Hadronic electromagnetic properties at finite lattice spacing

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(Received 2 March 2004; published 2 June 2004)

Electromagnetic properties of the octet mesons as well as the octet and decuplet baryons are augmented in quenched and partially quenched chiral perturbation theory to include $O(a)$ corrections due to lattice discretization. We present the results for the $SU(3)$ flavor group in the isospin limit as well as the results for $SU(2)$ flavor with nondegenerate quarks. These corrections will be useful for extrapolation of lattice calculations using Wilson valence and sea quarks, as well as calculations using Wilson sea quarks and Ginsparg-Wilson valence quarks.

DOI: 10.1103/PhysRevD.69.114503 PACS number(s): 12.38.Gc

I. INTRODUCTION

Lattice gauge theory can provide first principles calculations in the strongly interacting regime of QCD, where quarks and gluons are bound into color-neutral hadronic states. These calculations, however, are severely limited by the available computing power, necessitating the use of light quark masses m_q that are much larger than those in reality. Hence, one needs to extrapolate from the quark masses used on the lattice to those of nature.

A model independent way to do this extrapolation is to study QCD at hadronic scales through its low-energy effective theory, chiral perturbation theory (χPT). Since χPT provides a systematic expansion in terms of m_q/Λ_χ, where Λ_χ is the chiral symmetry breaking scale, one can, in principle, understand how QCD observables behave as functions of the quark mass. In order to address the quenched and partially quenched approximations employed by lattice calculations, χPT has been extended to quenched chiral perturbation theory (QχPT) [1–7] and partially quenched chiral perturbation theory (PQχPT) [8–16].

Recently, we considered the electromagnetic properties of the octet mesons and both the octet and decuplet baryons in QχPT and PQχPT [17–19]. Owing in part to the charge neutrality of singlet fields, the quenched results are not more singular in the chiral limit than their unquenched counterparts. We showed, however, that despite this similarity, the quenched results contain singlet contributions that have no analogue in χPT. Moreover, quenching closed quark loops alters the contribution from chiral logarithms. For the decuplet baryon form factors, for example, quenching completely removes these chiral logarithms. Many others have also observed that the behavior of meson loops near the chiral limit is misrepresented in QχPT; see for example [20–24]. On the other hand, PQχPT results are devoid of such complications and allow for a smooth limit to QCD.

Not only are lattice calculations limited to unphysically large quark masses; they are also severely restricted by two further parameters: the size L of the lattice, which is not considerably larger than the system under investigation, and the lattice spacing a, which is not considerably smaller than the relevant hadronic distance scale. To address the issue of finite lattice spacing, χPT has been extended (following the earlier work of [25–28]) in the meson sector to $O(a)$ for the Wilson action [29] and for mixed actions [30]. Corrections at $O(a^2)$ have also been pursued [31,32]. Corrections to baryon observables have only recently been investigated [33]. To consider finite lattice spacing corrections, one must formulate the underlying lattice theory and match the new operators that appear onto those in the chiral effective theory. This can be done by utilizing a dual expansion in quark mass and lattice spacing. Following [31,33], we assume a hierarchy of energy scales

$$m_q \ll \Lambda_\chi \ll \frac{1}{a} \tag{1}$$

and ignore finite volume effects. The small dimensionless expansion parameters are

$$\epsilon^2 \sim \begin{cases} m_q/\Lambda_\chi, \\ a\Lambda_\chi, \\ p^2/\Lambda_\chi^2, \end{cases} \tag{2}$$

where p is an external momentum. Thus we have a systematic way to calculate $O(a)$ corrections in χPT for the observables of interest.

In this work we investigate the $O(a)$ corrections to the electromagnetic properties of the meson and baryon octets, the baryon decuplet, and the decuplet to octet electromagnetic transitions in QχPT and PQχPT. We work up to next-to-leading order in the chiral expansion and to leading order in the heavy baryon expansion. The paper is structured as follows. First, in Sec. II, we review PQχPT at finite lattice spacing with mixed actions. Since the setup for QχPT parallels that of PQχPT, we will only highlight differences where appropriate. Next in Sec. III we calculate finite lattice spacing corrections to the charge radii of the octet mesons to $O(\epsilon^2)$. This is followed by the calculation of such corrections to the charge radii and magnetic moments of the octet baryons; the charge radii, magnetic moments, and electric quadrupole moments of the decuplet baryons; and the decuplet to octet electromagnetic transition moments

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(Secs. IV–VI). Corresponding results for the above electromagnetic observables in the $SU(2)$ flavor group are presented in Appendix A. In Appendix B we determine the $O(a)$ corrections in an alternative power counting scheme for coarser lattices where $\epsilon \sim a\Lambda_\chi$. A conclusion appears in Sec. VII.

II. PQχPT AT FINITE LATTICE SPACING

In partially quenched QCD (PQQCD) [8–15] the quark part of the Symanzik Lagrangian [25,34,35] to $O(a)$ is written as

$$\mathcal{L}=\overline{Q}(i\not{D}-m_Q)Q+a\overline{Q}\sigma^{\mu\nu}G_{\mu\nu}c_Q Q, \tag{3}$$

where the second term, the Pauli term, breaks chiral symmetry in the same way as the quark mass term. Here, the nine quarks of PQQCD are in the fundamental representation of the graded group $SU(6|3)$ [36–38] and appear in the vector

$$Q=(u,d,s,j,l,r,\tilde{u},\tilde{d},\tilde{s}), \tag{4}$$

which obeys the graded equal-time commutation relation

$$Q_i^\alpha(\mathbf{x})Q_j^{\beta\dagger}(\mathbf{y})-(-1)^{\eta_i\eta_j}Q_j^{\beta\dagger}(\mathbf{y})Q_i^\alpha(\mathbf{x})=\delta^{\alpha\beta}\delta_{ij}\delta^3(\mathbf{x}-\mathbf{y}), \tag{5}$$

where α and β are spin, and i and j are flavor indices. The remaining graded equal-time commutation relations can be written analogously. The different statistics for fermionic and bosonic quarks are incorporated in the grading factor

$$\eta_k=\begin{cases}1 & \text{for } k=1,2,3,4,5,6,\\ 0 & \text{for } k=7,8,9.\end{cases} \tag{6}$$

The quark mass matrix is given by

$$m_Q=\mathrm{diag}(m_u,m_d,m_s,m_j,m_l,m_r,m_u,m_d,m_s), \tag{7}$$

while the Sheikholeslami-Wohlert (SW) [39] coefficient matrix for mixed actions reads

$$c_Q=\mathrm{diag}(c^v,c^v,c^v,c^s,c^s,c^s,c^v,c^v,c^v). \tag{8}$$

If the quark Q_i is a Wilson fermion [40], then $(c_Q)_i=c_{\mathrm{sw}}$. Alternately, if Q_i is of the Ginsparg-Wilson variety [41] (e.g., Kaplan fermions [42] or overlap fermions [43]), then $(c_Q)_i=0$. Since one expects simulations to be performed with valence quarks that are all of the same species as well as sea quarks all of the same species, we have labeled the SW coefficients in Eq. (8) by valence (v) and sea (s) instead of flavor. In the limit $m_j=m_u$, $m_l=m_d$, and $m_r=m_s$ one recovers QCD at $O(a)$.

The light quark electric charge matrix $\mathcal{Q}$ is not uniquely defined in PQQCD [44]. By imposing the charge matrix $\mathcal{Q}$ to have vanishing supertrace, no new operators involving the singlet component are introduced. This can be accomplished by [45]

$$\mathcal{Q}=\mathrm{diag}\left(\frac{2}{3},-\frac{1}{3},-\frac{1}{3},q_j,q_l,q_r,q_j,q_l,q_r\right). \tag{9}$$

In addition to the SW term in Eq. (3), the vector-current operator of PQQCD also receives corrections at $O(a)$. There are three operator structures to consider [46]:

$$\mathcal{O}_0^\mu=a\overline{Q}\mathcal{Q}c_0 m_Q\gamma^\mu Q,$$

$$\mathcal{O}_1^\mu=a\overline{Q}\mathcal{Q}c_1(i\overleftrightarrow{D}^\mu)Q,$$

$$\mathcal{O}_2^\mu=aD_\nu(\overline{Q}\mathcal{Q}c_2\sigma^{\mu\nu}Q), \tag{10}$$

where $\overleftrightarrow{D}^\mu=\overleftarrow{D}^\mu-\overrightarrow{D}^\mu$ and D^μ is the gauge covariant derivative. The form of the matrices c_0, c_1, and c_2 in PQQCD is

$$c_j=\mathrm{diag}(c_j^v,c_j^v,c_j^v,c_j^s,c_j^s,c_j^s,c_j^v,c_j^v,c_j^v), \tag{11}$$

where c_j^v and c_j^s are the coefficients of the vector-current correction operator $\mathcal{O}_j^\mu$ for valence and sea quarks respectively. If the vector-current operator is $O(a)$ improved in the valence (sea) sector, then $c_j^v=0$ ($c_j^s=0$). The operator $\mathcal{O}_0^\mu$, which corresponds to a renormalization of the vector current, contains a factor of am_Q that renders it $O(\epsilon^4)$. Thus contributions to electromagnetic observables from $\mathcal{O}_0^\mu$ are neglected below. The equations of motion which follow from Eq. (3) can be used to show that the operator $\mathcal{O}_2^\mu$ is redundant up to $O(a^2)$ corrections. Therefore, we need not consider $\mathcal{O}_2^\mu$. For ease we define the matrix product $c_{1,\mathcal{Q}}=\mathcal{Q}c_1$.

A. Mesons

For massless quarks at zero lattice spacing, the Lagrangian in Eq. (3) exhibits a graded symmetry $SU(6|3)_L\otimes SU(6|3)_R\otimes U(1)_V$ that is assumed to be spontaneously broken down to $SU(6|3)_V\otimes U(1)_V$. The low-energy effective theory of PQQCD that results from expanding about the physical vacuum state is PQχPT. The emerging 80 pseudo-Goldstone mesons can be described at $O(\epsilon^2)$ by a Lagrangian which accounts now for the two sources of explicit chiral symmetry breaking [25,29,30]:

$$\mathcal{L}=\frac{f^2}{8}\mathrm{str}(D^\mu\Sigma^\dagger D_\mu\Sigma)+\lambda_m\mathrm{str}(m_Q\Sigma+m_Q^\dagger\Sigma^\dagger)$$
$$+a\lambda_a\mathrm{str}(c_Q\Sigma+c_Q^\dagger\Sigma^\dagger)+\alpha\partial^\mu\Phi_0\partial_\mu\Phi_0-\mu_0^2\Phi_0^2, \tag{12}$$

where

$$\Sigma=\exp\left(\frac{2i\Phi}{f}\right)=\xi^2, \tag{13}$$

$$\Phi=\begin{pmatrix}M & \chi^\dagger\\ \chi & \tilde{M}\end{pmatrix}, \tag{14}$$

$f=132$ MeV, and the gauge-covariant derivative is $D_\mu\Sigma=\partial_\mu\Sigma+ie\mathcal{A}_\mu[\mathcal{Q},\Sigma]$. The str() denotes a graded flavor trace. The M, $\tilde{M}$, and χ are matrices of pseudo-Goldstone bosons and pseudo-Goldstone fermions; see, for example, [45]. Ex-

panding the Lagrangian in Eq. (12) one finds that to lowest order mesons with quark content $Q\overline{Q}'$ have mass

$$m_{QQ'}^2=\frac{4}{f^2}[\lambda_m(m_Q+m_{Q'})+a\lambda_a(c_Q+c_{Q'})]. \tag{15}$$

The flavor singlet field is $\Phi_0=\mathrm{str}(\Phi)/\sqrt{6}$. It is rendered heavy by the $U(1)_A$ anomaly and can be integrated out in PQχPT, with its mass μ_0 taken on the order of the chiral symmetry breaking scale, $\mu_0\to\Lambda_\chi$. In this limit the propagator of the flavor singlet field is independent of the coupling α and deviates from a simple pole form [12,14]. In QχPT, the singlet must be retained.

B. Baryons

In PQQCD there are baryons with quark composition $Q_iQ_jQ_k$ that can contain all three types of quarks. The spin-1/2 baryons are embedded in the **240**-dimensional super-multiplet $\mathcal{B}_{ijk}$, which contains the familiar octet baryons, while the spin-3/2 baryons are embedded in the **138**-dimensional super-multiplet $\mathcal{T}^\mu_{ijk}$, which contains the familiar decuplet baryons [6,45].

At leading order in the heavy baryon expansion and at $O(a)$, the free Lagrangian for the $\mathcal{B}_{ijk}$ and $\mathcal{T}^\mu_{ijk}$ fields is given by [6,33]

$$\begin{aligned}\mathcal{L}=&\, i(\overline{\mathcal{B}}v\cdot\mathcal{D}\mathcal{B})+2\alpha_M(\overline{\mathcal{B}}\mathcal{B}\mathcal{M}_+)+2\beta_M(\overline{\mathcal{B}}\mathcal{M}_+\mathcal{B})\\&+2\sigma_M(\overline{\mathcal{B}}\mathcal{B})\mathrm{str}(\mathcal{M}_+)+2\alpha_A(\overline{\mathcal{B}}\mathcal{B}\mathcal{A}_+)+2\beta_A(\overline{\mathcal{B}}\mathcal{A}_+\mathcal{B})\\&+2\sigma_A(\overline{\mathcal{B}}\mathcal{B})\mathrm{str}(\mathcal{A}_+)-i(\overline{\mathcal{T}}^\mu v\cdot\mathcal{D}\mathcal{T}_\mu)+\Delta(\overline{\mathcal{T}}^\mu\mathcal{T}_\mu)\\&+2\gamma_M(\overline{\mathcal{T}}^\mu\mathcal{M}_+\mathcal{T}_\mu)-2\overline{\sigma}_M(\overline{\mathcal{T}}^\mu\mathcal{T}_\mu)\mathrm{str}(\mathcal{M}_+)\\&+2\gamma_A(\overline{\mathcal{T}}^\mu\mathcal{A}_+\mathcal{T}_\mu)-2\overline{\sigma}_A(\overline{\mathcal{T}}^\mu\mathcal{T}_\mu)\mathrm{str}(\mathcal{A}_+),\end{aligned} \tag{16}$$

where $\mathcal{M}_+=\frac{1}{2}(\xi^\dagger m_Q\xi^\dagger+\xi m_Q\xi)$ and $\mathcal{A}_+=\frac{1}{2}a(\xi^\dagger c_Q\xi^\dagger+\xi c_Q\xi)$. Here Δ is the mass splitting between the **240** and **138** in the chiral limit. The parenthesis notation used in Eq. (16) is defined in [6] so that the contraction of flavor indices maintains proper transformations under chiral rotations. Notice that the presence of the chiral symmetry breaking SW operator in Eq. (3) has led to new $O(a)$ operators (and new constants α_A, β_A, σ_A, γ_A, and $\overline{\sigma}_A$) in Eq. (16). The Lagrangian describing the interactions of the $\mathcal{B}_{ijk}$ and $\mathcal{T}^\mu_{ijk}$ with the pseudo-Goldstone mesons is

$$\begin{aligned}\mathcal{L}=&\,2\alpha(\overline{\mathcal{B}}S^\mu\mathcal{B}A_\mu)+2\beta(\overline{\mathcal{B}}S^\mu A_\mu\mathcal{B})+2\mathcal{H}(\overline{\mathcal{T}}^\nu S^\mu A_\mu\mathcal{T}_\nu)\\&+\sqrt{\frac{3}{2}}\mathcal{C}[(\overline{\mathcal{T}}^\nu A_\nu\mathcal{B})+(\overline{\mathcal{B}}A_\nu\mathcal{T}^\nu)].\end{aligned} \tag{17}$$

The axial-vector and vector meson fields A^μ and V^μ are defined by $A^\mu=(i/2)(\xi\partial^\mu\xi^\dagger-\xi^\dagger\partial^\mu\xi)$ and $V^\mu=\frac{1}{2}(\xi\partial^\mu\xi^\dagger+\xi^\dagger\partial^\mu\xi)$. The latter appears in Eq. (16) for the covariant derivatives of $\mathcal{B}_{ijk}$ and $\mathcal{T}_{ijk}$, which both have the form

$$\begin{aligned}(\mathcal{D}^\mu\mathcal{B})_{ijk}=&\,\partial^\mu\mathcal{B}_{ijk}+(V^\mu)_{il}\mathcal{B}_{ljk}+(-)^{\eta_i(\eta_j+\eta_m)}(V^\mu)_{jm}\mathcal{B}_{imk}\\&+(-)^{(\eta_i+\eta_j)(\eta_k+\eta_n)}(V^\mu)_{kn}\mathcal{B}_{ijn}.\end{aligned} \tag{18}$$

The vector S^μ is the covariant spin operator [47–49]. The parameters $\mathcal{C}$ and $\mathcal{H}$ entering in Eq. (17) are identical to those in QCD, while the parameters α and β can be related to the familiar constants D and F of QCD:

$$\alpha=\frac{2}{3}D+2F \quad\text{and}\quad \beta=-\frac{5}{3}D+F. \tag{19}$$

In QQCD these identifications cannot be made.

III. OCTET MESON ELECTROMAGNETIC PROPERTIES

The electromagnetic form factor $G(q^2)$ of an octet meson ϕ has the form

$$\langle\phi(p')|J^\mu|\phi(p)\rangle=G(q^2)(p+p')^\mu, \tag{20}$$

where $q^\mu=(p'-p)^\mu$. At zero momentum transfer $G(0)=Q$, where Q is the charge of ϕ. The charge radius r is related to the slope of $G(q^2)$ at $q^2=0$, namely

$$\langle r^2\rangle=6\frac{d}{dq^2}G(q^2)|_{q^2=0}. \tag{21}$$

Recall, at one-loop order in the chiral expansion the charge radii are $O(\epsilon^2)$.

There are two finite-a terms in the $O(\epsilon^4)$ Lagrangian [31]

$$\begin{aligned}\mathcal{L}=&\,\alpha_{A,4}\frac{8a\lambda_a}{f^2}\,\mathrm{str}(D_\mu\Sigma^\dagger D^\mu\Sigma)\,\mathrm{str}(c_Q\Sigma+c_Q^\dagger\Sigma^\dagger)\\&+\alpha_{A,5}\frac{8a\lambda_a}{f^2}\,\mathrm{str}(D_\mu\Sigma^\dagger D^\mu\Sigma(c_Q\Sigma+c_Q^\dagger\Sigma^\dagger))\end{aligned} \tag{22}$$

that contribute to meson form factors at the tree level. The new parameters $\alpha_{A,4}$ and $\alpha_{A,5}$ in Eq. (22) are finite lattice spacing analogues of the dimensionless Gasser-Leutwyler coefficients α_4 and α_5 of χPT [50]. The above terms contribute to meson form factors at $O(\epsilon^2)$ but their contributions are independent of q^2 and annihilated by the corresponding wave function renormalization, thus ensuring charge non-renormalization.

The SW term can potentially contribute at $O(\epsilon^2)$ when $\mathcal{A}_+$ is inserted into the kinetic term of the leading-order $\mathcal{L}$ in Eq. (12). Contributions to form factors from such terms vanish by charge non-renormalization. Insertions of $\mathcal{A}_+$ into the α_9 term of the Gasser-Leutwyler Lagrangian produces the $O(\epsilon^6)$ terms

$$\mathcal{L}=im_1\Lambda_\chi F_{\mu\nu}\,\mathrm{str}(\{\mathcal{Q}_+,\mathcal{A}_+\}D^\mu\Sigma D^\nu\Sigma^\dagger+\{\mathcal{Q}_+,\mathcal{A}_+\}D^\mu\Sigma^\dagger D^\nu\Sigma)$$
$$+im_2\Lambda_\chi F_{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+D^\mu\Sigma\mathcal{A}_+D^\nu\Sigma^\dagger+\mathcal{Q}_+D^\mu\Sigma^\dagger\mathcal{A}_+D^\nu\Sigma)$$
$$+im_3\Lambda_\chi F_{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+D^\mu\Sigma D^\nu\Sigma^\dagger+\mathcal{Q}_+D^\mu\Sigma^\dagger D^\nu\Sigma)\mathrm{str}(\mathcal{A}_+), \tag{23}$$

where we have defined $\mathcal{Q}_+=\frac{1}{2}(\xi^\dagger\mathcal{Q}\xi^\dagger+\xi\mathcal{Q}\xi)$. These terms contribute at $O(\epsilon^4)$ to the charge radii and can be ignored (see Appendix B for a discussion relating to larger lattice spacings).

Additionally we must consider the contribution from the vector-current correction operator $\mathcal{O}_1^\mu$ in Eq. (10). In the meson sector, the leading operators $\mathcal{O}_1^\mu$ in the effective theory can be ascertained by inserting $a\Lambda_\chi c_{1,\mathcal{Q}}$ in place of $\mathcal{Q}$ in the operators that contribute to form factors. The effective field theory operators must also preserve the charge of the meson ϕ. It is easiest to embed the operators $\mathcal{O}_1^\mu$ in a Lagrangian so that electromagnetic gauge invariance is manifest. To leading order, the contribution from $\mathcal{O}_1^\mu$ is contained in the term

$$\mathcal{L}=i\alpha_{A,9}a\Lambda_\chi F_{\mu\nu}\mathrm{str}(c_{1,\mathcal{Q}}\partial^\mu\Sigma\partial^\nu\Sigma^\dagger+c_{1,\mathcal{Q}}\partial^\mu\Sigma^\dagger\partial^\nu\Sigma). \tag{24}$$

Thus the correction to meson form factors from $\mathcal{O}_1^\mu$ is at $O(\epsilon^4)$.

The charge radius of the meson ϕ to $O(\epsilon^2)$ then reads

$$\langle r^2\rangle=\alpha_9\frac{24Q}{f^2}+\frac{1}{16\pi^2f^2}\sum_X A_X\log\frac{m_X^2}{\mu^2}, \tag{25}$$

where X corresponds to loop mesons having mass m_X [the masses implicitly include the finite lattice spacing corrections given in Eq. (15); otherwise the expression is identical to the $a=0$ result]. The coefficients A_X in PQχPT appear in Ref. [17]. In the case of QχPT, the coefficients $A_X=0$ for all loop mesons and there are no additional contributions from the singlet field at this order. Thus there is neither quark mass dependence nor lattice spacing dependence in the quenched meson charge radii at this order.

IV. OCTET BARYON ELECTROMAGNETIC PROPERTIES

Baryon matrix elements of the electromagnetic current J^μ can be parametrized in terms of the Dirac and Pauli form factors F_1 and F_2, respectively, as

$$\langle\overline{B}(p')|J^\mu|B(p)\rangle=\overline{u}(p')\left\{v^\mu F_1(q^2)+\frac{[S^\mu,S^\nu]}{M_B}q_\nu F_2(q^2)\right\}u(p) \tag{26}$$

with $q=p'-p$ and M_B is the degenerate octet baryon mass. The Dirac form factor is normalized at zero momentum transfer to the baryon charge: $F_1(0)=Q$. The electric charge radius $\langle r_E^2\rangle$ and magnetic moment μ can be defined in terms of these form factors by

$$\langle r_E^2\rangle=6\frac{dF_1(0)}{dq^2}+\frac{3}{2M_B^2}F_2(0) \tag{27}$$

and

$$\mu=F_2(0). \tag{28}$$

Recall, that the one-loop contributions in the chiral expansion to the charge radii are $O(\epsilon^2)$, while those to the magnetic moments are $O(\epsilon)$.

There are no finite-a operators in Eq. (16) that contribute to octet baryon form factors. As in the meson sector, however, the SW term could contribute when $\mathcal{A}_+$ is inserted into the Lagrangian. Here and henceforth we do not consider these insertions into the kinetic terms in Eq. (16) because their contributions alter the baryon charges and will be canceled by the appropriate wave function renormalization.

The SW term, however, does contribute when $\mathcal{A}_+$ is inserted into the charge radius and magnetic moment terms. For the charge radius, we then have the $O(a)$ terms

$$\mathcal{L}=\frac{b_1}{\Lambda_\chi}(-)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})}\overline{\mathcal{B}}^{kji}\{\mathcal{Q}_+,\mathcal{A}_+\}^{kk'}\mathcal{B}^{ijk'}v_\mu\partial_\nu F^{\mu\nu}+\frac{b_2}{\Lambda_\chi}\overline{\mathcal{B}}^{kji}\{\mathcal{Q}_+,\mathcal{A}_+\}^{ii'}\mathcal{B}^{i'jk}v_\mu\partial_\nu F^{\mu\nu}$$
$$+\frac{b_3}{\Lambda_\chi}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{B}^{i'j'k}v_\mu\partial_\nu F^{\mu\nu}+\frac{b_4}{\Lambda_\chi}(-)^{\eta_i(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}\mathcal{Q}_+^{jj'}\mathcal{A}_+^{ii'}\mathcal{B}^{i'j'k}v_\mu\partial_\nu F^{\mu\nu}$$
$$+\frac{b_5}{\Lambda_\chi}(-)^{\eta_j\eta_{j'}+1}\overline{\mathcal{B}}^{kji}(\mathcal{Q}_+^{ij'}\mathcal{A}_+^{ji'}+\mathcal{A}_+^{ij'}\mathcal{Q}_+^{ji'})\mathcal{B}^{i'j'k}v_\mu\partial_\nu F^{\mu\nu}+\frac{1}{\Lambda_\chi}[b_6(\overline{\mathcal{B}}\mathcal{B}\mathcal{Q}_+)+b_7(\overline{\mathcal{B}}\mathcal{Q}_+\mathcal{B})]v_\mu\partial_\nu F^{\mu\nu}\mathrm{str}(\mathcal{A}_+)$$
$$+\frac{b_8}{\Lambda_\chi}(\overline{\mathcal{B}}\mathcal{B})v_\mu\partial_\nu F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+\mathcal{A}_+), \tag{29}$$

which contribute at $O(\epsilon^4)$ to the charge radii and are thus neglected. Insertions of $\mathcal{A}_+$ into the magnetic moment terms produce

$$\begin{aligned}\mathcal{L}=&\, ib_1'(-)^{(\eta_i+\eta_j)(\eta_k+\eta_{k'})}\overline{\mathcal{B}}^{kji}[S_\mu,S_\nu]\{\mathcal{Q}_+,\mathcal{A}_+\}^{kk'}\mathcal{B}^{ijk'}F^{\mu\nu}+ib_2'\overline{\mathcal{B}}^{kji}[S_\mu,S_\nu]\{\mathcal{Q}_+,\mathcal{A}_+\}^{ii'}\mathcal{B}^{i'jk}F^{\mu\nu}\\&+ib_3'(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}[S_\mu,S_\nu]\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{B}^{i'j'k}F^{\mu\nu}+ib_4'(-)^{\eta_i(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}[S_\mu,S_\nu]\mathcal{Q}_+^{jj'}\mathcal{A}_+^{ii'}\mathcal{B}^{i'j'k}F^{\mu\nu}\\&+ib_5'(-)^{\eta_j\eta_{j'}+1}\overline{\mathcal{B}}^{kji}[S_\mu,S_\nu](\mathcal{Q}_+^{ij'}\mathcal{A}_+^{ji'}+\mathcal{A}_+^{ij'}\mathcal{Q}_+^{ji'})\mathcal{B}^{i'j'k}F^{\mu\nu}+i[b_6'(\overline{\mathcal{B}}[S_\mu,S_\nu]\mathcal{B}\mathcal{Q}_+)\\&+b_7'(\overline{\mathcal{B}}[S_\mu,S_\nu]\mathcal{Q}_+\mathcal{B})]F^{\mu\nu}\mathrm{str}(\mathcal{A}_+)+ib_8'(\overline{\mathcal{B}}[S_\mu,S_\nu]\mathcal{B})F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+\mathcal{A}_+),\end{aligned}\tag{30}$$

which are $O(\epsilon^2)$ corrections to the magnetic moments and can be discarded [33].

Finally we assess the contribution from the operator $\mathcal{O}_1^\mu$ in Eq. (10). As in the meson sector, the charge preserving operators can be constructed by the replacement $\mathcal{Q}\to a\Lambda_\chi c_{1,\mathcal{Q}}$ in leading-order terms. Again it is easier to embed these operators in $\mathcal{L}$ so that gauge invariance is transparent. For the charge radius, the leading vector-current correction operator is contained in the term

$$\mathcal{L}=\frac{a}{\Lambda_\chi}[c_{A,\alpha}(\overline{\mathcal{B}}\mathcal{B}c_{1,\mathcal{Q}})+c_{A,\beta}(\overline{\mathcal{B}}c_{1,\mathcal{Q}}\mathcal{B})]v_\mu\partial_\nu F^{\mu\nu},\tag{31}$$

which leads to $O(\epsilon^4)$ corrections. For the magnetic moment operator, such a replacement leads to

$$\mathcal{L}=\frac{ia}{2}[\mu_{A,\alpha}(\overline{\mathcal{B}}[S_\mu,S_\nu]\mathcal{B}c_{1,\mathcal{Q}})+\mu_{A,\beta}(\overline{\mathcal{B}}[S_\mu,S_\nu]c_{1,\mathcal{Q}}\mathcal{B})]F^{\mu\nu}\tag{32}$$

and corrections that are of higher order than the one-loop results [33]. See Appendix B for results in an alternate power counting scheme.

To $O(\epsilon^2)$ the baryon charge radii are thus

$$\begin{aligned}\langle r_E^2\rangle=&-\frac{6}{\Lambda_\chi^2}(Qc_-+\alpha_Dc_+)+\frac{3}{2M_B^2}(Q\mu_F+\alpha_D\mu_D)\\&-\frac{1}{16\pi^2f^2}\sum_X\left[A_X\log\frac{m_X^2}{\mu^2}-5\beta_X\log\frac{m_X^2}{\mu^2}\right.\\&\left.+10\beta_X'\mathcal{G}(m_X,\Delta,\mu)\right]\end{aligned}\tag{33}$$

and the magnetic moments to $O(\epsilon)$ read

$$\mu=(Q\mu_F+\alpha_D\mu_D)+\frac{M_B}{4\pi f^2}\sum_X[\beta_Xm_X+\beta_X'\mathcal{F}(m_X,\Delta,\mu)].\tag{34}$$

The a dependence is treated as implicit in the meson masses. The PQχPT coefficients A_X, β_X, and β_X' can be found in [17,45] along with the functions $\mathcal{F}(m_X,\Delta,\mu)$ and $\mathcal{G}(m_X,\Delta,\mu)$. The quenched charge radii at $O(\epsilon^2)$ are similar in form (although $A_X=0$ in QχPT) due to the lack of singlet contributions at this order. The QχPT coefficients β_X^Q and $\beta_X^{Q'}$ appear in [17,22]. The quenched magnetic moments, however, receive additional contributions from singlet loops. The relevant formulas of [22] are not duplicated here in the interests of space but only need trivial modification by taking into account the a dependence of meson masses.

V. DECUPLET BARYON ELECTROMAGNETIC PROPERTIES

Decuplet matrix elements of the electromagnetic current J^ρ can be parametrized as [18]

$$\langle\overline{T}(p')|J^\rho|T(p)\rangle=-\overline{u}_\mu(p')\mathcal{O}^{\mu\rho\nu}u_\nu(p),\tag{35}$$

where $u_\mu(p)$ is a Rarita-Schwinger spinor for an on-shell heavy baryon. The tensor $\mathcal{O}^{\mu\rho\nu}$ can be parametrized in terms of four independent, Lorentz invariant form factors:

$$\begin{aligned}\mathcal{O}^{\mu\rho\nu}=&\,g^{\mu\nu}\left\{v^\rho F_1(q^2)+\frac{[S^\rho,S^\tau]}{M_B}q_\tau F_2(q^2)\right\}\\&+\frac{q^\mu q^\nu}{(2M_B)^2}\left\{v^\rho G_1(q^2)+\frac{[S^\rho,S^\tau]}{M_B}q_\tau G_2(q^2)\right\},\end{aligned}\tag{36}$$

where the momentum transfer $q=p'-p$. The form factor $F_1(q^2)$ is normalized to the decuplet charge in units of e such that $F_1(0)=Q$.

The conversion from the covariant vertex functions used above to multipole form factors for spin-3/2 particles is explicated in [51]. For our calculations, the charge radius

$$\langle r_E^2\rangle=6\left\{\frac{dF_1(0)}{dq^2}-\frac{1}{12M_B^2}[2Q-3F_2(0)-G_1(0)]\right\},\tag{37}$$

the magnetic moment

$$\mu=F_2(0),\tag{38}$$

and the electric quadrupole moment

$$\mathbb{Q}=-\frac{1}{2}G_1(0).\tag{39}$$

The charge radii are $O(\epsilon^2)$ at next-to-leading order (NLO) in the chiral expansion, while the magnetic moments are $O(\epsilon)$ and the electric quadrupole moments are $O(\epsilon^0)$. At one-loop order in the chiral expansion, the magnetic octupole moment is zero.

There are no finite-a operators in Eq. (16) that contribute to decuplet baryon form factors. The SW term can potentially contribute when $\mathcal{A}_+$ is inserted into the Lagrangian. There are three such terms: the charge radius, magnetic moment, and electric quadrupole terms. Insertions of $\mathcal{A}_+$ into the charge radius term produces

$$\begin{aligned}\mathcal{L}=&\frac{d_1}{\Lambda_\chi}\overline{\mathcal{T}}^{\sigma,kji}\{\mathcal{Q}_+,\mathcal{A}_+\}^{ii'}\mathcal{T}_\sigma^{i'jk}v_\mu\partial_\nu F^{\mu\nu}\\&+\frac{d_2}{\Lambda_\chi}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{T}}^{\sigma,kji}\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{T}_\sigma^{i'j'k}v_\mu\partial_\nu F^{\mu\nu}\\&+\frac{d_3}{\Lambda_\chi}(\overline{\mathcal{T}}^\sigma\mathcal{Q}_+\mathcal{T}_\sigma)v_\mu\partial_\nu F^{\mu\nu}\mathrm{str}(\mathcal{A}_+)\\&+\frac{d_4}{\Lambda_\chi}(\overline{\mathcal{T}}^\sigma\mathcal{T}_\sigma)v_\mu\partial_\nu F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+\mathcal{A}_+).\end{aligned}\tag{40}$$

These contribute to decuplet charge radii at $O(\epsilon^4)$. As in the octet sector, insertions of $\mathcal{A}_+$ into the magnetic moment term, namely

$$\begin{aligned}\mathcal{L}=&id_1'\overline{\mathcal{T}}_\mu^{kji}\{\mathcal{Q}_+,\mathcal{A}_+\}^{ii'}\mathcal{T}_\nu^{i'jk}F^{\mu\nu}\\&+id_2'(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{T}}_\mu^{kji}\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{T}_\nu^{i'j'k}F^{\mu\nu}\\&+id_3'(\overline{\mathcal{T}}_\mu\mathcal{Q}_+\mathcal{T}_\nu)F^{\mu\nu}\mathrm{str}(\mathcal{A}_+)\\&+id_4'(\overline{\mathcal{T}}_\mu\mathcal{T}_\nu)F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+\mathcal{A}_+),\end{aligned}\tag{41}$$

produce $O(\epsilon^2)$ corrections. Likewise, insertions of $\mathcal{A}_+$ into the electric quadrupole term have the form[1]

$$\begin{aligned}\mathcal{L}=&\frac{d_1''}{\Lambda_\chi}\overline{\mathcal{T}}^{\{\mu,kji}\{\mathcal{Q}_+,\mathcal{A}_+\}^{ii'}\mathcal{T}^{\nu\},i'jk}v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{d_2''}{\Lambda_\chi}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{T}}^{\{\mu,kji}\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{T}^{\nu\},i'j'k}v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{d_3''}{\Lambda_\chi}(\overline{\mathcal{T}}^{\{\mu}\mathcal{Q}_+\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\,\mathrm{str}(\mathcal{A}_+)\\&+\frac{d_4''}{\Lambda_\chi}(\overline{\mathcal{T}}^{\{\mu}\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\,\mathrm{str}(\mathcal{Q}_+\mathcal{A}_+),\end{aligned}\tag{42}$$

and produce $O(\epsilon^2)$ corrections. All of these corrections are of higher order than the one-loop results.

Finally we assess the contribution from the operator $\mathcal{O}_1^\mu$ in Eq. (10). The effective operators can be constructed by replacing $\mathcal{Q}$ by $a\Lambda_\chi c_{1,\mathcal{Q}}$ in leading-order terms. Embedding these terms in a Lagrangian, we have

$$\begin{aligned}\mathcal{L}=&\frac{3ac_{A,c}}{\Lambda_\chi}(\overline{\mathcal{T}}^\sigma c_{1,\mathcal{Q}}\mathcal{T}_\sigma)v_\mu\partial_\nu F^{\mu\nu}+3ia\mu_{A,c}(\overline{\mathcal{T}}_\mu c_{1,\mathcal{Q}}\mathcal{T}_\nu)F^{\mu\nu}\\&-\frac{3a\mathbb{Q}_{A,c}}{\Lambda_\chi}(\overline{\mathcal{T}}^{\{\mu}c_{1,\mathcal{Q}}\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}.\end{aligned}\tag{43}$$

Each of these terms leads to corrections of higher order than the one-loop results and can be dropped. Thus at this order the only finite lattice spacing corrections to decuplet electromagnetic properties appear in the meson masses. For reference, the expressions are

$$\begin{aligned}\langle r_E^2\rangle=&Q\left(\frac{2\mu_c-1}{M_B^2}+\frac{\mathbb{Q}_c+6c_c}{\Lambda_\chi^2}\right)-\frac{1}{3}\frac{9+5\mathcal{C}^2}{16\pi^2f^2}\sum_X A_X\log\frac{m_X^2}{\mu^2}\\&-\frac{25}{27}\frac{\mathcal{H}^2}{16\pi^2f^2}\sum_X A_X\mathcal{G}(m_X,\Delta,\mu),\end{aligned}\tag{44}$$

$$\begin{aligned}\mu=&2\mu_cQ-\frac{M_B\mathcal{H}^2}{36\pi f^2}\sum_X A_X\mathcal{F}(m_X,\Delta,\mu)\\&-\frac{\mathcal{C}^2M_B}{8\pi f^2}\sum_X A_Xm_X,\end{aligned}\tag{45}$$

and

$$\begin{aligned}\mathbb{Q}=&-2Q\left(\mu_c+\mathbb{Q}_c\frac{2M_B^2}{\Lambda_\chi^2}\right)+\frac{M_B^2\mathcal{C}^2}{24\pi^2f^2}\sum_X A_X\log\frac{m_X^2}{\mu^2}\\&-\frac{M_B^2\mathcal{H}^2}{54\pi^2f^2}\sum_X A_X\mathcal{G}(m_X,\Delta,\mu).\end{aligned}\tag{46}$$

The coefficients A_X are tabulated in [18]. Extending the result to QχPT, where $A_X=0$, one must include additional contributions from singlet loops. With finite lattice spacing corrections, the expressions are identical to those in [18] except with masses given by Eq. (15). Thus for brevity we do not reproduce them here.

VI. DECUPLET TO OCTET ELECTROMAGNETIC TRANSITIONS

The decuplet to octet matrix elements of the electromagnetic current J^μ appear as [19]

$$\langle\overline{B}(p)|J^\mu|T(p')\rangle=\overline{u}(p)\mathcal{O}^{\mu\beta}u_\beta(p'),\tag{47}$$

where the tensor $\mathcal{O}^{\mu\beta}$ can be parametrized in terms of three independent, Lorentz invariant form factors:

$$\begin{aligned}\mathcal{O}^{\mu\beta}=&(q\cdot Sg^{\mu\beta}-S^\mu q^\beta)\frac{G_1(q^2)}{M_B}+(q\cdot vg^{\mu\beta}\\&-v^\mu q^\beta)q\cdot S\frac{G_2(q^2)}{(2M_B)^2}+(q^2g^{\mu\beta}-q^\mu q^\beta)S\cdot q\frac{G_3(q^2)}{4M_B^2\Delta},\end{aligned}\tag{48}$$

where the photon momentum $q=p'-p$. At next-to-leading order in the chiral expansion, we recall that $G_1(q^2)$ is $O(\epsilon)$ while $G_2(q^2)$ and $G_3(q^2)$ are $O(\epsilon^0)$. The conversion of

[1]The action of $\{\cdots\}$ on Lorentz indices produces the symmetric traceless part of the tensor, viz. $\mathcal{O}^{\{\mu\nu\}}=\mathcal{O}^{\mu\nu}+\mathcal{O}^{\nu\mu}-\frac{1}{2}g^{\mu\nu}\mathcal{O}^\alpha{}_\alpha$.

these vertex covariant functions to multipole form factors is detailed in [52]. The multipole moments up to $O(\epsilon)$ are[2]

$$G_{M1}(0)=\left(\frac{2}{3}-\frac{\Delta}{6M_B}\right)G_1(0)+\frac{\Delta}{12M_B}G_2(0), \tag{49}$$

$$G_{E2}(0)=\frac{\Delta}{6M_B}G_1(0)+\frac{\Delta}{12M_B}G_2(0), \tag{50}$$

$$G_{C2}(0)=\left(\frac{1}{3}+\frac{\Delta}{6M_B}\right)G_1(0)+\left(\frac{1}{6}+\frac{\Delta}{6M_B}\right)G_2(0)+\frac{1}{6}G_3(0). \tag{51}$$

There are no new finite-a operators in Eq. (16) that contribute to decuplet to octet transition form factors. Insertion of $\mathcal{A}_+$ into leading-order transition terms leads to corrections of $O(\epsilon^2)$ or smaller. For completeness the terms are

$$\begin{aligned}\mathcal{L}=&\, it_1\overline{\mathcal{B}}^{kji}S_\mu\mathcal{Q}_+^{il}\mathcal{A}_+^{li'}\mathcal{T}_\nu^{i'jk}F^{\mu\nu}+it_2\overline{\mathcal{B}}^{kji}S_\mu\mathcal{A}_+^{il}\mathcal{Q}_+^{li'}\mathcal{T}_\nu^{i'jk}F^{\mu\nu}\\&+it_3(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}S_\mu\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{T}_\nu^{i'j'k}F^{\mu\nu}\\&+it_4(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}S_\mu\mathcal{A}_+^{ii'}\mathcal{Q}_+^{jj'}\mathcal{T}_\nu^{i'j'k}F^{\mu\nu}\\&+it_5(\overline{\mathcal{B}}S_\mu\mathcal{Q}_+\mathcal{T}_\nu)F^{\mu\nu}\mathrm{str}(\mathcal{A}_+),\end{aligned} \tag{52}$$

for the magnetic dipole transition, and

$$\begin{aligned}\mathcal{L}=&\,\frac{t_1'}{\Lambda_\chi}\overline{\mathcal{B}}^{kji}S^{\{\mu}\mathcal{Q}_+^{il}\mathcal{A}_+^{li'}\mathcal{T}^{\nu\},i'jk}v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{t_2'}{\Lambda_\chi}\overline{\mathcal{B}}^{kji}S^{\{\mu}\mathcal{A}_+^{il}\mathcal{Q}_+^{li'}\mathcal{T}^{\nu\},i'jk}v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{t_3'}{\Lambda_\chi}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}S^{\{\mu}\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{T}^{\nu\},i'j'k}v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{t_4'}{\Lambda_\chi}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}S^{\{\mu}\mathcal{A}_+^{ii'}\mathcal{Q}_+^{jj'}\mathcal{T}^{\nu\},i'j'k}v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{t_5'}{\Lambda_\chi}(\overline{\mathcal{B}}S^{\{\mu}\mathcal{Q}_+\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\,\mathrm{str}(\mathcal{A}_+),\end{aligned} \tag{53}$$

for the quadrupole transition. Finally, insertion of $\mathcal{A}_+$ into the PQχPT term proportional to $i(\overline{\mathcal{B}}S^\mu\mathcal{Q}\mathcal{T}^\nu)\partial^\alpha\partial_\mu F_{\nu\alpha}$ leads to

$$\begin{aligned}\mathcal{L}=&\,\frac{it_1''}{\Lambda_\chi^2}\overline{\mathcal{B}}^{kji}S_\mu\mathcal{Q}_+^{il}\mathcal{A}_+^{li'}\mathcal{T}_\nu^{i'jk}\partial^\alpha\partial^\mu F^\nu{}_\alpha\\&+\frac{it_2''}{\Lambda_\chi^2}\overline{\mathcal{B}}^{kji}S_\mu\mathcal{A}_+^{il}\mathcal{Q}_+^{li'}\mathcal{T}_\nu^{i'jk}\partial^\alpha\partial^\mu F^\nu{}_\alpha\\&+\frac{it_3''}{\Lambda_\chi^2}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}S_\mu\mathcal{Q}_+^{ii'}\mathcal{A}_+^{jj'}\mathcal{T}_\nu^{i'j'k}\partial^\alpha\partial^\mu F^\nu{}_\alpha\\&+\frac{it_4''}{\Lambda_\chi^2}(-)^{\eta_{i'}(\eta_j+\eta_{j'})}\overline{\mathcal{B}}^{kji}S_\mu\mathcal{A}_+^{ii'}\mathcal{Q}_+^{jj'}\mathcal{T}_\nu^{i'j'k}\partial^\alpha\partial^\mu F^\nu{}_\alpha\\&+\frac{it_5''}{\Lambda_\chi^2}(\overline{\mathcal{B}}S_\mu\mathcal{Q}_+\mathcal{T}_\nu)\partial^\alpha\partial^\mu F^\nu{}_\alpha\,\mathrm{str}(\mathcal{A}_+),\end{aligned} \tag{54}$$

for the Coulomb quadrupole transition.

Similarly, constructing $\mathcal{O}_1^\mu$ in the effective theory by replacing $\mathcal{Q}$ with $a\Lambda_\chi c_{1,\mathcal{Q}}$ in the transition operators leads to terms of at least $O(\epsilon^2)$ which are contained in the terms

$$\begin{aligned}\mathcal{L}=&\,ia\mu_{A,T}\sqrt{\frac{3}{8}}(\overline{\mathcal{B}}S_\mu c_{1,\mathcal{Q}}\mathcal{T}_\nu)F^{\mu\nu}\\&+\frac{a\mathbb{Q}_{A,T}}{\Lambda_\chi}\sqrt{\frac{3}{2}}(\overline{\mathcal{B}}S^{\{\mu}c_{1,\mathcal{Q}}\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\\&+\frac{iac_{A,T}}{\Lambda_\chi^2}\sqrt{\frac{3}{2}}(\overline{\mathcal{B}}S^\mu c_{1,\mathcal{Q}}\mathcal{T}^\nu)\partial^\alpha\partial_\mu F_{\nu\alpha}.\end{aligned} \tag{55}$$

All of these corrections from effective $\mathcal{O}_1^\mu$ operators are of higher order than the one-loop results. Thus, at this order, the only finite lattice spacing corrections to the transition moments appear in the meson masses. For reference the expressions are

$$\begin{aligned}G_1(0)=&\,\frac{\mu_T}{2}\alpha_T\\&-4\pi\mathcal{H}\mathcal{C}\frac{M_B}{\Lambda_\chi^2}\sum_X\beta_X^T\int_0^1dx\left(1-\frac{x}{3}\right)\mathcal{F}(m_X,x\Delta,\mu)\\&+4\pi\mathcal{C}(D-F)\frac{M_B}{\Lambda_\chi^2}\sum_X\beta_X^B\int_0^1dx(1-x)\mathcal{F}(m_X,-x\Delta,\mu),\end{aligned} \tag{56}$$

$$\begin{aligned}G_2(0)=&\,\frac{M_B^2}{\Lambda_\chi^2}\Bigg\{-4\mathbb{Q}_T\alpha_T\\&+16\mathcal{H}\mathcal{C}\sum_X\beta_X^T\int_0^1dx\frac{x(1-x)}{3}\mathcal{G}(m_X,x\Delta,\mu)\\&-16\mathcal{C}(D-F)\sum_X\beta_X^B\int_0^1dxx(1-x)\mathcal{G}(m_X,-x\Delta,\mu)\Bigg\},\end{aligned} \tag{57}$$

[2]Here, we count $\epsilon\sim\Delta/M_B$.

and

$$G_3(0) = -16\frac{M_B^2}{\Lambda_\chi^2}\sum_X \int_0^1 dx x(1-x)\left(x-\frac{1}{2}\right)\frac{\Delta m_X}{m_X^2 - x^2\Delta^2} \times\left[\frac{1}{3}\mathcal{H}\mathcal{C}\beta_X^T\mathcal{R}\left(\frac{x\Delta}{m_X}\right) + \mathcal{C}(D-F)\beta_X^B\mathcal{R}\left(-\frac{x\Delta}{m_X}\right)\right]. \tag{58}$$

The coefficients β_B and β_T are tabulated in [19] along with the function $\mathcal{R}(x)$. Extending the result to QχPT, where the coefficients are replaced with their quenched counterparts β_B^Q and β_T^Q, one must include additional contributions from singlet loops. With finite lattice spacing corrections the expressions are identical to those in [19] except with masses given by Eq. (15). Thus for brevity we do not reproduce them here.

VII. CONCLUSIONS

Above we have calculated the finite lattice spacing corrections to hadronic electromagnetic observables in both QχPT and PQχPT for the $SU(3)$ flavor group in the isospin limit and the $SU(2)$ group with nondegenerate quarks. In the power counting scheme of [31,33], $O(a)$ corrections contribute to electromagnetic observables at higher order than the one-loop chiral corrections. Thus finite lattice spacing manifests itself only in the meson masses at this order.

In practice one should not adhere rigidly to a particular power-counting scheme. Each observable should be treated on a case by case basis. The actual size of a and additionally the size of counterterms are needed to address the relevance of $O(a)$ corrections for real lattice data. For this reason we have presented an exhaustive list of $O(a)$ operators relevant for hadronic electromagnetic properties. In an alternate power counting for a coarser lattice (as explained in Appendix B), some of the operators listed above contribute at the same order as the one-loop results in the chiral expansion.

The corrections detailed in Appendix B in the baryon sector may also be necessary if one goes beyond the heavy baryon limit (that is, including $1/M_B$ corrections). For example, in the case of the octet baryon magnetic moment [see Eq. (34)] at NLO in the heavy baryon expansion μ would be known to $O(\epsilon^2)$. Thus $O(a)$ corrections in the power counting of Eq. (2) are needed since they are also $O(\epsilon^2)$.

Knowledge of the low-energy behavior of PQQCD at finite lattice spacing is crucial to extrapolate lattice calculations from the quark masses used on a finite lattice to the physical world. The formal behavior of the PQQCD electromagnetic observables in the chiral limit has the same form as in QCD. Moreover, there is a well-defined connection to QCD and one can reliably extrapolate lattice results down to the quark masses of reality. For simulations using unimproved lattice actions (with Wilson quarks or mixed quarks), our results will aid in the continuum extrapolation and will help lattice simulations make contact with real-world data.

ACKNOWLEDGMENTS

We thank Gautam Rupak and Ruth Van de Water for helpful discussions, and Martin Savage and Steve Sharpe for critical comments on the manuscript. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-97ER4014.

APPENDIX A: HADRONIC ELECTROMAGNETIC PROPERTIES IN $SU(2)$ FLAVOR WITH NONDEGENERATE QUARKS AT FINITE LATTICE SPACING

In this appendix, we consider the case of $SU(2)$ flavor PQQCD[3] and summarize the changes needed to determine finite lattice spacing corrections to the electromagnetic properties of hadrons considered above. For the two flavor case, we keep the up and down valence quark masses nondegenerate and similarly for the sea quarks. Thus the quark mass matrix reads

$$m_Q^{SU(2)} = \mathrm{diag}(m_u, m_d, m_j, m_l, m_u, m_d), \tag{A1}$$

while the SW matrix is

$$c_Q^{SU(2)} = \mathrm{diag}(c^v, c^v, c^s, c^s, c^v, c^v). \tag{A2}$$

Defining ghost and sea quark charges is constrained only by the restriction that QCD be recovered in the limit of appropriately degenerate quark masses. Thus the most general form of the charge matrix is

$$\mathcal{Q}^{SU(2)} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, q_j, q_l, q_j, q_l\right), \tag{A3}$$

which is not supertraceless. Analogous to the three flavor case, the vector current will receive $O(a)$ corrections from the operators in Eq. (10) of which only the operator $\mathcal{O}_1^\mu$ is relevant. The coefficient matrix associated with this operator is

$$c_1^{SU(2)} = \mathrm{diag}(c_1^v, c_1^v, c_1^s, c_1^s, c_1^v, c_1^v). \tag{A4}$$

The $O(a)$ operators listed above in Secs. III–VI are the same for the $SU(2)$ flavor group; however, the coefficients have different numerical values. Additionally there are operators involving str($\mathcal{Q}_+^{SU(2)}$). These are listed for each electromagnetic observable below.

1. Octet mesons

In the meson sector, one has the additional term

$$\mathcal{L} = i m_4 \Lambda_\chi F_{\mu\nu}\,\mathrm{str}(\mathcal{A}_+ D^\mu \Sigma D^\nu \Sigma^\dagger + \mathcal{A}_+ D^\mu \Sigma^\dagger D^\nu \Sigma)\mathrm{str}(\mathcal{Q}_+^{SU(2)}). \tag{A5}$$

[3]For brevity we refer to $SU(4|2)$ PQQCD as $SU(2)$. The distinction will always be clear.

2. Octet baryons

In the octet baryon sector, there are terms which originate from $\mathcal{A}_+$ insertions,

$$\begin{aligned}\mathcal{L}=&\frac{1}{\Lambda_\chi}[b_9(\bar{\mathcal{B}}\mathcal{B}\mathcal{A}_+)+b_{10}(\bar{\mathcal{B}}\mathcal{A}_+\mathcal{B})]v_\mu\partial_\nu F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+\frac{b_{11}}{\Lambda_\chi}(\bar{\mathcal{B}}\mathcal{B})v_\mu\partial_\nu F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\mathrm{str}(\mathcal{A}_+)\\&+i[b_9'(\bar{\mathcal{B}}[S_\mu,S_\nu]\mathcal{B}\mathcal{A}_+)\\&+b_{10}'(\bar{\mathcal{B}}[S_\mu,S_\nu]\mathcal{A}_+\mathcal{B})]F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+ib_{11}'(\bar{\mathcal{B}}[S_\mu,S_\nu]\mathcal{B})F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\mathrm{str}(\mathcal{A}_+),\end{aligned}\tag{A6}$$

and additional vector-current correction operators

$$\begin{aligned}\mathcal{L}=&\frac{ac_{A,\gamma}}{\Lambda_\chi}(\bar{\mathcal{B}}\mathcal{B})v_\mu\partial_\nu F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}^{SU(2)}c_1^{SU(2)})\\&+\frac{ia\mu_{A,\gamma}}{2}(\bar{\mathcal{B}}[S_\mu,S_\nu]\mathcal{B})F^{\mu\nu}\mathrm{str}(\mathcal{Q}^{SU(2)}c_1^{SU(2)}).\end{aligned}\tag{A7}$$

3. Decuplet baryons

Next in the decuplet sector there are terms that result from $\mathcal{A}_+$ insertions,

$$\begin{aligned}\mathcal{L}=&\frac{d_5}{\Lambda_\chi}(\bar{\mathcal{T}}^\sigma\mathcal{A}_+\mathcal{T}_\sigma)v_\mu\partial_\nu F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+\frac{d_6}{\Lambda_\chi}(\bar{\mathcal{T}}^\sigma\mathcal{T}_\sigma)v_\mu\partial_\nu F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+^{SU(2)})\mathrm{str}(\mathcal{A}_+)\\&+id_5'(\bar{\mathcal{T}}_\mu\mathcal{A}_+\mathcal{T}_\nu)F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+id_6'(\bar{\mathcal{T}}_\mu\mathcal{T}_\nu)F^{\mu\nu}\mathrm{str}(\mathcal{Q}_+^{SU(2)})\,\mathrm{str}(\mathcal{A}_+)\\&+\frac{d_5''}{\Lambda_\chi}(\bar{\mathcal{T}}^{\{\mu}\mathcal{A}_+\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+\frac{d_6''}{\Lambda_\chi}(\bar{\mathcal{T}}^{\{\mu}\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\mathrm{str}(\mathcal{A}_+),\end{aligned}\tag{A8}$$

and also further vector-current correction operators

$$\begin{aligned}\mathcal{L}=&\frac{3ac_{A,\gamma}'}{\Lambda_\chi}(\bar{\mathcal{T}}^\sigma\mathcal{T}_\sigma)v_\mu\partial_\nu F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}^{SU(2)}c_1^{SU(2)})\\&+3ia\mu_{A,\gamma}'(\bar{\mathcal{T}}_\mu\mathcal{T}_\nu)F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}^{SU(2)}c_1^{SU(2)})\\&-\frac{3a\mathbb{Q}_{A,\gamma}}{\Lambda_\chi}(\bar{\mathcal{T}}^{\{\mu}\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\,\mathrm{str}(\mathcal{Q}^{SU(2)}c_1^{SU(2)}).\end{aligned}\tag{A9}$$

4. Baryon transitions

Finally for the transitions, there are only new $\mathcal{A}_+$ insertions

$$\begin{aligned}\mathcal{L}=&it_6(\bar{\mathcal{B}}S_\mu\mathcal{A}_+\mathcal{T}_\nu)F^{\mu\nu}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+\frac{t_6'}{\Lambda_\chi}(\bar{\mathcal{B}}S^{\{\mu}\mathcal{A}_+\mathcal{T}^{\nu\}})v^\alpha\partial_\mu F_{\nu\alpha}\,\mathrm{str}(\mathcal{Q}_+^{SU(2)})\\&+\frac{it_6''}{\Lambda_\chi^2}(\bar{\mathcal{B}}S_\mu\mathcal{A}_+\mathcal{T}_\nu)\partial^\alpha\partial^\mu F^\nu{}_\alpha\,\mathrm{str}(\mathcal{Q}_+^{SU(2)}).\end{aligned}\tag{A10}$$

For each electromagnetic observable considered above, contributions from all $O(a)$ operators in the effective theory are of higher order than the one-loop results in the chiral expansion. Thus one need only retain the finite lattice spacing corrections to the meson masses and use the previously found expressions for electromagnetic properties in $SU(2)$ PQχPT [17–19,33,53].

APPENDIX B: COARSE-LATTICE POWER COUNTING

In this appendix, we detail the $O(a)$ corrections to electromagnetic properties in an alternate power-counting scheme. We imagine a sufficiently coarse lattice, where $a\Lambda_\chi$ can be treated as $O(\epsilon)$, so that[4]

$$\epsilon^2\sim\begin{cases}m_q/\Lambda_\chi,\\a^2\Lambda_\chi^2,\\p^2/\Lambda_\chi^2.\end{cases}\tag{B1}$$

In this case, there are known additional $O(a^2)$ corrections [31] to the meson masses that are now at $O(\epsilon^2)$ and must be included in expressions for loop diagrams. The free Lagrangian for $\mathcal{B}_{ijk}$ and $\mathcal{T}^\mu_{ijk}$ fields contains additional terms of $O(a^2)$ that correct the baryon masses and modify the kinetic terms. Potential contributions due to the latter, whatever their form, must be canceled by wave function renormalization diagrams. The only contribution of $O(a^2)$ could come from tree-level electromagnetic terms but these are necessarily higher order. Thus in this power counting there are no unknown $O(a^2)$ corrections for electromagnetic properties.

The only possible corrections come from the $O(a)$ operators assembled above. A few of these do contribute at tree level and are spelled out below.

1. Octet mesons

The $O(a)$ corrections to the meson form factors are now $O(\epsilon^3)$ in the power counting. While the meson charge radii at NLO in the chiral expansion are at $O(\epsilon^2)$, further corrections in the chiral expansion are at $O(\epsilon^4)$. Thus one can use the $O(a)$ operators to completely deduce the charge radii to

[4]This power counting coupled with the chiral expansion is most efficient for valence Ginsparg-Wilson quarks where $O(a)$ corrections vanish. We thank Gautam Rupak for pointing this out.

TABLE I. The coefficients A and B for the octet baryons.

	A	B
p	1	0
n	$-\frac{1}{3}$	$-\frac{1}{3}$
Σ^+	1	0
Σ^0	$\frac{1}{6}$	$\frac{1}{6}$
Λ	$-\frac{1}{6}$	$-\frac{1}{6}$
$\Sigma^0\Lambda$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$
Σ^-	$-\frac{2}{3}$	$\frac{1}{3}$
Ξ^0	$-\frac{1}{3}$	$-\frac{1}{3}$
Ξ^-	$-\frac{2}{3}$	$\frac{1}{3}$

$O(\epsilon^3)$ [apart from $O(\epsilon^3)$ corrections to the meson masses]. These $O(a)$ operators are given in Eqs. (23) and (24) and yield a correction $\delta\langle r_E^2\rangle$ to the meson charge radii of the form

$$\delta\langle r_E^2\rangle = Q\frac{24a\Lambda_\chi}{f^2}[c^v(2m_1+m_2)+3c^s m_3+c_1^v\alpha_{A,9}]. \tag{B2}$$

Notice that there are no corrections associated with an unimproved current operator in the sea sector since c_1^s is absent.

In the case of $SU(2)$ flavor, there is an additional contribution from the operator in Eq. (A5). At the tree level, however, this operator vanishes. The only correction to Eq. (B2) in changing to $SU(2)$ flavor is to replace $3c^s$ with $2c^s$ which reflects the change in the number of sea quarks.

2. Octet baryons

For the octet baryon electromagnetic properties, the $O(a)$ corrections to the charge radii are now $O(\epsilon^3)$ and can be dropped as they are the same order as neglected $1/M_B$ corrections. The magnetic moments, however, do receive corrections from local operators. Specifically, the $O(a)$ operators which contribute to magnetic moments at $O(\epsilon)$ are insertion of $\mathcal{A}_+$ into the magnetic moment operator given in Eq. (30) and $\mathcal{O}_1^\mu$ corrections given in Eq. (32). Calculation of these corrections yields a shift in the magnetic moments:

$$\begin{aligned}\delta\mu = aM_B\Bigg\{ & c^v\left[A\left(b_1'+\frac{1}{2}b_4'\right)-B(2b_2'+b_3'-b_5')\right] \\ & +3c^s\left(\frac{1}{2}Ab_6'-Bb_7'\right)+C(c^s-c^v)q_{jlr}b_8' \\ & +\frac{c_1^v}{2}\left[\frac{1}{2}\mu_{A,\alpha}A-\mu_{A,\beta}B\right]\Bigg\},\end{aligned} \tag{B3}$$

where $q_{jlr}=q_j+q_l+q_r$. The coefficients A and B are listed for octet baryons in Table I, while $C=1$ for all octet magnetic moments and $C=0$ for the $\Lambda\Sigma^0$ transition moment. Notice that there are no corrections associated with an unimproved current operator in the sea sector.

In the case of $SU(2)$ flavor, there are additional contributions given in Eqs. (A6) and (A7). For the proton and neutron, we have

$$\begin{aligned}\delta\mu^{SU(2)} = aM_B\Bigg\{ & c^v\left[A\left(b_1'+\frac{1}{2}b_4'\right)-B(2b_2'+b_3'-b_5')\right. \\ & \left.+\frac{1}{3}(b_9'+b_{10}')\right]+2c^s\left(\frac{1}{2}Ab_6'-Bb_7'+\frac{1}{3}b_{11}'\right) \\ & +\left[c^s q_{jl}+c^v\left(\frac{1}{3}-q_{jl}\right)\right]b_8'+\frac{c_1^v}{2}\left[\frac{1}{2}A\mu_{A,\alpha}-B\mu_{A,\beta}\right. \\ & \left.+\left(\frac{1}{3}-q_{jl}\right)\mu_{A,\gamma}\right]+\frac{c_1^s}{2}q_{jl}\mu_{A,\gamma}\Bigg\},\end{aligned} \tag{B4}$$

where $q_{jl}=q_j+q_l$.

3. Decuplet baryons

For the decuplet baryon electromagnetic properties in coarse-lattice power counting, the $O(a)$ corrections to the charge radii are $O(\epsilon^3)$ and the corrections to the electric quadrupole moments are $O(\epsilon)$, both of which are higher order than the one-loop results. The magnetic moments, however, do receive corrections from local operators. Specifically, the $O(a)$ operators which contribute to magnetic moments at $O(\epsilon)$ are $\mathcal{A}_+$ insertions into the magnetic moment operator given in Eq. (41) and $\mathcal{O}_1^\mu$ correction operators given in Eq. (43). Calculation of these corrections yields a shift in the magnetic moments:

$$\begin{aligned}\delta\mu = 2aM_B\Bigg[& \frac{1}{3}c^v Q(2d_1'+d_2')+c^s Qd_3' \\ & +(c^s-c^v)q_{jlr}d_4'+c_1^v Q\mu_{A,c}\Bigg].\end{aligned} \tag{B5}$$

Notice that in $SU(3)$ $\mathrm{str}\,\mathcal{Q}=0$; hence there is no dependence on c_1^s in the above result.

In the case of $SU(2)$ flavor, there are additional contributions given in Eqs. (A8) and (A9). The corrections to the Δ quartet magnetic moments are then

$$\begin{aligned}\delta\mu^{SU(2)} = 2aM_B\Bigg\{ & \frac{1}{3}c^v(2Qd_1'+Qd_2'+d_5') \\ & +\frac{2}{3}c^s(Qd_3'+d_6')+\left[c^s q_{jl}+c^v\left(\frac{1}{3}-q_{jl}\right)\right]d_4' \\ & +c_1^v[Q\mu_{A,c}+(1-3q_{jl})\mu_{A,\gamma}']+3c_1^s q_{jl}\mu_{A,\gamma}'\Bigg\}.\end{aligned} \tag{B6}$$

4. Baryon transitions

For the decuplet to octet electromagnetic transitions in coarse-lattice power counting, the $O(a)$ corrections to $G_2(0)$ and $G_3(0)$ are $O(\epsilon)$ which are of higher order than the oneloop results. The $G_1(q^2)$ form factor does, however, receive corrections from local operators. Specifically these $O(a)$ operators which contribute to $G_1(0)$ at $O(\epsilon)$ are the

insertions of $\mathcal{A}_+$ into the magnetic dipole transition operator given in Eq. (52) and the vector-current corrections given in Eq. (55). Calculation of these corrections yields a shift of $G_1(0)$:

$$\delta G_1(0) = aM_B\alpha_T\sqrt{\frac{2}{3}}\left\{c^v\left(t_1+t_2+t_3-\frac{1}{2}t_4\right)+3c^s t_5+c_1^v\mu_{A,T}\sqrt{\frac{3}{8}}\right\}, \tag{B7}$$

where the transition coefficients α_T appear in [19]. Again, at this order the result is independent of $O(a)$ improvement to the electromagnetic current in the sea sector. In the case of $SU(2)$ flavor, there is an additional dipole operator given in Eq. (A10). At the tree level, however, this operator vanishes. The only correction to Eq. (B7) in changing to $SU(2)$ flavor is to replace $3c^s$ with $2c^s$ which reflects the change in the number of sea quarks.

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