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String production at the level of effective field theory

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Pair creation of strings in time-dependent backgrounds is studied from an effective field theory viewpoint, and some possible cosmological applications are discussed. Simple estimates suggest that excited strings may have played a significant role in preheating, if the string tension as measured in the four-dimensional Einstein frame falls a couple of orders of magnitude below the four-dimensional Planck scale.

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I. INTRODUCTION

In curing the nonrenormalizability of gravity, string theory introduces a large number of heavy states: the excited states of a string. The number of these states increases roughly as an exponential of their energy, in contrast to field theories, where the number of states rises as a power of the energy. This exponential increase is often described as a Hagedorn density of states. The extra states in string theory are conventionally assumed to lie near the Planck scale. Our grasp of the nonperturbative dynamics of string theory is far from complete, but one well-motivated conjecture is that interactions vastly reduce the degrees of freedom of the theory, so that the number of available states scales ‘‘holographically’’ as the area of a system rather than its volume. Disentangling this ‘‘more is less’’ paradox, elucidating the true degrees of freedom of string theory in a nonperturbative regime, and reconciling the renormalization group with holography may all be necessary steps before we can give a fully satisfactory string theoretic account of the very early universe. We are a long way from achieving this, but certainly there is ample reason to believe that theories with extended objects are needed to properly formulate quantum gravity, and that such theories often have a Hagedorn density of states over an energy range encompassing a great many states. Armed with no more than this, we would like to enquire what the possible consequences are for cosmology—in particular, for situations where the massive states may be produced through the usual ambiguity of the vacuum state in backgrounds with time evolution. We will start with an overview of pair production and the steepest descent method for estimating occupation numbers. Then we will move on to a possible application to the theory of preheating.

In regimes of parameter space where a spacetime description gives a good approximation of string dynamics, the on-shell constraint for a given string state boils down to a second order ordinary differential equation in time:

$$\ddot{\chi}+\omega(t)^2\chi=0, \tag{1}$$

where $\chi(t)$ is the wave function for the state in question. This is also the equation for a mode of a scalar field, possibly rescaled by some time-dependent factor to eliminate the $\dot{\chi}$ term. The function $\omega(t)$ must be determined for any given background and string state. Quantizing strings in a general spacetime background is fraught with difficulties because the spacetime equations of motion are already encoded in the conditions for conformal invariance on the worldsheet, and we seem to be missing some aspects of string dynamics which give rise to the weakly curved spacetime we observe. If we can quantize strings in a particular background, then $\omega(t)$ will be determined, and it will include contributions from the momentum of the string and its excitation state. In the following treatment, we will simply assume that the spectrum is known, and that it includes the familiar Hagedorn density of states. We are interested in establishing conditions under which the total string production is finite (without invoking a cutoff on the Hagedorn spectrum), and in estimating the total rate of string production when it is dominated by highly excited strings. For this reason, we take $\omega(t)^2$ to be large.

To speak meaningfully about string pair production, it is necessary at least to have an asymptotic ‘‘out’’ region where ω is slowly varying, in the sense that time derivatives of ω are much smaller than the power of ω with the same dimension. When there is such a region, one can compare the infinite order out adiabatic vacuum to the actual quantum state to determine occupation numbers for a given string state. What the actual quantum state is can be subtle, but if there is an asymptotic ‘‘in’’ region where again ω is slowly varying, one can follow the standard approach of letting the actual quantum state correspond to the in vacuum. Let us consider this optimal situation first and further assume that t runs from $-\infty$ to $+\infty$.

Two widespread analytical techniques for extracting approximate pair production rates from Eq. (1) are steepest descent contours, applicable when particle occupation numbers are small, and parametric resonance, applicable when $\omega(t)^2$ is an oscillatory function of time. We will be mainly interested in the former and will develop it in Sec. II. We will then give an application to the theory of preheating in Sec. III, where conventionally one needs parametric resonance to understand the physics—but, as we shall see, the steepest descent method is still suitable for discussing production of excited string states. The conclusion will be that there is a plausible regime of parameters where strings of some type played a significant roll in preheating, but that overproduction of superheavy dark matter is a potential problem.

After this work was complete, I learned of [1],[1] which overlaps significantly with the methodology developed in Sec. II.

[1] I thank E. Martinec for bringing this paper to my attention.

II. THE STEEPEST DESCENT METHOD

The steepest descent method is based on early work by [2] and has been developed by various authors. Our treatment will to an extent parallel that of [3]. The key assumption is that the occupation number $|\beta|^2$ for a given mode is always much less than 1. Here, β is a Bogliubov coefficient for comparing the in and out vacua.[2] Setting

$$\chi(t)=\frac{\alpha(t)}{\sqrt{2\omega(t)}}\ \exp-\left(i\int^t du\ \omega(u)\right)+\frac{\beta(t)}{\sqrt{2\omega(t)}}\ \exp\left(i\int^t du\ \omega(u)\right), \tag{2}$$

with the requirement $|\alpha(t)|^2-|\beta(t)|^2=1$, one may recast Eq. (1) as

$$\dot{\alpha}(t)=\frac{\dot{\omega}}{2\omega}\exp\left(2i\int^t du\ \omega((t))\right)\beta(t),$$

$$\dot{\beta}(t)=\frac{\dot{\omega}}{2\omega}\exp-\left(2i\int^t du\ \omega((t))\right)\alpha(t). \tag{3}$$

Using $\beta(t)\ll 1$ and $\alpha(t)\approx 1$, one quickly arrives at the general formula for $\beta=\beta(\infty)$:

$$\beta\approx\int_{-\infty}^{\infty} dt\,\frac{\dot{\omega}}{2\omega}\exp\left(-2i\int^t du\ \omega(u)\right). \tag{4}$$

Assuming that spacetime is weakly curved in the out region, that $|g_{tt}|\to 1$ there, and that ω approaches some constant ω_∞ for any given string state, the density of string states rises roughly as e^{ω_∞/T_H}, where T_H is the Hagedorn temperature,[3]

$$T_H=\frac{1}{2\pi\sqrt{\alpha' c_\perp/6}} \tag{5}$$

for closed strings, where the string tension is $\tau=1/(2\pi\alpha')$ and $c_\perp$ is the central charge of the transverse degrees of freedom ($c_\perp=12$ for the type II superstring), assumed here to be the same in the holomorphic and antiholomorphic sectors. This exponential behavior is modified by a power law that depends on the string theory in question as well as on the dimensionality of noncompact spacetime. Assuming $|\beta|^2$ can be approximated by some function of ω_∞, the total number of strings produced may be very roughly estimated to be

$$N_{\rm tot}\sim\int^\infty d\omega_\infty|\beta|^2 e^{\omega_\infty/T_H}. \tag{6}$$

In s-brane decay (with ω_∞ replaced by ω_0, as explained in footnote 3), the exponential growth of states is exactly canceled by exponential suppression of β at large ω, leaving a power law behavior that may or may not converge, depending on the dimension [7,5,6]. Our estimates will focus on the exponential behavior in more generic circumstances.

A useful bookkeeping device for exploring the high-energy properties of Eq. (1) is to rescale $t\to t/\sigma$, where σ is a dimensionless constant. A naive expectation, which will turn out to be close enough to the truth in some interesting cases, is that the on-shell condition for highly excited string states is given by the rescaled equation

$$\ddot{\chi}+\sigma^2\omega(t)^2\chi=0, \tag{7}$$

with σ^2 set equal to the excitation level N, and with ω remaining nearly the same for different string states. More precisely, the assumption is that *without* any rescaling of time, highly excited string states have $\omega\approx\sigma\bar{\omega}$ with $\sigma=\sqrt{N}$ and $\bar{\omega}$ nearly independent of the string state. Then Eq. (7) is correct, with $\omega\to\bar{\omega}$, though for a reason orthogonal to time rescaling. Granting such a setup, the total number of strings produced is

$$N_{\rm tot}\sim\int^\infty d\sigma|\beta(\sigma)|^2 e^{\sigma\bar{\omega}_\infty/T_H},$$

$$\beta(\sigma)\approx\int_{-\infty}^{\infty}dt\,\frac{\dot{\bar{\omega}}}{2\bar{\omega}}\exp\left(-2i\sigma\int^t du\ \bar{\omega}(u)\right). \tag{8}$$

In Eq. (8), $\bar{\omega}_\infty/T_H$ is a fixed number, independent of the string state. Certainly, Eq. (8) has been arrived at through a series of assumptions that are far from self-evident. However, it has some nice consequences that we believe are more general. First of all, $\bar{\omega}(t)^2$ must be infinitely differentiable on the real axis in order to avoid producing an infinite number of strings. The same arguments used in [8] to show that occupation numbers in the adiabatic vacuum of order A scale as $\omega^{-(A+1)}$ can be adapted to show that β scales as σ^{-n} for large σ when there is a discontinuity in the nth derivative of $\bar{\omega}(t)^2$. Probably it is also necessary for $\bar{\omega}(t)^2$ to be analytic for real t—we will have more to say about this point later. It is intriguing that analyticity is also characteristic of the statistical mechanics of systems with a finite number of degrees of freedom, hinting once again that the number of degrees of freedom for gravitationally coupled strings is finite.

To evaluate the expression for β in Eq. (4), a technique based on contour integration and steepest descent was developed in [3]. At least in simple circumstances, the singularities of the outer integrand are poles and branch points, occurring in the complex plane where $\omega(t)^2=0$ or ∞. These singularities are distributed symmetrically on either side of the real axis because $\omega(t)^2$ is real valued for real arguments. Singularities arising from simple zeros of $\omega(t)^2$ were treated in detail in [3], and through a steepest descent method the following estimate was obtained:

[2]In the standard analogy to one-dimensional scattering, where time is mapped to position and Eq. (1) is regarded as a time-independent Schrödinger equation with potential $-\omega^2(t)$, β is roughly the reflection amplitude.

[3]Unusual circumstances might invalidate this description of the Hagedorn density, for instance the boundary operator in [4–6] that gives a given open string state a mass that grows exponentially with time. In this particular circumstance, ω_∞ in the discussion above could be replaced by $\omega=\omega_0$ evaluated at the time-symmetric point of the full s-brane solution.

$$\beta\approx\frac{i\pi}{3}\exp\left(-2i\int_{-\infty}^{r}dt\,\omega(t)\right)\exp\left(-2i\int_{r}^{t_*}dt\,\omega(t)\right), \tag{9}$$

where $t_* = r - i\mu$ is the location of a zero of $\omega(t)^2$ in the lower half plane, and r and μ are real. If there are several zeros, one gets a sum of terms of the type appearing on the right hand side of Eq. (9). Assuming that one zero dominates, we may roughly estimate

$$|\beta|^2\approx\left(\frac{\pi}{3}\right)^2 e^{-\pi\mu\omega(r)}. \tag{10}$$

The factor of π in the exponent arises from approximating Re $\omega(t)$ along the line between r and t_* by an elliptical arc which passes through zero at t_* and has its apex at r. A better estimate may be obtained if more information is available for the function $\omega(t)^2$. With the estimate (10) in hand, we can return to Eq. (8) and obtain

$$N_{\rm tot}\sim\int^{\infty}d\sigma\, e^{\sigma[-\pi\mu\bar{\omega}(r)+\bar{\omega}_\infty/T_H]}. \tag{11}$$

Evidently, this converges provided the exponent is negative. If we wished to compute the total energy of the strings created, it would alter only the power law prefactor in the integrand of Eq. (11), and the criterion for convergence would still be that the exponent is negative.

We derived Eq. (11) on the understanding that $|g_{tt}|\to 1$ as $t\to\infty$. If this is not so, but there is still an appropriate asymptotic out region, then we need only replace $\bar{\omega}_\infty$ by $\sqrt{g_\infty^{tt}}\bar{\omega}_\infty$ (defined as a limit). Then the integral in Eq. (11) would converge if

$$\frac{\sqrt{g_\infty^{tt}}\bar{\omega}_\infty}{\pi\mu\bar{\omega}(r)}<T_H. \tag{12}$$

The left hand side acts in some rough sense like a temperature for the background in question. Similar results can be established for singularities of $\bar{\omega}(t)^2$ in the complex t plane: the result is some coefficient other than π in Eq. (12). We will encounter such an alteration of Eq. (12) at the end of this section.

The type of result expressed in Eq. (12) provides a good intuitive argument, albeit slightly circular, for why $\bar{\omega}(t)^2$ should be analytic on the real line: zeros and singularities of $\bar{\omega}(t)^2$ in the complex plane have to be far enough away from the real axis for Eq. (12) to be satisfied. So the radius of convergence of $\bar{\omega}(t)^2$ is finite everywhere on the real line.

As an interesting class of examples, consider a $k=0$ Friedmann-Robertson-Walker (FRW) cosmology:

$$ds^2=a(\eta)^2(-d\eta^2+d\vec{x}^2). \tag{13}$$

The modes of a conformally coupled scalar with mass m satisfy Eq. (1) with t replaced by η and $\omega^2=k^2+m^2a^2$.[4] Assuming that the metric in Eq. (13) is the string frame metric, and that the curvature is substringy, the mass spectrum is approximately given by $m^2=N/\alpha'$, where N is the level.[5] For highly excited string states, neglecting k is a good approximation, though not a uniform one if $a(\eta)$ becomes arbitrarily small in the far past. Let us assume that $a(\eta)$ approaches nonzero constant values, $a_{-\infty}$ or a_∞, as $\eta\to\pm\infty$. The scaling analysis above is appropriate, with $\bar{\omega}(\eta)=a(\eta)/\sqrt{\alpha'}$. Assuming that a single zero $\eta_*=r-i\mu$ of $a(\eta)^2$ dominates, the condition (12) for there to be a finite number of strings created is

$$\frac{1}{\pi\mu a(r)}<T_H. \tag{14}$$

Specializing further, let us consider one of the classic exactly solvable problems[6]

$$a(\eta)^2=A+B\tanh(\rho\eta). \tag{15}$$

The exact result leads to exponentially suppressed particle production for large masses:

$$|\beta|^2=\frac{\sinh^2[\pi(\omega_\infty-\omega_{-\infty})/2\rho]}{\sinh(\pi\omega_\infty/\rho)\sinh(\pi\omega_{-\infty}/\rho)}\approx e^{-2\pi\omega_{-\infty}/\rho}$$
$$=e^{-2\pi\sigma\bar{\omega}_{-\infty}/\rho}, \tag{16}$$

where $\omega(\eta)=\sqrt{k^2+a(\eta)^2m^2}$ and $\omega_{\mp\infty}$ are the limits of $\omega(\eta)$ in the far past and future, and the approximate equality holds good in the limit where $\omega_{\pm\infty}\gg\rho$. The zero of $a(\eta)^2$ in the lower half plane closest to the real axis is $\eta_*=-i\pi/2\rho+(1/2\rho)\log[(A-B)/(A+B)]$, so from Eq. (10) we obtain

$$|\beta|^2\approx e^{-c_1 2\pi\sigma\bar{\omega}_{-\infty}/\rho}\quad\text{where}\quad c_1=\frac{\pi}{4}\sqrt{1+B/A}. \tag{17}$$

Because $0\leq B/A\leq 1$, we have $\pi/4\leq c_1<\pi/\sqrt{8}$, indicating fairly good agreement with the exact result (perfect agreement would be $c_1=1$). A criterion of the form (12) or (14) emerges immediately from estimating the total number of strings produced from Eqs. (11) and (16), only with a factor of c_1 multiplying the left hand side. Thus we conclude from this example that the steepest descent method gives a reasonable approximation of the criterion for finiteness of the number of strings produced.

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[4]Different choices of the parameter ξ in the term $\xi\chi^2R$ that controls the coupling to the Ricci scalar result in finite shifts in ω^2. As long as ξ does not grow too quickly as one goes to more highly excited states, it should not affect the analysis at the level we are working at.

[5]Neither the zero point nor the normalization of N quite agrees with the conventional definition of the excitation level in type II string theory. This minor discrepancy is of no consequence as long as we correctly keep track of the normalization of T_H.

[6]This treatment is similar to the one in [3].

Let us conclude this section with a background that includes de Sitter space in a certain limit, but has well-defined in and out regions away from this limit. The geometry is defined by Eq. (13) with

$$a(\eta)^2 = A + B\frac{\eta}{\sqrt{\eta^2 + 1/\rho^2}}, \tag{18}$$

where A, B, and ρ are constants. If $A > B > 0$, this background is qualitatively similar to Eq. (15), but in the limit where $A = B > 0$, the metric interpolates between the $k=0$ patch of dS_4 in the far past and flat space in the far future. To be precise, for $A = B$ and $\eta \ll -\rho$, we have

$$ds^2 \approx \frac{L^2}{\eta^2}(-d\eta^2 + d\vec{x}^2) \quad \text{where} \quad L = \sqrt{\frac{A}{2\rho^2}}. \tag{19}$$

Although the background defined by Eqs. (13) and (18) is not motivated by string theory considerations, it might be interesting as a simple ‘‘regulation’’ of de Sitter space: one can for example define an S matrix for $A > B > 0$ and then take the limit $A \to B$ to investigate aspects of quantum field theory in de Sitter space.

The singularity of $a(\eta)^2$ in the lower half plane closest to the real axis is a branch cut starting at $\eta_* = -i/\rho$. [There is also a zero at $\eta = -(i/\rho)(A/\sqrt{A^2 - B^2})$, but this is always further from the real axis, and it disappears altogether in the $A = B$ limit.] The condition (14) becomes $T_s < T_H$, where

$$T_s = \frac{1}{\pi \mu a(r)} = \frac{\rho}{\pi\sqrt{A}}. \tag{20}$$

As remarked earlier, T_s is an effective temperature for the spacetime. Interestingly, the estimate (20) does not depend on B at all—although, clearly, B must control a prefactor on the total number of strings produced that vanishes as $B \to A$. In this limit, T_s as estimated in Eq. (20) differs by $\sqrt{2}$ from the temperature of the de Sitter horizon in the far past, which is $T_{dS} = 1/(2\pi L)$ with L as defined in Eq. (19). The discrepancy is entirely due to the crudeness of estimating the integral in the second factor of Eq. (9) using an elliptical arc passing through zero: evaluating that integral exactly for $A = B$ changes the estimate (20) so that $T_s = T_{dS}$ exactly.

It is tempting to interpret the divergence in Eq. (6) that arises when Eq. (10) is violated as dual to the development of an open string tachyon stretched in the complex plane between the singularity point t_* and its complex conjugate $\overline{t_*}$. Such an interpretation has been offered in time-dependent backgrounds resulting from well-defined stringy constructions [6], and the idea of D-branes in imaginary time has been further developed in [9].

In general, one should consider the possibility that the singularities of $\overline{\omega}(t)$ [or, more precisely, of the integrand in Eq. (8)] in the complex t plane are not pointlike or even branch cuts, but cover finite regions of the plane. This could happen, for instance, if we replaced the $a(\eta)^2$ in Eq. (15) by a continuous sum over different values of ρ. A criterion like (12) should still apply, where μ is roughly the closest approach of the singular region to the real axis.

III. STRINGS AND PREHEATING

A widely studied application of particle creation in cosmology is the theory of preheating (see, for example, [10–12]), whereby coherent fluctuations of the inflaton ϕ around its minimum lead to an oscillating mass term for another bosonic field χ through a term in the action proportional to $\phi^2\chi^2$. The resulting variation in $\omega(t)^2$ can set up a parametric resonance, which produces exponentially growing particle occupation numbers for χ. The variation of $\omega(t)^2$ cannot be expected to be perfectly periodic, since the universe is expanding and there may be significant deviations from pure quadratic behavior in the inflaton potential in the region where the inflaton oscillates. This limits the amplification that parametric resonance can provide. One must then ask under what conditions parametric resonance is ‘‘efficient,’’ so that an appreciable fraction of the energy stored in the inflaton oscillations will be converted to χ particles via preheating. Oversimplifying a little, the answer is that the mass of χ must vary by a large factor: the maximum mass in each cycle of oscillation must be many times greater than the minimum mass. In such a situation, there is a ‘‘broad resonance’’ [11], and a variety of complicated effects like back reaction and rescattering become significant. In [12], more precise analytic criteria were developed to decide whether a resonance is efficient. Let us explore how the criteria for efficient extraction of energy from the inflaton oscillations might be altered due to the presence of a Hagedorn density of states.

Occupation numbers of excited string states should be small even if strings do participate significantly in preheating. The reason is that resonances become exponentially narrow, and the rate of growth of occupation numbers within a resonance becomes exponentially slow, as one increases the tree-level mass. There can still be a large total number of strings produced because of the competing exponential growth of the density of states. To distinguish approximately where the competing exponentials have equal ‘‘strength,’’ it is sufficient to work with $|\beta|^2 \ll 1$ for all individual string modes, except perhaps for those that are massless at tree level. Our plan of attack, then, is to use the methods developed in Sec. II to study production of the massive excited states.

It is very plausible that the effective string tension, as measured in the four-dimensional Einstein frame, varies as the inflaton varies. Let us as usual assume a $k=0$ FRW cosmology. When the string frame metric is weakly curved, the spectrum is given approximately by $g^{tt}_{\rm str}\omega^2 = g^{xx}_{\rm str}k^2 + m^2 = g^{xx}_{\rm str}k^2 + N/\alpha'$, where as usual N is the excitation level. The four-dimensional Einstein metric is related to the four-dimensional part of the string metric by a conformal transformation: $ds^2_{4E} = e^{\gamma\varphi/M_{\rm Pl,4}} ds^2_{\rm str}$ where $M_{\rm Pl,4}$ is the four-dimensional Planck mass, φ is a canonically normalized scalar which is some combination of the dilation and the volume of the internal manifold, and γ is some constant, presumably $O(1)$ in generic compactifications.[7] We may adjust the zero point of φ so that $\varphi=0$ at the minimum around which one oscillates during reheating. Evidently, $g_{4E}^{tt}\omega^2 = g_{4E}^{xx}k^2 + e^{\gamma\varphi/M_{\text{Pl},4}}N/\alpha'$. Using the usual ansatz $ds_{4E}^2 = -dt^2 + a(t)^2 d\vec{x}^2$, we end up with

$$\omega(t)^2 = k^2 a(t)^2 + e^{\gamma\varphi(t)/M_{\text{Pl},4}}\frac{N}{\alpha'}. \tag{21}$$

Now, φ is *not* necessarily the inflaton: there are many scalars in four dimensions that specify the size and shape of the internal manifold. But it seems safe to assume, at least on grounds of genericity, that there is some overlap between φ and the inflaton ϕ. If ϕ oscillates with frequency Ω between slowly varying extremes φ_{min} and φ_{max}, then we may approximate (21) by

$$\omega(t)^2 = \frac{\omega_+^2 + \omega_-^2}{2} + \frac{\omega_+^2 - \omega_-^2}{2}\cos\Omega t,$$

$$\omega_-^2 = k^2 a(t)^2 + e^{\gamma\varphi_{\text{min}}/M_{\text{Pl},4}}\ \frac{N}{\alpha'}, \tag{22}$$

$$\omega_+^2 = k^2 a(t)^2 + e^{\gamma\varphi_{\text{max}}/M_{\text{Pl},4}}\ \frac{N}{\alpha'}.$$

Neglecting derivatives of slowly varying quantities like Ω and $\omega_\pm$, the wave function of a given bosonic string mode, suitably scaled by some time-dependent factor, satisfies Mathieu's equation

$$\left(\frac{d^2}{dz^2} + A - 2q\cos 2z\right)\chi = 0 \tag{23}$$

with $z = \Omega t/2$ and A and q specified by

$$A \pm 2q = A_\pm = \left(\frac{\omega_\pm}{2\Omega}\right)^2. \tag{24}$$

For (A,q) in special regions of the plane, solutions to Eq. (23) exist whose average value grows with z as $e^{\nu z}$, where ν is the imaginary part of the so-called Floquet exponent. These solutions are a manifestation of parametric resonance, but we will not need them for the reasons reviewed above. Our estimates will be based wholly on the steepest descent method, which should be reliable because $|\beta|^2$ has to be small near the boundary of the region where string production is important for reheating.

After the usual "rescaling" $\omega \to \sigma\bar{\omega}$, we find ourselves in the situation described in Eqs. (7)–(12). In Eqs. (22)–(24), we have neglected the damping of the inflaton oscillations due to expansion and to dissipation of energy into the string states. Including these effects will make the first oscillation produce more strings than any of the subsequent ones. To be more precise: the zeros of $\omega(t)^2 = \sigma^2\bar{\omega}(t)^2$ as given in Eq. (22) are

$$t_{*,n}^{\pm} = \frac{2\pi n}{\Omega} \pm \frac{i}{\Omega}\log\frac{\bar{\omega}_+ + \bar{\omega}_-}{\bar{\omega}_+ - \bar{\omega}_-}, \tag{25}$$

but we are going to assume that $t_* = t_{*,0}^- = r - i\mu$ makes the biggest contribution to string production. In order for the formal expression (8) to converge, Eq. (12) must be satisfied, which is to say

$$\bar{\omega}(r)\mu = \frac{\bar{\omega}_-}{\Omega}\log\frac{\bar{\omega}_+ + \bar{\omega}_-}{\bar{\omega}_+ - \bar{\omega}_-} > \frac{\sqrt{g_\infty^{tt}}\,\bar{\omega}_\infty}{\pi T_H}. \tag{26}$$

The right hand side is a quantity of order unity. We should regard it as uncertain by at least a factor of 2, given that Eq. (22) is only a rough guess for $\omega(t)^2$, and the factor of $1/\pi$ is only approximate, as the analysis of the example (15) showed.

If the bound (26) is satisfied with room to spare, then the highly excited string states are hardly produced at all. Conversely, if it is violated, a huge number of excited string states are produced, rendered finite, if need be, by a cutoff on the Hagedorn spectrum. A cutoff at an energy Λ limits the number of strings produced to something like e^{Λ/T_H}, which is very large if Λ is considerably higher than T_H. So it seems likely that the string production is more likely to be limited by the available energy in the coherent inflaton oscillations. Thus a violation of Eq. (26) is very likely to mean that excited string states will play a vital role in preheating, in the sense that a significant fraction of the energy in the inflaton oscillations goes into excited strings. We obtained Eq. (26) from a single oscillation of the inflaton. Further oscillations of the inflaton are then presumably rapidly damped, and even if a finite number of zeros of $\omega(t)^2$ contribute significantly to the total string production, the result will probably be to change only a prefactor in the expression for the total number of strings produced: Eq. (26), which comes from the exponent of Eq. (11), is less likely to be affected.[8]

[7]The expression $ds_{4E}^2 = e^{\gamma\varphi/M_{\text{Pl},4}}ds_{\text{str}}^2$ is imprecise because the combination of scalars in the exponent may not be an eigenvector of the scalar kinetic operator in the four-dimensional Einstein frame. But the oversimplification will not matter in the subsequent treatment.

[8]We might worry that parametric resonance could alter Eq. (26). But the way that occupation numbers $|\beta|^2$ significantly larger than (10) arise in parametric resonance is by having $|\alpha|^2$ significantly larger than 1, so that the approximation $\alpha(t)\approx 1$ that led to Eq. (4) fails. This is related to having exponential growth $e^{\nu z}\approx e^{\nu\Omega t/2}$ of certain mode functions. Occupation numbers of order unity for excited string states should result in a huge divergence in the total number of strings produced. Exponentially suppressed occupation numbers for highly excited strings mean that the approximations that led to Eq. (26) should be valid. One can try to test this self-consistent reasoning further by adapting the estimate of [12] to a Hagedorn spectrum of bosonic states. The result is $dN_{\text{tot}}/dt \sim \int^\infty d\sigma\,\nu(\sigma)e^{\sigma\bar{\omega}/T_H}$, where $\nu(\sigma)$ is the Floquet exponent for a string state at level σ^2, as determined by the A and q appearing in a rescaled version of Eq. (24). This expression leads to agreement with Eq. (26) up to factors of order unity, but it is hard to evaluate those factors.

Neglecting the logarithm in Eq. (26) as well as various factors of order unity leads to the approximate convergence condition

$$\sqrt{\tau_-} \gtrsim m_{\text{inflaton}}, \tag{27}$$

where τ_- is the minimum string tension as measured in four-dimensional Einstein frame. (The point here is that $\overline{\omega}_- \propto \sqrt{\tau_-}$ while Ω is roughly the inflaton mass at its minimum.) The criterion for there to be copious string production due to coherent oscillations of the inflaton is that the inequality (27) should be violated. This happens in a considerably broader regime of parameters than the regime that appears in the standard analysis of preheating. In the standard analysis, in order for the resonance to be "efficient," the minimum mass of the additional scalar χ must be many times smaller than the inflaton mass, so that during many oscillations of the inflaton, the parameters of the Mathieu equation fall in the range $2q < A < 2q + \sqrt{bq}$ for some constant b of order unity. In contrast, for us, the analogous region of efficient preheating is $2q < A < 2xq$ for some $x > 1$. Of course, from a purely quantum field theoretic standpoint, it may seem contrived for the masses of an entire Hagedorn spectrum to oscillate in unison—but that is where the perspective of string theory makes natural things that otherwise would seem very special.

Three potential problems with the idea that strings could have played a significant role in preheating are as follows, in increasing order of seriousness. First, the Hagedorn spectrum might be cut off at a sufficiently low scale to invalidate the above analysis. Second, the fundamental string tension is supposed to be near the Planck scale, much higher than typical values for the inflaton mass, so despite the considerations of the previous paragraph, it may still seem quite unnatural to have (27) violated. And third, superheavy stable particles could be produced in unacceptable numbers, some of them with fractional charge. Let us examine these issues more closely.

(1) Cutoff on the spectrum: Since we believe that the number of degrees of freedom is thinned out dramatically by interactions at high energies, we should introduce some energy cutoff Λ and keep only those states with energy less than Λ. If Λ is only slightly above the string scale $1/\sqrt{\alpha'}$, of course all of our arguments collapse, and Eq. (6) is not a good way at all to estimate the number of string quanta created. But, in fact, there is reason to think that $\Lambda \gg 1/\sqrt{\alpha'}$: for example, as noted in [13], the typical size of a string is greater than the ten-dimensional Schwarzschild radius associated with its total mass until $N \sim 1/g_s^8$, where N is the excitation level and g_s is the string coupling. It would be consistent with the spirit of holography to put Λ at about the energy scale of strings at this high excitation level: $\Lambda \sim 1/(\sqrt{\alpha'} g_s^4)$, which is indeed a fairly high scale if g_s is even moderately small.[9] Since the total energy that can go into the strings is finite in preheating, but scales as an exponential of the typical energy of the string modes produced, it might be that total energy provides a process-dependent cutoff that is often lower than Λ.

(2) Heaviness of fundamental strings: in conventional scenarios, the square root of the string tension is roughly five orders of magnitude larger than the inflaton mass (10^{18} GeV as compared to 10^{13} GeV, roughly speaking). In order to violate Eq. (27) without making the inflaton mass much bigger than it is usually assumed to be, the string coupling needs to become very small during the inflaton oscillations—or, more precisely, the four-dimensional Planck scale, which scales as

$$M_{\text{Pl},4} \sim (\text{Vol}\,\text{CY}_3)^{1/2} g_s^{-1} \alpha'^{-2}, \tag{28}$$

must attain values much larger than $1/\sqrt{\alpha'}$. This is not inconceivable, but it does seem contrived. It has been pointed out to me [14] that an inflaton mass during preheating substantially larger than 10^{13} GeV might be arranged in string theory. This lies outside the scope of the current investigation.

(3) Production of superheavy stable particles: Unless the compactification manifold is simply connected (or unless open strings can propagate throughout ten-dimensional spacetime), there will be stable states corresponding to strings wound around noncontractible cycles. Such states can have fractional electric charge [15], and they would be very heavy, with mass $m \sim \ell/\alpha'$, where ℓ is the length of the noncontractible cycle. The trouble is that there is a Hagedorn spectrum of excited strings with any given winding number, and it is not clear that their production is suppressed strongly enough compared to strings with no nontrivial topology which can decay into string modes that are massless at the tree level. Excited string states with winding number would relax to stable, superheavy particles. The bounds on the density of such particles are stringent because they would contribute to dark matter density. Tracing back the upper bound on dark matter density today to the epoch of preheating leads to the conclusion that superheavy stable particles should comprise a very small fraction of the total energy at that time—perhaps on the order of 10^{-17} [3].

We can estimate how rapidly such particles would be created as follows. The energy of states with nontrivial winding is

[9]It is argued in [13] that gravitational self-interaction of strings compresses the spectrum and makes the density of states rise faster than in the free theory, until at some high scale string states might be in one-to-one correspondence with the microstates of black holes. Such a superexponential increase in the number of states one can actually produce in the way we have discussed in this paper is disastrous, since almost no background will have a finite total number of quanta produced. Presumably this is where a total reorganization of concepts is needed, perhaps with holography playing a central role.

$$\omega^2 = \omega_0^2 + m^2, \tag{29}$$

where ω_0^2 is the energy of a string state with no winding, but with the same momenta in the noncompact dimensions and at the same excitation level. The density of the states with winding should be roughly the same, as a function of ω_0, as the density of states without winding.[10] Thus the calculations leading up to Eq. (11) can be adapted to an approximate treatment of production of heavy charged particles by replacing the density of states

$$e^{\omega/T_H} \to \frac{\omega}{\sqrt{\omega^2 - m^2}} e^{\sqrt{\omega^2 - m^2}/T_H}. \tag{30}$$

If production of highly excited string states dominates, then roughly an equal number are produced with winding as without, simply because $\sqrt{\omega^2 - m^2} \approx \omega$ for these highly excited strings. Prefactors on the density of states might suppress the production of strings with winding by a few orders of magnitude—but suppression by a factor of 10^{-17} seems difficult. This is the relevant factor because we are operating on the assumption that excited strings do suck out a significant fraction of the energy of the coherent oscillations of the inflaton field. Naturally, if this assumption is lifted, there is no pressing problem with overproduction of superheavy stable particles.[11]

The considerably more stringent limits on fractionally charged particles have not been invoked in the discussion of the previous two paragraphs because it is not clear to me that fractionally charged particles are a universal aspect of string models with nontrivial topology in the extra dimensions.

In the analysis following Eq. (30), we have assumed that string production is not somehow limited to modes less energetic than ℓ/α'. Intuitively speaking, the point is that the spectrum of strings with winding defines the same temperature (despite the presence of a gap) as the strings without winding. We have also neglected the annihilation of strings wound in one direction with strings wound in the other. Some estimates of the cross section might be made, but annihilation processes do not seem likely to generate a suppression factor anywhere close to 10^{-17}. Thus we regard the overproduction of winding states as a major peril if highly excited strings are assumed to play any role at all in preheating.

[10]This is easily seen to be so in the case of a circle compactification when there is a winding number but no Kaluza-Klein momentum. I doubt the Calabi-Yau case would be much different, but I have not carried out the computations explicitly.

[11]In fact, if we assume that the minimum string tension (as measured in the four-dimensional Einstein frame) is about an order of magnitude higher than the inflaton mass, then the factor $e^{-\pi\mu\omega(r)}$ suppressing string production [cf. Eq. (10)] can be arranged to be around 10^{-17}. This might suggest a new twist on the idea of string-motivated superheavy dark matter.

IV. NONPERTURBATIVELY CONSTRUCTED STRINGS

Points 2 and 3 from the end of the previous section may incline us to doubt that excited strings can plausibly have anything to do with preheating: on the one hand, they are probably too heavy to be produced in significant numbers, and on the other hand, if they are, there is a tendency to overproduce superheavy stable particles.

Strings in four dimensions arising from branes wrapped on cycles of an internal manifold offer the possibility of ameliorating the problems described above. Regarding problem 2, the key point is that strings coming from wrapped branes become tensionless at special points in the moduli space which are a finite distance from generic points and typically occur at real codimension 2. Regarding problem 3, the helpful feature is that degenerating cycles can occur in an isolated nearly singular region of the internal manifold, so that no state with nontrivial topology is ever light. We will expand on these points somewhat in the following paragraphs and in the process do another simple estimate of string production.

First, to see that wrapped branes must be included on an equal footing with other states in the spectrum, recall, for example, the duality between the heterotic string on T^3 and M theory on K3, where when cycles of the K3 shrink, wrapped M2-branes become light and provide the W bosons of enhanced gauge symmetry that can be seen by other means on the heterotic side [16,17].

Arbitrarily light strings can arise in various ways [18–21]. A typical situation is for the effective tension in four dimensions to be

$$\tau_{\text{eff}} \approx M|\varphi| \quad \text{with } M \sim \frac{M_{\text{Pl},4}}{g_s}, \tag{31}$$

where φ is a canonically normalized complex scalar field. The precise dependence of M on g_s may depend on details, but some inverse dependence on g_s is to be expected from states that descend from string solitons. Let us consider D3-branes wrapped on a shrinking S^2 within a Calabi-Yau manifold as a definite example. The complex scalar φ in this case is proportional to $\int_{S^2}(J_2 + iB_2)$. The real part of this integral is the volume of the S^2, and the imaginary part is an axion whose associated instanton is a fundamental string wrapping the S^2.[12] The action includes the standard Dirac-Born-Infeld (DBI) term,

$$S = -\tau_{\text{D3}} \int d^4\xi \sqrt{G_{\mu\nu} + B_{\mu\nu}} + \cdots. \tag{32}$$

Integrating this on the S^2 and using $\tau_{\text{D3}} \sim 1/(\alpha'^2 g_s)$ gives the Nambu-Goto action for a string extended in other directions, with a tension given by Eq. (31). The overall normalization of the tension definitely depends on more data than

[12]There are well-understood worldsheet instanton corrections to the metric on the Kahler moduli space which we will be neglecting. These singularities are logarithmic and do not change the fact that the point at which wrapped D3-branes become tensionless is at a finite distance from generic points. They may slightly alter Eq. (31).

we have specified so far (for instance, the total volume of the Calabi-Yau manifold), but Eq. (31) should be right up to factors of order unity provided one is not in a peculiar corner of the moduli space (such as a Calabi-Yau manifold which is many times larger than the fundamental string scale).

An S^2 can indeed degenerate at a finite distance in moduli space in a Calabi-Yau compactification and in an isolated part of the Calabi-Yau manifold. A famous example is the resolved conifold [22–24]. It is also possible for degenerate spheres to arise along a two-dimensional locus within the Calabi-Yau manifold: this locus might be a Riemann surface of nontrivial topology, which resurrects the potential hazard of overproduction of superheavy stable particles.

At least in certain circumstances, it is clear that nonperturbatively constructed strings have the same Hagedorn behavior as fundamental strings. The cleanest evidence comes from NS5-branes, where for many coincident branes, the Hagedorn temperature of the fractional instanton strings is exactly the Hawking temperature for the near-extremal supergravity solution [25]; and from noncommutative open strings, where the effective string tension of open strings is lowered by applying an electric field, and the Hagedorn behavior is simple to understand [26]. In both cases, $T_H \sim \sqrt{\tau_{\text{eff}}}$. We will assume that this relation continues to hold, and that the arguments that led to Eq. (6) still apply.

Assume now that the scalar φ has some overlap with the dilaton, and that it is undergoing coherent oscillations just after the end of inflation. It is unnatural to suppose that φ passes exactly through zero, but it might reasonably be assumed to pass near zero in the complex plane. Perhaps naively, we will assume that $\dot{\varphi}$ is nearly constant while $|\varphi|$ is small. Let φ_0 be the closest approach of φ to the origin. Then the effective string tension varies with time like this:

$$\tau_{\text{eff}} \approx M|\varphi_0 + \dot{\varphi}t| = M\sqrt{|\varphi_0|^2 + (|\dot{\varphi}|t)^2}. \tag{33}$$

Proceeding in the same spirit as in the previous sections, we suppose that modes of the string in four dimensions have a dispersion relationship $\omega(t)^2 = k^2 + N\tau_{\text{eff}}$, where N is something like the level, and we are not trying to keep track of the expansion of the universe during the brief moment when the effective string tension becomes small. The Hagedorn temperature is $T_H \sim \sqrt{\tau_{\text{eff}}}$. The frequencies $\omega(t)$ (and also the Hagedorn temperature) are slowly varying in the far future and the far past, in the sense that derivatives of these quantities are much less than the appropriate power of the undifferentiated quantities. So asymptotic in and out vacua can be defined, at least in the approximation where we neglect what happens when the scalar rolls out of the realm of validity of Eq. (31), and particle production can be estimated in the same way that led to Eq. (10). If we define $\overline{\omega}(t) = \sqrt{\tau_{\text{eff}}(t)}$, then the result is that an integral of the type found in Eq. (11) converges provided a criterion similar to (26) holds. Dropping various factors of order unity, that convergence criterion is $\overline{\omega}(r)\mu \gtrsim 1$. The solution to $\tau_{\text{eff}}(t) = 0$ in the lower half of the complex plane is $t_* \equiv r - i\mu = -i|\varphi_0|/|\dot{\varphi}|$. Thus the convergence criterion is $\sqrt{M|\varphi_0|^3}/|\dot{\varphi}| \gtrsim 1$. We can roughly estimate $\dot{\varphi} \sim M_{\text{Pl},4} m_{\text{inflaton}}$, on the assumption that the coherent oscillations of φ are of amplitude $M_{\text{Pl},4}$ and of frequency m_{inflaton}. Recalling that the minimum tension is $\tau_{\text{eff},-} \approx M|\varphi_0|$, we can summarize the convergence criterion as follows:

$$\begin{aligned} 1 \lesssim \overline{\omega}(r)\mu = \frac{\sqrt{M\varphi_0^3}}{\dot{\varphi}} = \frac{\tau_{\text{eff},-}^{3/2}}{M\dot{\varphi}} \\ \sim \frac{\tau_{\text{eff},-}^{3/2}}{MM_{\text{Pl},4}m_{\text{inflaton}}} \sim 10^4 \left(\frac{\tau_{\text{eff},-}}{M_{\text{Pl},4}^2}\right)^{3/2} \end{aligned} \tag{34}$$

or equivalently,

$$\sqrt{\tau_{\text{eff},-}} \gtrsim \frac{1}{20} M_{\text{Pl},4},$$

where we have assumed $M \sim 10 M_{\text{Pl},4}$ and $m_{\text{inflaton}} \sim 10^{-5} M_{\text{Pl},4}$. The estimates leading to Eq. (34) are inevitably fairly crude as long as we have not specified what the inflaton is and how much it overlaps with the scalar φ controlling the effective string tension. The result in Eq. (34) is rather different from Eqs. (26) and (27). Intuitively, this is because the harmonic dependence that led to Eq. (26) is the smoothest interpolation between a given maximum and minimum in a specified time interval, whereas Eq. (33), though still analytic, implies almost a discontinuity in the first derivative of $\omega(t)^2$.

Evidently, the nonperturbatively constructed strings do not have to dip very far below the Planck scale in order to violate the convergence criterion (34). As before, the exponential nature of the divergence then would seem likely to be efficient in sucking energy out of the coherent motion of the inflaton. More precisely, it would be efficient in sucking out the energy from the coherent motion of φ, which is probably not the only component of the inflaton. This might be modeled in a Hartree approximation by a damping term for the evolution of one scalar which acts only near a certain value of that scalar, while other scalars, coupled to the first by terms in the potential, are damped only by the usual $3H\dot{\phi}$ term.

V. CONCLUSIONS

The convergence criterion on the total number of strings produced, which we have variously stated as Eqs. (12), (26), and (34), amounts to the statement that in the appropriate string frame, where the string tension is by definition constant, curvatures are substringy—only it is crucial that not only are the second derivatives of the string metric bounded by the string scale, but all derivatives are bounded in an appropriate way to ensure a uniform radius of convergence, of order the string length, for the time-varying energies of string modes. In the context of preheating, a violation of this bound on curvatures, either for fundamental strings or for strings constructed as wrapped branes, has the simple interpretation that the production of highly excited string states efficiently drains the energy of coherent inflaton oscillations. This is somewhat in analogy with the standard theory of preheating, but copious string production occurs in a larger region of parameters [as measured in the (A,q) plane of parameters for Mathieu functions]. Unfortunately, it is not particularly clear to me what distinctive experimental signatures should be expected if highly excited strings played a significant role in preheating. The strings decay into states that are massless at the tree level, and presumably these can then thermalize. However, if excited strings states are created in appreciable numbers, it seems that stable superheavy particles are likely to be overproduced, if they exist in the string spectrum. This problem can be avoided by considering strings in four dimensions that arise as branes wrapped on cycles of an internal manifold—cycles which shrink to produce a pointlike singularity on that manifold.

The tension of such nonperturbatively constructed strings vanishes at certain points in moduli space, and near such points the dependence of the tension on canonically normalized scalars may reasonably be assumed to be Eq. (31). This dependence leads to a convergence criterion (34) which is considerably more restrictive than the one derived in Eqs. (26) and (27) assuming harmonic dependence—meaning that the minimum tension does not have to be as low, assuming the dependence (31), in order to violate the convergence criteria and produce a large number of excited strings. In fact, as we see in Eq. (34), the effective string tension needs to dip only a couple of orders of magnitude below the four-dimensional Planck scale in order to have copious string production, with otherwise rather standard assumptions about the amplitude and frequency of inflaton oscillations just after the end of inflation.

In summary, it is quite plausible that excited strings (probably of nonperturbative origin) play a role in postinflationary cosmology.

A much studied scenario in which light strings appear is ekpyrosis [27,28], in which either an M5-brane wrapped on a holomorphic cycle of a Calabi-Yau manifold collides with the E_8 boundary of spacetime constructed in [29], or the two E_8 boundaries collide with one another. In the first case, M2-branes stretched between the M5-brane and the boundary behave as strings that become light at the moment of collision. In the second case, M2-branes stretched between the two E_8 boundaries become light, and they become precisely the perturbative heterotic strings in the limit where the two boundaries coincide. The results of this paper bear on ekpyrosis in two ways.

(1) We have noted in Sec. IV a natural way of avoiding or "regulating" the collision: one simply needs to assume a nonzero value for the pseudoscalar superpartner of the real scalar controlling the distance between the branes/boundaries. In the case of colliding boundaries, this pseudoscalar is just the axion arising from the Neveu-Schwarz $B_{\mu\nu}$ field of the heterotic string with both indices in four dimensions. If this axion exists in the spectrum, it is in fact unnatural to suppose that it is zero during ekpyrosis.

(2) If ekpyrosis does occur, presumably in a "regulated" way as explained in the previous point, then estimates of the creation of light strings proceed in parallel to our calculations in Sec. IV. If excited strings are produced during ekpyrosis, the problem of overproducing stable superheavy particles seems likely to recur. Specifically, if an M5-brane wrapped on a holomorphic curve hits an E_8 boundary, then the light strings can wrap cycles of the holomorphic curve (which is a Riemann surface embedded in the Calabi-Yau manifold), unless that curve is topologically an S^2. And if the two boundaries collide, then the light strings can wrap on the Calabi-Yau manifold, unless the Calabi-Yau is simply connected, which would considerably reduce the particle-phenomenological appeal of the setup.

It does not help to assume that the strings are light only for a brief instant of time. In fact, according to (34), larger $\dot{\varphi}$ (meaning a briefer collision) causes more string production, not less: it is then easier to violate the inequality in (34).

The reason that it is easy to offer the plausible interpretation of draining energy from coherent inflaton oscillations when strings are copiously produced just after inflation is that a significant fraction of the energy is kinetic (relating to the time derivative of the inflation). A much thornier question is what happens if a string becomes lighter than the Hubble scale during slow-roll inflation, when most of the energy is stored in the scalar potential. I have speculated elsewhere [30] that light strings might limit the rate of inflation to the string scale, and that there could be a fixed point of forward time evolution where a string tension is vanishing at the same time that the expansion of the universe is slowing to something less fast than exponential. An observation that tells against this idea is that string production should result in a new source of positive energy density, and, unlike the case of preheating, there is no obvious way to suck this energy away from the scalar potential energy. Granting the Friedman equation $H^2 = \rho/(3M_{\mathrm{Pl},4}^2)$, it is hard to see what mechanism due to light strings will drive ρ and therefore H to zero. Perhaps this question must wait on some treatment more firmly based on the first principles of string theory.

Indeed, an objection that could be raised to the entire enterprise of this paper is that α' corrections to the spacetime equations of motion become significant when curvatures are at the string scale, so it is not clear that we have adequately addressed classical effects before delving into the quantum effect of particle creation. There are at least three reasons to continue thinking about string creation in general backgrounds: (1) Nonperturbative strings do not have a well-understood connection to spacetime equations of motion; (2) worldsheet techniques have not so far provided a particularly large class of well-understood time-dependent backgrounds; (3) because copious string creation occurs, in some heuristic sense, at the same order in α' as corrections to the classical spacetime equations, string production probably does have an important role to play in our understanding of general time-dependent backgrounds.

Let us end with some open questions.

Our estimates for string production were fairly crude in places, starting with the neglect of power law corrections to the exponential growth in the density of states, and including also the neglect of various factors of order unity that enter into the exponential behavior in Eq. (11). Is it possible to give more precise estimates?

Can one give an explicit string theory construction of time-dependent backgrounds in which highly excited string states are produced? This is surely complicated by the fact that back reaction from such strings is likely to be important.

Might the *CP* violation that results from turning on an axion to ''get around'' a point of tensionless strings be of interest in baryogenesis?

If highly excited strings are produced when inflation is over or nearly over, is there some definite effect on the spectrum of fluctuations? How do decay products of highly excited strings thermalize?

If we assume that the end of inflation involves oscillations in a complex moduli space with tensionless strings on some real codimension 2 loci, how much isocurvature perturbation is likely to be generated? Is the scenario plausibly consistent with existing bounds?

We hope to return to some of these questions in the future.

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