Phenomenology of the heavy B_H in a littlest Higgs model

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(Received 14 November 2003; revised manuscript received 11 February 2004; published 14 June 2004)

We comprehensively study in the littlest Higgs model the phenomenology of the heavy $U(1)$ gauge boson B_H, which is the lightest of the newly introduced heavy gauge bosons and toplike vector quarks. Some unexpected behavior is that in the parameter space where the mass of B_H is minimized the corrections to the electroweak precision observables are also suppressed. For the global symmetry breaking scale $f \lesssim 3$ TeV, the B_H is light enough to be produced at a 500 GeV linear collider. We show that this light B_H is not excluded by the direct search for the neutral heavy gauge boson at the Fermilab Tevatron in a large portion of the parameter space. Furthermore, even the light B_H with a mass around 200 GeV yields negligible contributions to the muon anomalous magnetic moment, which is consistent with the current inconclusive status of the theoretical calculation of $(g-2)_\mu$ in the standard model. The effects of the littlest Higgs model on the $e^+e^- \to \mu^+\mu^-$ process are also studied; this is one of the most efficient processes to probe the B_H.

DOI: 10.1103/PhysRevD.69.115010 PACS number(s): 12.60.Cn, 13.40.Em, 13.66.Hk

I. INTRODUCTION

The standard model (SM) of particle physics has provided an excellent effective field theory of high energy phenomena up to energies of the order of 100 GeV. A direct and important question is what is the cutoff scale of this effective description. The Higgs boson mass may hold the key because of its quadratic sensitivity to UV physics. The naturalness argument suggests that the cutoff scale of the SM is not much above the electroweak scale: New physics will appear around TeV energies. A supersymmetry (SUSY) model is one of the best motivated candidates as the cutoff scale is naturally replaced by the soft SUSY breaking scale. However, the minimal supersymmetric SM has already required some amount of fine-tuning due to its prediction of the upper bound on the Higgs boson mass, which confronts the experimental lower bound. Brane world scenarios with large or warped extra dimensions have also been suggested to understand the hierarchy problem as a geometrical stabilization problem. However, those extra-dimensional theories are not weakly coupled at the TeV scale.

Recently, new models, dubbed the ''little Higgs'' models, have drawn a lot of interest; they remain weakly coupled at the TeV scale with the one-loop stabilized Higgs potential. The original idea, dating back to 1970s, is that the lightness of the Higgs boson is attributed to its being a pseudo Goldstone boson [1]. The model was not phenomenologically viable due to the remaining quadratic divergence of the radiative correction to the Higgs boson mass. A new ingredient, the collective symmetry breaking idea, was discovered through dimensional deconstruction [2,3]. It ensures that the Higgs boson mass is radiatively generated at the two-loop level [4–6]: The one-loop level quadratic divergences from the SM gauge boson (top quark) loops are canceled by those from new heavy gauge boson (fermion) loops. The cancellation occurs between the SM particles and the new particles with the same statistics, unlike the cancellations in supersymmetric theories; it is due to the exactly opposite coupling. One of the simplest and phenomenologically viable models is the so called ''littlest Higgs'' model, described by the global symmetry breaking pattern of $SU(5)/SO(5)$ [7]. The extended gauge symmetry $[SU(2)\otimes U(1)]^2$ determines the heavy gauge boson sector, including the mass spectra and new gauge coupling structure. The fermionic sector is rather model dependent, e.g., the $U(1)_{1,2}$ charge assignment and Yukawa couplings [8,9]. Later, the idea was realized in other simple nonlinear sigma models [7,10–15]. The challenging problem of obtaining an UV-complete theory has been discussed in Refs. [16,17].

In this paper, we concentrate on the littlest Higgs model [7]. As one of the simplest realizations of the little Higgs idea, it is the minimal extension of the SM to date which stabilizes the electroweak scale and remains weakly coupled at the TeV scale. The model predicts the presence of new heavy gauge bosons ($W_H^\pm$, W_H^3, and B_H) and a new heavy toplike vector quark T. The minimality of the littlest Higgs model would leave characteristic signatures at present and future collider experiments. Since the tree level corrections to the electroweak precision data constrain the new particles to be heavier than a few TeV, a 500 GeV linear collider (LC) was not expected to efficiently test the model. In the literature, the phenomenologies of the littlest Higgs model at the CERN Large Hadronic Collider (LHC) have been studied, showing that the LHC has the potential to detect the new particles [18–20]. In the littlest Higgs model, however, we find that the global symmetry structure $SU(5)/SO(5)$ allows a substantially lighter B_H, light enough to be produced on shell at a 500 GeV LC. Moreover, as will be shown below, the B_H becomes lighter as the model parameters minimize the corrections to the electroweak precision measurements.

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The presence of a heavy and neutral gauge boson with mass about a few hundreds GeV can be dangerous to other low energy observables. We study the constraints from the direct search at the Tevatron and the one-loop contributions to the muon anomalous magnetic moment. Another issue here concerns the collider signatures of the B_H. As one of the cleanest signals, the $e^+e^- \to \mu^+\mu^-$ process is to be discussed.

The paper is organized as follows. In Sec. II, we briefly review the littlest Higgs model. We point out the preferred parameter space by considering the tree level expressions of the SM gauge boson masses and couplings. In Sec. III, the physical properties of the B_H are studied, with a focus on its mass and decay patterns. The direct search bounds from the Tevatron data are also discussed. In Sec. IV, the one-loop corrections from the new gauge boson loop to the muon anomalous magnetic moment are calculated. The numerical result is to be compared with the latest experimental data. In Sec. V, we study the effects of the littlest Higgs model on the process $e^+e^- \to \mu^+\mu^-$. We summarize our results in Sec. VI.

II. LITTLEST HIGGS MODEL

At the TeV scale, the littlest Higgs model is embedded into a nonlinear σ model in the coset space of $SU(5)/SO(5)$. The leading two-derivative term for the sigma field Σ is

$$\mathcal{L}_\Sigma = \frac{1}{2}\frac{f^2}{4}\mathrm{Tr}|\mathcal{D}_\mu \Sigma|^2. \tag{1}$$

The local gauge symmetry $[SU(2)\otimes U(1)]^2$ is also assumed and is manifest in the covariant derivative of the sigma field, given by

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - i\sum_{j=1}^{2}[g_j W_j^a (Q_j^a \Sigma + \Sigma Q_J^{aT}) + g_j' B_j (Y_j \Sigma + \Sigma Y_j^T)]. \tag{2}$$

The generators of two $SU(2)$'s are

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & \\ & \mathbf{0}_{3\times 3} \end{pmatrix}, \quad Q_2^a = \begin{pmatrix} \mathbf{0}_{3\times 3} & \\ & -\sigma^{a*}/2 \end{pmatrix}, \tag{3}$$

and those of two $U(1)$'s are

$$Y_1 = \mathrm{diag}(-3,-3,2,2,2)/10, \quad Y_2 = \mathrm{diag}(-2,-2,-2,3,3)/10. \tag{4}$$

At the scale $\Lambda_S \sim 4\pi f$, a symmetric tensor of the $SU(5)$ global symmetry develops an order f vacuum expectation value (VEV), of which the direction is into the Σ_0:

$$\Sigma_0 = \begin{pmatrix} & & \mathbf{1}_{2\times 2} \\ & 1 & \\ \mathbf{1}_{2\times 2} & & \end{pmatrix}. \tag{5}$$

Two symmetry breakings occur.

(1) The global $SU(5)$ symmetry is broken into $SO(5)$, which leaves 14 massless Goldstone bosons: They transform under the electroweak gauge group as a real singlet $\mathbf{1}_0$, a real triplet $\mathbf{3}_0$, a complex doublet $\mathbf{2}_{\pm 1/2}$, and a complex triplet $\mathbf{3}_{\pm 1}$.

(2) The assumed gauge symmetry $[SU(2)\otimes U(1)]^2$ is also broken into its diagonal subgroup $SU(2)_L \otimes U(1)_Y$, identified as the SM gauge group. The gauge fields $\vec{W}'^{\mu}$ and B'^{μ} associated with the broken gauge symmetries become massive by eating the Goldstone bosons of $\mathbf{1}_0$ and $\mathbf{3}_0$.

The nonlinear sigma fields are then parameterized by the Goldstone fluctuations:

$$\Sigma = \Sigma_0 + \frac{2i}{f}\begin{pmatrix} \phi^\dagger & \frac{h^\dagger}{\sqrt{2}} & \mathbf{0}_{2\times 2} \\ \frac{h^*}{\sqrt{2}} & 0 & \frac{h}{\sqrt{2}} \\ \mathbf{0}_{2\times 2} & \frac{h^T}{\sqrt{2}} & \phi \end{pmatrix} + \mathcal{O}\left(\frac{1}{f^2}\right), \tag{6}$$

where h is a doublet and ϕ is a triplet under the unbroken $SU(2)_L$. A brief comment is that this Higgs triplet, developing a nonzero VEV, may explain neutrino mass terms through its Yukawa coupling with leptons in a SM gauge invariant way [21]. Lepton Yukawa coupling has some freedom since it is insensitive to the quadratic divergence of the Higgs boson mass for a cutoff scale around 10 TeV.

The gauge fields $\vec{W}'^{\mu}$ and B'^{μ} associated with the broken gauge symmetries are related to the SM gauge fields by

$$W^\mu = sW_1^\mu + cW_2^\mu, \quad W'^\mu = -cW_1^\mu + sW_2^\mu,$$
$$B^\mu = s'B_1^\mu + c'B_2^\mu, \quad B'^\mu = -c'B_1^\mu + s'B_2^\mu, \tag{7}$$

with mixing angles of

$$c = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad c' = \frac{g_1'}{\sqrt{g_1'^2 + g_2'^2}}. \tag{8}$$

The SM gauge couplings are then $g = g_1 s = g_2 c$ and $g' = g_1' s' = g_2' c'$. At the scale f, the SM gauge bosons remain massless, while the heavy gauge bosons acquire masses of order f:

$$m_{W'} = \frac{g}{2sc}f, \quad m_{B'} = \frac{g'}{2\sqrt{5}s'c'}f. \tag{9}$$

The presence of $\sqrt{5}$ in the denominator of $m_{B'}$ leads to a relatively light new neutral gauge boson. It is to be compared with the $SU(6)/Sp(6)$ case of $m_{B'} = g'f/(2\sqrt{2}s'c')$.

Even though the Higgs boson at tree level remains massless as a Goldstone boson, its mass is radiatively generated because any nonlinearly realized symmetry is broken by the gauge, Yukawa, and self-interactions of the Higgs field. Early attempts at constructing the Higgs boson as a pseudo Goldstone boson suffered from the same quadratic divergence as in the SM. Little Higgs models introduce a collective symmetry breaking: Only when multiple gauge symmetries are broken is the Higgs boson mass radiatively generated; the loop corrections to the Higgs boson mass occur at least at the two-loop level. Phenomenologically the one-loop quadratic divergence induced by the SM particles is canceled by that induced by the new particles as in SUSY models. However, the cancellations in little Higgs models are due to the exactly opposite coupling, which is provided by a larger symmetry structure. For example, the $B^{\mu}B_{\mu}h^2$ and $B'^{\mu}B'_{\mu}h^2$ couplings are

$$\mathcal{L}_{\Sigma}(B\cdot B)\supset g'^2 B_{\mu}B^{\mu}\mathrm{Tr}\left[\frac{1}{4}h^{\dagger}h\right]-g'^2 B'_{\mu}B'^{\mu}\,\mathrm{Tr}\left[\frac{1}{4}h^{\dagger}h\right]. \tag{10}$$

It is to be compared with SUSY models where the cancellation occurs due to the different spin statistics between the SM particle and its superpartner.

Since a more severe quadratic divergence comes from the top quark loop, another top-quark-like fermion is also required. In addition, this new fermion is naturally expected to be heavy with mass of order *f*. As a minimal extension, we introduce a vectorlike fermion pair $\tilde{t}$ and $\tilde{t}'^{c}$ with the SM quantum numbers $(\mathbf{3},\mathbf{1})_{Y_i}$ and $(\bar{\mathbf{3}},\mathbf{1})_{-Y_i}$. With $\chi_i=(b_3,t_3,\tilde{t})$ and antisymmetric tensors of ϵ_{ijk} and ϵ_{xy}, the following Yukawa interaction is chosen in the littlest Higgs model:

$$\mathcal{L}_Y=\frac{1}{2}\lambda_1 f\sum_{i,j,k=1}^{3}\sum_{x,y=4}^{5}\epsilon_{ijk}\epsilon_{xy}\chi_i\Sigma_{jx}\Sigma_{ky}u_3'^{c}+\lambda_2 f\tilde{t}\;\tilde{t}'^{c}+\mathrm{H.c.} \tag{11}$$

$$\supset -i\lambda_1\left(\sqrt{2}h^0 t_3+if\tilde{t}-\frac{i}{f}h^0h^{0*}\tilde{t}\right)u_3'^{c}+\mathrm{H.c.} \tag{12}$$

As Eq. (12) shows, the quadratic divergence from the top quark loop is canceled by that from the heavy top quark loop. In addition, this cancellation is stable against radiative corrections.

Electroweak symmetry breaking is induced by the remaining Goldstone bosons h and ϕ. Through radiative corrections, the gauge, Yukawa, and self-interactions of the Higgs field generate a Higgs potential [22]. Even though the quadratic divergence of the Higgs boson mass coefficient $-\mu^2$ vanishes at the one-loop level, μ^2 has log-divergent one-loop and quadratically divergent two-loop contributions. We treat μ^2 as a free parameter of order 100 GeV. For positive μ^2, the h and ϕ fields develop VEVs of $\langle h^0\rangle=v/\sqrt{2}$ and $\langle\phi^0\rangle=v'$, which trigger the electroweak symmetry breaking. Now the SM W and Z bosons acquire masses of order v, and small (of order v^2/f^2) mixing between $W^{\pm}$ and $W'^{\pm}$ (Z and W'^3) occurs. In the following, we denote the mass eigenstates of the SM gauge fields by $W_L^{\pm}$ and Z_L and the heavy gauge bosons by $W_H^{\pm}$, W_H^3, and B_H.

Some phenomenological discussions are in order here. First, the requirement of a positive mass squared of the Higgs triplet constrains v' to be $v'/v<v/(4f)$. Second, since the $U(1)_{\mathrm{QED}}$ symmetry remains intact, the gauge couplings of the photon are the same as in the SM. For the Yukawa interaction, we assume that Eq. (11) is valid for the SM light fermions including leptons, except that their corresponding extra vectorlike fermions are absent. The $U(1)_{1,2}$ charges of the SM fermions are chosen generation independently by requiring the fermion couplings to be invariant under $[SU(2)\otimes U(1)]^2$ and anomaly-free [23]. In other little Higgs models, several alternatives for the $U(1)_{1,2}$ charge choice exist [8].

The question of consistency with the electroweak precision data merits some discussions. The absence of custodial $SU(2)$ global symmetry in this model yields weak isospin violating contributions to the electroweak precision observables. In the early study, global fits to the experimental data put rather severe constraints on the $f>4$ TeV at 95% C.L. [24,25]. However, their analyses are based on a simple assumption that the SM fermions are charged only under $U(1)_1$. If the SM fermions are charged under $U(1)_1\otimes U(1)_2$, the bounds become relaxed: The substantial parameter space allows $f\simeq 1$–2 TeV [8,11,26]. If only the $U(1)_Y$ is gauged, the experimental constraints are looser [17,26].

To illustrate the preferred parameter space consistent with the low energy data, we present the SM Z boson mass:

$$M_{Z_L}^2=m_z^2\left[1+\Delta\left(\frac{1}{4}+c^2(1-c^2)-\frac{5}{4}(c'^2-s'^2)^2\right)+8\Delta'\right], \tag{13}$$

where $m_z=gv/(2c_W)$, $\Delta=v^2/f^2\ll 1$, $\Delta'=v'^2/v^2\ll 1$. The gauge couplings of the Z_L with the charged leptons, in the form of $\gamma^{\mu}(g_-^Z P_-+g_+^Z P_+)$ with $P_{\pm}=(1\pm\gamma^5)/2$, are

$$g_+^Z=\frac{e}{s_W c_W}\left[-\frac{1}{2}+s_W^2+\Delta\left\{\frac{c^2}{2}(c^2-1/2)-\frac{5}{4}(c'^2-s'^2)(c'^2-2/5)\right\}\right], \tag{14}$$

$$g_-^Z=\frac{e}{s_W c_W}\left[s_W^2+\frac{5}{2}\Delta(c'^2-s'^2)(c'^2-2/5)\right].$$

Here the QED bare coupling e^2 is the running coupling at the Z pole, and the bare value of s_W^2 is related to the measured value of s_0^2 by

$$\frac{1}{s_W c_W}=\frac{1}{s_0 c_0}\left[1-\frac{\Delta}{2}\left\{c^2 s^2-\frac{5}{4}(c'^2-s'^2)^2\right\}-2\Delta'\right]. \tag{15}$$

The main corrections to the low energy observables in Eqs. (13)–(15) are proportional to c^2 or $(c'^2-s'^2)$. In the parameter space around $c\ll 1$ and $c'=1/\sqrt{2}$, therefore, the new contributions are suppressed: *f* about 2 TeV is allowed in the region of $c\in[0,0.5]$ and $c'\in[0.62,0.73]$ [26].

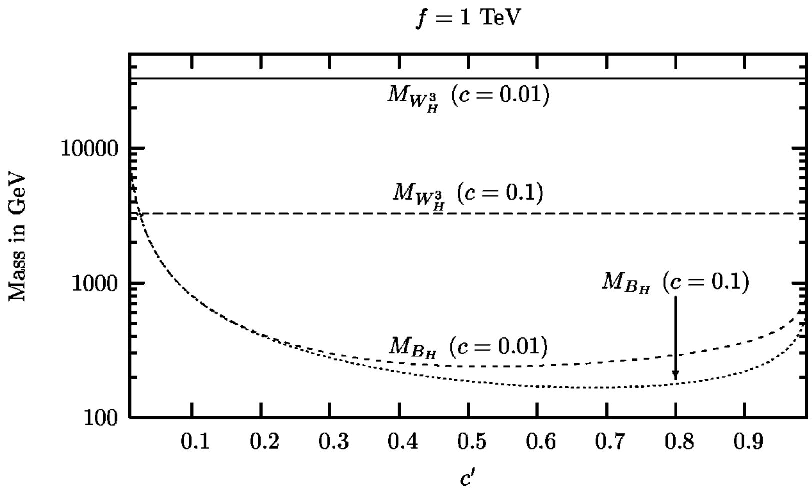

FIG. 1. The masses of B_H and W_H^3 in GeV as a function of c' for $c=0.01$, 0.1. The f is set to be 1 TeV.

III. PROPERTIES OF THE B_H

Among various little Higgs models such as the $SU(6)/Sp(6)$ model [8,11], the $SU(4)^4/SU(3)^3$ model [12], and the $SU(4)_L \times U(1)_4$ model [27], the littlest Higgs model can be distinguished by the presence of a relatively light B_H. From Eq. (9), the mass ratio of the B_H to the W_H^3 is

$$\frac{M_{B_H}^2}{M_{W_H^3}^2}=\frac{s_W^2}{5c_W^2}\frac{s^2c^2}{s'^2c'^2}+\mathcal{O}\left(\frac{v^2}{f^2}\right)\sim 0.06\left(\frac{s^2c^2}{s'^2c'^2}\right). \quad (16)$$

In general the B_H is substantially lighter than the $W_H^3(W_H)$. In the parameter space preferred by the electroweak precision data ($c \ll 1$ and $c' \sim 1/\sqrt{2}$), this ratio is further reduced.

In Fig. 1, we present M_{B_H} and $M_{W_H^3}$ as a function of c' for $f=1$ TeV (M_{B_H} and $M_{W_H^3}$ increase linearly with f). In most of the parameter space, the B_H is much lighter than the W_H^3: Around $c'=1/\sqrt{2}$ and $c \ll 1$, the mass difference is maximized. Since the B_H is mainly the B', its mass depends weakly on the value of c, the mixing parameter between $SU(2)_1$ and $SU(2)_2$ gauge bosons. M_{B_H} for $c=0.3$ is practically identical with that for $c=0.1$. On the contrary, $M_{W_H^3}$ is sensitive to c, while almost insensitive to c': The mass of W_H^3 increases as c decreases; for $c<0.1$, the W_H^3 becomes too heavy to be probed even at the LHC.

Note that in the preferred parameter space the B_H becomes light enough to be produced at a future linear collider with 500 GeV c.m. energy.

Figure 2 illustrates contours for $M_{B_H}=200,300,500$ GeV in the parameter space of (c',f). The value of c, which affects M_{B_H} little, is set to be 0.1. In particular, the region around $c'=1/\sqrt{2}$ allows, for $f \lesssim 3$ TeV, the on-shell production of the B_H at a 500 GeV LC.

We review the gauge boson-fermion couplings for B_H^μ in the form of $\gamma^\mu(g_v+g_a\gamma^5)$ with the anomaly cancellation condition [23]

$$g_v^{B_H\text{-}\bar{u}\text{-}u}=+\frac{g'}{12s'c'}(2-5c'^2),$$

$$g_a^{B_H\text{-}\bar{u}\text{-}u}=+\frac{g'}{20s'c'}(2-5c'^2),$$

$$g_v^{B_H\text{-}\bar{d}\text{-}d}=-\frac{g'}{60s'c'}(2-5c'^2),$$

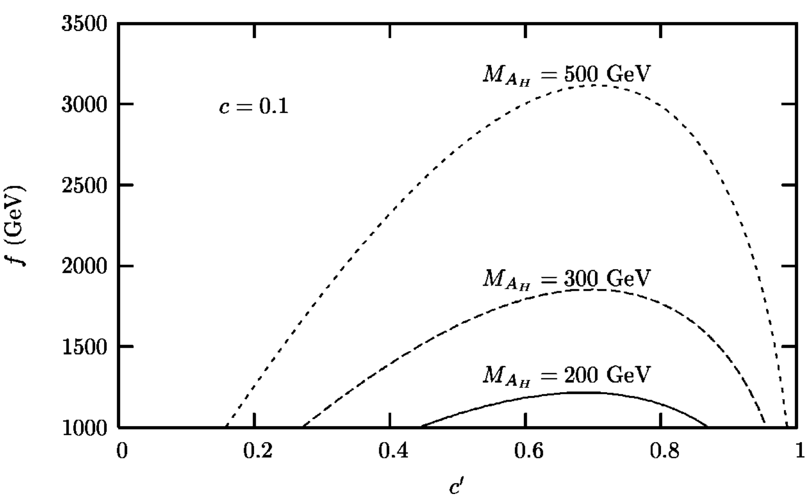

FIG. 2. In the parameter space of (c',f), contours for $M_{B_H}=200,300,500$ GeV. The value of c is fixed to be 0.1.

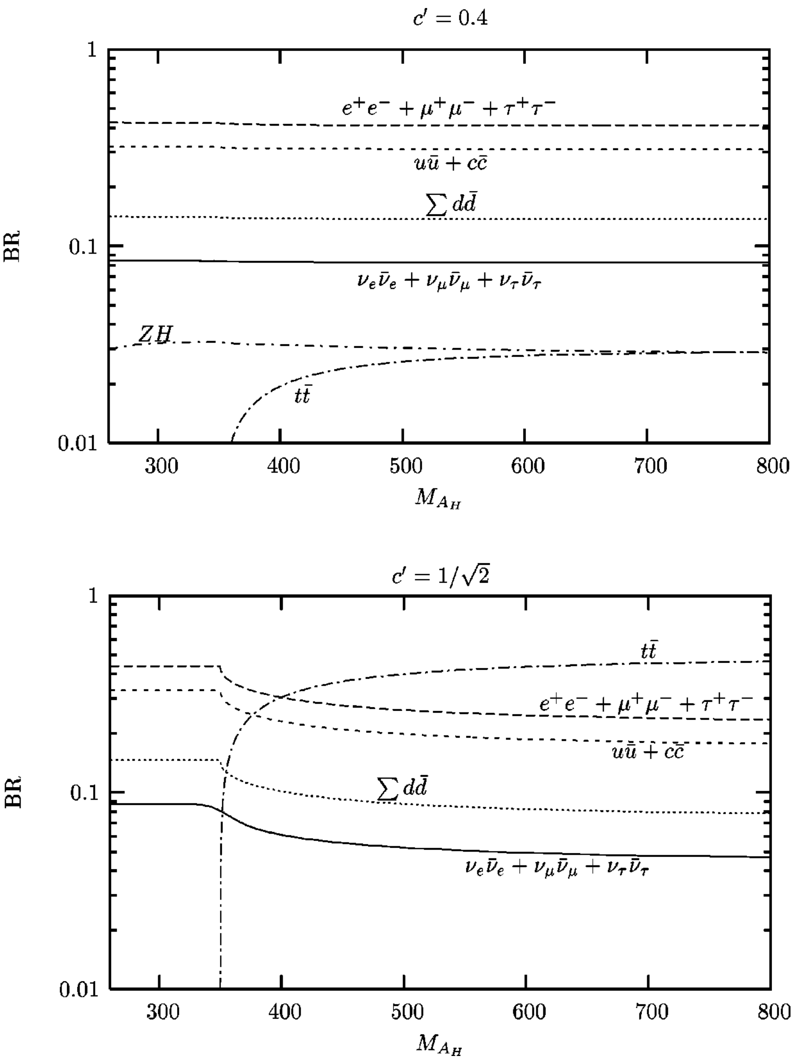

FIG. 3. The branching ratios of the B_H as a function of M_{B_H} for $c'=0.4$ and $c'=1/\sqrt{2}$. The Higgs boson mass is 120 GeV.

$$g_a^{B_H\text{-}\bar{d}\text{-}d} = -\frac{g'}{20s'c'}(2-5c'^2),$$

$$g_v^{B_H\text{-}e^+\text{-}e^-} = -\frac{3g'}{20s'c'}(2-5c'^2),$$

$$g_a^{B_H\text{-}\bar{e}^+\text{-}e^-} = -\frac{g'}{20s'c'}(2-5c'^2). \tag{17}$$

An apparently special point at $c'=\sqrt{2/5}\approx 0.63$ exists where all the gauge couplings of the B_H with light fermions vanish. Note that $c'=\sqrt{2/5}$ yields $(c'^2-s'^2)^2=0.04$, implying negligible contributions to the electroweak precision observables in Eqs. (13)–(15). An exception is the right-handed top quark coupling of[1]

$$g_+^{B_H\text{-}t\text{-}\bar{t}} = \frac{g'}{s'c'}\left(\frac{4}{3}-\frac{5}{6}c'^2-\frac{1}{5}\,\frac{\lambda_1^2}{\lambda_1^2+\lambda_2^2}\right). \tag{18}$$

The additional term is attributed to the proposed top Yukawa coupling in Eq. (11): The physical mass eigenstates, the $SU(2)$-singlet top quark t_R^c, and the heavy top quark T_R, are mixtures of weak eigenstates $u_3'^c$ and $\tilde{t}'^c$ through

$$t_R^c = \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}}(-\lambda_1\tilde{t}'^c+\lambda_2 u_3'^c),$$

$$T_R = \frac{1}{\sqrt{\lambda_1^2+\lambda_2^2}}(-\lambda_1\tilde{t}'^c+\lambda_2 u_3'^c). \tag{19}$$

In the special case of $c'=\sqrt{2/5}$, the top quark is the only fermion interacting with the B_H.

The heavy B_H then decays into a fermion pair and Zh. Decay into a SM W_L pair is suppressed by a factor of $(v/f)^4$. The partial decay rates are

[1]The left-handed gauge coupling of the top quark is $g_-^{B_H\text{-}t\text{-}\bar{t}} = g'/s'c'(1/15-1/6c'^2)$.

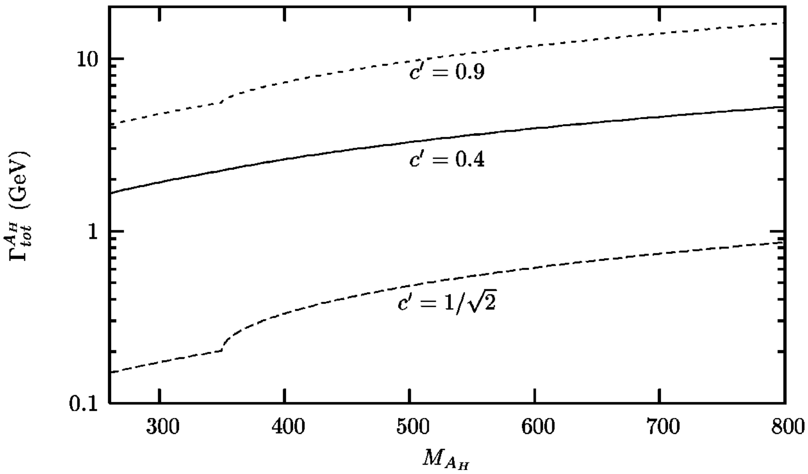

FIG. 4. The total decay rate of the B_H as a function of M_{B_H} for $c'=0.4$ and $c'=1/\sqrt{2}$. The Higgs boson mass is 120 GeV.

$$\Gamma(B_H\to f\bar{f}) = \frac{N_c}{12\pi}[(g_v^{B_H\text{-}\bar{f}\text{-}f})^2(1+2r_f) + (g_a^{B_H})^2(1-4r_f)]\sqrt{1-4r_f}M_{B_H}, \quad (20)$$

$$\Gamma(B_H\to Zh) = \frac{g'^2(c'^2-s'^2)}{384\pi c's'}\lambda^{1/2} \times[(1+r_Z-r_h)^2+8r_Z]M_{B_H},$$

where N_c is the color factor, $r_i = m_i^2/M_{B_H}^2$, and $\lambda = 1+r_Z^2+r_h^2+2r_Z+2r_h+2r_Zr_h$. Note that for $c'=1/\sqrt{2}$ the B_H decay into Zh is prohibited. In Fig. 3, we show the branching ratios (BRs) of the B_H as a function of M_{B_H} for $c'=0.4$ and $c'=1/\sqrt{2}$. The two top quark Yukawa couplings λ_1 and λ_2 in Eq. (11) are assumed to be equal to each other. Except for the narrow region around $c'=1/\sqrt{2}$, the BR patterns are almost the same: The decay into a charged lepton pair is dominant. If $c'=1/\sqrt{2}$, the B_H-Z-h coupling vanishes and the B_H gauge coupling with a lepton pair is suppressed since then c' is near $\sqrt{2/5}$. In this case, the decay into a top quark pair becomes dominant if kinematically allowed. In Fig. 4, we show the total decay rate of the B_H as a function of M_{B_H} for $c'=0.4, 1/\sqrt{2}, 0.9$. In particular, the $c'=1/\sqrt{2}$ case yields a very narrow resonance peak, raising the possibility that the resonance signal might be missed.

The presence of a quite light B_H seems incompatible with the current direct searches for a new neutral vector boson at the Fermilab Tevatron [28]. A rough estimate of $M_{B_H} \gtrsim 375$ GeV was guessed in Ref. [8,11]. However the B_H couplings with light fermions vanish at $c'=\sqrt{2/5}$ as can be seen in Eq. (17). A substantially light B_H in the parameter space around $c'=\sqrt{2/5}$ can survive. In Fig. 5, we plot $\sigma(B_H)\cdot \text{BR}(B_H\to e^+e^-,\mu^+\mu^-)$ as a function of M_{B_H}. For B_H production, we consider $p\bar{p}$ collisions with $\sqrt{s}=1.8$ TeV and the Martin-Roberts-Stirling (MRS) parton distribution functions [29]. We set $f=1.5$ TeV and $c=0.1$. We also consider the value of c' only in the range of $c'\in[0.6,0.8]$, which leads to $(c'^2-s'^2)^2 \leqslant 0.1$, suppressing the corrections to the electroweak precision observables. The limited range of c' yields a rather narrow mass spectrum of B_H around 244.7–257.1 GeV. Note that there is a twofold ambiguity in the value of c' with the given M_{B_H}, f, and c, as can be seen in Fig. 1. We define $c'_{(\text{min})}$ at which the M_{B_H} is minimized: For $f=1.5$ TeV and $c=0.1$, $c'_{(\text{min})}\approx 0.694$. The solid line in Fig. 5 is for $c'>c'_{(\text{min})}$ and the dashed line is for $c'\leqslant c'_{(\text{min})}$. The dotted line shows the CDF 95% C.L. upper limit on $\sigma(Z')\cdot \text{BR}(Z'\to e^+e^-,\mu^+\mu^-)$ [28]. It is clear that

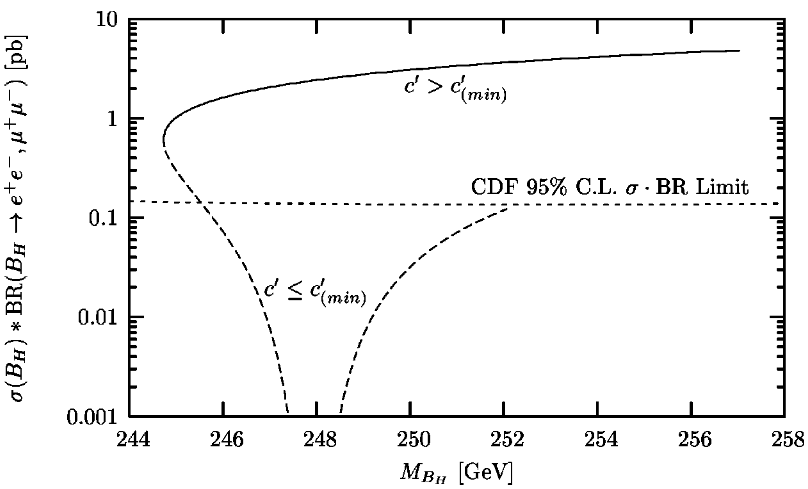

FIG. 5. Limits on B_H production from $\sigma(B_H)\cdot \text{BR}(B_H\to e^+e^-,\mu^+\mu^-)$. We set $f=1.5$ TeV, $c=0.1$, and $c'\in[0.6,0.8]$. Here $c'_{(\text{min})}$ denotes the c' value at which the M_{B_H} is minimized with the given f and c.

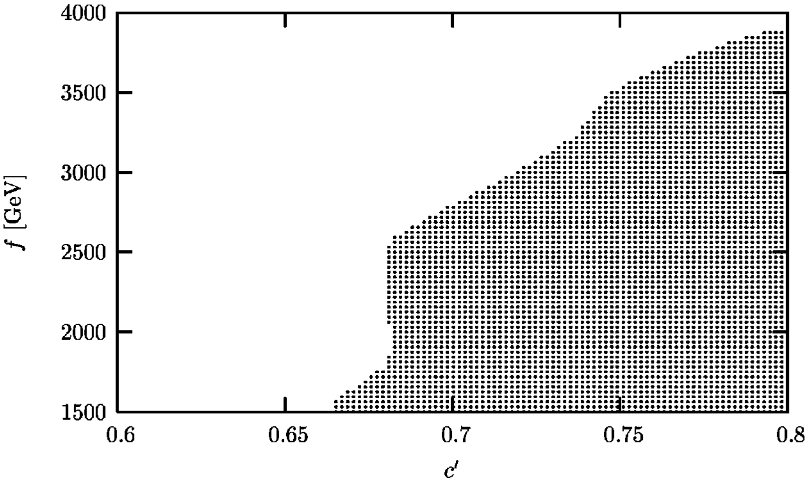

FIG. 6. The dotted region is the excluded area in the parameter space of (c',f) by the CDF 95% C.L. upper limit on $\sigma(Z')\cdot\mathrm{BR}(Z'\to e^+e^-,\mu^+\mu^-)$. The value of c is 0.1.

as the c' approaches $\sqrt{2/5}$ (in the $c'\leqslant c'_{(\mathrm{min})}$ case) the suppressed gauge couplings $g_{v,a}^{B_H\text{-}\bar f\text{-}f}$ reduce the B_H production as well as its BR into e^+e^- and $\mu^+\mu^-$. The direct searches at the Tevatron constrain the value of c', not the mass of the heavy neutral gauge boson directly. A relatively light B_H with mass about 250 GeV in the littlest Higgs model is phenomenologically viable in a substantial portion of the parameter space around $c'=\sqrt{2/5}$.

Figure 6 illustrates the excluded (dotted) area in the (c',f) parameter space from the direct search at the Tevatron. The value of c is fixed 0.1. A large portion of the parameter space consistent with the electroweak precision data can accommodate the Tevatron direct searches for new gauge bosons decaying into dileptons.

IV. ANOMALOUS MAGNETIC MOMENT OF THE MUON AND THE LITTLE HIGGS MODEL

In this section, we study the one-loop level contribution of the littlest Higgs model by calculating its effects on the anomalous magnetic moment of the muon. The present status of the theoretical evaluation of the $(g-2)_\mu$ in the SM is not conclusive because of the inconsistent values between the hadronic vacuum polarizations based on e^+e^- and τ data [30,31]. Comparison with the experimental value implies

$$a_\mu^{\mathrm{expt}}-a_\mu^{\mathrm{SM}}(e^+e^-)=(35.5\pm 11.7)\times 10^{-10}\quad [3\sigma],$$
$$a_\mu^{\mathrm{expt}}-a_\mu^{\mathrm{SM}}(\tau)=(10.3\pm 10.7)\times 10^{-10}\quad [1\sigma]. \tag{21}$$

In the littlest Higgs model, one-loop corrections come from the Feynman diagrams mediated by the B_H, W_H^3, and $W_H^\pm$ as depicted in Fig. 7. Since each contribution to Δa_μ is inversely proportional to the gauge boson mass squared and the B_H is much lighter than the W_H^3 or $W_H^\pm$ at least by an order of magnitude, we consider only the B_H contribution of [32]

$$\Delta a_{B_H}=\frac{1}{12\pi^2}\left(\frac{m_\mu}{M_{B_H}^2}\right)^2[(g_v^{B_H\text{-}e^+\text{-}e^-})^2-5(g_a^{B_H\text{-}e^+\text{-}e^-})^2]. \tag{22}$$

Figure 8 shows Δa_{B_H} as a function of c' for fixed $M_{B_H}=200,300,500$ GeV. Since we require $f>1$ TeV, a limited space of c' is presented. The contribution to Δa_μ increases as M_{B_H} decreases and c' deviates from the value of $\sqrt{2/5}$. In the whole parameter space, Δa_{B_H} is quite safe from the recent experimental data in Eq. (21). Therefore, it is concluded that the light B_H is not inconsistent with the current status of the theoretical and experimental value of the muon anomalous magnetic moment.

V. $e^+e^-\to\mu^+\mu^-$

In Sec. III, we have shown that, as the B_H becomes lighter, the corrections to the electroweak precision observables become smaller. This is contrary to the usual case in which the *heavier* new particle suppresses the corrections to the low energy observables. It is also shown that in a major portion of parameter space the B_H dominant decay mode is into a charged lepton pair. Therefore, one of the most efficient processes for probing the model is the process $e^+e^-\to\mu^+\mu^-$.

This process has two SM s-channel Feynman diagrams mediated by the photon and Z boson. In the littlest Higgs model, two additional s-channel diagrams contribute, mediated by the B_H and W_H^3. The corresponding helicity amplitude $\mathcal{M}_{\lambda_e\bar\lambda_e\lambda_\mu\bar\lambda_\mu}$, where λ_l $(\bar\lambda_l)$ is the polarization of l^- (l^+), can be simplified by $\mathcal{M}_{\lambda_e\lambda_\mu}$ since $\lambda_l=-\bar\lambda_l$ with the

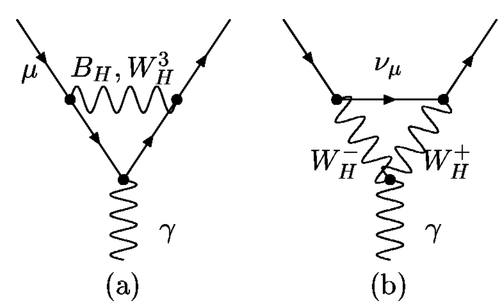

FIG. 7. Feynman diagrams for one-loop contributions of heavy gauge bosons. (a) shows the contributions from B_H,W_H^3 and (b) shows the contribution from W_H.

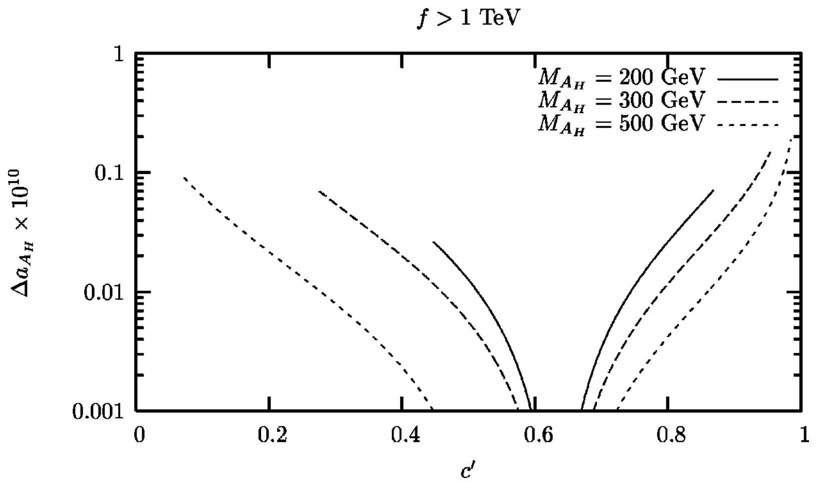

FIG. 8. The contribution to the muon anomalous magnetic moment due to the B_H as a function of c' for $M_{B_H}=200,300,500$ GeV.

lepton mass neglected. We have

$$\mathcal{M}_{\lambda_e\lambda_\mu} = -(1+\lambda_e\lambda_\mu\cos\theta)\sum_{V_j} g^{V_j}_{\lambda_e} g^{V_j}_{\lambda_e}\mathcal{D}_{V_j}, \tag{23}$$

where θ is the scattering angle of the muon with respect to the electron beam, $V_j = A, Z, B_H, W_H^3$, $g^{V_j}_{\lambda_e} \equiv g^{V_j - l^+ - l^-}_{\lambda_e}$, and $\mathcal{D}_{V_j}$ is the propagation factor of

$$\mathcal{D}_{V_j} = \frac{s}{s - M^2_{V_j} + iM_{V_j}\Gamma_{V_j}}, \tag{24}$$

and

$$g^{W^3_H}_{+} = -\frac{gc}{2s}, \quad g^{W^3_H}_{-} = 0. \tag{25}$$

In Fig. 9, we present the total cross section as a function of $\sqrt{s}$ for $c'=0.4$ and $c'=1/\sqrt{2}$. We set $M_{B_H}=400$ GeV and $c=0.1$. If c' deviates sizably from the critical point of $\sqrt{2/5}$ (see the $c'=0.4$ case), the coupling B_H-l^+-l^- is large enough to yield substantial deviations from the SM results even outside the resonance peak. In the parameter region of the suppressed B_H-l^+-l^- coupling (see the $c'=1/\sqrt{2}$ case), only around the resonance peak can a significant new signal be produced.

VI. SUMMARY AND CONCLUSION

The littlest Higgs model could be an alternative model for new physics beyond the standard model which solves the little hierarchy problem. From the extension in the gauge sector, we expect a new set of gauge bosons. The B_H is shown to be the lightest of all and, moreover, becomes lighter in the parameter space preferred by the electroweak precision measurements. We checked the consistency of the parameter space against the direct searches for a new neutral gauge boson at the Tevatron and the one-loop induced anomalous magnetic moment of the muon. Its numerical value is $\Delta a_\mu \leqslant 0.1\times 10^{-10}$ in the whole parameter region. Then we study the on-shell production and decay of the B_H in future linear colliders. The B_H mainly decays into a lepton pair but for $c'=\sqrt{2/5}$ it decays into a top-quark pair and Zh if the kinematics allows. The high energy process of $e^+e^- \to \mu^+\mu^-$ is also studied, and the resonance structure of the B_H production is shown for various parameters of the model. In conclusion, the B_H of the littlest Higgs model can be light enough to be produced in a future linear collider.

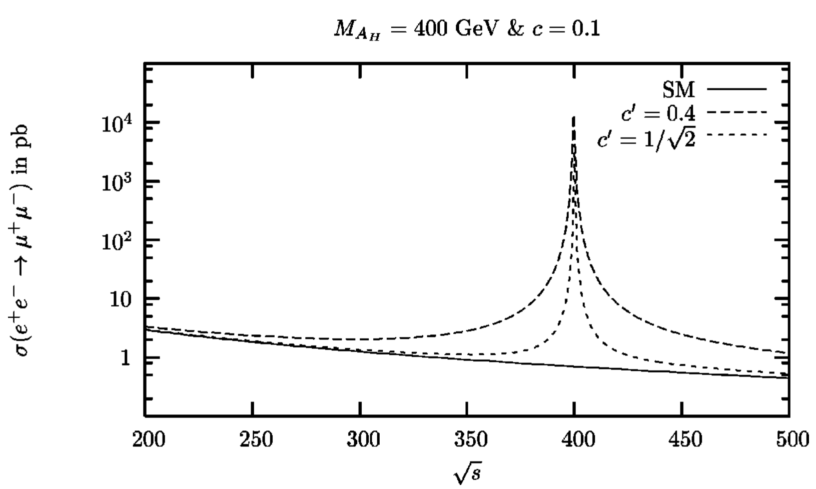

FIG. 9. The total cross section of the process as a function of $\sqrt{s}$ with $M_{B_H}=400$ GeV and $c=0.1$. We consider two cases of $c'=0.4$ and $c'=1/\sqrt{2}$.

ACKNOWLEDGMENT

The authors thank Professor J. E. Kim, K. Choi, and E. J. Chun for valuable comments on our work. S.C. also thanks Professor T. Moroi and M. Yamaguchi for valuable discussions during the KIAS-KAIST joint workshop. The work of J.S. was supported by the faculty research fund of Konkuk University.

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