

$D^0\text{--}\bar{D}^0$ mass difference from a dispersion relation

Adam F. Falk,¹ Yuval Grossman,^{2,3,4} Zoltan Ligeti,⁵ Yosef Nir,⁶ and Alexey A. Petrov⁷

¹*Department of Physics and Astronomy, The Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218, USA*

²*Department of Physics, Technion–Israel Institute of Technology, Technion City, 32000 Haifa, Israel*

³*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA*

⁴*Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064, USA*

⁵*Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720, USA*

⁶*Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

⁷*Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA*

(Received 24 February 2004; published 29 June 2004)

We study the standard model prediction for the mass difference between the two neutral D meson mass eigenstates, Δm . We derive a dispersion relation based on heavy quark effective theory that relates Δm to an integral of the width difference of heavy mesons, $\Delta\Gamma$, over varying values of the heavy meson mass. Modeling the m_D dependence of certain D decay partial widths, we investigate the effects of SU(3) breaking from phase space on the mass difference. We find that Δm may be comparable in magnitude to $\Delta\Gamma$ in the standard model.

DOI: 10.1103/PhysRevD.69.114021

PACS number(s): 14.40.Lb, 11.30.Hv, 12.39.Hg, 13.20.Fc

I. INTRODUCTION

The mixing and decay of K , B , and D mesons are sensitive probes of physics beyond the standard model. Among the many processes that one might study, flavor-changing neutral current D decays and $D^0\text{--}\bar{D}^0$ mixing provide unique information, because in the standard model (SM) they occur via loop diagrams involving intermediate down-type quarks. In particular, because of severe Cabibbo-Kobayashi-Maskawa (CKM) and Glashow-Iliopoulos-Maiani (GIM) suppressions, the mixing of D mesons is expected to be quite slow, and thus the D system is one of the most intriguing probes of new physics in low energy experiments [1].

We begin by recalling the formalism for heavy meson mixing. Using standard notation, the expansion of the off-diagonal terms in the neutral D mass matrix to second order in perturbation theory is given by

$$\begin{aligned} \left(M - \frac{i}{2}\Gamma\right)_{12} &= \frac{1}{2m_D} \langle D^0 | \mathcal{H}_w^{\Delta C=2} | \bar{D}^0 \rangle + \frac{1}{2m_D} \\ &\times \sum_n \frac{\langle D^0 | \mathcal{H}_w^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_w^{\Delta C=1} | \bar{D}^0 \rangle}{m_D - E_n + i\epsilon}. \end{aligned} \quad (1)$$

The first term represents the $\Delta C=2$ contributions that are local at the scale $\mu \sim m_D$. It contributes only to M_{12} , and is expected to be very small unless it receives a large enhancement from new physics. The second term in Eq. (1) comes from the double insertion of $\Delta C=1$ operators in the SM Lagrangian and it contributes to both M_{12} and Γ_{12} . It is dominated by the SM contributions even in the presence of new physics. Two physical parameters that characterize the mixing are

$$x = \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta\Gamma}{2\Gamma}, \quad (2)$$

where Δm and $\Delta\Gamma$ are the mass and width differences of the two neutral D meson eigenstates and Γ is their average width. Because of the GIM mechanism the mixing amplitude is proportional to differences of terms suppressed by $m_{d,s,b}^2/m_W^2$, and so $D^0\text{--}\bar{D}^0$ mixing is very slow in the SM [2]. The contribution of the b quark is further suppressed by the small CKM elements $|V_{ub}V_{cb}^*|^2/|V_{us}V_{cs}^*|^2 = \mathcal{O}(10^{-6})$, and can be neglected.

Thus, the D system essentially involves only the first two generations, and therefore CP violation is absent both in the mixing amplitude and in the dominant tree-level decay amplitudes and will be neglected hereafter. Once the contribution of b quarks is neglected, the mixing vanishes in the flavor SU(3) limit, and it only arises at second order in SU(3) breaking if SU(3) breaking can be treated analytically [3]:

$$x, y \sim \sin^2 \theta_C \times [\text{SU(3) breaking}]^2, \quad (3)$$

where θ_C is the Cabibbo angle. Precise calculations of x and y in the SM are not possible at present, because the charm mass is neither heavy enough to justify inclusive calculations, nor is it light enough to allow a few exclusive channels to give a reliable estimate.

According to Eq. (3), computing x and y in the SM requires a calculation of SU(3) violation in the decay rates. There are many sources of SU(3) violation, most of them involving nonperturbative physics in an essential way. In Ref. [3], SU(3) breaking arising from phase space differences was studied; computing them in two-, three-, and four-body D decays, it was found that y could naturally be at the level of one percent. This result can be traced to the fact that the SU(3) cancellation between the contributions of members of the same multiplet can be badly broken when decays to the heaviest members of a multiplet have small or vanishing phase space. This effect is manifestly not included in the operator product expansion– (OPE-) based calculations of $D^0\text{--}\bar{D}^0$ mixing, which cannot address threshold effects.

The purpose of the present paper is to address the following question: if the dominant SU(3) breaking mechanism is indeed the one studied in Ref. [3], and it gives rise to y at the percent level, then can x naturally be comparably large? This is particularly relevant because the present experimental upper bounds on x and y are at a few times 10^{-2} level [4,5] and are expected to significantly improve (for a review of the experimental situation, see Ref. [6]). To interpret the results from future measurements of x and y , and possibly establish the presence of new physics, we need to know the allowed range in the SM. In particular, since new physics can only contribute to x , an experimental observation of $x \gg y$ would imply a large new physics contribution to D^0 - \bar{D}^0 mixing. Although y is determined by SM processes, its value still affects the sensitivity to new physics [7].

In this paper we study the SM predictions for x/y due to SU(3) breaking from final state phase space differences. In Sec. II we derive a dispersion relation using heavy quark effective theory (HQET) that relates Δm to $\Delta \Gamma$. To compute Δm , we need a calculation of $\Delta \Gamma$ for varying heavy meson mass, so we review its calculation from Ref. [3] in Sec. III. In Sec. IV, we calculate Δm and present numerical results.

We find that despite the fact that SU(3) breaking in phase space affects x in a different way than it affects y , the final estimates of x and y are comparable. We present our conclusions in Sec. V and discuss the implications of our findings for experimental searches for new physics in D^0 - \bar{D}^0 mixing.

II. DERIVATION OF THE DISPERSION RELATION

We start by reviewing the relevant formalism for D^0 - \bar{D}^0 mixing. Equation (1) implies that the mass eigenstates are linear combinations of the weak interaction eigenstates, $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$. Since we neglect the effects of intermediate states containing a b quark, $|D_{1,2}\rangle$ are also CP eigenstates, $CP|D_{\pm}\rangle = \pm|D_{\pm}\rangle$. Their mass and width differences are

$$\Delta m \equiv m_{D_+} - m_{D_-} = 2M_{12}, \quad \Delta \Gamma \equiv \Gamma_{D_+} - \Gamma_{D_-} = 2\Gamma_{12}. \quad (4)$$

Neglecting the small contribution from the local $\Delta C=2$ operators, Eq. (1) gives

$$\Delta m = \frac{1}{2m_D} \text{P} \sum_n \frac{\langle D^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | \bar{D}^0 \rangle + \langle \bar{D}^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | D^0 \rangle}{m_D - E_n},$$

$$\Delta \Gamma = \frac{1}{2m_D} \sum_n [\langle D^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | \bar{D}^0 \rangle + \langle \bar{D}^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | D^0 \rangle] (2\pi) \delta(m_D - E_n), \quad (5)$$

where P denotes the principal value prescription, the sum is over all intermediate states, n , and it implicitly includes $(2\pi)^3 \delta^3(\vec{p}_D - \vec{p}_n)$.

To derive a dispersion relation between Δm and $\Delta \Gamma$, consider the following correlator:

$$\Sigma_{p_D}(q) = i \int d^4z \langle \bar{D}(p_D) | T[\mathcal{H}_w(z) \mathcal{H}_w(0)] | D(p_D) \rangle \times e^{i(q-p_D) \cdot z}. \quad (6)$$

Here p_D is a label given by the momentum of the on-shell D meson state (satisfying $p_D^2 = m_D^2$) and $q - p_D$ is an auxiliary four-vector that inserts external momentum to the weak interaction (see Fig. 1). There is no simple physical interpretation

of Σ except at $q = p_D$, where $\Sigma_{p_D}(p_D)$ is related to physical properties of D mesons. Inserting a complete set of states in Eq. (6) and comparing with Eq. (5), we find

$$-\frac{1}{2m_D} \Sigma_{p_D}(p_D) = \left(\Delta m - \frac{i}{2} \Delta \Gamma \right). \quad (7)$$

The correlator $\Sigma_{p_D}(q)$ is an analytic function of q (but not of p_D) with a cut in the complex q^0 plane for $q^0 > \sqrt{|\vec{q}|^2 + 4m_\pi^2}$ for a fixed \vec{q} .

To write the dispersion relation in terms of physical quantities, i.e., to give $\Sigma_{p_D}(q)$ for $q \neq p_D$ a physical interpretation, we need to eliminate the heavy quark mass dependence from Eq. (6).¹ The momentum of a heavy meson H containing a heavy quark Q can be written as $p_H^\mu = m_H v^\mu$, with $v^2 = 1$. We can decompose Q as

$$Q(z) = e^{-im_Q v \cdot z} h_v^{(Q)}(z) + e^{+im_Q v \cdot z} \tilde{h}_v^{(Q)}(z) + \dots, \quad (8)$$

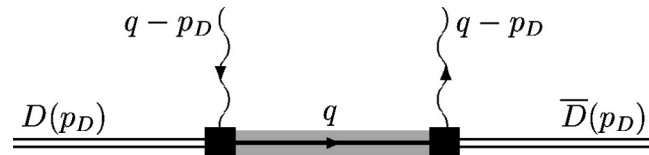


FIG. 1. The correlator in Eq. (6). The black boxes denote the weak Hamiltonian, the wavy lines show external momenta inserted, and the gray area represents hadronic intermediate states.

¹The method of using HQET to derive a dispersion relation in the heavy quark mass was developed first in Ref. [8], where it was used to study the inclusive nonleptonic heavy meson decay rate.

where the HQET fields $h_v^{(Q)}$ and $\tilde{h}_v^{(Q)}$, respectively, annihilate a heavy Q quark and create a heavy \bar{Q} antiquark with four-velocity v . Here and in the rest of this section the ellipses denote terms suppressed by a relative factor of Λ_{QCD}/m_c . The $\Delta C=1$ weak Hamiltonian contributing to neutral D meson mixing is

$$\begin{aligned}\mathcal{H}_w &= \frac{4G_F}{\sqrt{2}} V_{cq_1} V_{uq_2}^* \sum_i C_i \mathcal{O}_i \\ &= \hat{\mathcal{H}}_w [e^{-im_c v \cdot z} h_v^{(c)} + e^{im_c v \cdot z} \tilde{h}_v^{(c)}] + \dots,\end{aligned}\quad (9)$$

where $q_{1,2}=d$ or s , and the four-quark operators, suppressing their Dirac structures, are of the form

$$\mathcal{O}_i \sim \bar{q}_1 q_2 \bar{u} c = e^{-im_c v \cdot z} \bar{q}_1 q_2 \bar{u} h_v^{(c)} + e^{im_c v \cdot z} \bar{q}_1 q_2 \bar{u} \tilde{h}_v^{(c)} + \dots\quad (10)$$

In Eq. (9) $\hat{\mathcal{H}}_w$ contains the light quark fields, the Wilson coefficients, and summation over operators. We also replace the QCD states $|D\rangle$ by HQET states $|H(v)\rangle$,

$$|D(p=m_D v)\rangle = \sqrt{m_D} |H(v)\rangle + \dots\quad (11)$$

The new states have a normalization that is independent of the heavy quark mass [9]. Then Eq. (6) yields

$$\begin{aligned}\Sigma_{p_D}(q) &= im_D \int d^4 z \langle \bar{H}(v) | T \{ [e^{-im_c v \cdot z} \hat{\mathcal{H}}_w h_v^{(c)}(z) \\ &\quad + e^{im_c v \cdot z} \hat{\mathcal{H}}_w \tilde{h}_v^{(c)}(z)] [\hat{\mathcal{H}}_w h_v^{(c)}(0) \\ &\quad + \hat{\mathcal{H}}_w \tilde{h}_v^{(c)}(0)] \} e^{i(q-p_D) \cdot z} | H(v) \rangle + \dots\end{aligned}\quad (12)$$

The only nonzero contributions to this correlator involve a single h and \tilde{h} field each,

$$\begin{aligned}\Sigma_{p_D}(q) &= im_D \int d^4 z \langle \bar{H}(v) | \{ e^{-im_c v \cdot z} \\ &\quad \times T[\hat{\mathcal{H}}_w h_v^{(c)}(z), \hat{\mathcal{H}}_w \tilde{h}_v^{(c)}(0)] \\ &\quad + e^{im_c v \cdot z} T[\hat{\mathcal{H}}_w \tilde{h}_v^{(c)}(z), \hat{\mathcal{H}}_w h_v^{(c)}(0)] \} \\ &\quad \times e^{i(q-p_D) \cdot z} | H(v) \rangle + \dots\end{aligned}\quad (13)$$

The two terms in Eq. (13) behave differently in the HQET limit $m_c \rightarrow \infty$ with q fixed. The term proportional to $\exp[i(q-p_D-m_c v) \cdot z]$ oscillates infinitely rapidly and is integrated out at the heavy scale. It should be removed from the effective theory and replaced by a local $\mathcal{H}_w^{\Delta C=2}$ contribution that can be included as a matrix element of $\Delta C=2$ operators. Such contributions are estimated to give rise to x and y at or below the 10^{-3} level [10–12],² and since we are interested in the question whether x could be near the percent level, we can neglect them.

²In the OPE-based calculations, because m_c/Λ_{QCD} is not very large and subleading terms in the Λ_{QCD}/m_c expansion are enhanced by Λ_{QCD}/m_s [10], such terms dominate the short distance contribution [10–12].

By contrast, the term proportional to $\exp[i(q-p_D+m_c v) \cdot z]$ becomes independent of m_c as $m_c \rightarrow \infty$. Recalling that $p_D = m_D v$, we have

$$\begin{aligned}\Sigma_{p_D}(q) &= im_D \int d^4 z \langle \bar{H}(v) | T[\hat{\mathcal{H}}_w \tilde{h}_v^{(c)}(z), \hat{\mathcal{H}}_w h_v^{(c)}(0)] \\ &\quad \times e^{i(q-\bar{\Lambda} v) \cdot z} | H(v) \rangle + \dots,\end{aligned}\quad (14)$$

where $\bar{\Lambda} = m_D - m_c + \mathcal{O}(\Lambda_{\text{QCD}}^2/m_c)$. It is convenient to define

$$\begin{aligned}\bar{\Sigma}_v(q) &= i \int d^4 z \langle \bar{H}(v) | T[\hat{\mathcal{H}}_w \tilde{h}_v^{(Q)}(z), \hat{\mathcal{H}}_w h_v^{(Q)}(0)] \\ &\quad \times e^{i(q-\bar{\Lambda} v) \cdot z} | H(v) \rangle,\end{aligned}\quad (15)$$

which is manifestly independent of the heavy quark mass. It follows that

$$\Sigma_{p_D}(q) = m_D \bar{\Sigma}_v(q) + \dots,\quad (16)$$

and Eq. (7) becomes to leading order in Λ_{QCD}/m_c

$$\bar{\Sigma}_v(q) = -2\Delta m(E) + i\Delta\Gamma(E),\quad (17)$$

where $E \equiv \sqrt{q^2}$. The correlator $\bar{\Sigma}_v(q)$ is the two-point function for a static neutral heavy meson of mass E , with $\Delta m(E)$ and $\Delta\Gamma(E)$ the corresponding mass and width differences. Thus, unlike $\Sigma_{p_D}(q)$, $\bar{\Sigma}_v(q)$ has a physical interpretation for any value of q . Equation (17) shows that $\bar{\Sigma}_v(q)$ only depends on q^2 . Choosing a frame in which $\vec{q} = \vec{0}$, we can use the analyticity of $\bar{\Sigma}_v(q)$ to write a dispersion relation,

$$\bar{\Sigma}_v(m_D, \vec{0}) = \frac{1}{\pi} \int_{2m_\pi}^{\infty} dE \frac{\text{Im} \bar{\Sigma}_v(E, \vec{0})}{E - m_D + i\epsilon}.\quad (18)$$

Using Eq. (17), we obtain

$$\Delta m = -\frac{1}{2\pi} \text{P} \int_{2m_\pi}^{\infty} dE \left[\frac{\Delta\Gamma(E)}{E - m_D} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{E}\right) \right].\quad (19)$$

Eq. (19) is the main result of this section. It expresses Δm_D in terms of a weighted integral of the width difference of heavy mesons, $\Delta\Gamma(E)$, over varying heavy meson masses, E . The heavy quark limit was essential in deriving this relation, since $\bar{\Sigma}_v(q)$ has a physical interpretation for arbitrary q , while for $q \neq p_D$, $\Sigma_{p_D}(q)$ does not. The $\mathcal{O}(\Lambda_{\text{QCD}}/E)$ error in the integrand is a consequence of our reliance on this limit, and the resulting correction is $\mathcal{O}(1)$ in the small E region. Dispersion relations for Δm_D were considered previously in Ref. [13], where $\text{Im} \Sigma(s)$ (with a different definition of Σ) was modeled, but it does not have a physical interpretation for $s \neq m_D^2$.

To calculate x/y using the dispersion relation, we need to know $\Delta\Gamma$ as a function of the heavy meson mass. Examining Eq. (19), we expect that values of E close to m_D give the largest contribution to x . In the next section we recall the

calculation of $\Delta\Gamma(E)$ performed in Ref. [3]. If $\Delta\Gamma(E)$ is a decreasing function of E at least as a positive power, $1/E^a$ with $a>0$, then the dispersion relation does not require subtraction in order to converge. In the model we consider, $\Delta\Gamma(E)$ actually falls off as $\sim 1/E^2$, and we will argue that some kind of decreasing behavior is likely to hold model independently.

III. CALCULATION OF THE LIFETIME DIFFERENCE

The computation of x using Eq. (19) requires us to know $\Delta\Gamma$ for a heavy meson of varying mass. The calculation of $\Delta\Gamma$ cannot at present be done from first principles. In Ref. [3] $\Delta\Gamma$ was computed using a simple model in which SU(3) breaking was taken into account in calculable phase space differences, but neglected in the incalculable hadronic matrix elements. This approach was motivated by the fact that phase space differences alone can explain the experimental data in several cases; for example, the ratio $\Gamma(D_2^* \rightarrow D\pi)/\Gamma(D_2^* \rightarrow D^*\pi)$ [14], the large SU(3) breaking in $\Gamma(D \rightarrow K^*\ell\bar{\nu})/\Gamma(D \rightarrow \rho\ell\bar{\nu})$ [15], and the lifetime ratio τ_{D_s}/τ_{D^0} [16]. It certainly cannot explain all SU(3) violation, for example, $\Gamma(D \rightarrow \pi\pi)/\Gamma(D \rightarrow KK)$. The generic conclusion of Ref. [3] was that if multibody final states close to the D threshold have significant branching ratios, then they can give rise to sizable contributions to $\Delta\Gamma$ that are absent in the OPE-based calculations. Our purpose in the next section will be to see whether the same mechanism can also give rise to x at or near the percent level. Here we review the analysis of Ref. [3].

We denote a set of final states F belonging to a certain representation R of SU(3) by F_R . For example, for two pseudoscalar mesons in the octet, the possible representations for $F = PP$ are $R=8$ and 27 . In Ref. [3] it was shown that y_{F_R} , the value that y would take if elements of F_R were the only channels open for D^0 decay, can be expressed as

$$y_{F_R} = \frac{\sum_{n \in F_R} \langle \bar{D}^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | D^0 \rangle}{\sum_{n \in F_R} \langle D^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | D^0 \rangle} = \frac{\sum_{n \in F_R} \langle \bar{D}^0 | \mathcal{H}_w | n \rangle \langle n | \mathcal{H}_w | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \rightarrow n)}. \quad (20)$$

The derivation of this relation assumes the absence of CP violation, so that $\langle \bar{D}^0 | \mathcal{H}_w | n \rangle$ is related to $\langle D^0 | \mathcal{H}_w | \bar{n} \rangle$, and uses the fact that both $|n\rangle$ and $|\bar{n}\rangle$ belong to the same SU(3) multiplet. When the SU(3) breaking in the matrix elements is neglected, Eq. (20) gives a calculable contribution to y_{F_R} without any hadronic parameters. The numerator contains a combination of Clebsch-Gordan and CKM coefficients that

ensures that y_{F_R} is proportional to $m_s^2 \sin^2 \theta_C$ when the sum over all members of any given multiplet F_R is performed, as required by Eq. (3).

As an example, the contribution of the multiplet containing two pseudoscalar mesons in an SU(3) octet is given by

$$y_{(PP)_8} = \sin^2 \theta_C \left[\frac{1}{2} \Phi(\eta, \eta) + \frac{1}{2} \Phi(\pi^0, \pi^0) + \Phi(\pi^+, \pi^-) + \Phi(K^+, K^-) + \frac{1}{3} \Phi(\eta, \pi^0) - \frac{1}{3} \Phi(\eta, K^0) - 2\Phi(K^+, \pi^-) - \Phi(K^0, \pi^0) \right] \left[\frac{1}{6} \Phi(\eta, \bar{K}^0) + \Phi(K^-, \pi^+) + \frac{1}{2} \Phi(\bar{K}^0, \pi^0) \right]^{-1} + \mathcal{O}(\sin^4 \theta_C), \quad (21)$$

where $\Phi(n)$ is the phase space factor for $D \rightarrow n$ decay. Then y can be computed as the sum of the y_{F_R} 's weighted with the D^0 decay rate to each representation,

$$y = \frac{1}{\Gamma} \sum_{F_R} y_{F_R} \left[\sum_{n \in F_R} \Gamma(D^0 \rightarrow n) \right]. \quad (22)$$

The y_{F_R} were computed for all PP , PV , and VV representations and for the fully symmetric $3P$ and $4P$ final states [3]. The contribution of poles corresponding to nearby K resonances was shown to be small [3,17]. Assuming that the values of $y_{(4P)_R}$ for $R=8, 27, 27'$ are typical for all R , it was found that the $4P$ final states give a contribution to $\Delta\Gamma$ at the percent level. The result is large because many of the decays in question are close to or above threshold, so the SU(3) cancellation in these multiplets is largely ineffective, yielding $y_{(4P)_R} = \mathcal{O}(0.1)$ [3]. Moreover, the D^0 branching ratio to four pseudoscalars is approximately 10%. We shall now use this model of SU(3) breaking, together with some assumptions about the energy dependences of the relevant decay rates, to compute x/y .

IV. CALCULATION OF THE MASS DIFFERENCE

The crucial difference between the calculation of x and y is that once we assume that the only source of SU(3) breaking is from the final state phase space differences, the hadronic matrix elements cancel in y , but not in x . As determined by Eq. (19), x depends on $\Delta\Gamma(E)$, and so the E dependence of the hadronic matrix elements does affect x . Using Eq. (19), we find for x/y ,

$$r_{F_R} \equiv \frac{x_{F_R}}{y_{F_R}} = -\frac{1}{\pi} \mathbf{P} \int_{2m_\pi}^{\infty} \frac{dE}{E - m_D} \frac{y_{F_R}(E)}{y_{F_R}(m_D)} \frac{\Gamma_{F_R}(E)}{\Gamma_{F_R}(m_D)}. \quad (23)$$

We will quote our results in terms of r_{F_R} . To proceed further we need to understand or make some assumptions about the

E dependence of the decay rate to the final state F , $\Gamma_F(E)$. We define the dimensionless function

$$g_F(E) \propto \frac{\Gamma_F(E)}{\Gamma_F(m_D)}, \quad (24)$$

and we will study the E dependence of this quantity. Note that the constant of proportionality in Eq. (24) cancels in the ratio r_F . Moreover, g_F is expected to depend only on the final state F , and not on the SU(3) representation R .

One can reconstruct x from x_{F_R} using a relation analogous to Eq. (22). Below we calculate r_{F_R} for several final states and then estimate the total x . First we will study $F=PP$, because it is a simple case that is interesting to understand in detail. Then we will turn to $F=4P$, because it is the final state that can give $y \sim 1\%$.

A. Two-body $D \rightarrow PP$ decays

For decays to two pseudoscalar mesons, it is possible to develop a reasonable model of $g_{PP}(E)$. When $m_H \gg \Lambda_{\text{QCD}}$, we may approximate the $H \rightarrow \pi\pi$ amplitude with its factorized form. Here $A(H \rightarrow \pi\pi) \sim G_F V_{\text{CKM}} m_H^2 f_\pi F_{H \rightarrow \pi}$, where f_π is the pion decay constant and $F_{H \rightarrow \pi}$ is the $H \rightarrow \pi$ form factor at $q^2 = m_\pi^2$. It has been shown that, as $m_H \rightarrow \infty$, $F_{H \rightarrow \pi} \propto (\Lambda/m_H)^{3/2+X}$ [18], where X arises from summing Sudakov logarithms of the form $\exp[C\alpha_s(m_H)\ln^2(m_H/\Lambda)] \sim (\Lambda/m_H)^X$ with $X = -2\pi C/\beta_0$. Since $\Gamma \propto |A|^2/m_H$, we conclude that

$$g_{PP}(E \gg \Lambda_{\text{QCD}}) \propto E^{-2X}. \quad (25)$$

The existing calculations suggest that $|X| \ll 1$ [19], so we set $X=0$ hereafter.

In the $E \rightarrow 0$ limit our calculation is necessarily unreliable, as the derivation of Eq. (19) relied on HQET. Nevertheless, as a model we will take the behavior of the $K \rightarrow \pi\pi$ amplitude in chiral perturbation theory. At leading order, this transition is mediated by an operator of the form $\text{Tr}(\partial_\mu \Sigma^\dagger \mathcal{O} \partial^\mu \Sigma)$, where $\Sigma = \exp[2i\mathcal{M}/f]$ and \mathcal{M} is the meson octet. Since this term has two derivatives, it implies that the decay amplitude is proportional to m_K^2 . Since this is the only dependence on m_K in the amplitude, the E dependence of the rate is

$$g_{PP}(E \rightarrow 0) \propto E^3. \quad (26)$$

Based on these considerations, we employ the following simple model for $g_{PP}(E)$:

$$g_{PP}(E) = \begin{cases} E^3/(m_1^2 m_2) & \text{for } E < m_1, \\ E/m_2 & \text{for } m_1 < E < m_2, \\ 1 & \text{for } E > m_2, \end{cases} \quad (27)$$

where $m_{1,2}$ are free parameters. The overall normalization cancels in the results. This model allows for a ‘‘chiral’’ region, $E < m_1$, an ‘‘intermediate’’ region, $m_1 < E < m_2$, and a ‘‘high energy’’ region, $E > m_2$. In our calculations m_1 is al-

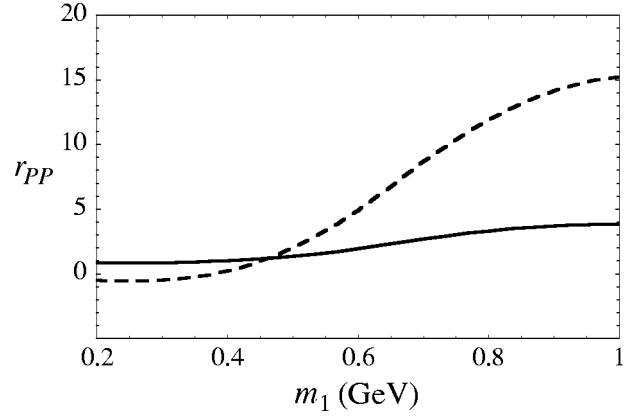


FIG. 2. Predictions for $r_{(PP)_8}$ (solid curve) and $r_{(PP)_{27}}$ (dashed curve) as functions of m_1 .

lowed to vary in the range 0.2–1.0 GeV, and m_2 in the range 1.5–10 GeV. As we emphasized above, our derivation relies on HQET, so any strong dependence on scales below ~ 1 GeV would signal an irreducible lack of reliability.

In Fig. 2 we plot $r_{(PP)_8}$ (solid) and $r_{(PP)_{27}}$ (dashed) as a function of m_1 , for $m_2 = 2$ GeV. In this case all members of the final state representations are kinematically allowed and have large phase space, so we find that the result is dominated by cancellations below the scale m_D . Therefore r_{PP} is sensitive to the shape of $g_{PP}(E)$ at low energies, i.e., the value of m_1 , but changing m_2 to 3 or 4 GeV has little effect on r_{PP} . Because of the strong dependence on m_1 , we should not trust this result. However, since y for these representations is very small, $y_{(PP)_8} = -0.018\%$ and $y_{(PP)_{27}} = -0.0034\%$ [3], these final states do not give sizable contributions to x in any case.

When we consider decays to the lightest pseudoscalar octet, the dependence of these pseudo-Goldstone boson masses on m_s is given by (for $m_{u,d}=0$)

$$m_\pi^2 = 0, \quad m_K^2 = \mu m_s, \quad m_\eta^2 = \frac{4}{3} \mu m_s, \quad (28)$$

where μ is a hadronic scale. We can then expand $\Delta\Gamma(E)$ for large E as

$$\Delta\Gamma_{PP}(E) = [\Gamma_{PP}(E)|_{m_s \rightarrow 0}] \left(c_0 + \frac{c_1}{E^2} + \frac{c_2}{E^4} + \dots \right). \quad (29)$$

Because SU(3) breaking in our approach comes from phase space differences, the coefficients c_i depend quadratically on the masses of the final state particles. Since in Eq. (28) m_s is always accompanied by μ and $\Delta\Gamma$ must be suppressed by m_s^2 , we conclude that $c_0 = c_1 = 0$. The coefficient c_2 can be proportional to $\mu^2 m_s^2$ and is the leading nonvanishing term, implying a $1/E^4$ suppression of $\Delta\Gamma_{PP}(E)$ compared to $\Gamma_{PP}(E)$. However, the actual π , K , and η masses do not exactly satisfy Eq. (28) in the $m_{u,d}=0$ limit, or the Gell-Mann–Okubo (GMO) relation, $3m_\eta^2 = 4m_K^2 - m_\pi^2$. Violating the GMO relation is equivalent to adding a small term to m_K^2

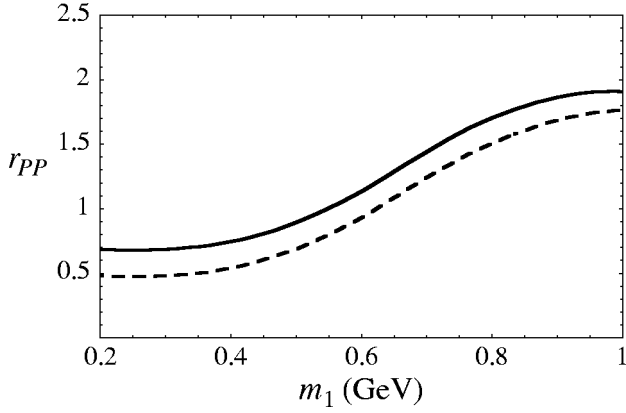
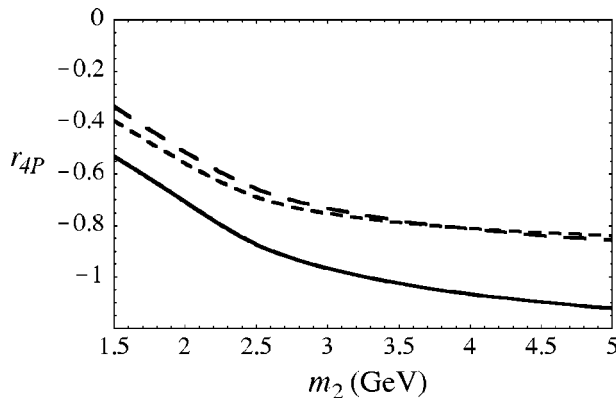


FIG. 3. Predictions for $r_{(PP)_8}$ (solid curve) and $r_{(PP)_{27}}$ (dashed curve) as functions of m_1 , imposing the GMO relation on the π , K , and η masses.

or to m_η^2 of the form εm_s^2 . This changes the asymptotic behavior of $\Delta\Gamma(E)$, because now we can have $c_1 \sim \varepsilon m_s^2$. Since the $D \rightarrow PP$ decay is far from threshold, the SU(3) cancellation in this channel is very sensitive to the pseudoscalar meson masses. This can be verified analytically by expanding Eq. (21). As shown in Fig. 3 (again for $m_2 = 2$ GeV), imposing the GMO relation on the π , K , and η masses decreases r_{PP} significantly, in such a manner that y_{PP} increases by roughly the same factor, while $|x_{PP}|$ is approximately stable at the $(5-8) \times 10^{-4}$ level. As discussed in Ref. [3], our results have little sensitivity to including or neglecting π - η - η' mixing.

By contrast, for final states including vector mesons or heavier pseudoscalar representations, the masses of the mesons depend linearly on m_s . Thus, for these final states, $\Delta\Gamma_F(E)/\Gamma_F(E)$ is simply proportional to m_s^2/E^2 for large E , and there is no strong dependence on the precise values of the hadron masses. This is the minimal suppression of $\Delta\Gamma_F(E)/\Gamma_F(E)$ consistent with group theory, i.e., Eq. (3), and our phase space model for SU(3) violation indeed gives such an effect. These results imply that the dispersion relation in Eq. (19) converges for any final state F , for which $\Gamma_F(E)$ does not increase as E^2 or faster. This is very likely to be true for all final states [recall that $\Gamma_{PP}(E) \sim \text{constant}$ for large E].



B. Four-body $D \rightarrow 4P$ decays

Now we turn to the $4P$ final state in the fully symmetric 8, 27, and 27' representations of SU(3). We know even less about $g_{4P}(E)$ than about $g_{PP}(E)$, so we use two models to attempt to bracket roughly the uncertainties,

$$g_{4P}(E) = g_{PP}(E) \quad \text{and} \quad g'_{4P}(E) = \begin{cases} E/m_1 & \text{for } E < m_1 \\ 1 & \text{for } m_1 < E < m_2 \\ m_2/E & \text{for } E > m_2. \end{cases} \quad (30)$$

The choice of $g'_{4P}(E)$ allows for the possibility that $\Gamma(H \rightarrow 4P)$ may start to fall for large m_H instead of remain constant. This alternative is motivated by the argument that because the quasi-two-body picture holds only in a small part of phase space, in most of the phase space the opening of many decay channels will reduce the rate.

The left plot in Fig. 4 shows $r_{(4P)_8}$ (solid curve), $r_{(4P)_{27}}$ (long dashed curve), and $r_{(4P)_{27'}}$ (short dashed curve), as functions of m_2 , using $g_{4P}(E)$ with $m_1 = 0.8$ GeV. For $m_1 < 1$ GeV there is no dependence on m_1 . The dependence of the curves on m_2 is negligible for $m_2 \gtrsim 3$ GeV. If we use $g'_{4P}(E)$ instead, shown in the right plot in Fig. 4, then $r_{(4P)_R}$ changes roughly by a factor of two. We have explored other forms of $g_{4P}(E)$ as well, and we find that these two cases cover a reasonable range of predictions.

In contrast to $D \rightarrow PP$ decays, for the $4P$ final state there is no strong dependence on the π , K , and η masses. Because the decay is close to threshold, the dispersion integral is dominated by E near m_D , where some of the $4P$ final states are kinematically forbidden, and so the sensitivity to the pseudoscalar meson masses is reduced. Imposing the GMO relation makes only a small difference; for example, for the $(4P)_8$ representation the value $r_{(4P)_8} = -0.98$ obtained with the $g_{4P}(E)$ model, $m_1 = 0.8$ GeV, $m_2 = 3$ GeV, and the physical meson masses (corresponding to the solid curve in the left plot in Fig. 4), would change to $r_{(4P)_8} = -0.87$ if the GMO relation were imposed.

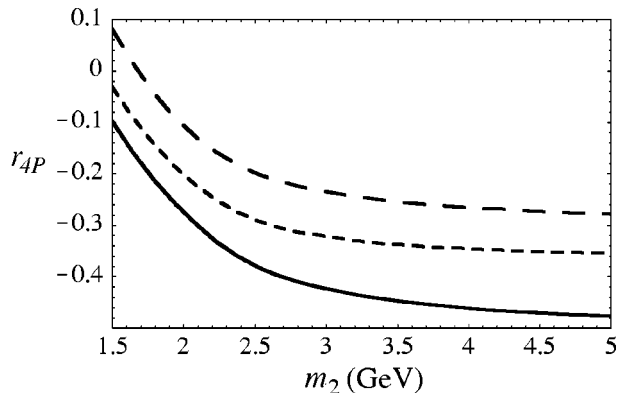


FIG. 4. Predictions for $r_{(4P)_8}$ (solid curve), $r_{(4P)_{27}}$ (long dashed curve), and $r_{(4P)_{27'}}$ (short dashed curve), as functions of m_2 for the models $g_{4P}(E)$ (left figure) and $g'_{4P}(E)$ (right figure) in Eq. (30).

V. DISCUSSION AND CONCLUSIONS

It is likely that the dominant contributions to the mass and width differences in the D system have a long distance origin in the SM. Therefore, naively one would expect x and y to be of the same order of magnitude. We have derived a new dispersion relation (19) and used it to study this question. Our dispersion relation has the useful property that it relates the mass difference in the heavy neutral meson system at fixed heavy meson mass to the physical width difference of heavy mesons with varying mass.

The advantage of using a dispersion relation that relates x to y is that we can use existing models for y to calculate x . Our dispersion relation is likely to converge without any subtraction, because the SU(3) breaking required to yield non-zero mixing introduces an m_s^2/E^2 suppression in $y(E)$. We have used a model in which SU(3) breaking arises from phase space differences, which may give a reasonable approximation to $y(E)$ only when E is not very large. Since the derivation of the dispersion relation employed the heavy quark limit, it is essential not to interpret our analysis as a precise calculation for x . Instead, we used this model only to get a rough and qualitative prediction about the likely relation of x to y .

To make numerical predictions we needed the heavy mass dependence of heavy meson partial widths to certain final states, which introduces some additional model dependence in our results. (For decays to two pseudoscalars, there are limits in which one can draw firmer conclusions about the mass dependence, which we have incorporated into the model.) We calculated the ratio x/y for PP and $4P$ final states. Our conclusion is that it is indeed likely that in the standard model, x is not much smaller than y in the D system. In our numerical study, we found that for the $4P$ final state, x/y varies roughly between -0.1 and -1 . We conclude that if y is in the ballpark of $+1\%$ as expected if the $4P$ final states dominate y [3], then we should expect $|x|$ between 10^{-3} and 10^{-2} , and that x and y are of opposite sign. This estimate has a large uncertainty, and we can trust it only at the order of magnitude level. We have explored the sensitivity of this qualitative result to a number of the assumptions we have made, and have found that changing the details of the model does not significantly alter our conclusions. Furthermore, including some SU(3) breaking in the matrix elements cancels to some extent in x/y and does not induce dramatic changes.

The significance of our result is clear only in the context of the experimental situation. The current bounds on x and y are at the level of a few percent, and the central question is

whether their actual observation at or just below this level could be interpreted as a clear signal of physics beyond the standard model. We would argue that our analysis has taught us that, without further refinement, the answer is no. We have identified a real effect that could plausibly give x and y at the percent level, albeit with very large uncertainties.

In general, an observation of $x \gg y$ would be an indication for new physics, but this could only be established if y were very small, at the 10^{-3} level. Such a situation could arise if new physics enhanced x but not y . Yet since one cannot exclude the possibility of cancellation between different SM contributions to y , even this outcome would not admit an unambiguous interpretation.

However, if x were indeed enhanced by new physics, such new physics may also introduce a sizable new CP violating phase which may be observable. Thus, we would argue that in $D^0\text{-}\bar{D}^0$ mixing, the only single measurement that could establish by itself the presence of new physics would be the observation of CP violation, which is very small in the standard model independent of hadronic effects.

ACKNOWLEDGMENTS

It is a pleasure to thank Mark Wise, as usual, for helpful conversations. We are grateful to the Aspen Center for Physics for its hospitality while portions of this work were completed. A.F. was supported in part by the U.S. National Science Foundation under Grant PHY-9970781. Y.G. was supported in part by the Department of Energy, contract DE-AC03-76SF00515 and by the Department of Energy under grant no. DE-FG03-92ER40689. Z.L. was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and by a DOE Outstanding Junior Investigator program. The work of Y.G. and Z.L. was also supported in part by the United States–Israel Binational Science Foundation (BSF) through Grant No. 2000133. Y.N. is supported by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, by EEC RTN contract HPRN-CT-00292-2002, by a Grant from the G.I.F., the German-Israeli Foundation for Scientific Research and Development, by the Minerva Foundation (München), and by a grant from the United States–Israel Binational Science Foundation (BSF), Jerusalem, Israel. A.P. was supported in part by the U.S. National Science Foundation under Grant PHY-0244853, and by the U.S. Department of Energy under Contract DE-FG02-96ER41005.

-
- [1] G. Burdman and I. Shipsey, *Annu. Rev. Nucl. Part. Sci.* **53**, 431 (2003).
 - [2] For a survey of predictions, see A.A. Petrov, hep-ph/0311371; H.N. Nelson, hep-ex/9908021.
 - [3] A.F. Falk, Y. Grossman, Z. Ligeti, and A.A. Petrov, *Phys. Rev. D* **65**, 054034 (2002).
 - [4] BABAR Collaboration, B. Aubert *et al.*, *Phys. Rev. Lett.* **91**,

- 121801 (2003); Belle Collaboration, K. Abe *et al.*, *ibid.* **88**, 162001 (2002); hep-ex/0308034; CLEO Collaboration, S.E. Csorna *et al.*, *Phys. Rev. D* **65**, 092001 (2002); FOCUS Collaboration, J.M. Link *et al.*, *Phys. Lett. B* **485**, 62 (2000); E791 Collaboration, E.M. Aitala *et al.*, *Phys. Rev. Lett.* **83**, 32 (1999); *Phys. Rev. D* **57**, 13 (1998).
- [5] BABAR Collaboration, B. Aubert *et al.*, *Phys. Rev. Lett.* **91**,

- 171801 (2003); CLEO Collaboration, R. Godang *et al.*, *ibid.* **84**, 5038 (2000).
- [6] B.D. Yabsley, Int. J. Mod. Phys. A **19**, 949 (2004).
- [7] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir, and A.A. Petrov, Phys. Lett. B **486**, 418 (2000).
- [8] B. Grinstein, Phys. Lett. B **529**, 99 (2002).
- [9] A.V. Manohar and M.B. Wise, *Heavy Quark Physics*, Cambridge Monographs on Particle Physics, Nuclear Physics, and Cosmology Vol. 10 (Cambridge University Press, Cambridge, England, 2000).
- [10] H. Georgi, Phys. Lett. B **297**, 353 (1992).
- [11] T. Ohl, G. Ricciardi, and E.H. Simmons, Nucl. Phys. **B403**, 605 (1993).
- [12] I.I.Y. Bigi and N.G. Uraltsev, Nucl. Phys. **B592**, 92 (2001).
- [13] J.F. Donoghue, E. Golowich, B.R. Holstein, and J. Trampetic, Phys. Rev. D **33**, 179 (1986); P. Colangelo, G. Nardulli, and N. Paver, Phys. Lett. B **242**, 71 (1990); G. Burdman, hep-ph/9508349.
- [14] N. Isgur and M.B. Wise, Phys. Rev. Lett. **66**, 1130 (1991); A.F. Falk and M.E. Luke, Phys. Lett. B **292**, 119 (1992).
- [15] Z. Ligeti, I.W. Stewart, and M.B. Wise, Phys. Lett. B **420**, 359 (1998).
- [16] S. Nussinov and M.V. Purohit, Phys. Rev. D **65**, 034018 (2002).
- [17] E. Golowich and A.A. Petrov, Phys. Lett. B **427**, 172 (1998).
- [18] V.L. Chernyak and I.R. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990); J. Charles, A. Le Yaouanc, L. Oliver, O. Pene, and J.C. Raynal, Phys. Rev. D **60**, 014001 (1999); C.W. Bauer, D. Pirjol, and I.W. Stewart, *ibid.* **67**, 071502 (2003).
- [19] R. Akhouri, G. Sterman, and Y.P. Yao, Phys. Rev. D **50**, 358 (1994); C.W. Bauer, S. Fleming, and M.E. Luke, *ibid.* **63**, 014006 (2001).