B_s^0-$\overline{B}_s^0$ mixing and the $B_s \to J/\psi\phi$ asymmetry in supersymmetric models

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We analyze supersymmetric contributions to B_s mixing and their impact on mixing-induced *CP* asymmetries, using the mass insertion approximation. We discuss in particular the correlation of supersymmetry (SUSY) effects in the *CP* asymmetries of $B_s \to J/\psi\phi$ and $B_d \to \phi K_S$ and find that the mass insertions dominant in B_s mixing and $B_d \to \phi K_S$ are $(\delta_{23}^d)_{LL,RR}$ and $(\delta_{23}^d)_{LR,RL}$, respectively. We show that models with dominant $(\delta_{23}^d)_{LR,RL}$ can accommodate a negative value of $S_{\phi K_S}$, in agreement with the Belle measurement of that observable, but yield a B_s mixing phase too small to be observed. On the other hand, models with dominant $(\delta_{23}^d)_{LL,RR}$ predict sizable SUSY contributions to both ΔM_s and the mixing phase, but do not allow the asymmetry in $B_d \to \phi K_S$ to become negative, except for small values of the average down squark mass, which, in turn, entail a value of ΔM_s too large to be observed at the Fermilab Tevatron and CERN LHC. We conclude that the observation of B_s mixing at hadron machines, together with the confirmation of a negative value of $S_{\phi K_S}$, disfavors models with a single dominant mass insertion.

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I. INTRODUCTION

The impressive performance of the *B* factory experiments BaBar and Belle provides the basis for scrutinizing tests of the standard model (SM) picture of flavor structure and *CP* violation in the quark sector, and opens the possibility of probing virtual effects from new physics at low energies. In the supersymmetric extension of the SM, a new source of flavor violation arises from the fact that, in general, the rotation that translates flavor eigenstates into mass eigenstates will not be the same for quark and squark fields, which implies the appearance of a new squark mixing matrix or, alternatively, that of off-diagonal squark mass terms in a basis where the quarks are mass eigenstates and both quark and squark fields have undergone the same rotation—the so-called super Cabibbo-Kobayashi-Maskawa (CKM) basis. A convenient tool for studying the impact of this new source of flavor violation is the mass-insertion approximation (MIA), which was first introduced in [1] and since then has been widely used as a largely model-independent tool for analyzing and constraining supersymmetry (SUSY) effects in *B* physics. In the super-CKM basis the couplings of fermions and their SUSY partners to neutral gauginos are flavor-diagonal and flavor-violating SUSY effects are encoded in the nondiagonal entries of the sfermion mass matrix. The sfermion propagators are expanded in a series in $\delta = \Delta^2/\tilde{m}_{\tilde{q}}^2$, where Δ^2 are the off-diagonal entries and $\tilde{m}_{\tilde{q}}$ is the average sfermion mass. We assume $\Delta^2 \ll \tilde{m}_{\tilde{q}}^2$, so that the first term in the expansion is sufficient, and also that the diagonal sfermion masses are nearly degenerate.

Flavor-changing box and penguin processes as observed at the *B* factories are very sensitive to flavor-violating effects beyond the SM, and the constraints on or measurement of nondiagonal squark masses will help to discriminate among various soft SUSY breaking mechanisms. In summer 2002, the BaBar and Belle Collaborations reported the first measurements of the mixing-induced *CP* asymmetry $S_{\phi K_S}$ in $B_d \to \phi K_S$, which at the quark level is $b \to s\overline{s}s$ and thus a pure penguin process, which is expected to exhibit, in the SM, the same mixing-induced *CP* asymmetry as observed in $B_d \to J/\psi K_S$ [2]. The experimental results, however, updated in summer 2003, paint a slightly different picture:

$$S_{J/\psi K_S} = 0.736 \pm 0.049 \quad \text{(BaBar and Belle) [3,4]} \tag{1}$$

$$S_{\phi K_S} \overset{2002}{=} -0.39 \pm 0.41 \ [5,6]$$
$$\overset{2003}{=} \begin{cases} -0.96 \pm 0.50^{+0.09}_{-0.11}, & \text{Belle [7]}, \\ +0.45 \pm 0.43 \pm 0.07, & \text{BaBar [8]}. \end{cases} \tag{2}$$

Although the experimental situation in $B_d \to \phi K_S$ is not yet conclusive, the deviation of $S_{\phi K_S}$ from $S_{J/\psi K_S}$ may constitute a first potential glimpse at physics beyond the SM, and it is both worthwile and timely to pursue any interpretion of these results in terms of new physics and to analyze their impact on future measurements to be performed at the *B* factories or at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC); see e.g. [9–12].

In the framework of the MIA, the measurement of $S_{J/\psi K_S}$, which is in agreement with the SM expectation, indicates that $(\delta_{13}^d)_{AB}$, $A,B=L,R$, is small [13], whereas the result for $S_{\phi K_S}$ indicates a relatively large $(\delta_{23}^d)_{AB}$. Furthermore, by including the constraints on $(\delta_{23}^d)_{AB}$ from $b\to s\gamma$, it was found [9] that, for average squark masses of order 500 GeV, only models with dominant $(\delta_{23}^d)_{LR,RL}$ can accommodate a negative value of $S_{\phi K_S}$.

δ_{23}^d insertions also determine the size of SUSY contributions to B_s mixing and, as a consequence, the mixing-induced *CP* asymmetries in tree-level dominated decays like e.g. $B_s\to J/\psi\phi$, which is one of the benchmark channels to be studied at hadron machines. Within the SM, the B_s mixing phase is very small, and consequently $S_{J/\psi\phi}$ is expected to be of $\mathcal{O}(10^{-2})$. In SUSY, on the other hand, the third-to-second generation $(b\to s)$ box diagram may carry a sizable *CP* violating phase, which is described in terms of the same mass insertion $(\delta_{23}^d)_{AB}$ governing the *CP* asymmetry $S_{\phi K_S}$. It is therefore both important and instructive to analyze all $b\to s$ transitions in the same framework, paying particular attention to the correlations between observables. This is the subject of this paper.

Our paper is organized as follows. In Sec. II, we recall the master formulas determining B_s mixing and the *CP* asymmetry in $B_s\to J/\psi\phi$ and discuss the SM expectations for the B_s mixing parameters and the experimental reach for B_s mixing at hadron colliders. In Sec. III, we discuss the dominant SUSY contributions to B_s mixing in the framework of the mass insertion approximation. In Sec. IV, we present numerical results and discuss the correlation between the constraints from $b\to s\gamma$ and $S_{\phi K_S}$, obtained previously in Ref. [9], and B_s mixing. Section V contains a summary and conclusions.

II. B_s MIXING AND THE MIXING-INDUCED *CP* ASYMMETRY IN $B_s\to J/\psi\phi$

A. Master formulas and new physics effects

Let us begin by recalling[1] the master formulas for B_s mixing and the resulting mixing-induced asymmetry in $B_s\to J/\psi\phi$. As for B_d, the mixing angles p and q between the flavor and mass eigenstates in the B_s system can be expressed in terms of the B_s^0-$\overline{B}_s^0$ transition matrix element M_{12}:

$$\frac{q}{p}=\sqrt{\frac{M_{12}^*}{M_{12}}}, \tag{3}$$

where we have used $\Delta\Gamma_s\ll\Delta M_s$ and $\Delta\Gamma_s\ll\Gamma_s^{\rm tot}$. The resulting mass and width differences between mass eigenstates are given by

$$\Delta M_s=-2M_{12},\quad \Delta\Gamma_s=2\Gamma_{12}\cos\zeta_B, \tag{4}$$

where $\zeta_B\equiv\arg(\Gamma_{12}/M_{12})$. Γ_{12} can be computed from diagrams with two insertions of the $\Delta B=1$ Hamiltonian and is dominated by the tree contribution. SUSY effects are very small, so to very good accuracy one can set

$$\Gamma_{12}=\Gamma_{12}^{\rm SM}. \tag{5}$$

In the SM, M_{12} is dominated by top quark exchange; the mixing phase in the Wolfenstein parametrization and its size are given by[2]

$$\arg M_{12}^{\rm SM}=2\arg(V_{tb}V_{ts}^*)=-2\lambda^2\eta=\mathcal{O}(10^{-2}). \tag{6}$$

In SUSY, there are new contributions to M_{12} induced by e.g. gluino and chargino box diagrams, which potentially carry a large phase and which we parametrize as

$$\sqrt{\frac{M_{12}}{M_{12}^{\rm SM}}}\equiv r_s e^{i\beta_s}, \tag{7}$$

which entails

$$\Delta M_s=r_s^2\Delta M_s^{\rm SM},\quad \Delta\Gamma_s\simeq\Delta\Gamma_s^{\rm SM}\cos 2\beta_s, \tag{8}$$

assuming $\beta_s\gg\arg M_{12}^{\rm SM}$. The above result implies that new physics contributions will always lead to a decrease of $\Delta\Gamma_s$, as was first discussed in Ref. [14].

Let us now discuss the effect of SUSY on the mixing-induced *CP* asymmetry in the tree-dominated decay $B_s\to J/\psi\phi$, which is expected to be very small in the SM and hence highly susceptible to large or even moderate new *CP* violating phases. Although the final state $J/\psi\phi$ is not a *CP* eigenstate, but a superposition of *CP* odd and even states which can be disentangled by an angular analysis of their decay products [15,16], the advantage of that channel over the similar process $B_s\to J/\psi\eta(')$ is the comparatively clean although still challenging reconstruction of the ϕ via $\phi\to K^+K^-$, whereas the $\eta(')$ is even more elusive. Once the *CP* waves have been identified, the analysis of $B_s\to J/\psi\phi$ proceeds largely along the same lines as that of $B_d\to J/\psi K_S$, except for the fact that, in contrast to B_d mixing, the width difference $\Delta\Gamma_s$ cannot be neglected and entails a slight modification of the formula for the asymmetry. Without a separation of the final state *CP* waves, the mixing asymmetry still depends on hadronic parameters describing the polarization amplitudes $A_{0,\parallel,\perp}$ characteristic for the final state ($A_{0,\parallel}$ for *CP* even and $A_\perp$ for *CP* odd). One finds, assuming no direct *CP* violation,

$$S_{J/\psi\phi}\sin\Delta M_s t=\frac{\Gamma(\overline{B}_s^0\to J/\psi\phi)-\Gamma(B_s^0\to J/\psi\phi)}{\Gamma(\overline{B}_s^0\to J/\psi\phi)+\Gamma(B_s^0\to J/\psi\phi)}$$

[1]Here we use the convention $|B_s\rangle_1=p|B_s^0\rangle+q|\overline{B}_s^0\rangle$ and $|B_s\rangle_2=p|B_s^0\rangle-q|\overline{B}_s^0\rangle$ where we define $\mathbf{CP}|P\rangle=+|P\rangle$ and $\Delta M_s=M_2-M_1$ and $\Delta\Gamma_s=\Gamma_1-\Gamma_2$.

[2]Note that we can write $\arg M_{12}^{\rm SM}$ in a different phase convention but its size is always the same.

$$=\frac{D\,\mathrm{Im}\left[\frac{q}{p}\overline{\rho}_{\mathrm{odd}}\right]+\mathrm{Im}\left[\frac{q}{p}\overline{\rho}_{\mathrm{even}}\right]}{DF_{\mathrm{odd}}(t)+F_{\mathrm{even}}(t)}\sin\Delta M_s t \tag{9}$$

where

$$F_{\mathrm{odd,even}}(t)=\cosh\left(\frac{\Delta\Gamma_s}{2}t\right)+\mathrm{Re}\left[\frac{q}{p}\overline{\rho}_{\mathrm{odd,even}}\right]\sinh\left(\frac{\Delta\Gamma_s}{2}t\right) \tag{10}$$

and D encodes the polarization amplitudes:

$$D\equiv\frac{|A_\perp|^2}{|A_\parallel|^2+|A_0|^2}. \tag{11}$$

D, as a hadronic quantity, comes with a certain theoretical uncertainty. Reference [17], for instance, quotes $D\approx 0.3\pm 0.2$.

The parameter $\overline{\rho}$ is defined as

$$\overline{\rho}_{\mathrm{odd,even}}=\frac{A(\overline{B}_s^0\to J/\psi\phi)_{\mathrm{odd,even}}}{A(B_s^0\to J/\psi\phi)_{\mathrm{odd,even}}} \tag{12}$$

and can be computed from the $\Delta B=1$ effective Hamiltonian, yielding

$$\overline{\rho}_{\mathrm{odd,even}}=\mp\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}=\xi_{\mathrm{odd,even}} \tag{13}$$

with $\xi_{\mathrm{even}}=+1$ and $\xi_{\mathrm{odd}}=-1$. Accordingly, we have

$$\frac{q}{p}\overline{\rho}_{\mathrm{odd,even}}\simeq\xi_{\mathrm{odd,even}}e^{-2i\beta_s}. \tag{14}$$

B. Estimate of ΔM_s^{SM} and $\Delta\Gamma_s^{\mathrm{SM}}$

In order to estimate ΔM_s^{SM}, one usually uses the ratio $\Delta M_s^{\mathrm{SM}}/\Delta M_d^{\mathrm{SM}}$, in which all short-distance effects cancel:

$$\frac{\Delta M_s^{\mathrm{SM}}}{\Delta M_d^{\mathrm{SM}}}=\frac{M_{B_s}}{M_{B_d}}\frac{B_{B_s}f_{B_s}^2}{B_{B_d}f_{B_d}^2}\frac{|V_{ts}|^2}{|V_{td}|^2}. \tag{15}$$

The remaining ratio of hadronic parameters has been calculated on the lattice yielding [18]

$$\frac{B_{B_s}(m_b)f_{B_s}^2}{B_{B_d}(m_b)f_{B_d}^2}=(1.15\pm 0.06^{+0.07}_{-0.00})^2,$$

where the asymmetric error is due to the effect of chiral logarithms in the quenched approximation. In many SUSY models the dominant new contributions to B_d mixing involve transitions between the third and the first generations and are thus suppressed by the corresponding CKM matrix elements, so that B_d mixing is saturated by the SM contribution [11,13,19,20] and we can assume $\Delta M_d=\Delta M_d^{\mathrm{SM}}$. ΔM_d is measured from the time dependence of B_d mixing and is rather precisely known [21]:

$$(\Delta M_d)_{\mathrm{expt}}=(0.489\pm 0.008)\mathrm{ps}^{-1}.$$

As for $|V_{ts}|^2/|V_{td}|^2$, one has to use a value that is not contaminated by new physics. Stated differently, one needs a measurement of the angle α^{SM} or γ^{SM} from pure SM processes. Various strategies for a clean determination of these angles have been proposed (see Ref. [22]) and are expected to yield stringent constraints in the near future. For the time being, however, one has to resort to a different method and exploit the very basic fact that a triangle is completely determined by three parameters, which in our case are the base, of length 1, the left side, which is determined by $|V_{ub}/V_{cb}|$, and the angle β^{SM} between the base and the right side. The essential assumptions that enter here are (i) that the determination of $|V_{cb}|$ and $|V_{ub}|$ from semileptonic decays is free of new physics, which is a model-independent assumption as these are tree processes, and (ii) that β as measured from $B_d\to J/\psi K_S$ is actually β^{SM}—which, as mentioned above, is indeed the case in many SUSY models, but is a more model-dependent statement than (i). Using

$$\sin 2\beta=0.736\pm 0.049\ [3,4], \tag{16}$$

$$|V_{ub}/V_{cb}|=0.090\pm 0.025\ [21], \tag{17}$$

one obtains an allowed region for the position of the apex of the unitarity triangle which is shown as the shaded area in Fig. 1(a). The allowed values of γ^{SM} are $45°<\gamma^{\mathrm{SM}}<100°$. $|V_{ts}/V_{td}|$ can be read off the figure as a function of γ^{SM} from the right side of the triangle and translated into an allowed region for ΔM_s^{SM} as shown in Fig. 1(b), where we also include the error from $B_{B_s}f_{B_s}^2/(B_{B_d}f_{B_d}^2)$. As can be seen from this figure, the current experimental bound $\Delta M_s>13\ \mathrm{ps}^{-1}$ [21] does not yet exclude any value of γ^{SM} between 45° and 100°.

Let us now turn to $\Delta\Gamma_s^{\mathrm{SM}}$. A recent estimate including next to leading order (NLO) QCD corrections and lattice results for the hadronic parameters yields [23]

$$\frac{\Delta\Gamma_s^{\mathrm{SM}}}{\Gamma_s^{\mathrm{tot}}}=(0.12\pm 0.06). \tag{18}$$

At present, there is no experimental bound.

C. Observability of the B_s^0-$\overline{B}_s^0$ oscillation

A convenient measure of the frequency of the oscillation is the parameter x_s, defined as

$$x_s\equiv\frac{\Delta M_s}{\Gamma_{B_s}};$$

x_s indicates the observability of the oscillation, which is governed by $\sin(x_s t/\tau_s)$; it is evident that the experimental resolution of rapid oscillations with $x_s\gg 1$ is extremely difficult.

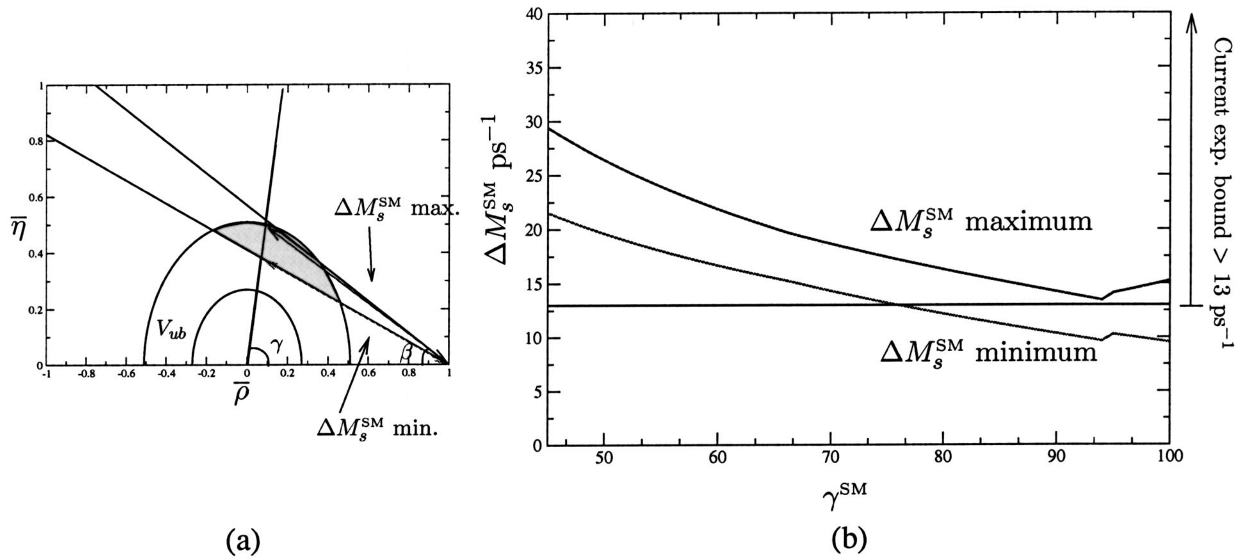

FIG. 1. (a) Allowed region (shaded area) for the apex of the SM unitarity triangle, using the constraints from $|V_{ub}/V_{cb}|$ and $\sin 2\beta$. (b) $\Delta M_s^{\rm SM}$ as function of $\gamma^{\rm SM}$ as determined from (a).

The current experimental lower bound is $x_s > 19$; recent studies of the experimental reach of the BTeV [24] and the LHC [25] experiments indicate that x_s can be measured up to values $x_s \approx 90$ (note that the corresponding parameter in the B_d system, x_d, has been measured to be 0.73). The performance of ATLAS, CMS and LHCb in analyzing $B_s \to J/\psi\phi$ has also been studied, which allows the determination of the correlation between the new physics mixing phase $\sin 2\beta_s$ and the frequency x_s [25]. Although the sensitivity to $\sin 2\beta_s$ gets worse as x_s increases, values of $\sin 2\beta_s$ as small as $\mathcal{O}(10^{-2})$ are within experimental reach for moderate $x_s < 40$.

Let us now discuss the correlation between $2\beta_s$ and x_s in terms of contributions from beyond the SM. For later convenience, we parametrize the new physics contributions as

$$R \equiv \frac{M_{12}^{\rm NP}}{M_{12}^{\rm SM}}, \tag{19}$$

which implies

$$2\beta_s = \arg[1+R], \quad x_s = \frac{\Delta M_s^{\rm SM}}{\Gamma_s}|1+R|. \tag{20}$$

In Fig. 2(a) we plot the correlation between $2\beta_s$ and x_s for different values of $|R| \in \{0.3,0.5,0.8,1,3,5\}$ varying the phase $\arg R$ between 0 and 2π. The value of $\Delta M_s^{\rm SM}$ is chosen to be 25 ps^{-1}. The figure shows that the current experimental bound on x_s has already excluded some phase region for $0.5 < |R| < 1$. In view of the limitation of the experimental resolution, $x_s < 90$, it is clear that new physics can be resolved only if it is not too large, i.e. $|R| < 4$. As for the mixing phase, $2\beta_s$, small $|R| \ll 1$ will result in small $2\beta_s$ that cannot be distinguished from the SM expectation, unless $\arg R$ is very close to zero or π. For large SUSY contributions $|R| > 1$, on the other hand, $\sin 2\beta_s \simeq 1$ is very possible.

Let us now discuss new physics effects on $\Delta\Gamma_s$. As discussed in [14,15], $\Delta\Gamma_s$ is always reduced by new physics due to the factor $\cos 2\beta_s$ in Eq. (8). In Fig. 2(b), we plot $\Delta\Gamma_s/\Delta\Gamma_s^{\rm SM}$ in terms of $\arg R$ for different values of $|R|$. As can be seen from this figure, $\Delta\Gamma_s$ can even become zero for large values of $|R|$ and $\arg R = \pm\pi/2$.

Finally, let us discuss the effect of $\Delta\Gamma_s$ on the time-dependent asymmetry Eq. (9). In Fig. 3 we show the time-dependent asymmetry of $B_s \to J/\psi\phi$ for the parameter set $\Delta M_s = 25$ ps^{-1}, $\Delta\Gamma_s^{\rm SM}/\Gamma_s^{\rm tot} = 0.12$, $D = 0.33$, $|R| = 1$ and $\arg R = \pi/2$. Note that the maximum of the $\sin \Delta M_s t$ curve slowly decreases with t, which is the effect of the denominator of Eq. (9). Although this effect is rather small, it may be used to determine $\Delta\Gamma_s$ once experimental data become available in a sufficiently large range of t.

III. SUSY CONTRIBUTIONS TO B_s MIXING

The mass difference in the B_s system and the time-dependent asymmetry $S_{J/\psi\phi}$ depend essentially on M_{12} which can be computed from the effective $\Delta B = 2$ Hamiltonian $H_{\rm eff}^{\Delta B=2}$. In supersymmetric theories $H_{\rm eff}^{\Delta B=2}$ is generated by the SM box diagrams with W exchange and box diagrams mediated by charged Higgs boson, neutralino, gluino and chargino exchange. The Higgs boson contributions are suppressed by the quark masses and can be neglected. The neutralino diagrams are also heavily suppressed compared to the gluino and chargino ones, due to the electroweak neutral couplings to fermion and sfermions. Thus, the B^0-$\overline{B}^0$ transition matrix element is to good accuracy given by

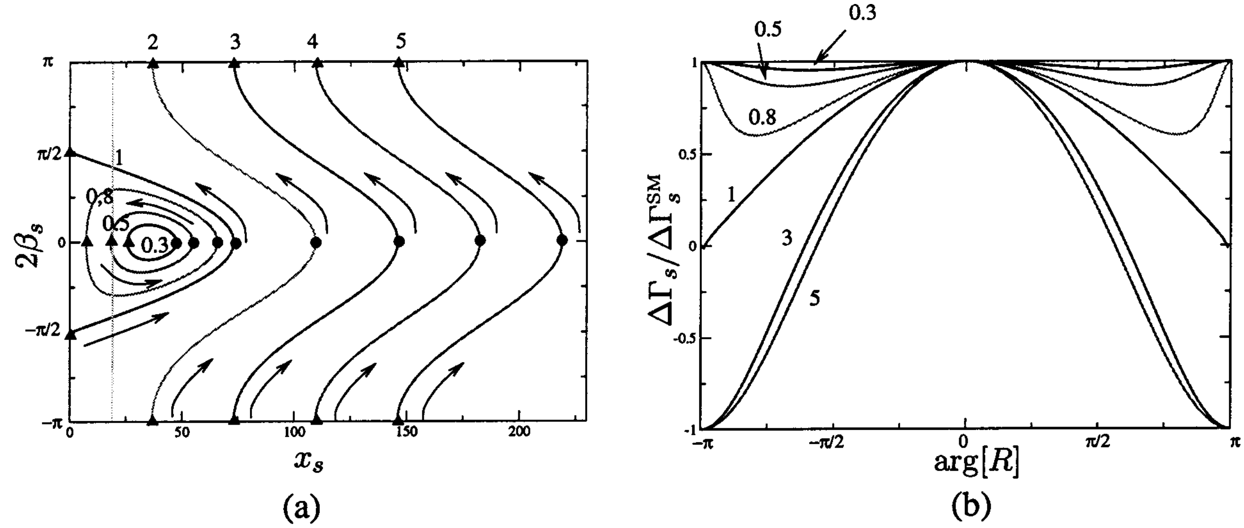

FIG. 2. (a) Correlation between x_s and $2\beta_s$ for $|R| \in \{0.3,0.5,0.8,1,3,5\}$ and $\arg R \in [0,2\pi]$, where R parametrizes the new physics contributions to M_{12}, Eqs. (19),(20). The numbers in the figure represent the values of $|R|$ and the circles and triangles indicate $\arg R = 0$ and π, respectively. The value of $\arg R$ increases in the direction of the arrow. The perpendicular line is the current experimental lower bound of x_s. (b) New physics in $\Delta\Gamma_s$. The numbers in the figure represent the values of $|R|$. $|\Delta\Gamma_s|$ is always reduced by new physics and can even become zero.

$$M_{12} = M_{12}^{\rm SM} + M_{12}^{\tilde{g}} + M_{12}^{\tilde{\chi}^+}, \tag{21}$$

where $M_{12}^{\rm SM}$, $M_{12}^{\tilde{g}}$ and $M_{12}^{\tilde{\chi}^+}$ indicate the SM, gluino and chargino contributions, respectively. The SM contribution is known at NLO accuracy in QCD [26] and is given by

$$M_{12}^{\rm SM} = \left(\frac{G_F}{4\pi}\right)^2 (V_{tb}^* V_{ts})^2 S_0(x_t)\,\eta_{2B}[\alpha_s(\mu)]^{-6/23} \times\left[1+\frac{\alpha_s(\mu)}{4\pi}J_5\right]\left(-\frac{4}{3}m_{B_s}f_{B_s}^2 B_1(\mu)\right), \tag{22}$$

where $S_0(x_t)$ is given by

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3} \tag{23}$$

with $x_t = (m_t/m_W)^2$. Contributions from virtual u and c quarks are suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism. The short-distance QCD corrections are encoded in η_{2B} and J_5, with $\eta_{2B} = 0.551$ and $J_5 = 1.627$ [26].

Including gluino and chargino exchanges, $H_{\rm eff}^{\Delta B=2}$ takes the form

$$H_{\rm eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu)\tilde{Q}_i(\mu) + \text{H.c.}, \tag{24}$$

where $C_i(\mu)$, $\tilde{C}_i(\mu)$, $Q_i(\mu)$ and $\tilde{Q}_i(\mu)$ are the Wilson coefficients and effective operators, respectively, normalized at the scale μ, with

$$Q_1 = \overline{s_L^\alpha}\gamma_\mu b_L^\alpha \overline{s_L^\beta}\gamma^\mu b_L^\beta,$$

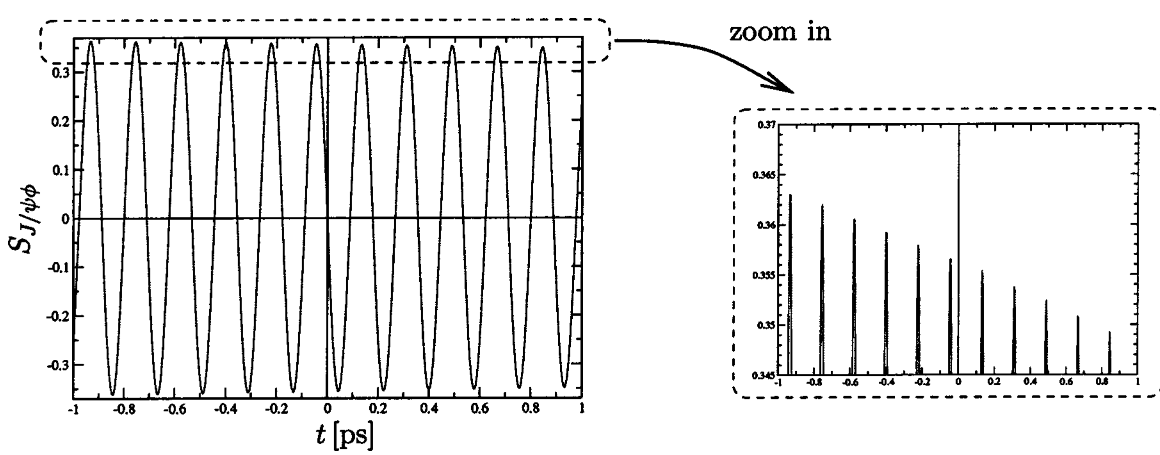

FIG. 3. The time-dependent asymmetry of $B_s \to J/\psi\phi$ according to Eq. (9); parameters as given in the text.

$$Q_2=\overline{s_R^\alpha}b_L^\alpha\overline{s_R^\beta}b_L^\beta,$$

$$Q_3=\overline{s_R^\alpha}b_L^\beta\overline{s_R^\beta}b_L^\alpha,$$

$$Q_4=\overline{s_R^\alpha}b_L^\alpha\overline{s_L^\beta}b_R^\beta,$$

$$Q_5=\overline{s_R^\alpha}b_L^\beta\overline{s_L^\beta}b_R^\alpha. \tag{25}$$

The operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging $L\leftrightarrow R$.

In the MIA, the gluino contributions to the Wilson coefficients at the SUSY scale M_S are given by [27]

$$C_1^{\tilde{g}}(M_S)=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}[24xf_6(x)+66\tilde{f}_6(x)](\delta_{23}^d)_{LL}^2, \tag{26}$$

$$C_2^{\tilde{g}}(M_S)=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}204xf_6(x)(\delta_{23}^d)_{RL}^2, \tag{27}$$

$$C_3^{\tilde{g}}(M_S)=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}36xf_6(x)(\delta_{23}^d)_{RL}^2, \tag{28}$$

$$C_4^{\tilde{g}}(M_S)=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}\{[504xf_6(x)-72\tilde{f}_6(x)]\times(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}-132\tilde{f}_6(x)\times(\delta_{23}^d)_{LR}(\delta_{23}^d)_{RL}\}, \tag{29}$$

$$C_5^{\tilde{g}}(M_S)=-\frac{\alpha_s^2}{216m_{\tilde{q}}^2}\{[24xf_6(x)+120\tilde{f}_6(x)]\times(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}-180\tilde{f}_6(x)\times(\delta_{23}^d)_{LR}(\delta_{23}^d)_{RL}\}, \tag{30}$$

where $x=m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and $m_{\tilde{q}}$ is the average down squark mass. Explicit expressions for $f_6(x)$ and $\tilde{f}_6(x)$ can be found in [27]. The Wilson coefficients $\tilde{C}_{1,2,3}$ are obtained by interchanging $L\leftrightarrow R$ in the mass insertions appearing in $C_{1,2,3}$. Note that the coefficient of the mass insertion $(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}$ in $C_4^{\tilde{g}}$ is much larger than the coefficients of the other mass insertions, which renders ΔM_{B_s} and $S_{J/\psi\phi}$ very sensitive to these insertions.

The chargino contributions to the relevant Wilson coefficients, at leading order in the MIA, next-to-leading order in the Wolfenstein parameter λ and including the effects of a potentially light right top squark, are given by [19]

$$\begin{aligned}C_1^{\tilde{\chi}^+}(M_S)=\frac{\alpha^2}{48m_{\tilde{q}}^2}\sum_{i,j}\{&|V_{i1}|^2|V_{j1}|^2[(\delta_{32}^u)_{LL}^2\\&+2\lambda(\delta_{31}^u)_{LL}(\delta_{32}^u)_{LL}]L_2(x_i,x_j)\\&-2Y_t|V_{i1}|^2V_{j1}V_{j2}^*[(\delta_{32}^u)_{LL}(\delta_{32}^u)_{RL}\\&+\lambda(\delta_{32}^u)_{LL}(\delta_{31}^u)_{RL}]R_2(x_i,x_j,z)\\&+Y_t^2V_{i1}V_{i2}^*V_{j1}V_{j2}^*[(\delta_{32}^u)_{RL}^2\\&+2\lambda(\delta_{32}^u)_{RL}(\delta_{31}^u)_{RL}]\tilde{R}_2(x_i,x_j,z)\},\end{aligned} \tag{31}$$

$$\begin{aligned}C_3^{\tilde{\chi}^+}(M_S)=\frac{\alpha^2}{12m_{\tilde{q}}^2}\sum_{i,j}&U_{i2}U_{j2}V_{j1}V_{i1}[(\delta_{32}^u)_{LL}^2\\&+2\lambda(\delta_{32}^u)_{LL}(\delta_{31}^u)_{LL}]L_0(x_i,x_j),\end{aligned} \tag{32}$$

where $x_i=m_{\chi_i^+}^2/m_{\tilde{q}}^2$, $z=m_{\tilde{t}_R}^2/m_{\tilde{q}}^2$ and the functions $R_2(x,y,z)$, $\tilde{R}_2(x,y,z)$, $L_0(x,y)$ and $L_2(x,y)$ are given in [19]. $U_{i,j}$ and $V_{i,j}$ are the unitary matrices that diagonalize the chargino mass matrix and Y_t is the top Yukawa coupling (for more details, see [19]). Note that, neglecting the effect of the Yukawa couplings of the light quarks, the chargino contributions to C_4 and C_5 are negligible and that charginos do not contribute to $C_2(M_S)$ and $\tilde{C}_2(M_S)$ due to the color structure of the diagrams; nonzero values at lower scales are however induced by QCD mixing effects.

To obtain the Wilson coefficients at the scale $\mu\sim m_b$ one has to solve the corresponding renormalization group equations, which to LO accuracy was done in Ref. [13], with the result

$$C_r(\mu)=\sum_i\sum_s(b_i^{(r,s)}+\eta c_i^{(r,s)})\eta^{a_i}C_s(M_S), \tag{33}$$

where $\eta=\alpha_s(M_S)/\alpha_s(\mu)$. The coefficients $b_i^{(r,s)}$, $c_i^{(r,s)}$ and a_i are given in Ref. [13].

In order to calculate M_{12}, we also need the matrix elements of the effective operators Q_i and $\tilde{Q}_i$ over B_s meson states. As usual, the matrix elements are expressed in terms of the decay constant f_{B_s}, using the vacuum insertion approximation; terms neglected in this approximation are included in a bag factor B_i which is expected to be of order one. One has

$$\langle\overline{B_s}^0|Q_1|B_s^0\rangle\equiv-\frac{1}{3}m_{B_s}f_{B_s}^2B_1(\mu), \tag{34}$$

$$\langle\overline{B_s}^0|Q_2|B_s^0\rangle\equiv\frac{5}{24}\left(\frac{m_{B_s}}{m_b(\mu)+m_s(\mu)}\right)^2m_{B_s}f_{B_s}^2B_2(\mu), \tag{35}$$

$$\langle\overline{B_s}^0|Q_3|B_s^0\rangle\equiv-\frac{1}{24}\left(\frac{m_{B_s}}{m_b(\mu)+m_s(\mu)}\right)^2m_{B_s}f_{B_s}^2B_3(\mu), \tag{36}$$

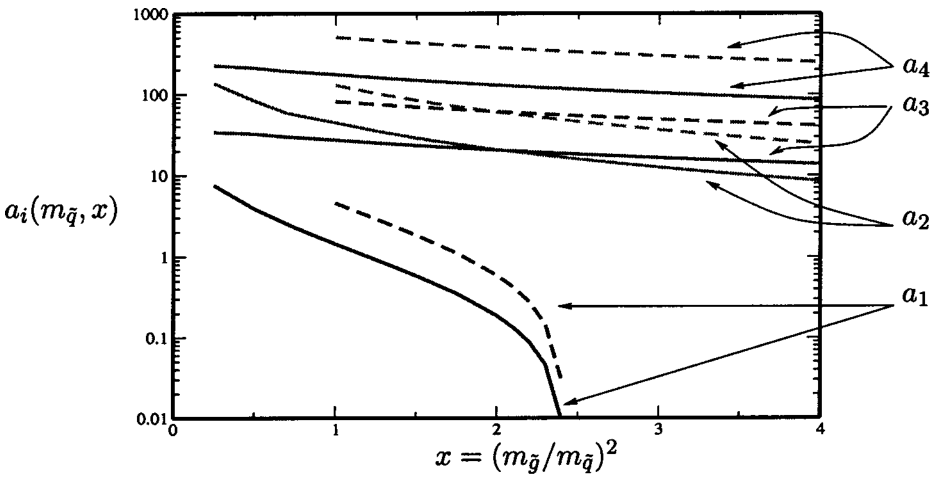

FIG. 4. $a_i(m_{\tilde{q}},x)$ defined in Eq. (39) as a function of $x = (m_{\tilde{g}}/m_{\tilde{q}})^2$ for $m_{\tilde{q}} = 500$ GeV (solid lines) and 300 GeV (dashed lines).

$$\langle \overline{B_s}^0 | Q_4 | B_s^0 \rangle \equiv -\frac{1}{4}\left(\frac{m_{B_s}}{m_b(\mu)+m_s(\mu)}\right)^2 m_{B_s} f_{B_s}^2 B_4(\mu), \quad (37)$$

$$\langle \overline{B_s}^0 | Q_5 | B_s^0 \rangle \equiv -\frac{1}{12}\left(\frac{m_{B_s}}{m_b(\mu)+m_s(\mu)}\right)^2 m_{B_s} f_{B_s}^2 B_5(\mu); \quad (38)$$

the matrix elements of $\tilde{Q}_i$ are the same as for Q_i. The hadronic parameters f_{B_s} and B_i have been calculated on the lattice, yielding[3] $B_1(m_b) = 0.86(2)(^{+5}_{-4})$, $B_2(m_b) = 0.83(2)\times(4)$, $B_3(m_b) = 1.03(4)(9)$, $B_4(m_b) = 1.17(2)(^{+5}_{-7})$, and $B_5(m_b) = 1.94(3)(^{+23}_{-7})$ [28]; as we shall see in the next section, we do not need a numerical value for f_{B_s}.

IV. NUMERICAL ANALYSIS AND DISCUSSION

Let us now proceed to the numerical analysis of the impact of SUSY effects on ΔM_{B_s} and $\sin 2\beta_s$, which is most conveniently done by studying the ratio R, Eq. (19), of intrinsically supersymmetric to SM contributions to M_{12}. We start with the gluino contributions, which, as discussed in the previous section, depend on the average down squark mass and on the ratio $x = (m_{\tilde{g}}/m_{\tilde{q}})^2$. In terms of the mass-insertion parameters δ_{23}^d, R can be written as

$$\begin{aligned} R_{\tilde{g}} \equiv \frac{M_{12}^{\tilde{g}}}{M_{12}^{\mathrm{SM}}} &\simeq a_1(m_{\tilde{q}},x)[(\delta_{23}^d)_{LL}^2 + (\delta_{23}^d)_{RR}^2] + a_2(m_{\tilde{q}},x) \\ &\times[(\delta_{23}^d)_{LR}^2 + (\delta_{23}^d)_{RL}^2] + a_3(m_{\tilde{q}},x)[(\delta_{23}^d)_{LR}(\delta_{23}^d)_{RL}] \\ &+ a_4(m_{\tilde{q}},x)[(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}] \end{aligned} \quad (39)$$

with $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$. The coefficients $a_i(m_{\tilde{q}},x)$ depend implicitly on the Wilson coefficients and matrix elements defined in the previous section. Let us pause here for a moment and consider what range of values for δ_{23}^d we actually do expect. Although our analysis is model independent, we may nevertheless get some guidance for what to expect by looking at various SUSY models. As an example, we give numerical results obtained from three different kinds of SUSY models with $m_{\tilde{q}} \sim m_g \sim 500$ GeV in the following. Let us start with the minimal supergravity model. In this model, universality at the grand unified theory scale is assumed and the running to the M_W scale leads only to small off diagonal entries: $(\delta_{23}^d)_{LL} \simeq 0.009 + 0.001i$ and $(\delta_{23}^d)_{RR,LR,RL} \simeq 0$. On the other hand, in the SUSY SO(10) model considered in Ref. [11] where large low energy neutrino mixings originate from a large mixing in the charged leptons and right handed down quarks (for different SUSY SO(10) models, see for example, [29,30]), one gets the following mass insertions: $(\delta_{23}^d)_{RR} \simeq 0.5 + 0.5i$ and $(\delta_{23}^d)_{LL,LR,RL} \simeq 0$. The models with non-universal A terms also have a chance to enhance the LR and RL mass insertions. A specific example discussed in Ref. [31] gives the relevant mass insertions as $(\delta_{23}^d)_{LR} \simeq 0.002 + 0.005i$ and $(\delta_{23}^d)_{LL,RR,RL} \simeq 0$.

As can be seen from the above examples, a single mass insertion is dominant in many models. This implies that, for $(\delta_{23}^d)_{LL,RR}$ $[(\delta_{23}^d)_{LR,LR}]$ dominated models, only the term proportional to $a_1(m_{\tilde{q},x})$ $[a_2(m_{\tilde{q},x})]$ contributes to R. We would also like to mention that $(\delta_{23}^d)_{AB}$ is already constrained by $B(b \to s\gamma)$, which yields $|(\delta_{23}^d)_{LL,RR}| < 1$ and $|(\delta_{23}^d)_{LR,RL}| < \mathcal{O}(10^{-2})$ [32].

Numerical results for the x dependence of $a_i(m_{\tilde{q}},x)$ are given in Fig. 4, for two representative values of the down squark mass, $m_{\tilde{q}} = \{300,500\}$ GeV. In order to obtain this result, we have set $M_S = m_{\tilde{q}}$ and used the following input parameters:

$$V_{ts} = 0.0412, \quad m_t = (174 \pm 5)\ \text{GeV}, \quad \alpha_s(M_Z) = 0.119,$$

$$m_b(m_b) = 4.2\ \text{GeV}, \quad \mu = m_b,$$

$$m_s(2\ \text{GeV}) = (100 \pm 20)\ \text{MeV}.$$

The impact of the theoretical uncertainties of m_t and m_s on a_i is very small, and also the variation with $\mu \sim m_b$ does not exceed a few percent. The main source of uncertainty of $a_i(m_{\tilde{q}},x)$ comes from the B_i parameters: although the factor

[3]The overall sign is different from the one in [28], which is due to the different sign choice of the CP transformation; we chose $\mathbf{CP}|P^0\rangle = +|\overline{P}^0\rangle$.

B_1 cancels in a_1, the other a_i carry a $\sim 20\%$ uncertainty from B_i/B_1. Note that $R_{\tilde{g}}$ is independent of f_{B_s}.

Let us continue with the discussion of the results depicted in Fig. 4. The solid and dashed lines refer to $m_{\tilde{q}}=500$ GeV and 300 GeV, respectively. We see that all a_i are monotonically decreasing functions in x and are by about a factor 3 larger for $m_{\tilde{q}}=300$ GeV than for $m_{\tilde{q}}=500$ GeV. Note also that $a_1(m_{\tilde{q}},x)$ becomes negative for large values of x. It is also evident that $a_4(m_{\tilde{q}},x)$ is largest, in agreement with the remark in the previous section, so that the dominant contribution to B_s mixing through gluino exchange is expected to be due to LL and RR mass insertions. Although $a_{2,3}(m_{\tilde{q}},x) \sim \mathcal{O}(10)$ are also large, the constraint from $B(b\to s\gamma)$ on the helicity-flip mass insertions $(\delta_{23}^d)_{LR,RL}$ renders their contributions to B_s mixing negligible.

As an explicit example for the relative size of the a_i, we choose $m_{\tilde{q}}=500$ GeV and $x=1$, which yields

$$\begin{aligned} R_{\tilde{g}}(m_{\tilde{q}}=500\ \mathrm{GeV},x=1) &\simeq 1.44[(\delta_{23}^d)_{LL}^2+(\delta_{23}^d)_{RR}^2]+27.57[(\delta_{23}^d)_{LR}^2+(\delta_{23}^d)_{RL}^2] \\ &\quad -44.76[(\delta_{23}^d)_{LR}(\delta_{23}^d)_{RL}]-175.79[(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}]. \end{aligned} \tag{40}$$

Using the constraints from $b\to s\gamma$, $|(\delta_{23}^d)_{LR(RL)}|<10^{-2}$ and $|(\delta_{23}^d)_{LL(RR)}|<1$, it is evident that helicity-flipping mass insertions contribute $\mathcal{O}(10^{-3})$ to $R_{\tilde{g}}$, whereas single LL or RR mass insertions can yield $\mathcal{O}(1)$ contributions.

In Sec. II, we have already discussed the dependence of ΔM_s and $\sin 2\beta_s$ on R; cf. Fig. 2(a). The constraint from $b\to s\gamma$ implies that LR and RL mass insertions alone cannot generate a value of $2\beta_s$ larger than $\sim\mathcal{O}(10^{-3})$, which is too small to be observed at the Tevatron or the LHC. LL and RR mass insertions, on the other hand, can result in sizable—and measurable—values of the B_s mixing phase: for instance, $(\delta_{23}^d)_{LL}=1\times e^{i\pi/4}$ yields $\Delta M_s/\Delta M_s^{\mathrm{SM}}=1.75$ and $\sin 2\beta_s=0.82$, while for $(\delta_{23}^d)_{LL}\simeq(\delta_{23}^d)_{RR}=0.1\times e^{i\pi/10}$ one finds $\Delta M_s/\Delta M_s^{\mathrm{SM}}=1.12$ and $\sin 2\beta_s=-0.93$. Note that for the same mass insertion, i.e. $(\delta_{23}^d)_{LL}=1\times e^{\pi/4}$, the smaller squark mass, $m_{\tilde{q}}=300$ GeV, accompanied by $x=1$ gives about 3 times larger $|R|$, i.e. $|R|>4$, which is beyond the experimental reach at the LHC, as discussed in Sec. II.

Let us now turn to the chargino contributions. The chargino mediated processes depend on five relevant SUSY low energy parameters: $m_{\tilde{q}}$, $m_{\tilde{t}_R}$, M_2, μ and $\tan\beta$. With $m_{\tilde{t}_R}=150$ GeV, $m_{\tilde{q}}=200$ GeV, $M_2=\mu=300$ GeV and $\tan\beta=5$, we find

$$\begin{aligned} \frac{M_{12}^{\tilde{\chi}^+}}{M_{12}^{\mathrm{SM}}} &\simeq 10^{-4}(\delta_{31}^u)_{LL}(\delta_{32}^u)_{LL}+2\times10^{-4}(\delta_{32}^u)_{LL}^2+9.8 \\ &\quad\times10^{-8}(\delta_{32}^u)_{LL}(\delta_{31}^u)_{RL}+2\times10^{-7}(\delta_{32}^u)_{LL}(\delta_{32}^u)_{RL} \\ &\quad+2.4\times10^{-7}(\delta_{31}^u)_{RL}(\delta_{32}^u)_{RL}+5.4\times10^{-7}(\delta_{32}^u)_{RL}, \end{aligned} \tag{41}$$

which is obviously much smaller than the gluino contribution. Even though the chargino contributions are very sensitive to the value of $\tan\beta$, an increase of $\tan\beta$ to 50 entails an enhancement of only the first two terms in Eq. (41) from 10^{-4} to 10^{-2}—still not large enough to distinguish ΔM_s and $\sin 2\beta_s$ from the SM prediction.

Let us finally discuss the implication of the experimental data of the CP asymmetry in the $B_d\to\phi K_s$ process, $S_{\phi K_s}$. As the underlying quark-level process is a $b\to s$ transition, it is clear that this process is governed by the same mass insertions, $(\delta_{23}^d)_{AB}$. Since a possible hint of new physics may already have been seen in this mode, it is very interesting to analyze the implications of the experimental data on $S_{\phi K_s}$ for B_s mixing. Let us first recall the main result of the supersymmetric contributions to $S_{\phi K_S}$ previously obtained in Ref. [9]: the mixing CP asymmetry is given by

$$S_{\phi K_S}=\frac{\sin 2\beta+2R_\phi\cos\delta\sin(\theta_\phi+2\beta)+R_\phi^2\sin(2\theta_\phi+2\beta)}{1+2R_\phi\cos\delta\cos\theta_\phi+R_\phi^2}, \tag{42}$$

where δ is the difference of the strong phase between SM and SUSY, but assumed to be $\delta=0$ in the following (see [10] for a more detailed discussion). R_ϕ is the absolute value of the ratio between SM and SUSY decay amplitudes and θ_ϕ is its phase, that is

$$R_\phi e^{i\theta_\phi}\equiv\left(\frac{A^{\mathrm{SUSY}}}{A^{\mathrm{SM}}}\right)_{\phi K_S}. \tag{43}$$

For $m_{\tilde{g}}\simeq m_{\tilde{q}}=500$ GeV, we obtain

$$\begin{aligned} R_\phi e^{i\theta_\phi} &\simeq 0.23(\delta_{LL}^d)_{23}+97.4(\delta_{LR}^d)_{23}+97.4(\delta_{RL}^d)_{23} \\ &\quad+0.23(\delta_{RR}^d)_{23}. \end{aligned} \tag{44}$$

Considering the same constraint from $b\to s\gamma$, we arrive at the conclusion that the LR or RL mass insertion gives the largest contribution to $S_{\phi K_s}$ while the LL or RR contribution is subdominant. In Ref. [9], we found that it is very difficult to get a negative $S_{\phi K_S}$ from LL or RR mass insertion dominated models without decreasing $m_{\tilde{q}}$.

The most interesting result we would like to emphasize here is that B_s mixing and $S_{\phi K_S}$ are dominated by different mass insertions: LL, RR and LR, RL, respectively. In Table I, we present our results for ΔM_{B_s}, $\sin 2\beta_s$ and $S_{\phi K_s}$ for various sets of the mass insertions with $m_{\tilde{q}}=m_{\tilde{g}}=\{300\ \mathrm{GeV},500\ \mathrm{GeV}\}$.[4] As we mentioned above, the LL and RR mass insertions may lower the value of $S_{\phi K_s}$ and make it comparable to experiment if the SUSY masses are light enough (see the first and second rows for $m_{\tilde{q}}=m_{\tilde{g}}=300$ and 500 GeV). In this case, however, ΔM_s becomes

[4] In this table, the phases are chosen to be negative so that $S_{\phi K_S}$ becomes less than $S_{J/\psi K_S}$ (see the more detailed discussion in [9]).

TABLE I. Numerical results for ΔM_{B_s}, $\sin 2\beta_s$ and $S_{\phi K_s}$ for some representative values of $(\delta_{32}^d)_{AB}$ $(A,B=L,R)$ for $m_{\tilde{q}}=m_{\tilde{g}}\in\{300,500\}$ GeV.

$m_{\tilde{q}}=m_{\tilde{g}}=500$ GeV						
Mass insertion sets				Results		
$\delta_{LL(RR)}$	$\delta_{RR(LL)}$	$\delta_{LR(RL)}$	$\delta_{RL(LR)}$	$\Delta M_s(\text{ps}^{-1})$	$\sin 2\beta_s$	$S_{\phi K_S}$
$1\times e^{-i\pi/2}$	0	0	0	10.7	0	0.50
$1\times e^{-i\pi/4}$	0	0	0	43.5	-0.82	0.59
0	0	$0.01\times e^{-i\pi/2}$	0	24.9	0	-0.36
0	0	$0.01\times e^{-i\pi/4}$	0	25.0	-2.8×10^{-3}	0.19
$1\times e^{-i\pi/2}$	$1\times e^{-i\pi/2}$	0	0	4.39×10^3	0	0.25
$0.1\times e^{-i\pi/4}$	$0.1\times e^{-i\pi/4}$	0	0	50	0.87	0.70
$m_{\tilde{q}}=m_{\tilde{g}}=300$ GeV						
Mass insertion sets				Results		
$\delta_{LL(RR)}$	$\delta_{RR(LL)}$	$\delta_{LR(RL)}$	$\delta_{RL(LR)}$	$\Delta M_s(\text{ps}^{-1})$	$\sin 2\beta_s$	$S_{\phi K_S}$
$1\times e^{-i\pi/2}$	0	0	0	87.6	0	0.05
$1\times e^{-i\pi/4}$	0	0	0	115	-0.98	0.37
0	0	$0.01\times e^{-i\pi/2}$	0	24.8	0	-0.76
0	0	$0.01\times e^{-i\pi/4}$	0	25.0	-8.3×10^{-3}	-0.15
$1\times e^{-i\pi/2}$	$1\times e^{-i\pi/2}$	0	0	1.26×10^4	0	-0.52
$0.1\times e^{-i\pi/4}$	$0.1\times e^{-i\pi/4}$	0	0	128	0.98	0.65

so large that it may not be resolved experimentally.[5] On the other hand, although *LR* or *RL* dominated models can explain the experimental data of $S_{\phi K_s}$ and also predict $\Delta M_s \sim \Delta M_s^{\text{SM}}$, which is good news for the experimental side, in this case $\sin 2\beta_s$ is too small to be observed (see the third and fourth rows for $m_{\tilde{q}}=m_{\tilde{g}}=300$ and 500 GeV). Therefore, if the B_s oscillations are resolved experimentally with $x_s<90$ and an observable $\sin 2\beta_s$ and also a negative $S_{\phi K_s}$ is confirmed, *the SUSY models with combined mass insertions effects would be preferred to the ones with a single dominant mass insertion.* An example of the former class of models could result in, for instance, the following mass insertions $(\delta_{32}^d)_{AB}$:

$$|(\delta_{23}^d)_{LL}|\simeq 0.02,$$

$$|(\delta_{23}^d)_{RR}|\simeq 0.5,$$

$$|(\delta_{23}^d)_{LR}|\simeq|(\delta_{32})_{RL}|\simeq 0.005,$$

$$\arg[(\delta_{23}^d)_{LL}]\simeq\arg[(\delta_{23}^d)_{RR}]\simeq -\frac{\pi}{4},$$

$$\arg[(\delta_{23}^d)_{LR}]\simeq\arg[(\delta_{23}^d)_{RL}]\simeq -\frac{\pi}{2},$$

which lead to

$$\Delta M_s \simeq 40 \ \ \text{ps}^{-1},$$

$$\sin 2\beta_s \simeq 0.86,$$

$$S_{\phi K_S}\simeq -0.7.$$

Such nonuniversal soft SUSY breaking terms (*LR* and *RL* of order 10^{-3} and large *RR*) are possible in models derived from string theory, as discussed in, for instance, Ref. [31].

V. CONCLUSIONS

We have studied supersymmetric contributions to B_s mixing and the mixing-induced *CP* asymmetry of $B_s \to J/\psi\phi$ in the mass insertion approximation, including constraints from other $b\to s$ processes, in particular $b \to s\gamma$ and $B_d \to \phi K_s$. The SM predictions for these quantities are $S_{J/\psi\phi}\simeq 10^{-2}$ and $\Delta M_s = 10$–30 ps^{-1}, depending on the value of γ. We have shown that in SUSY these predictions can change quite drastically, which is mainly due to gluino exchange contributions, whereas the chargino contributions to these processes are negligible. We find that values $S_{J/\psi\phi}\simeq \mathcal{O}(1)$ and $\Delta M_s = 10$–10^4 ps^{-1} are quite possible. We also find that unlike their effects on the *CP* asymmetry of $B_d \to \phi K_s$, the mass insertions $(\delta_{23}^d)_{LR(RL)}$ do not provide significant contributions to these processes, whereas $(\delta_{23}^d)_{LL(RR)}$ imply a large ΔM_s and $\sin 2\beta_s$. We have argued that a clean measurement of the B_s^0-$\overline{B}_s^0$ oscillation and a significant deviation of $S_{\phi K_s}$ from $S_{J/\psi K_s}$ would exclude SUSY models with a single dominant mass insertion, which predict either small oscillation and negative $S_{\phi K_s}$ or large oscillation and $S_{\phi K_s} \simeq S_{J/\psi K_s}$.

[5] Note that if $x_s>90$, we cannot find the value of ΔM_s experimentally; however, we can still find whether there is a new physics effect or not.

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