde{\omega}_R^2-\frac{\eta^2}{4}}. \tag{A5}$$

2. Effective potential and critical temperature

We shall first give the finite temperature effective potential of the standard model at one-loop order, assuming that the Higgs boson mass is much smaller than other particle masses of the standard model such as the top quark, the weak boson, thus $m_H \ll m_t, m_Z, m_W$. This is taken as a necessary condition for the first order phase transition. This condition, however, is not met in reality. Nevertheless, we take this case as an important illustration for more complicated cases.

The zero-temperature part of the effective potential [30] is

$$V_0(\phi)=\frac{m_H^2}{8v^2}(\phi^2-v^2)^2+2Bv^2\phi^2-\frac{3}{2}B\phi^4+B\phi^4\log\left(\frac{\phi^2}{v^2}\right),$$

$$B=\frac{3}{64\pi^2v^4}(2m_W^4+m_z^2-4m_t^2)\simeq -0.005, \tag{A6}$$

where $v=246$ GeV is the vacuum expectation value (VEV) of the Higgs field, and m_a is the mass of a particle species a. The finite temperature effect is then given by the free energy [31]

$$V_T(\phi)=\frac{T^4}{2\pi^2}\left[6I_B\left(\frac{m_W\phi}{vT}\right)+3I_B\left(\frac{m_z\phi}{vT}\right)-6I_F\left(\frac{m_t\phi}{vT}\right)\right],$$

$$I_{B,F}(a^2)=\int_0^\infty dr r^2\log[1\mp\exp(-\sqrt{r^2+a^2})]. \tag{A7}$$

The Higgs contribution has been neglected, due to the assumption of $m_H \ll$ masses of other particles.

According to Ref. [31], we may make the high-temperature expansion of the effective potential with $T \ll v$, to get

$$V(\phi)=V_0(\phi)+V_T(\phi)\simeq D(T^2-T_0^2)\phi^2-ET\phi^3+\frac{\lambda(T)}{4}\phi^4, \tag{A8}$$

where the dimensionless coefficients are given by

$$D=\frac{2m_W^2+m_z^2+2m_t^2}{8v^2}\simeq 0.2,$$

$$E=\frac{2m_W^3+m_z^3}{4\pi v^3}\simeq 0.01,$$

$$T_0^2=\frac{m_H^2-8Bv^2}{4D}\simeq 1.25[m_H^2+(50\ \text{GeV})^2], \tag{A9}$$

$$\lambda(T)=\frac{m_H^2}{2v^2}-\frac{3}{16\pi^2v^4}\left(2m_W^4\log\frac{m_W^2}{a_BT^2}+m_z^4\log\frac{m_z^2}{a_BT^2}-4m_t^4\log\frac{m_t^2}{a_FT^2}\right).$$

Here $a_B=2\log 4\pi-2\gamma\simeq 3.91$, $a_F=2\log\pi-2\gamma\simeq 1.14$. The critical temperature is computed from these, to give

$$T_c^2=\frac{T_0^2}{1-E^2/[\lambda(T)D]}\simeq\frac{T_0^2}{1-(8\ \text{GeV}/m_H)^2}. \tag{A10}$$

At the value of T_c two local minima coincide, while T_0 indicates where the symmetric phase of $\varphi=0$ disappears as the local minimum. It is thus important to note that the condition $T_c>T_0$ for the first order phase transition is met.

3. Friction coefficient in the standard model

The friction coefficient η is computed using the method of Refs. [32,33]. According to Ref. [33], it is possible to identify the two-body bilinear operator of standard model fields as the environment variable $Q(\omega)$. The most dominant contribution to the friction η comes from the top loop diagram where the top-Yukawa coupling is of the form $y_t\phi\bar{t}t/\sqrt{2}$. Thus, the Langevin equation of motion for the zero-mode Higgs field ϕ_0 becomes

$$\ddot{\phi}_0+V'(\phi_0)+2\int_0^t ds\ \mathrm{Im}\,\alpha(t-s)\phi_0(s)=F(t), \tag{A11}$$

$$\langle\{F(t),F(s)\}\rangle_{\mathrm{env}}=\mathrm{Re}\,\alpha(t-s), \tag{A12}$$

$$\alpha(x_0)=\frac{y_t^2}{2}\int d^3x\langle\bar{t}t(x)\bar{t}t(0)\rangle_{\mathrm{env}}=\int d\omega\frac{r_H(\omega)}{e^{\beta\omega}-1}e^{i\omega x_0}. \tag{A13}$$

The real-time Green's function $\langle\bar{t}t(x)\bar{t}t(0)\rangle_{\mathrm{env}}$ can be obtained by the analytic continuation of the imaginary-time Green's function [21], and it is easy to calculate the imaginary-time Green's function by using the Matsubara formalism. The necessary spectral weight $r_H(\omega)$ is given by

$$r_H(\omega)=\frac{y_t^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{\omega^2}{4p^2}\delta(\omega-2p)(1-2n_p), \tag{A14}$$

where $p=|\vec{p}|$ is the momentum magnitude and $n_p=(e^{\beta p}+1)^{-1}$, assuming that the top quark is massless in the symmetric phase around $T\approx T_0$.

In the vicinity of the false vacuum, namely, $\varphi\approx 0$, the behavior of the zero-mode Higgs field exhibits exponentially dumped oscillation. The solution of the homogeneous zero-mode Higgs field equation (A11) is approximately obtained by

$$\begin{aligned}\phi_0(t)&\simeq 2\int_0^\infty d\omega\frac{r_H(\omega)\sin(\omega t)}{(\omega^2-\omega_p^2)+\pi^2r_H^2(\omega)}\\&\simeq 2\int_0^\infty d\omega\frac{r_H(\omega_p)\sin(\omega t)}{(\omega^2-\omega_p^2)+\pi^2r_H^2(\omega_p)}\\&=\sin(\omega_pt)\exp\left(-\frac{\pi r_H(\omega_p)}{2\omega_p}t\right),\end{aligned} \tag{A15}$$

where ω_p is the curvature of the false vacuum [$\omega_p^2=V''(0)$], and the friction coefficient in this case is computed from Eq. (A14). The result is

$$\eta=\frac{\pi}{\omega_p}r(\omega_p)=\frac{y_t^2}{32\pi}\omega_p\tanh\frac{\beta\omega_p}{4}. \tag{A16}$$

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