Exotic baryon mass spectrum and the 10−8 and $\overline{10}$−8 mass difference in the Skyrme model

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The **8**, **10**, and $\overline{\mathbf{10}}$ baryon mass spectra as a function of the Skyrme charge e and the $SU(3)_f$ symmetry breaking parameters are given in tabular form. We also estimate the decuplet-octet and the antidecuplet-octet mass difference. A comparison with existing literature is given.

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Recently we applied the concept of a minimal SU(3) extended Skyrme model to nonleptonic hyperon and Ω^- decays [1], producing reasonable agreement with experiment. This concept uses only one free parameter, the Skyrme charge e, and the flavor symmetry breaking (SB) term, proportional to λ_8 in the kinetic and mass terms. The main aim of this Brief Report is the application of the same concept in an attempt to predict the baryonic decuplet-octet (Δ) and antidecuplet-octet ($\overline{\Delta}$) mass difference as well as to evaluate the mass spectrum for octet, decuplet, and the recently discovered antidecuplet baryons.

The experimental discovery [2] and the later confirmation [3] of the exotic, presumably spin-1/2, baryon of positive strangeness, Θ^+, was recently supported by the first observation of Θ^+ in hadron-hadron interactions [4], and by the NA49 Collaboration [5] discovery of the exotic isospin-3/2 baryon with strangeness -2, $\Xi^{--}_{3/2}$. In this way, the antidecuplet and possibly other states of the higher $SU(3)_f$ representation moved from pure theory into the real world of particle physics.

The first successful prediction of mass of one member of the $\overline{\mathbf{10}}$ baryons, known as the pentaquark or Θ^+ baryon, in the framework of the Skyrme model was presented in Ref. [6]. Later, many authors used different types of quark, chiral soliton, lattice QCD, diquark, etc., models [7–23] to estimate the higher SU(3) representation ($\overline{\mathbf{10}}$, **27**, etc.,) mass spectrum, relevant mass differences, and other baryon properties.

In this Brief Report, as in Ref. [1], we use the minimal SU(3) extension of the Skyrme Lagrangian introduced in [24]:

$$\mathcal{L}=\mathcal{L}_\sigma+\mathcal{L}_{\rm Sk}+\mathcal{L}_{\rm WZ}+\mathcal{L}_{\rm SB}, \tag{1}$$

where $\mathcal{L}_\sigma$, $\mathcal{L}_{\rm Sk}$, $\mathcal{L}_{\rm WZ}$, and $\mathcal{L}_{\rm SB}$ denote the σ-model, Skyrme, Wess-Zumino, and symmetry breaking Lagrangians [25–33], respectively.

For profile function $F(r)$ we use the arctan ansatz [34,35]:

$$F(r)=2\arctan\left[\left(\frac{r_0}{r}\right)^2\right]. \tag{2}$$

Here r_0—the soliton size—is the variational parameter and the second power of r_0/r is determined by the long-distance behavior of the equations of motion. After rescaling $x=ref_\pi$, we obtain the ratio $r/r_0=x/x_0$. The quantity x_0 has the meaning of a dimensionless size of a soliton [or rather in units of $(ef_\pi)^{-1}$]. The advantage of using Eq. (2) is that all integrals involving the profile function $F(x/x_0)$ can be evaluated analytically.

The SU(3) extension of the Skyrme Lagrangian $\mathcal{L}$ uses the set of parameters $\hat{x},\beta',\delta'$ introduced in [24]:

$$\hat{x}=\frac{2m_K^2f_K^2}{m_\pi^2f_\pi^2}-1,\quad \beta'=\frac{f_K^2-f_\pi^2}{4(1-\hat{x})},$$

$$\delta'=\frac{m_\pi^2f_\pi^2}{4}=\frac{m_K^2f_K^2}{2(1+\hat{x})}. \tag{3}$$

The δ' term is required to split pseudoscalar meson masses, while the β' term is required to split pseudoscalar decay constants.

Including the previously introduced arctan ansatz for the profile function $F(r)$, we calculate the SU(3) extended classical soliton mass $\mathcal{E}_{\rm csol}$, the decuplet-octet mass splitting Δ, the antidecuplet-octet mass splitting $\overline{\Delta}$, i.e., the moment of inertia λ_c for rotation in coordinate space, and the moment of inertia λ_s for flavor rotations in the direction of the strange degrees of freedom, except for the eighth direction [24,35], and the symmetry breaking (SB) quantity γ. The quantity γ is the coefficient in the SB piece $\mathcal{L}_{\rm SB}=-\frac{1}{2}\gamma(1-D_{88})$ of a total collective Lagrangian $\mathcal{L}$ and is linear in the SB parameter $(1-\hat{x})$. The above-mentioned quantities are given by the following equations:

$$\mathcal{E}_{\rm csol}=3\sqrt{2}\pi^2\frac{f_\pi}{e}\left[x_0+\frac{15}{16x_0}+\frac{2}{f_\pi^2}\left(3\beta' x_0+\frac{4}{3}\frac{\delta'}{e^2f_\pi^2}x_0^3\right)\right], \tag{4}$$

$$\Delta=\frac{3}{2\lambda_c(x_0)},\quad \overline{\Delta}=\frac{3}{2\lambda_s(x_0)}, \tag{5}$$

$$\lambda_c=\frac{\sqrt{2}\pi^2}{3e^3f_\pi}\left[6\left(1+2\frac{\beta'}{f_\pi^2}\right)x_0^3+\frac{25}{4}x_0\right],\tag{6}$$

$$\lambda_s=\frac{\sqrt{2}\pi^2}{4e^3f_\pi}\left[4\left(1-2(1+2\hat{x})\frac{\beta'}{f_\pi^2}\right)x_0^3+\frac{9}{4}x_0\right],\tag{7}$$

$$\gamma=4\sqrt{2}\pi^2\frac{1-\hat{x}}{ef_\pi}\left(\beta'x_0-\frac{4}{3}\frac{\delta'}{e^2f_\pi^2}x_0^3\right).\tag{8}$$

It is important to note that nowadays everybody agrees that the SU(3) extended Skyrme model classical soliton mass $\mathcal{E}_{\rm csol}$ receives too large value. The consequence of this is an unrealistic baryonic mass spectrum. The $\mathcal{E}_{\rm csol}$ is connected with octet mass mean $\mathcal{M}_{\mathbf{8}}$. From experiment we know $\mathcal{M}_{\mathbf{8}}=\frac{1}{8}\Sigma_{B=1}^{8}M_B^{\mathbf{8}}=1151$ MeV. Taking all that into account it is more appropriate to express mass formulas by $\mathcal{M}_{\mathbf{8}}$ instead by $\mathcal{E}_{\rm csol}$. However, we are using the result for the classical soliton mass (4) to obtain x_0, by minimalizing $\mathcal{E}_{\rm csol}$:

$$x_0^2=\frac{15}{8}\left[1+\frac{6\beta'}{f_\pi^2}+\sqrt{\left(1+\frac{6\beta'}{f_\pi^2}\right)^2+\frac{30\delta'}{e^2f_\pi^4}}\right]^{-1}.\tag{9}$$

The dimensionless size of the Skyrmion x_0 includes the dynamics of SB effects which takes place within the Skyrmion. It is clear from the above equation that a Skyrmion effectively shrinks when one ‘‘switches on’’ the SB effects and it shrinks more when the Skyrme charge e receives smaller values.

To obtain the **8**, **10**, and $\overline{\mathbf{10}}$ absolute mass spectra, we use the following definition of the mass formulas:

$$M_B^{\mathbf{8}}=\mathcal{M}_{\mathbf{8}}-\frac{1}{2}\delta_B^{\mathbf{8}}\gamma(x_0),$$

$$M_B^{\mathbf{10}}=\mathcal{M}_{\mathbf{8}}+\frac{3}{2\lambda_c(x_0)}-\frac{1}{2}\delta_B^{\mathbf{10}}\gamma(x_0),$$

$$M_B^{\overline{\mathbf{10}}}=\mathcal{M}_{\mathbf{8}}+\frac{3}{2\lambda_s(x_0)}-\frac{1}{2}\delta_B^{\overline{\mathbf{10}}}\gamma(x_0),\tag{10}$$

where $\mathcal{M}_{\mathbf{8}}$ is defined earlier and the splitting constants $\delta_B^{\mathbf{R}}$ are given in Eqs. (17)–(19) of Ref. [11]. Also, from experiment we know $\mathcal{M}_{\mathbf{10}}=\frac{1}{10}\Sigma_{B=1}^{10}M_B^{\mathbf{10}}=1382$ MeV.

Formulas (10) imply equal spacing for antidecuplets. From the existing experiments ($\Theta^+=1540$ MeV and $\Xi_{3/2}^{--}=1861$ MeV) we estimate that spacing to be $\overline{\delta}=(1861-1540)/3=107$ MeV. Next we estimate the masses of antidecuplets, $N^*=1647$ MeV, $\Sigma_{\overline{\mathbf{10}}}^{*}=1754$ MeV, and the $\overline{\mathbf{10}}$ mean mass $\mathcal{M}_{\overline{\mathbf{10}}}=\frac{1}{10}\Sigma_{B=1}^{10}M_B^{\overline{\mathbf{10}}}=1754$ MeV. Finally we obtain the antidecuplet-octet mass splittings $\overline{\Delta}_{\rm exp}=\mathcal{M}_{\overline{\mathbf{10}}}-\mathcal{M}_{\mathbf{8}}=603$ MeV. However, the decuplet-octet mass splittings $\Delta_{\rm exp}=231$ MeV represent the true experimental value.

Now we calculate the mass splittings Δ and $\overline{\Delta}$ for (i) SB with the approximation $f_\pi=f_K=93$ MeV ($\beta'=0$, $\delta'=4.12\times10^7$ MeV^4) and for (ii) SB with $f_\pi=93$ MeV, $f_K=113$ MeV, ($\beta'=-28.6$ MeV^2 and $\delta'=4.12\times10^7$ MeV^4). The results are presented in Table I.

TABLE I. The mass splittings Δ and $\overline{\Delta}$ for cases (i) and (ii) as a function of e.

	(i)			(ii)			
Mass Spl.\ e	3.4	4.2	4.6	3.4	4.2	4.6	Expt.
Δ (MeV)	129	229	294	128	227	291	231
$\overline{\Delta}$ (MeV)	354	621	795	273	474	604	603

We have chosen the three values of Skyrme charge $e=3.4$, 4.2, 4.6. The reason for this lies in the fact that in our minimal approach, case (ii), $e=3.4$ gives the best fit for the nucleon axial coupling constant $g_A=1.25$ [1], $e=4.2$ fits nicely $\Delta_{\rm exp}$, and $e=4.6$ gives the best fit for $\overline{\Delta}_{\rm exp}$. However, from Table I, we see that a certain middle value of e ($=4.2$) supports also case (i)—i.e., is in good agreement with experiment.

The **8**, **10**, and $\overline{\mathbf{10}}$ baryon mass spectra (10) as a function of the SB effects and the Skyrme charge e are given in Table II. Since we are using the most simple version of the total Lagrangian (1)—i.e., we omit vector meson effects, the so-called static kaon fluctuations [24], and other fine-tuning effects in the expressions (4)–(10)—our results given in Tables I and II do agree roughly with other Skyrme-model-based estimates [6–11]. In particular, our approach is similar to the one in Refs. [9,10]. The main difference is that our Lagrangian is simpler—i.e., contains only SB proportional to λ_8—and that we are using the arctan ansatz approximation for the profile function $F(r)$. Comparing the pure Skyrme model prediction of Ref. [10] (fits A and B in Table 2) with our results for $e=4.2$, presented in Table II, we have found up to 8% differences. One of the reasons is due to the fact that fits A and B in Table 2 of Ref. [10] were obtained for different e's: i.e., $e=3.96$ and $e=4.12$. Also, from our Table II one can see that for $e=4.2$, case (ii), the mass spectrum

TABLE II. The **8**, **10**, and $\overline{\mathbf{10}}$ baryon mass spectra (MeV) for cases (i) and (ii) as a functions of e.

	(i)			(ii)			
Mass\ e	3.4	4.2	4.6	3.4	4.2	4.6	Expt. [36]
N	934	1024	1051	793	934	977	939
Λ	1079	1109	1118	1032	1079	1093	1116
Σ	1223	1193	1184	1270	1223	1209	1193
Ξ	1295	1236	1218	1390	1295	1267	1318
Δ	1190	1327	1403	1130	1287	1370	1232
Σ^*	1280	1380	1445	1279	1378	1442	1385
Ξ^*	1371	1433	1487	1428	1468	1514	1530
Ω	1461	1486	1529	1578	1558	1587	1672
Θ^+	1325	1666	1862	1125	1444	1611	1540 [2,3]
N^*	1415	1719	1904	1274	1535	1683	–
$\Sigma_{\overline{\mathbf{10}}}^{*}$	1505	1772	1946	1424	1625	1755	–
$\Xi_{3/2}^{--}$	1595	1825	1988	1573	1715	1828	1861 [5]

differs from the experiment $<\sim 8\%$ for Ω^-, Θ^+, and $\Xi^{--}_{3/2}$. All other estimated masses are $<\sim 5\%$ different from experiment. From Table II we conclude that in our minimal approach the best fit for **8**, **10**, and $\overline{\mathbf{10}}$ baryon mass spectra, as a function of e and for $f_\pi \neq f_K$, would lie between $e \simeq 4.2$ and $e \simeq 4.6$.

Symmetry breaking effects are generally very important and do improve theoretical estimates of the quantities like Δ, $\overline{\Delta}$, the baryon mass spectrum, etc. Our Tables I and II show implicitly that inclusion of additional contributions, like vector meson contributions, the so-called static kaon fluctuations [24], and other fine-tuning effects into the SB Lagrangian [9], does not change the results dramatically. On the contrary, the main effect comes from the famous D_{88} term. The differences between f_π and f_K and the e dependence are important. All other contributions represent the fine-tuning effects of the order of a few percent [37]. This is important for understanding the overall picture of the baryonic mass spectrum as well as for further study of other nonperturbative, higher-dimensional operator matrix elements in the Skyrme model [1,38].

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