

**Nonsingular FRW cosmology and nonlinear electrodynamics**C. S. Camara,<sup>1,\*</sup> M. R. de Garcia Maia,<sup>1,†</sup> J. C. Carvalho,<sup>1,‡</sup> and J. A. S. Lima<sup>1,2,§</sup><sup>1</sup>*Departamento de Física, Universidade Federal do Rio Grande do Norte, 59072-970 Natal RN Brazil*<sup>2</sup>*Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Universidade de São Paulo, Rua do Matão, 1226 - Cidade Universitária, 05508-900, São Paulo, SP, Brazil*

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The possibility to avoid the cosmic initial singularity as a consequence of nonlinear effects on the Maxwell electromagnetic theory is discussed. For a flat Friedmann-Robertson-Walker (FRW) geometry we derive the general nonsingular solution supported by a magnetic field plus a cosmic fluid and a nonvanishing vacuum energy density. The nonsingular behavior of solutions with a time-dependent  $\Lambda(t)$  term are also examined. As a general result, it is found that the functional dependence of  $\Lambda(t)$  can uniquely be determined only if the magnetic field remains constant. All these models are examples of bouncing universes which may exhibit an inflationary dynamics driven by the nonlinear corrections of the magnetic field.

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**I. INTRODUCTION**

A fundamental difficulty underlying the standard Friedmann-Robertson-Walker (FRW) cosmology is the prediction of an initial singular state where all curvature invariants and some material quantities such as pressure, energy density, and temperature become infinite [1]. Generically, “the breakdown of the laws of physics at a singularity” is a clear manifestation of a mathematical inconsistency and physical incompleteness of any cosmological model. In this way, although strongly supported by the recent observations at low and intermediate redshifts, the present big-bang picture with dark energy must be improved at very high redshifts.

In order to solve such a problem several attempts based on many disparate mechanisms have been proposed in the literature. Some earlier approaches trying to develop a well behaved and more complete cosmological description include quadratic Lagrangians and other alternative theories for the gravitational field [2], a creation-field cosmology [3], a huge vacuum energy density at very early times [4], non-minimal couplings [5], nonequilibrium thermodynamic effects [6], and quantum-gravitational phenomena closely related to a possible spontaneous birth of the Universe [7].

More recently, a new interesting mechanism aiming at avoiding the cosmic singularity has been discussed by De Lorenci *et al.* [8] through a nonlinear extension of the Maxwell electromagnetic theory. The associated Lagrangian and the resulting electrodynamics can theoretically be justified based on different arguments. For example, the nonlinear terms can be added to the standard Maxwell Lagrangian by imposing the existence of symmetries such as parity conservation, gauge invariance, Lorentz invariance, etc. [9,10], as well as by the introduction of first-order quantum corrections

to the Maxwell electrodynamics [11,12]. It is worth notice that nonlinear corrections may also be important to avoid the black-hole singularity. Actually, an exact regular black-hole solution has been recently obtained with basis on the Einstein-dual nonlinear electrodynamics as proposed by Salazar, Garcia, and Plebansky [13,14]. Note, however, that the purpose of the present work is not to give a detailed description of the possible nonlinear theories and their physical effects. Our basic aim is rather modest—we will try to gain new insights into the possibilities beyond Maxwell theory based on the simplest nonlinear electromagnetic Lagrangian and its connection to the cosmic singularity problem.

In this concern, it has been found [8] that the primordial singularity can be removed because the nonlinear corrections reinforce the negative pressure at the early stages of the Universe. The authors also argued that the nonsingular behavior is unaffected by the presence of other ultrarelativistic components obeying the equation of state  $p_{(ur)} = \rho_{(ur)}/3$ .

In the present work, we extend the analysis by De Lorenci *et al.* [8] using different ingredients. We first obtain the behavior of the scale function for a spatially flat FRW geometry ( $\Lambda = 0$ ), thereby showing that their results is a particular case of the general solution. It is also analyzed how this solution is modified by the presence of a  $\Lambda$  term (vacuum energy density) which may be constant or a time varying quantity. Solutions with constant vacuum energy density also lead to a nonsingular universe, and the same happens with a time-dependent  $\Lambda(t)$  term. However, there are singular solutions where the decaying vacuum supplies the energy to a constant cosmological magnetic field. In this case, the time dependence of the cosmological term can uniquely be determined and corresponds to a slightly modification of the more frequent forms suggested in the literature [15–17]. The main physical restrictions, including the time interval where the nonlinear corrections must be important, are also discussed.

The paper is organized as follows. In Sec. II, we write down the Einstein field equations (EFE’s) for a flat FRW geometry supported by an energy momentum derived from the extended (nonlinear) electromagnetic Lagrangian. In Sec. III, we generalize the solution derived in Ref. [8] which as-

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sumes a vanishing cosmological term, a spatially flat geometry, and a time-dependent magnetic field. In Sec. IV, we obtain a new solution that takes into account the presence of a constant  $\Lambda$ . The behavior involving a time-dependent  $\Lambda$  is discussed in Sec. V. Finally, in the conclusion section, we summarize the basic results and also present some suggestions for future work. In what follows, Greek indices run from 0 to 3, Latin indices run from 1 to 3. Unlike of Ref. [8], we adopt the International System of Units (see the Appendix on units and dimensions of Ref. [18] for further details).

## II. BASIC EQUATIONS

As is widely known, the Lagrangian density for free fields in the Maxwell electrodynamics may be written as

$$\mathcal{L}_{(MAXWELL)} = -\frac{1}{4\mu_0} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{4\mu_0} F, \quad (1)$$

where  $F^{\mu\nu}$  is the electromagnetic field strength tensor and  $\mu_0$  is the magnetic permeability. The canonical energy-momentum tensor is then given by

$$T_{\mu\nu}^{(MAXWELL)} = \left(\frac{1}{\mu_0}\right) \left[ F_{\mu\alpha} F^{\alpha}_{\nu} + \frac{1}{4} F g_{\mu\nu} \right]. \quad (2)$$

In this work we consider the extended Lagrangian density to the electromagnetic field,

$$\mathcal{L} = -\frac{1}{4\mu_0} F + \omega F^2 + \eta F^{*2}, \quad (3)$$

where  $\omega$  and  $\eta$  are arbitrary constants,

$$F^* \equiv F^*_{\mu\nu} F^{\mu\nu}, \quad (4)$$

and  $F^*_{\mu\nu}$  is the dual of  $F_{\mu\nu}$ . As one may check, the corresponding energy-momentum tensor becomes

$$T_{\mu\nu} = -4 \frac{\partial \mathcal{L}}{\partial F} F^{\alpha}_{\mu} F_{\alpha\nu} + \left( \frac{\partial \mathcal{L}}{\partial F^*} F^* - \mathcal{L} \right) g_{\mu\nu}. \quad (5)$$

Let us now consider the above expressions in the context of a homogeneous and isotropic FRW flat model,

$$ds^2 = c^2 dt^2 - a^2(t) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]. \quad (6)$$

Naturally, electromagnetic fields may be the source of the above background only if the fields are considered in its average properties [19]. Now, applying the standard spatial averaging process we set

$$\langle E_i \rangle = 0, \quad (7)$$

$$\langle B_i \rangle = 0, \quad (8)$$

$$\langle E_i E_j \rangle = -\frac{1}{3} E^2 g_{ij}, \quad (9)$$

$$\langle B_i B_j \rangle = -\frac{1}{3} B^2 g_{ij}, \quad (10)$$

$$\langle E_i B_j \rangle = 0. \quad (11)$$

Equations (7)–(11) imply that

$$\langle F_{\mu\alpha} F^{\alpha}_{\nu} \rangle = \frac{2}{3} \left( \epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) \frac{U_{\mu} U_{\nu}}{c^2} + \frac{1}{3} \left( \epsilon_0 E^2 - \frac{B^2}{\mu_0} \right) g_{\mu\nu}, \quad (12)$$

where  $U_{\mu}$  is the four velocity. Under such conditions the average value of the energy-momentum tensor takes the perfect fluid form, namely

$$\langle T_{\mu\nu} \rangle = (\rho + p) \frac{U_{\mu} U_{\nu}}{c^2} - p g_{\mu\nu}, \quad (13)$$

where the density  $\rho$  and pressure  $p$  have the well-known form

$$\rho = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{B^2}{\mu_0} \right), \quad (14)$$

$$p = \frac{1}{3} \rho. \quad (15)$$

In order to analyze the modifications implied by the use of the modified Lagrangian (3), we also assume that for the stochastically defined electromagnetic fields only the average value of the squared magnetic field  $B^2$  survives at the very early Universe, i.e., we use Eqs. (7)–(11) with  $\langle E^2 \rangle = 0$ . In this concern, we remark that a homogeneous electric field in a plasma must give rise to an electric current of charged particles and then rapidly decay. Indeed, unlike what happens with the magnetic field, at present there is no basis whatsoever to presume the existence of an overall electric field. Indeed, since the late 1960s, it has been recognized that cosmological models with an overall electric field bears hardly any relation at all to reality (see for instance, Zeldovich and Novikov [20]). Naturally, this does not mean that the  $\langle E^2 \rangle$  term appearing in our stochastic approach can be neglected in comparison with  $\langle B^2 \rangle$ , but one may expect that its influence might be small for some special regimes, as for example, when the plasma may be treated with basis on the magnetohydrodynamics approximation. For a dense ionized gas, for example, the collision frequency can be so high that the electric field and its momenta may arise only as a consequence of the motion of the fluid, or as a result of the external charges distribution or (not “frozen in”) time-varying magnetic fields. Such a behavior may happen in the primeval plasma (below Planck’s temperature) because the Debye screening radius  $\sim (T/n)^{1/2}$  is very small in comparison with the macroscopic relevant scale for nonsingular world models, namely, the Hubble radius  $cH(t)^{-1}$ . Keeping these remarks in mind, we return to the basic equations by assuming that  $\langle E^2 \rangle$  has been neglected so that Eq. (13) still holds, but the energy density and pressure now read

$$\rho = \frac{1}{2\mu_0} B^2 (1 - 8\mu_0 \omega B^2), \quad (16)$$

$$p = \frac{1}{6\mu_0} B^2 (1 - 40\mu_0 \omega B^2) = \frac{1}{3} \rho - \frac{16}{3} \omega B^4. \quad (17)$$

Note that the weak energy condition ( $\rho > 0$ ) is obeyed for

$$B < \frac{1}{2\sqrt{2\mu_0\omega}}, \quad (18)$$

whereas the pressure will reach negative values only if

$$B > \frac{1}{2\sqrt{10\mu_0\omega}}. \quad (19)$$

On the other hand, there is a widespread belief that the early Universe evolved through some phase transitions, thereby yielding a vacuum energy density which at present is at least 118 orders of magnitude smaller than in the Planck time [21]. Such a discrepancy between theoretical expectation (from the modern microscopic theory of particles and gravity) and empirical observations constitutes a fundamental problem in the interface uniting astrophysics, particle physics, and cosmology, which is often called “the cosmological constant problem” [21,22]. This “puzzle” together with the observations of type-Ia supernovae [23] suggesting that the cosmic bulk of energy is repulsive and appears like a dark energy (probably of primordial origin) stimulated the recent interest for more general models containing an extra component and accounting for the present accelerated stage of the Universe [24]. A possible class of such cosmologies is provided by phenomenological models driven by a constant or a time-dependent  $\Lambda(t)$  term (see, for instance, Refs. [14–16,24–30] and references therein). The effective time-dependent cosmological term may be regarded as a second fluid component with energy density,  $\rho_\Lambda(t) = \Lambda(t)c^4/8\pi G$ , which transfers energy continuously to the material medium. The conditions under which this kind of cosmology can be described by a scalar field coupled to a perfect fluid has also been discussed in the literature [24–30]. For the sake of generality, we focus our attention to a decaying  $\Lambda$  model, but now in the presence of the primeval magnetic field. In the background defined by Eq. (6), the EFE read

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3c^2} \rho + \frac{\Lambda(t)c^2}{3}, \quad (20)$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda(t)c^2}{3} - \frac{4\pi G}{3c^2} (\rho + 3p), \quad (21)$$

while the energy conservation law can be written as

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\frac{\dot{\Lambda}c^4}{8\pi G}, \quad (22)$$

where an overdot means derivative with respect to the cosmic time  $t$ . As we shall see, for a constant magnetic field it is not necessary to assume a phenomenological expression for  $\Lambda(t)$  as usually done since it can be uniquely fixed from the above set of equations.

Replacing Eqs. (16) and (17) in the Einstein equations (20)–(22) we get

$$\frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3c^2} \frac{B^2}{\mu_0} (1 - 8\mu_0 \omega B^2) + \frac{\Lambda c^2}{3}, \quad (23)$$

$$\frac{\ddot{a}}{a} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3c^2} \frac{B^2}{\mu_0} (1 - 24\mu_0 \omega B^2), \quad (24)$$

$$\frac{B}{\mu_0} (1 - 16\mu_0 \omega B^2) \left( \dot{B} + 2\frac{\dot{a}}{a} B \right) = -\frac{\dot{\Lambda} c^4}{8\pi G}. \quad (25)$$

Inserting Eqs. (16) and (20) into Eq. (21) we find

$$a\ddot{a} + \dot{a}^2 - \frac{2}{3}\Lambda c^2 a^2 - \left( \frac{64\pi G \omega}{3c^2} \right) B^4 a^2 = 0. \quad (26)$$

The term proportional to  $B^4$  comes from the nonlinear correction in the Lagrangian. Note also that if the coupling constant  $\omega$  is zero, the standard FRW differential equation for a radiation filled universe plus a cosmological  $\Lambda$  term is recovered. By solving any two of the above equations one may discuss if the nonlinear terms added to the Maxwell Lagrangian may alter the primeval singular state. In order to compare with some previous results presented in the literature, we start our analysis by considering some interesting particular cases.

### III. NONSINGULAR MODELS WITH $\Lambda = 0$

This is the case studied by De Lorenci *et al.* [8] where a particular nonsingular solution was found. In what follows the corresponding general solution is given and we also show how to recover the quoted result. First of all, we remark that if  $B$  is a time-dependent quantity and  $\Lambda$  remains constant, Eq. (25) can be easily integrated to give

$$B(t) = B_0 \left( \frac{a_0}{a} \right)^2, \quad (27)$$

where  $B_0$  is a constant of integration. In this paper, the subscript 0 does *not* mean the present day value of a quantity. It only indicates its value at an arbitrary time  $t_0$  which appears in the general solution of  $a(t)$  as a second integration constant. This constant  $t_0$  was arbitrarily chosen in Ref. [8]. Thus  $B_0 = B(t_0)$  if  $a_0 = a(t_0)$  (in the quoted paper  $a_0$  was also normalized to unit). We stress that the scaling solution (27) holds even for a nonvanishing constant  $\Lambda$ . Inserting  $B(a)$  into Eq. (26) one finds

$$a\ddot{a} + \dot{a}^2 - \left( \frac{64\pi G \omega B_0^4 a_0^8}{3c^2} \right) a^{-6} = 0. \quad (28)$$

As one may check, the general solution of the above equation is

$$a(t) = a_0 [4\alpha_0^2(t-t_0)^2 + 4\alpha_0\beta_0(t-t_0) + 1]^{1/4}, \quad (29)$$

where we have defined

$$\alpha_0 \equiv \sqrt{\frac{4\pi G}{3\mu_0 c^2}} B_0, \quad (30)$$

$$\beta_0 \equiv \pm \sqrt{1 - 8\mu_0 \omega B_0^2}. \quad (31)$$

In order to compare with the results of Ref. [8] we recast Eq. (29) in the form

$$a(t) = a_0 (4\alpha_0^2 t^2 + 4\alpha_0 \gamma_0 t + \delta_0)^{1/4}, \quad (32)$$

with

$$\gamma_0 \equiv \beta_0 - 2\alpha_0 t_0, \quad (33)$$

$$\delta_0 \equiv 4\alpha_0 t_0 (\alpha_0 t_0 - \beta_0) + 1. \quad (34)$$

The linear term in  $t$  inside the parentheses of Eq. (32) does not appear in the solution given by the authors of Ref. [8]. This happens because the arbitrary integration constant  $t_0$  was chosen to be

$$t_0 = \frac{\beta_0}{2\alpha_0} = \frac{1}{2B_0} \sqrt{\frac{3\mu_0 c^2 (1 - 8\mu_0 \omega B_0^2)}{4\pi G}}. \quad (35)$$

The time behavior of the magnetic field is readily obtained from Eqs. (27) and (32). One finds

$$B(t) = \frac{B_0}{(4\alpha_0^2 t^2 + 4\alpha_0 \gamma_0 t + \delta_0)^{1/2}}, \quad (36)$$

with the energy density and pressure defined by Eqs. (16) and (17), respectively. Note also that the Hubble parameter can be written as

$$H = \frac{\dot{a}}{a} = \frac{\alpha_0 [2\alpha_0(t-t_0) + \beta_0]}{[4\alpha_0^2(t-t_0)^2 + 4\alpha_0\beta_0(t-t_0) + 1]}, \quad (37)$$

which becomes for  $t = t_0$

$$H_0 \equiv H(t_0) = \alpha_0 \beta_0 = B_0 \sqrt{\frac{4\pi G (1 - 8\mu_0 \omega B_0^2)}{3\mu_0 c^2}}. \quad (38)$$

(In Ref. [8], the notation  $H$  is used to represent the magnetic field.) From Eq. (32) we see that, for large  $t$ , we recover the classical solution for radiation dominated universes,  $a(t)$

$\propto t^{1/2}$ . Alternatively, we observe from the relation (16) that this solution is recovered for values of  $t$  where the magnetic field obeys the condition

$$8\mu_0 \omega B^2 \ll 1. \quad (39)$$

The most interesting feature of Eq. (32) is that the quadratic function inside the parentheses does not have real roots  $\omega > 0$ , being positive for any  $t$ . Therefore the model is nonsingular with  $a(t)$  reaching the minimum value  $a_{min} = a_0 (8\mu_0 \omega B_0^2)^{1/4}$  at a time  $t_{min} = -\gamma_0/2\alpha_0 = t_0 - \beta_0/2\alpha_0$ . It thus follows that the universe is a bouncing one: it begins arbitrarily large at  $t \ll t_{min}$ , decreases until its minimal value at  $t_{min}$ , and then begins to expand. The values of the magnetic field and energy density at  $t_{min}$  are

$$B(t_{min}) = \frac{1}{2\sqrt{2\mu_0 \omega}}, \quad (40)$$

$$\rho(t_{min}) = 0. \quad (41)$$

Before we proceed further, it is worth notice that if Eq. (32) correctly describes the evolution of the Universe in the distant past, it implies the existence of an inflationary era ( $\ddot{a} > 0$ ) on the interval

$$t_{min} - t_I < t < t_{min} + t_I, \quad (42)$$

where

$$t_I = \sqrt{\frac{3\mu_0^2 \omega c^2}{\pi G}}. \quad (43)$$

Figure 1 shows the scale factor, the magnetic field, the energy density, and the pressure as a function of time for a definite value of  $B_0$ . From relations (36) and (39) we stress that the classical solution is recovered for times much larger than  $t_{min}$  with the Universe entering in the standard radiation phase. As shown in Fig. 1, the nonlinear corrections are relevant only for  $8\mu_0 \omega B^2 \gtrsim 1/10$  or  $t \lesssim t_{min} + 3\sqrt{8\mu_0 \omega B_0^2}/2\alpha_0$ .

#### IV. NONSINGULAR MODELS FOR $\Lambda \neq 0$

For constant  $\Lambda$ , it is easy to see that Eq. (26) becomes

$$a\ddot{a} + \dot{a}^2 - \frac{2}{3}\Lambda c^2 a^2 - \left( \frac{64\pi G \omega B_0^4 a_0^8}{3c^2} \right) a^{-6} = 0. \quad (44)$$

Let us now search for an exact description in the presence of a cosmological constant. By combining Eqs. (23) and (27) for a constant  $\Lambda$  we find

$$\dot{Z}^2 = 16[\lambda Z^2 + \alpha_0^2(Z - 8\mu_0 \omega B_0^2)], \quad (45)$$

where the auxiliary scale factor  $Z$  and  $\lambda$  term are defined by

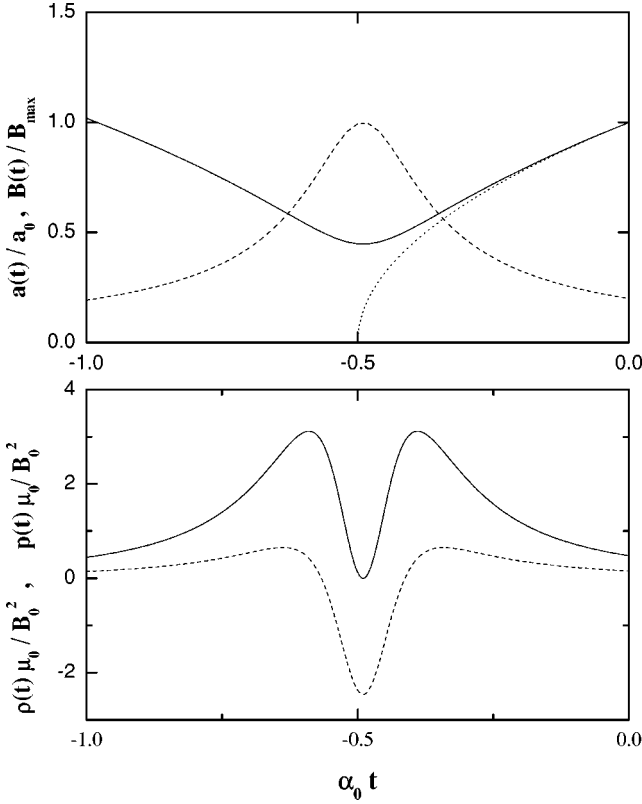


FIG. 1. The upper panel shows the scale factor (solid line), the magnetic field (dashed line), and the classical solution (dotted line) for ( $\omega=0$ ). The lower panel shows the energy density (solid line) and the pressure (dashed line) for the model with  $\Lambda=0$  and  $2B_0\sqrt{2\omega\mu_0}=0.2$ .

$$Z \equiv \left(\frac{a}{a_0}\right)^4, \quad (46)$$

$$\lambda \equiv \frac{\Lambda c^2}{3}. \quad (47)$$

Equation (45) can be easily integrated to give

$$a(t) = a_0 \left(\frac{1}{4\lambda}\right)^{1/4} \left[ C_0 e^{4\sqrt{\lambda}(t-t_0)} + \frac{D_0}{C_0} e^{-4\sqrt{\lambda}(t-t_0)} - 2\alpha_0^2 \right]^{1/4}, \quad (48)$$

where

$$C_0 \equiv \alpha_0^2 + 2\lambda + 2\sqrt{\lambda(\lambda + \alpha_0^2 - 8\alpha_0^2\mu_0\omega B_0^2)}, \quad (49)$$

$$D_0 \equiv \alpha_0^2(\alpha_0^2 + 32\lambda\mu_0\omega B_0^2). \quad (50)$$

The magnetic field is

$$B(t) = 2B_0\sqrt{\lambda} \left[ C_0 e^{4\sqrt{\lambda}(t-t_0)} + \frac{D_0}{C_0} e^{-4\sqrt{\lambda}(t-t_0)} - 2\alpha_0^2 \right]^{-1/2}. \quad (51)$$

It is straightforward to see that the term inside the square brackets of Eq. (48) is positive for all  $t$  and that the scale factor reaches its minimum value

$$a_{min} = a_0 \left[ \frac{\alpha_0}{2\lambda} (\sqrt{\alpha_0^2 + 32\lambda\mu_0\omega B_0^2} - \alpha_0) \right]^{1/4} \quad (52)$$

at

$$t_{min} = t_0 + \frac{1}{8\sqrt{\lambda}} \ln \left( \frac{D_0}{C_0} \right). \quad (53)$$

As in the previous case, the universe bounces at  $t_{min}$  and, if the solution would effectively hold near  $t_{min}$ , an inflationary phase would take place for all values of  $t$  such that

$$C_0^2 x^4 - 8\alpha_0^2 C_0 x^3 + 14D_0 x^2 - 8\alpha_0^2 \frac{D_0}{C_0} x + \frac{D_0^2}{C_0^2} > 0, \quad (54)$$

where

$$x \equiv e^{4\sqrt{\lambda}(t-t_0)}. \quad (55)$$

The magnetic field at  $t_{min}$  is

$$B(t_{min}) = \left[ \frac{\Lambda\mu_0 c^4}{2\pi G \left( \sqrt{1 + \frac{8\Lambda\mu_0^2\omega c^4}{\pi G}} - 1 \right)} \right]^{1/2}. \quad (56)$$

From relations (39) and (51) the de Sitter classical solution is recovered for

$$t \ll t_0 + \frac{1}{8\sqrt{\lambda}} \ln \left( \frac{\alpha_0^4}{C_0^2} \right) \quad (57)$$

and

$$t \gg t_0 + \frac{1}{8\sqrt{\lambda}} \ln \left[ \frac{(\alpha_0^2 + 32\mu_0\omega B_0^2\lambda)^2}{C_0^2} \right]. \quad (58)$$

Similarly to what happens with solution (29), the classical solution is recovered for times much larger than  $t_{min}$ . In Figs. 2 and 3 we show the scale factor, the magnetic field, the energy density, and the pressure as a function of time, for some values of  $\omega$ ,  $\Lambda$ , and  $B_0$ . In analogy with the solution (32), we have that the nonlinear corrections are relevant only for

$$8\mu_0\omega B^2 \geq 1/10$$

or

$$t \leq t_0 + \frac{1}{8\sqrt{\lambda}} \ln \left[ \frac{(\alpha_0^2 + 160\mu_0\omega B_0^2\lambda)^2}{C_0^2} \right] + \frac{1}{8\sqrt{\lambda}} \ln \left[ \left( 1 + \sqrt{1 - \frac{\alpha_0^2}{\alpha_0^2 + 160\mu_0\omega B_0^2\lambda}} \right)^2 \right].$$

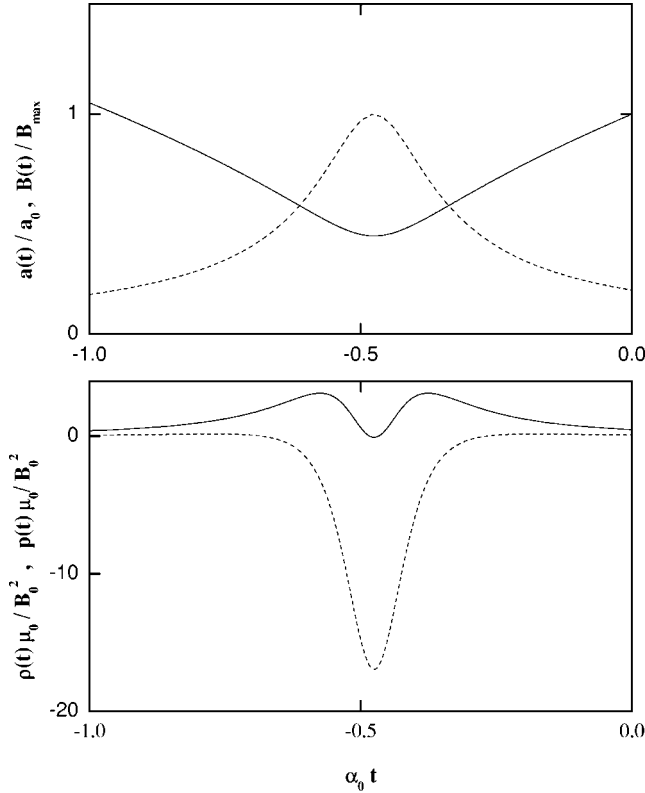


FIG. 2. The upper panel shows the scale factor (solid line) and the magnetic field (dashed line). The lower panel shows the energy density (solid line) and the pressure (dashed line) for the model with a constant nonvanishing  $\Lambda$ . The values for  $\Lambda$  and  $B_0$  are such that  $\sqrt{\lambda}/\alpha_0=0.4$  and  $2B_0\sqrt{2\omega\mu_0}=0.2$ .

### V. NONSINGULAR MODELS FOR A TIME-DEPENDENT $\Lambda$

The possible cosmological consequences of a decaying vacuum energy density or  $\Lambda(t)$  cosmologies are still under debate in the recent literature [14–30]. Such models may also be described in terms of a scalar field coupled to a fluid component. Another important motivation is its connection with the cosmological constant problem. On general grounds, one may expect that a decaying vacuum energy must play an important role in the Universe evolution (mainly in the very early Universe) and, probably, more interesting, it may indicate suggestive ways to solve the  $\Lambda$  problem, as for instance, by describing the effective regimes that should be provided by fundamental physics. In the majority of the papers dealing with a time-varying  $\Lambda$ , the decaying law is defined *a priori*, i.e., in a phenomenological way. The most commonly postulated decaying laws are those in which  $\Lambda(t)$  decreases as some power either of the scale factor  $a(t)$  or the Hubble parameter  $H$  (see Ref. [17] for a quick review). Some authors have also considered scaling laws formed by a combination of both quantities [16]. As remarked before, these proposals are in accordance but do not explain the difference of more than 100 orders of magnitude between the cosmological constant value at the beginning of Universe (provided by particles physics) and its actual value estimated from cosmology. In general, the EFE's imply that once  $\Lambda(t)$  is given one may integrate them for

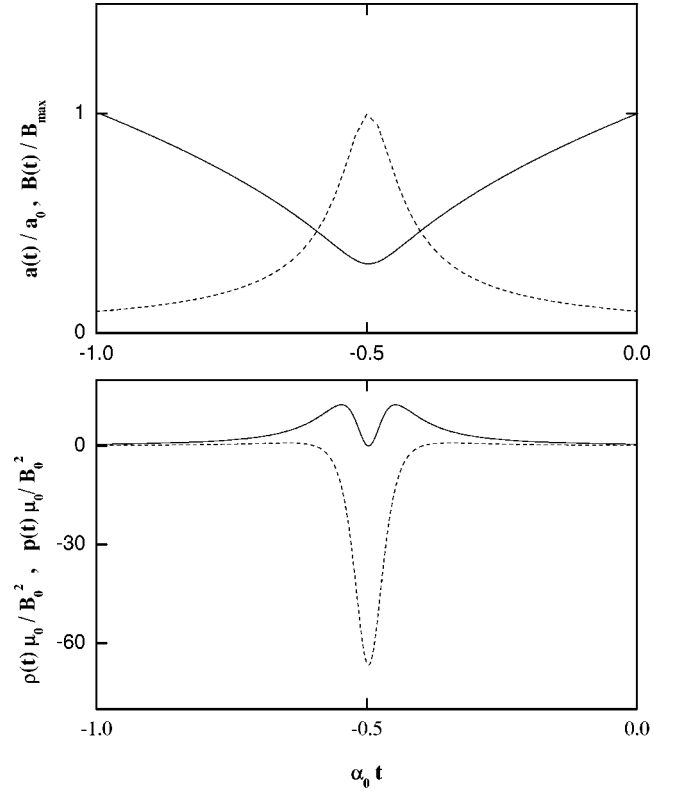


FIG. 3. As in Fig. 2 but for  $\sqrt{\lambda}/\alpha_0=0.01$  and  $2B_0\sqrt{2\omega\mu_0}=0.1$ .

obtaining  $B(t)$  and  $a(t)$ . Conversely, for a given dependence of  $B(t)$ , a unique time dependence for  $\Lambda(t)$  is readily fixed by the field equations.

Let us first analyze phenomenological models with a cosmological term defined by  $\Lambda=3\beta c^{-2}H^2$ , where  $\beta$  is a positive parameter smaller than unity [16,30]. The differential equation driving the scale factor is readily derived by combining relations (16), (17), (20), and (21). One finds

$$\dot{H} + 4(1-\beta)H^2 - (1-\beta)\frac{\alpha_0^2}{1-\beta_*^2} \times \left[ 1 - \sqrt{1 - \frac{4(1-\beta_*^2)}{\alpha_0^2}H^2} \right] = 0, \quad (59)$$

where  $H=\dot{a}/a$ . The constant  $\alpha_0$  is defined by relation (30) whereas  $\beta_*$  is given by

$$\beta_* \equiv \pm \sqrt{1 - 8\mu_0\omega(1-\beta)B_0^2}. \quad (60)$$

Now, separating the variables in Eq. (59) one finds

$$\int_{H_0}^H \left\{ \frac{\alpha_0^2}{1-\beta_*^2} \left[ 1 - \sqrt{1 - \frac{4(1-\beta_*^2)}{\alpha_0^2}H^2} \right] - 4H^2 \right\}^{-1} dH = (1-\beta)(t-t_0). \quad (61)$$

A simple integration results,

$$H = \frac{\dot{a}}{a} = \frac{\alpha_0[2\alpha_0(1-\beta)(t-t_0) + \beta_*]}{[4\alpha_0^2(1-\beta)^2(t-t_0)^2 + 4\alpha_0\beta_*(1-\beta)(t-t_0) + 1]}, \quad (62)$$

and integrating again, we obtain for the scale factor

$$a(t) = a_0[4\alpha_0^2(1-\beta)^2(t-t_0)^2 + 4\alpha_0(1-\beta)\beta_*(t-t_0) + 1]^{1/4(1-\beta)}. \quad (63)$$

This solution is nonsingular for  $\omega > 0$ , with  $a(t)$  reaching its minimal value,  $a_{min} = a_0[8\mu_0\omega(1-\beta)B_0^2]^{1/4(1-\beta)}$ , at a time  $t_{min} = t_0 - \beta_*/2\alpha_0(1-\beta)$ . It thus follows that the universe is a bouncing one. It begins arbitrarily large at  $t \ll t_{min}$ , decreases until its minimal value at  $t_{min}$ , and then begins the expansion phase. For completeness, the expression for the magnetic field is

$$B(t) = \frac{B_0(1-\beta)^{1/2}}{[4\alpha_0^2(1-\beta)^2(t-t_0)^2 + 4\alpha_0(1-\beta)\beta_*(t-t_0) + 1]^{1/2}}, \quad (64)$$

and at  $t_{min}$ , it is readily checked that  $B(t_{min}) = 1/2\sqrt{2\mu_0\omega}$  and  $\rho(t_{min}) = 0$ . As one should expect, in the limit  $\beta = 0$ , all the results above for a time-dependent  $\Lambda$  term reduce those of  $\Lambda = 0$  [see Eqs. (27)–(34)]. Before we proceed further, it is worthwhile to notice the existence of an inflationary era ( $\ddot{a} > 0$ ) which depends on the value of the  $\beta$  parameter. For  $1/2 < \beta \leq 1$  the universe always evolves through an accelerated expansion state. However, if  $0 < \beta < 1/2$ , it inflates on the interval  $t_{min} - t_I < t < t_{min} + t_I$ , where

$$t_I = \sqrt{\frac{3\mu_0^2\omega(1-\beta)^2c^2}{\pi G(1-2\beta)}}. \quad (65)$$

In Figs. 4 and 5 we show the time dependence of the scale factor,  $\Lambda$  term, magnetic field, energy densities, and pressure for some selected values of  $B_0$  and  $\beta$ . The main conclusion is that the singularity must be avoided for a generic time-dependent  $\Lambda$ . As shown in Figs. 4 and 5, the nonlinear corrections are relevant only for  $8\mu_0\omega B^2 \geq 1/10$  or  $t \leq t_{min} + 3\sqrt{8\mu_0\omega(1-\beta)B_0^2}/2\alpha_0(1-\beta)$ .

At this point one may ask by the inverse treatment, i.e., if the singularity is avoided for a given  $B(t)$ . Such a question is immediately answered by examining the simplest case, namely, the one for which the magnetic field remains constant in the course of the evolution. This possibility is clearly allowed by the generalized energy conservation law [see Eq. (25)]. The energy density of the magnetic field is kept constant because energy is continuously drained from the decaying vacuum component to the magnetic field. Actually, if the

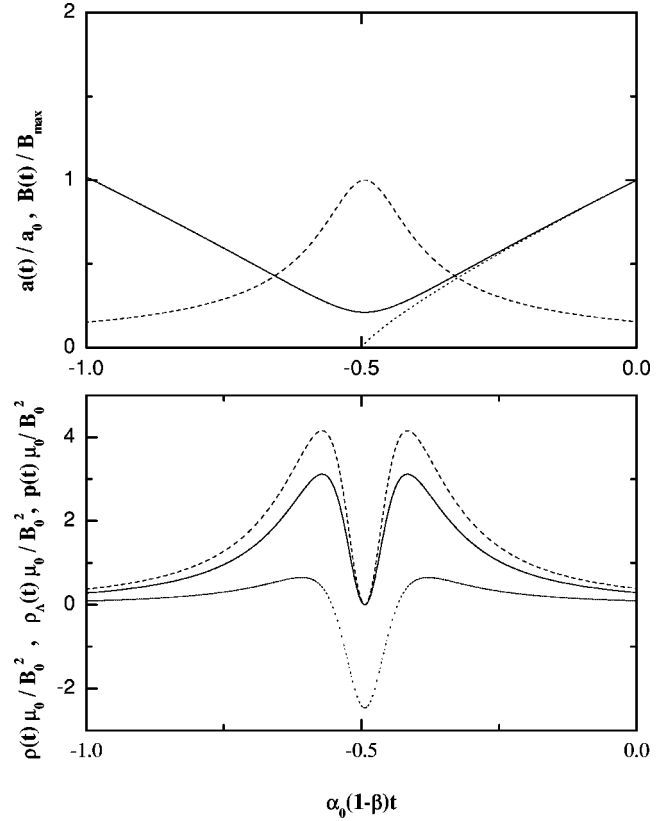


FIG. 4. The upper panel shows the scale factor (solid line), the magnetic field (dashed line), and the classical solution (dotted line) for ( $\omega = 0$ ). The lower panel shows the energy density of magnetic field (solid line), the energy density of the  $\Lambda$  term (dashed line), and the pressure (dotted line) for the model with  $\Lambda = (3\beta/c^2)H^2$ ,  $\beta = 0.4$ , and  $2B_0\sqrt{2\omega\mu_0} = 0.2$ .

$\Lambda$  term is maintained constant the unique solution with a constant magnetic field is the trivial one ( $\mathbf{B} = 0$ ). Unlike the previous solutions, we note that there is no an analogous classical solution for constant magnetic field. Therefore we will analyze such a possibility regardless of any constraint on its domain of validity.

If  $B(t) = B_0 = \text{const}$ , the energy conservation law yields

$$\Lambda(t) = \Lambda_0 + 3K_0 \ln\left(\frac{a}{a_0}\right), \quad (66)$$

where  $\Lambda_0 \equiv \Lambda(t_0)$  and

$$K_0 \equiv -\frac{16\pi G}{3c^4} \frac{B_0^2}{\mu_0} (1 - 16\mu_0\omega B_0^2). \quad (67)$$

Substituting Eq. (66) into Eq. (23), we get for the scale factor

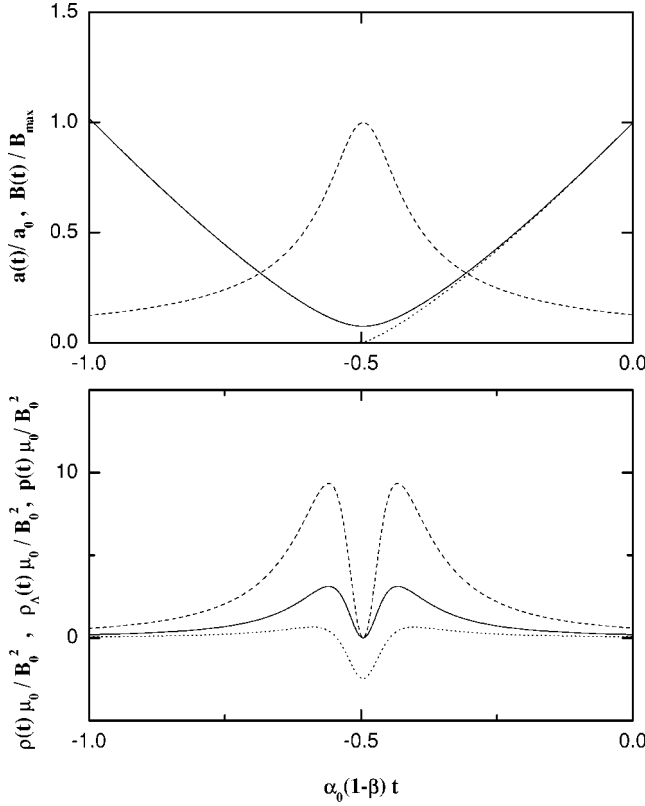


FIG. 5. The upper panel shows the scale factor (solid line), the magnetic field (dashed line), and the classical solution (dotted line) for  $(\omega=0)$ . The lower panel shows the energy density of magnetic field (solid line), the energy density of the  $\Lambda$  term (dashed line), and the pressure (dotted line) for the model with  $\Lambda=(3\beta/c^2)H^2$ ,  $\beta=0.6$ , and  $2B_0\sqrt{2\omega\mu_0}=0.2$ .

$$\begin{aligned} a(t) &= a_0 \exp\left[\frac{K_0 c^2}{4}(t-t_0)^2 + H_0(t-t_0)\right] \\ &= a_0 \exp\left(\frac{K_0 c^2}{4}t^2 + \beta_1 t + \beta_0\right), \end{aligned} \quad (68)$$

where

$$H_0 \equiv \pm \sqrt{\frac{\Lambda_0 c^2}{3} + \frac{4\pi G}{3c^2} \frac{B_0^2}{\mu_0} (1 - 8\mu_0 \omega B_0^2)}, \quad (69)$$

$$\beta_1 \equiv -\left(\frac{K_0 c^2}{2}t_0 - H_0\right), \quad \text{and} \quad \beta_0 \equiv t_0\left(\frac{K_0 c^2}{4}t_0 - H_0\right). \quad (70)$$

The Hubble parameter is

$$H(t) = \frac{K_0 c^2}{2}(t-t_0) + H_0 = \frac{K_0 c^2}{2}t + \beta_1, \quad (71)$$

and we have  $H=0$  for

$$t_c = -\frac{2\beta_1}{K_0 c^2} = t_0 - \frac{2H_0}{K_0 c^2}. \quad (72)$$

At this point, the scale factor reaches the value

$$a(t_c) = a_0 e^{-H_0^2/K_0 c^2}. \quad (73)$$

The behavior of the solution will depend on the sign of the constant  $K_0$  (for  $K_0=0$  we get the de Sitter solution). For  $K_0>0$ , that is, for  $B_0>1/(4\sqrt{\mu_0\omega})$ , the universe is always accelerated ( $\ddot{a}>0$ ) and has a minimum size at  $t_c$ .

A much more interesting solution is the one corresponding to  $K_0<0$  [ $B_0<1/(4\sqrt{\mu_0\omega})$ ]. For this range of  $B_0$ ,  $a(t)$ ,  $\ddot{a}>0$  for  $t<t_c - \sqrt{-2/K_0 c^2}$  and  $t>t_c + \sqrt{-2/K_0 c^2}$  and  $a(t)$  has a maximum at  $t_c$ . It is worth notice that the time interval  $\Delta t_{(NI)}$ , prior to  $t_c$ , for which the solution is *not* inflationary depends on the value of  $B_0$  as

$$\Delta t_{(NI)} = 2 \sqrt{-\frac{2}{K_0 c^2}} = \frac{2}{B_0} \sqrt{\frac{3\mu_0 c^2}{8\pi G(1-16\mu_0\omega B_0^2)}}. \quad (74)$$

The cosmological term  $\Lambda$  dominates the dynamics of the universe for values of  $t$  where

$$\Lambda c^4 > 8\pi G\rho. \quad (75)$$

For both models ( $K_0>0$  and  $K_0<0$ ), the condition (75) is satisfied by  $t<t_3$  and  $t>t_4$  where

$$t_3 = \frac{-2H_0 c - 2 \sqrt{\frac{16\pi G\rho}{3}}}{K_0 c^3} \quad (76)$$

and

$$t_4 = \frac{-2H_0 c + 2 \sqrt{\frac{16\pi G\rho}{3}}}{K_0 c^3}. \quad (77)$$

In Fig. 6, we show the scale factor and the cosmological term as a function of time for  $K_0>0$  and some values of  $B_0$  and  $\lambda_0 = \Lambda_0 c^2/3$ . The same quantities have also been plotted for  $K_0<0$  in Fig. 7.

Naturally, if one expects any such model to properly describe the evolution of the real universe, it would be advisable to take into account other matter fields, such as ultrarelativistic matter, scalar fields, or dust. In Ref. [8], it was demonstrated, for the case  $\Lambda=0$ ,  $B=B(t)$ , that the presence of ultrarelativistic matter with an equation of state  $p_{(ur)} = \rho_{(ur)}/3$  would just amount for a reparametrization of the constants  $B_0$  and  $\omega$ .



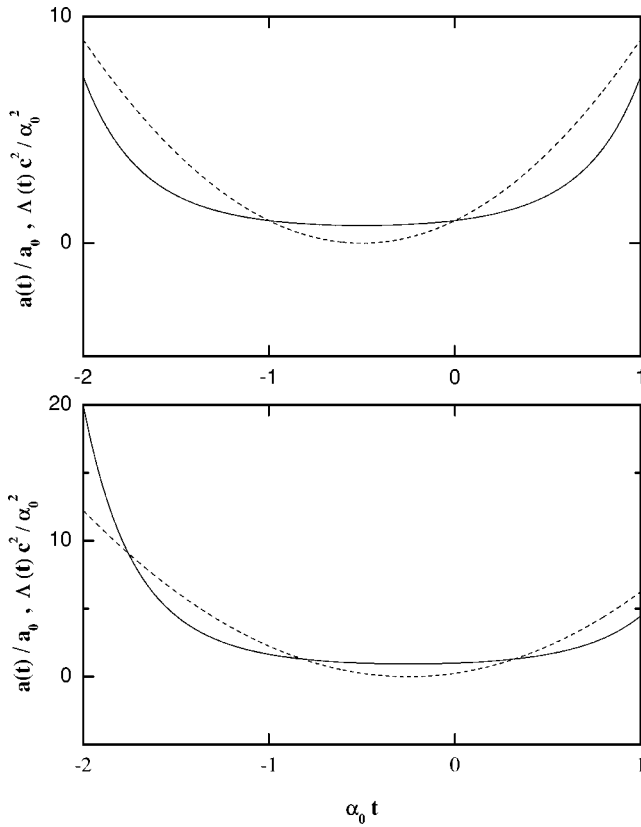


FIG. 6. The scale factor (solid line) and the cosmological term (dashed line) for the model with constant magnetic field, time-dependent  $\Lambda$ ,  $K_0 > 0$  ( $2B_0\sqrt{2\omega\mu_0} = 1$ ). In the upper panel  $\sqrt{\lambda_0}/\alpha_0 = 1$  and the lower panel is for  $\sqrt{\lambda_0}/\alpha_0 = 0.5$ .

At this point, we would like to stress the mathematical consistence of the whole set of solutions derived in the present work. In general, there are three unknown functions: the scale factor  $a$ , the magnetic field  $B$ , and the cosmological term  $\Lambda$  (constant or time dependent). As one may check for each case, the number of unknown functions and equations coincide, the unique exception is related to models containing a variable  $\Lambda(t)$  term for which the phenomenological law  $\Lambda = 3\beta c^{-2}H^2$ , has been considered (see Refs. [16,18]).

**VI. CONCLUSION**

We have examined whether nonlinear corrections to the Maxwell electrodynamics may avoid the cosmic singularity occurring in flat FRW universes. In brief, the answer is positive. We show that by discussing a large class of analytical cosmological models under three different assumptions. In the first case, the cosmological  $\Lambda$  term is identically zero and the dynamics is driven by a time-dependent magnetic field. This class generalizes the particular solution previously found by De Lorenci *et al.* [8], and confirms their statement concerning the avoidance of the initial singularity. In principle, since the solutions are nonsingular, they potentially solve the horizon problem. We have also examined if the basic features of such models remain true if new ingredients

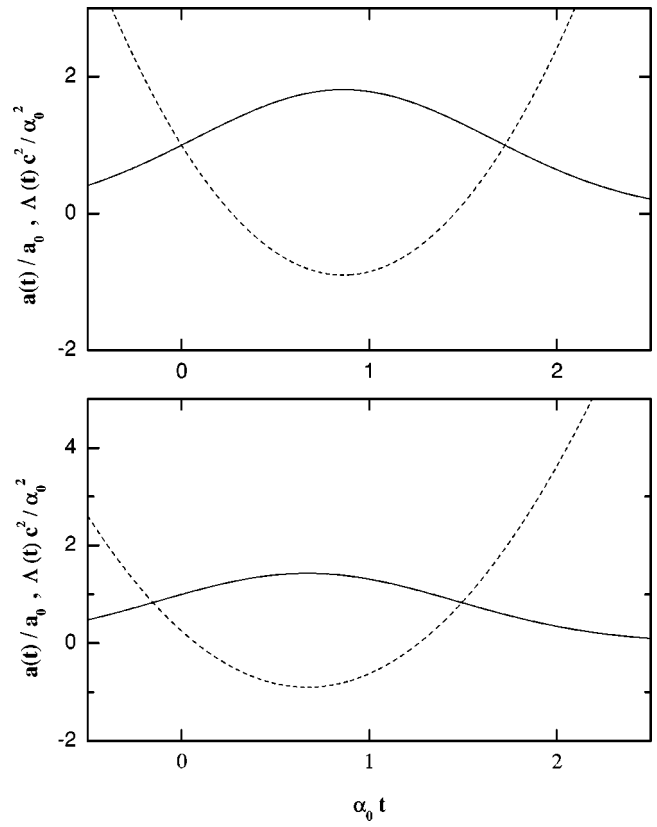


FIG. 7. As in Fig. 4 but for  $K_0 < 0$  ( $2B_0\sqrt{2\omega\mu_0} = 0.1$ ). In the upper panel  $\sqrt{\lambda_0}/\alpha_0 = 1$  and the lower panel is for  $\sqrt{\lambda_0}/\alpha_0 = 0.5$ .

are introduced in the matter content. In this concern, models with a constant and time-dependent  $\Lambda$  term were studied with some detail. Again, for both cases, the universe is also nonsingular, bouncing at a critical time when the scale factor reaches its minimum value.

For a decaying vacuum energy density we discuss two different scenarios. In the first one, it was phenomenologically described by  $\Lambda(t) \sim H^2$  as assumed by several authors [16,17]. These models are nonsingular and resemble the solutions with no  $\Lambda$ . The second scenario is a rather curious solution which describes a universe driven by a constant magnetic field. The time behavior of the cosmological term is now uniquely determined by the EFE's as a logarithm of the scale factor. It should be interesting to examine if such results are maintained in the presence of other matter fields, as well as for universes with nonzero curvature.

Finally, in analogy with the cosmological case, one may ask if nonlinear terms in the Maxwell Lagrangian may remove the physical singularity present in a charged black hole (Reissner-Nordstrom solution). This problem will be discussed in a forthcoming communication.

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