Analytic estimates of the QCD corrections to neutrino-nucleus scattering

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We study the QCD corrections to neutrino deep-inelastic scattering on a nucleus, and analytically estimate their size. For an isoscalar target, we show that the dominant QCD corrections to the ratio of the neutral- to charged-current events are suppressed by $\sin^4\theta_W$, where θ_W is the weak mixing angle. We then discuss the implications for the NuTeV determination of $\sin^2\theta_W$.

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I. INTRODUCTION

For more than three decades, neutrino deep-inelastic scattering has been an essential source of information regarding both the electroweak interactions and the structure of the nucleons. A very important quantity measured in neutrino (antineutrino) deep-inelastic scattering is the ratio R^ν ($R^{\bar\nu}$) of the total cross sections for the neutral- and charged-current processes. The most precise measurements to date of R^ν and $R^{\bar\nu}$ have been performed by the NuTeV Collaboration [1], which led to a determination of $\sin^2\theta_W$ (θ_W is the weak mixing angle) with uncertainty of less than a percent. Such a precision makes the inclusion of QCD corrections a necessary part of the determination of $\sin^2\theta_W$.

The next-to-leading order (NLO) QCD corrections, i.e., of order α_s, to neutrino-nucleon cross sections have been known for a long time [2–4], and the order α_s^2 corrections have also been computed [5–7]. However, to our knowledge, a careful analysis of the size of even the NLO QCD corrections to R^ν and $R^{\bar\nu}$ has not yet been performed. Part of the reason is the observation that the NLO QCD corrections to the Paschos-Wolfenstein ratio of differences of cross sections [8], $R_{\rm PW}$, cancel for an isoscalar target [9]. Most discussions of perturbative QCD corrections to the NuTeV determination of $\sin^2\theta_W$ have been concentrated on $R_{\rm PW}$ [10–12]. However, the relation between the NLO QCD corrections to $R_{\rm PW}$ and those to R^ν and $R^{\bar\nu}$ is not clear. In fact, it has been often claimed that the NLO QCD corrections to R^ν and $R^{\bar\nu}$ are expected to be as large as 10% (see [10,14–16]), given that the expansion parameter of the perturbative series is typically α_s/π, where α_s is evaluated at a scale of about 20 GeV^2. The NuTeV analysis takes into account a variety of corrections to the cross sections, including a partial, phenomenological description of the QCD corrections. However, the latter might differ from the result of a systematic expansion in α_s, and therefore it is essential to know how large these corrections are.

In this paper we derive an analytic, approximate expression for the NLO QCD corrections to R^ν and $R^{\bar\nu}$. We show that these are suppressed by an additional factor of $\sin^4\theta_W$. This conclusion is consistent from an order-of-magnitude point of view with the numerical results presented in Ref. [13]. We then address the issue of how these corrections might change the NuTeV result for $\sin^2\theta_W$. A definitive statement will require a re-analysis, including full NLO effects by the NuTeV Collaboration.

We emphasize that there are several kinds of QCD corrections that may affect the NuTeV analysis. First there are perturbative QCD corrections to the differential cross section, which are computable in the standard model, and are the focus of this paper. Second, there are nonperturbative effects, such as higher twist effects, which have been included in the NuTeV analysis (see section 5.1.12 of [17]). Third, there are corrections to the parton distribution functions (PDF's), which are being studied by various groups [18–20], and are not discussed here.

In Sec. II we review the lowest order differential cross section, and in Sec. III we present the order α_s corrections to the differential cross section. We then integrate (in Sec. IV) the differential cross section and use (in Sec. V) some perturbative expansions to obtain analytical expressions for the order α_s corrections to R^ν and $R^{\bar\nu}$. We estimate in Sec. VI the impact of the perturbative QCD corrections on the determination of $\sin^2\theta_W$, and we comment on our results in Sec. VII.

II. ν-NUCLEUS CROSS SECTION AT LEADING ORDER

We consider neutrino deep-inelastic scattering on a nucleus, ignoring the Fermi motion of the nucleons. In the laboratory frame, the inclusive ν_μ-nucleus collision is described by three kinematic variables: the squared momentum transfer Q^2, the energy E_ν of the incoming neutrino, and the inelasticity parameter y, which is the fraction of the lepton energy lost in the laboratory frame. In the parton model, Q^2 may be expressed in terms of the fraction x of the nucleon momentum, averaged over the entire nucleus:

$$Q^2 = 2xyM_N E_\nu . \tag{2.1}$$

Here M_N is the average nucleon mass in the nucleus; we are neglecting the parton mass, and both x and y range from 0 to 1.

To be specific, we will concentrate on an iron nucleus, but our considerations apply to any target which is approximately isoscalar. Neglecting the muon mass, there are three structure functions that contribute to the ν_μ-nucleon differential cross sections in the laboratory frame,

$$\frac{d\sigma^{\mathrm{C,N}}(\nu_\mu \mathrm{Fe})}{dxdy} = \frac{xM_N E_\nu G_{\mathrm{F}}^2}{\pi(1+Q^2/M_{W,Z}^2)^2}\left[\frac{y^2}{2}\mathcal{F}_1^{\mathrm{C,N}} + \left(1-y-\frac{xyM_N}{2E_\nu}\right)\mathcal{F}_2^{\mathrm{C,N}} + y\left(1-\frac{y}{2}\right)\mathcal{F}_3^{\mathrm{C,N}}\right], \tag{2.2}$$

where G_{F} is the Fermi constant, and the inclusive cross sections for the charged- and neutral-current processes, $\nu_\mu \mathrm{Fe} \to \mu^- X$ and $\nu_\mu \mathrm{Fe} \to \nu_\mu X$, are labeled, respectively, by $\sigma^{\mathrm{C}}(\nu_\mu \mathrm{Fe})$ and $\sigma^{\mathrm{N}}(\nu_\mu \mathrm{Fe})$. Note that instead of the structure functions introduced here, $\mathcal{F}_i \equiv \mathcal{F}_i(x,Q^2)$ with $i=1,2,3$, which are convenient for the discussion of NLO corrections, the textbooks typically use $F_1 = \mathcal{F}_1/2, F_2 = x\mathcal{F}_2, F_3 = \mathcal{F}_3$.

The structure functions can be written as expansions in several small parameters,

$$\mathcal{F}_i^{\mathrm{C,N}} = \mathcal{F}_{i\ \mathrm{LO}}^{\mathrm{C,N}} + \delta\mathcal{F}_i^{\mathrm{C,N}} + O\left(\frac{\alpha}{\pi\sin^2\theta_W}\right) + O\left(\frac{M_N^2}{Q^2}\right) + O\left(\frac{m_c^2}{Q^2}\right) + \dots . \tag{2.3}$$

The first term of the expansion is due to a W or Z exchange without any radiative corrections and in the limit where the momentum transfer is much larger than the mass of any particle in the initial or final state. For the charged-current process,

$$\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{C}} = \mathcal{F}_{2\ \mathrm{LO}}^{\mathrm{C}} = 2(d+s+\bar{u}+\bar{c})$$
$$\mathcal{F}_{3\ \mathrm{LO}}^{\mathrm{C}} = 2(d+s-\bar{u}-\bar{c}), \tag{2.4}$$

where $q \equiv q(x,Q^2)$, with $q=u,d,s,c$, is the probability distribution, averaged over the entire nucleus, for finding the parton q with momentum fraction x inside a nucleon of the iron nucleus, when the squared momentum transfer is Q^2.

We have included only quarks of the lighter two generations, because for the b quark the PDF is sufficiently small to be neglected at the NuTeV energies, and the deviations from unitarity of the diagonal block of the Cabibbo-Kobayashi-Maskawa (CKM) matrix associated with the first two generations are of order 10^{-3} ($|V_{ts}|^2$ or $|V_{cb}|^2$).

The leading-order structure functions for the neutral-current process are

$$\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{N}} = \mathcal{F}_{2\ \mathrm{LO}}^{\mathrm{N}} = 2(g_L^{u2}+g_R^{u2})(u+c+\bar{u}+\bar{c}) + 2(g_L^{d2}+g_R^{d2})(d+s+\bar{d}+\bar{s})$$
$$\mathcal{F}_{3\ \mathrm{LO}}^{\mathrm{N}} = 2(g_L^{u2}-g_R^{u2})(u+c-\bar{u}-\bar{c}) + 2(g_L^{d2}-g_R^{d2})(d+s-\bar{d}-\bar{s}). \tag{2.5}$$

As usual, $g_L^{u,d}, g_R^{u,d}$ are the quark couplings to the weak bosons, which depend on the electric charge, $Q^{u,d}$, and on the weak mixing angle, θ_W:

$$g_L^{u,d} = \pm\frac{1}{2} - Q^{u,d}\sin^2\theta_W,$$
$$g_R^{u,d} = -Q^{u,d}\sin^2\theta_W. \tag{2.6}$$

The $\bar{\nu}_\mu$-nucleus differential cross sections are obtained from the ν_μ-nucleus ones by interchanging the q and $\bar{q}$ distributions.

The term $\delta\mathcal{F}_i^{\mathrm{C,N}}$ in Eq. (2.3) represents the NLO QCD corrections, and is of order $O(\alpha_s/\pi)$, where $\alpha_s(Q^2) \approx 0.2$ for the average momentum transfer at NuTeV. Therefore, these corrections are *a priori* expected to be large, and their impact on the ratios of neutral- to charged-current events, $R^\nu, R^{\bar{\nu}}$, are the focus of this paper.

The electroweak corrections, encoded in the third term of the expansion (2.3), come from loops involving electroweak gauge bosons, the top quark, and the Higgs boson, as well as from the emission of a real photon. The photon corrections, although not enhanced by a $1/\sin^2\theta_W$ factor, turn out to dominate because their contributions to the charged- and neutral-current processes are substantially different, and lead to a shift of a few percent in the values of R^ν and $R^{\bar{\nu}}$ at NuTeV [17]. The target mass corrections are of order $M_N^2/Q^2 \approx M_N/E_\nu$, so that we expect them to be at most as large as a few percent. A recent discussion of the target mass corrections is given in Ref. [13]. The charm mass affects mainly the charged-current scattering off the strange sea, and accounts for a shift of about 2% in R^ν and $R^{\bar{\nu}}$ [17]. Details of how all the above corrections have been included in the NuTeV analysis can be found in Ref. [17].

III. NEXT-TO-LEADING ORDER QCD CORRECTIONS TO THE ν-Fe DIFFERENTIAL CROSS SECTIONS

It is convenient to compute the QCD corrections to the parton-level cross sections in the deep inelastic scattering (DIS) scheme, where only the $\mathcal{F}_1$ and $\mathcal{F}_3$ structure functions change [3]. The NLO QCD corrections to the $\mathcal{F}_1$ structure functions are due to one-loop contributions involving a gluon, and from the emission or absorption of a real gluon, which includes scattering off the gluon sea:

$$\delta\mathcal{F}_1^{\mathrm{C}} = -\frac{4\alpha_s}{3\pi}\int_x^1 dz\left[\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{C}}\left(\frac{x}{z},Q^2\right) + 6(1-z)g\left(\frac{x}{z},Q^2\right)\right],$$
$$\delta\mathcal{F}_1^{\mathrm{N}} = -\frac{4\alpha_s}{3\pi}\int_x^1 dz\left[\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{N}}\left(\frac{x}{z},Q^2\right) + 6(g_L^2+g_R^2)(1-z)g\left(\frac{x}{z},Q^2\right)\right], \tag{3.1}$$

where $g(x,Q^2)$ is the gluon distribution function, and

$$g_{L,R}^2 \equiv (g_{L,R}^u)^2 + (g_{L,R}^d)^2. \tag{3.2}$$

The $\mathcal{F}_3$ structure functions at NLO does not get a contribution from scattering off the gluon sea, and has a similar form for the charged and neutral currents,

$$\delta\mathcal{F}_3^{\mathrm{C,N}} = -\frac{2\alpha_s}{3\pi}\int_x^1 dz\left(1+\frac{1}{z}\right)\mathcal{F}_{3\ \mathrm{LO}}^{\mathrm{C,N}}\left(\frac{x}{z},Q^2\right). \tag{3.3}$$

These expressions apply to the $\overline{\nu}_\mu$-nucleus processes as well, with the only difference that the q and $\overline{q}$ distributions have to be interchanged in the expressions for the leading-order structure functions given in Eqs. (2.4) and (2.5).

Although corrections due to electromagnetic radiation, electroweak loops, target mass, and fermion masses are important for the lowest-order cross sections, as discussed in Sec. II, they can be neglected in the computation of the order-α_s corrections. Formally, they represent higher-order terms in the expansion (2.3). For example, the parton level processes $\nu g \to \nu c\overline{c}$ and $\nu_\mu g \to \mu^- c\overline{s}$ are suppressed at small Q^2, which is an order $(\alpha_s/\pi)(m_c^2/Q^2)$ effect.

IV. TOTAL CROSS SECTIONS FOR ν-Fe SCATTERING

In this section we derive some analytical, approximate expressions for the total cross sections in neutrino deep-inelastic scattering. We begin by expanding the gauge boson propagator in powers of $Q^2/M_{W,Z}^2$, and use Eq. (2.1):

$$\frac{1}{(1+Q^2/M_{W,Z}^2)^2} \approx 1 - \frac{4xyM_N E_\nu}{M_{W,Z}^2} + O\left(\frac{(xyM_N E_\nu)^2}{M_{W,Z}^4}\right). \tag{4.1}$$

This enables us to take advantage of the following identity:

$$\int_0^1 dx x^{n-1}\int_x^1 \frac{dz}{z} f(z) q\left(\frac{x}{z}\right) = q^{(n)}\int_0^1 dz z^{(n-1)} f(z), \tag{4.2}$$

where $f(z)$ is any non-singular function, and

$$q^{(n)} \equiv \int_0^1 dx x^{n-1} q(x) \tag{4.3}$$

is the nth moment of the $q(x)$ parton distribution.

In what follows we will keep only the leading term of the expansion shown in Eq. (4.1). Furthermore, when computing the $\delta\mathcal{F}_i$ corrections to the structure functions, given in Eqs. (3.1) and (3.3), the evolution of the quark and gluon PDF's, $q_j(x,Q^2)$ and $g(x,Q^2)$, may be approximated by taking the PDF's at the average Q^2, labeled $\overline{Q^2}$, as long as the range of Q^2 is not too large. The error on the cross section, due to this approximation of the NLO QCD corrections, is of the order of $\alpha_s^2(\overline{Q^2})\ln(Q^2/\overline{Q^2})$.

As a result, the integration over x and y of the differential cross sections given in Eq. (2.2) yields

$$\sigma^{\mathrm{C,N}}(\nu_\mu \mathrm{Fe}) = \frac{M_N E_\nu G_\mathrm{F}^2}{6\pi}(\mathcal{F}_1^{\mathrm{C,N}(2)} + 3\mathcal{F}_2^{\mathrm{C,N}(2)} + 2\mathcal{F}_3^{\mathrm{C,N}(2)}). \tag{4.4}$$

The second moments of the structure functions are given by

$$\mathcal{F}_i^{\mathrm{C,N}(2)} = \int_0^1 dx x \mathcal{F}_i^{\mathrm{C,N}}(x) = \mathcal{F}_{i\ \mathrm{LO}}^{\mathrm{C,N}(2)} + \delta\mathcal{F}_i^{\mathrm{C,N}(2)} + O\left(\frac{Q^2}{M_{W,Z}^2}, \frac{\alpha}{\pi\sin^2\theta_W}, \frac{M_N^2}{Q^2}, \frac{m_c^2}{Q^2}\right), \tag{4.5}$$

for $i=1,2,3$. The second moments of the lowest-order structure functions, $\mathcal{F}_{i\ \mathrm{LO}}^{\mathrm{C,N}(2)}$, are obtained simply by taking the second moments of the PDF's in Eqs. (2.4) and (2.5).

The second moments of the $\delta\mathcal{F}_i^{\mathrm{C,N}}$ corrections to the structure functions are given by

$$\delta\mathcal{F}_1^{\mathrm{C}(2)} = -\frac{4\alpha_s}{9\pi}\left[\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{C}(2)} + \frac{3}{2}g^{(2)}\right],$$

$$\delta\mathcal{F}_1^{\mathrm{N}(2)} = -\frac{4\alpha_s}{9\pi}\left[\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{N}(2)} + \frac{3}{2}(g_L^2+g_R^2)g^{(2)}\right],$$

$$\delta\mathcal{F}_3^{\mathrm{C,N}(2)} = -\frac{5\alpha_s}{9\pi}\mathcal{F}_{3\ \mathrm{LO}}^{\mathrm{C,N}(2)}, \tag{4.6}$$

where $g^{(2)}$ is the second moment of the gluon distribution function. Recall that these results are obtained in the DIS scheme, where $\delta\mathcal{F}_2^{\mathrm{C,N}(2)}=0$.

V. ESTIMATE OF THE NEUTRAL-CURRENT TO CHARGED-CURRENT EVENT RATIO

Although an analysis of the data involving the NLO QCD corrections to the differential cross sections [Eqs. (3.1) and (3.3)] is required for a precise determination of the shift in $\sin^2\theta_W$, we now show that it is also possible to estimate theoretically this shift.

A. General results

The approximate expressions that we obtained for the total cross sections, Eq. (4.4), have the same E_ν dependence for both the neutral-current and charged-current events. Therefore, the ratio of neutral- to charged-current events is independent of the neutrino flux, and is given by the ratio of total cross sections. At leading order in α_s, α, and the various mass ratios, this is

$$R_0^\nu = \frac{2\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{N}(2)} + \mathcal{F}_{3\ \mathrm{LO}}^{\mathrm{N}(2)}}{2\mathcal{F}_{1\ \mathrm{LO}}^{\mathrm{C}(2)} + \mathcal{F}_{3\ \mathrm{LO}}^{\mathrm{C}(2)}} = g_L^2 + r g_R^2 - \left(g_L^{u\,2} - \frac{g_R^{d\,2}}{3}\right)\frac{q_-}{q_0} + \left(\frac{g_L^{d\,2}}{3} - g_R^{u\,2}\right)\frac{\overline{q_-}}{q_0}, \tag{5.1}$$

where we have introduced two linear combinations of second moments,

$$q_0 \equiv d^{(2)} + s^{(2)} + \frac{1}{3}(\overline{u^{(2)}} + \overline{c^{(2)}}),$$

$$q_- \equiv d^{(2)} - u^{(2)} + s^{(2)} - c^{(2)}. \tag{5.2}$$

The ratio r of the total cross sections for the $\bar{\nu}$Fe and νFe charged-current processes at leading order, is simply

$$r = \frac{\overline{q_0}}{q_0}. \tag{5.3}$$

The ratio of neutral- to charged-current events is changed by the NLO QCD effects to

$$R^\nu = \frac{\sigma^{\rm N}(\nu_\mu {\rm Fe})}{\sigma^{\rm C}(\nu_\mu {\rm Fe})} = R_0^\nu + \delta R_1^\nu + \delta R_3^\nu + O\left(\frac{Q^2}{M_{W,Z}^2}, \frac{\alpha}{\pi \sin^2\theta_W}, \frac{M_N^2}{Q^2}, \frac{m_c^2}{Q^2}\right). \tag{5.4}$$

The shift in R^ν from order α_s corrections to $\mathcal{F}_i$, $i=1,3$, follows from Eq. (4.4):

$$\delta R_i^\nu = c_i \frac{\delta \mathcal{F}_i^{{\rm N}(2)} - R_0^\nu \delta \mathcal{F}_i^{{\rm C}(2)}}{2\mathcal{F}_{1\ {\rm LO}}^{{\rm C}(2)} + \mathcal{F}_{3\ {\rm LO}}^{{\rm C}(2)}}, \tag{5.5}$$

where $c_1 = 1/2$ and $c_3 = 1$.

The above equation, along with the expressions for the second moments of the leading-order structure functions [see Eqs. (2.4) and (2.5)] and their NLO corrections given in Eq. (4.6) lead to an analytic formula for the shift in R^ν in terms of measured quantities. This involves only two more linear combinations of second moments:

$$q_1 \equiv d^{(2)} + s^{(2)} + \overline{u^{(2)}} + \overline{c^{(2)}} + \frac{3}{4} g^{(2)},$$

$$q_3 \equiv d^{(2)} - \overline{u^{(2)}} + s^{(2)} - \overline{c^{(2)}}. \tag{5.6}$$

The final result is

$$\delta R_1^\nu = -\frac{2\alpha_s}{27\pi}\left\{ g_R^2 (1-r)\frac{q_1}{q_0} - \frac{q_-}{q_0}\left[g_L^{u2} + g_R^{u2} - \left(g_L^{u2} - \frac{g_R^{d2}}{3}\right)\frac{q_1}{q_0}\right] + \frac{\overline{q_-}}{q_0}\left[g_L^{d2} + g_R^{d2} - \left(\frac{g_L^{d2}}{3} - g_R^{u2}\right)\frac{q_1}{q_0}\right]\right\},$$

$$\delta R_3^\nu = \frac{5\alpha_s}{27\pi}\left\{ g_R^2 (1+r)\frac{q_3}{q_0} + \frac{q_-}{q_0}\left[g_L^{u2} - g_R^{u2} - \left(g_L^{u2} - \frac{g_R^{d2}}{3}\right)\frac{q_3}{q_0}\right] + \frac{\overline{q_-}}{q_0}\left[g_L^{d2} - g_R^{d2} + \left(\frac{g_L^{d2}}{3} - g_R^{u2}\right)\frac{q_3}{q_0}\right]\right\}. \tag{5.7}$$

For $\bar{\nu}$Fe scattering, the ratio of neutral- to charged-current events at leading order, $R_0^{\bar{\nu}}$, is given by the right-hand side of Eq. (5.1) with the following substitutions: $r \to 1/r, q_0 \to \overline{q_0}$, $q_- \leftrightarrow \overline{q_-}$. This is shifted at NLO by QCD effects by $\delta R_1^{\bar{\nu}} + \delta R_3^{\bar{\nu}}$, where $\delta R_{1,3}^{\bar{\nu}}$ are obtained from Eqs. (5.7) by performing the same substitutions as above, and in addition $q_1 \to \overline{q_1}, q_3 \to \overline{q_3}$.

B. Origin of the $\sin^4\theta_W$ suppression

Before evaluating the size of the NLO corrections given in Eq. (5.7), there is an important observation to be made. In the ''enhanced isospin symmetry'' limit, where

$$d^{(2)} = u^{(2)}, \quad \overline{d^{(2)}} = \overline{u^{(2)}}, \quad s^{(2)} = c^{(2)}, \quad \overline{s^{(2)}} = \overline{c^{(2)}}, \tag{5.8}$$

so that $q_- = \overline{q_-} = 0$, Eq. (5.7) implies that $\delta R_{1,3}^\nu$ are parametrically of the order of $g_R^2 \alpha_s/\pi$. Given that

$$g_R^2 = (5/9)\sin^4\theta_W \approx 2.76 \times 10^{-2}, \tag{5.9}$$

the NLO QCD corrections to R^ν are suppressed by a factor of approximately 30 compared to the naive expectation of α_s/π. It is therefore interesting to understand the origin of this suppression.

To this end, notice that in the limit where the quark masses are ignored, the cross section for the neutral-current process can be written as a sum of cross sections for neutrino scattering off left- and right-handed quarks:

$$\sigma^{\rm N}(\nu_\mu {\rm Fe}) = \sigma_{0L}^{\rm N} + \sigma_{0R}^{\rm N} + \delta\sigma_L^{\rm N} + \delta\sigma_R^{\rm N}, \tag{5.10}$$

where the subscript 0 refers to the leading order terms, and $\delta\sigma$ are the QCD corrections. If the enhanced isospin symmetry were exact, then

$$\frac{\sigma_{0L}^{\rm N}}{\sigma_0^{\rm C}} = \frac{\delta\sigma_L^{\rm N}}{\delta\sigma^{\rm C}} = g_L^2, \tag{5.11}$$

so that

$$R^\nu = \frac{\sigma_{0L}^{\rm N}}{\sigma_0^{\rm C}} + \frac{\sigma_{0R}^{\rm N} + \delta\sigma_R^{\rm N}}{\sigma_0^{\rm C} + \delta\sigma^{\rm C}}. \tag{5.12}$$

This equation shows that the QCD corrections to R^ν would vanish *to all orders* if the neutral-current involving the right-handed quarks were not present [in the limit where the quarks are massless and the enhanced isospin symmetry, Eq. (5.8), is exact]. The factor of g_R^2 is a consequence of this fact.

In reality isospin symmetry is broken due to the different number of neutrons and protons in the target, as well as by the quark mass differences and electromagnetic interactions. Therefore, in addition to the terms of order $g_R^2\alpha_s/\pi$, R^ν gets corrections of order $(q_-/q_0)\alpha_s/\pi$, as can be seen in Eq. (5.7). For an approximately isoscalar target such as iron, the terms of order $g_R^2\alpha_s/\pi$ dominate, albeit by a small margin, as discussed in the next subsection.

C. Size of the corrections to R^ν and $R^{\bar{\nu}}$

The nine second moments, $u^{(2)}$, $d^{(2)}$, $s^{(2)}$, $c^{(2)}$, $\overline{u}^{(2)}$, $\overline{d}^{(2)}$, $\overline{s}^{(2)}$, $\overline{c}^{(2)}$ and $g^{(2)}$, are given by an average over the second moments of the nucleon PDF's inside the iron nucleus, with corrections due to nuclear interactions. They are evaluated at an average Q^2. For NuTeV, the average value for Q^2 is 25.6 GeV^2 for the ν_μ beam and 15.4 GeV^2 for the $\bar{\nu}_\mu$ beam. We choose $\overline{Q^2}$ to be around 20 GeV^2.

The PDF's used in the NuTeV analysis come from a fit to the charged-current differential cross sections measured by the CCFR experiment [21] with the same iron target. The fit and the Monte Carlo simulation used for extracting $\sin^2\theta_W$ employ the same cross section model, which is described in Ref. [17]. At $\overline{Q^2}=20$ GeV^2, the fit gives the following values for the second moments [22]: $u^{(2)}\approx 0.196$, $d^{(2)}\approx 0.204$, $\overline{u}^{(2)}\approx\overline{d}^{(2)}\approx 0.032$, $s^{(2)}\approx\overline{s}^{(2)}\approx 0.013$, $c^{(2)}\approx\overline{c}^{(2)}\approx 0.006$, and $g^{(2)}\approx 0.498$. This fit assumed $s(x,Q^2)=\overline{s}(x,Q^2)$, $c(x,Q^2)=\overline{c}(x,Q^2)$, and isospin symmetry in the sense that the only difference between the u and d distributions is due to the different number of protons and neutrons in the iron nucleus.

An asymmetry of order a few percent between the s and $\overline{s}$ distributions, and isospin-breaking effects, due to the up-down quark mass splitting and electroweak interactions, expected to be of order $(m_d-m_u)/\Lambda_{\rm QCD}$, i.e. also a few percent, would be important for the leading order $R_0^{\nu,\bar{\nu}}$ ratios [10], but can be neglected in the estimate of the NLO corrections. Also, the shifts $\delta R_{1,3}^{\nu,\bar{\nu}}$ are only mildly sensitive to the choice of a different set of PDF's. The main reason is that only five independent combinations of second moments appear in Eqs. (5.7): $r\approx 0.49, q_1/q_0\approx 2.74, q_3/q_0\approx 0.78, q_-/q_0\approx 0.07, \overline{q}_-/q_0\approx 0.03$. The other relevant combinations of second moments can be expressed in terms of these. For example,

$$\frac{\overline{q}_1}{\overline{q}_0}=\frac{q_1-q_-+\overline{q}_-}{rq_0},$$

$$\frac{\overline{q}_3}{\overline{q}_0}=\frac{-q_3+q_-+\overline{q}_-}{rq_0}. \tag{5.13}$$

Note that q_-/q_0 and $\overline{q}_-/q_0$ have values comparable with g_R, and therefore the terms proportional with $(q_-/q_0)\alpha_s/\pi$ and $(\overline{q}_-/q_0)\alpha_s/\pi$ cannot be neglected in Eq. (5.7). Nonetheless, these terms are also multiplied by factors of the order of $g_L^{u2}\approx 0.12$ and $g_L^{d2}\approx 0.18$, so that the isospin symmetric corrections of order $g_R^2\alpha_s/\pi$ dominate.

For $\alpha_s=0.2$ and $\sin^2\theta_W=0.2227$ we obtain the following values for the shifts in R^ν due to NLO QCD corrections to the $\mathcal{F}_1$ and $\mathcal{F}_3$ structure functions [see Eqs. (5.7)]:

$$\delta R_1^\nu\approx -2.5\times 10^{-4},$$

$$\delta R_3^\nu\approx 4.6\times 10^{-4}. \tag{5.14}$$

In the case of the $\bar{\nu}$ beam, the results are

$$\delta R_1^{\bar{\nu}}\approx 6.1\times 10^{-4},$$

$$\delta R_3^{\bar{\nu}}\approx -9.9\times 10^{-4}. \tag{5.15}$$

These corrections are of the order of the standard deviations quoted by the NuTeV Collaboration [1] for the measured $R_{\rm exp}^\nu$ and $R_{\rm exp}^{\bar{\nu}}$: 7×10^{-4} and 16×10^{-4}, respectively. Note though that the measured quantities ($R_{\rm exp}^\nu$ and $R_{\rm exp}^{\bar{\nu}}$) are ratios of the numbers of short and long events observed in the NuTeV detector, and therefore differ from the ratios of neutral- and charged-current events (R^ν and $R^{\bar{\nu}}$) due to the experimental cuts, backgrounds and detector acceptance. A discussion of these effects, albeit primarily in the context of QCD corrections to $R_{\rm PW}$, is given in Ref. [12].

Comparing our results given in Eqs. (5.14) and (5.15) with the numerical results given in Ref. [13] we observe that the size of the effect is of the same order of magnitude, but the sign of $\delta R^\nu=\delta R_1^\nu+\delta R_3^\nu$ is opposite. The various approximations that we have employed in obtaining the analytical expression for δR^ν, such as ignoring the charm mass and the evolution of the PDF's, which introduce errors of the order of $(\alpha_s/\pi)(m_c^2/\overline{Q^2})$ and $\alpha_s^2(\overline{Q^2})\ln(Q^2/\overline{Q^2})$, respectively, do not seem to be sufficient to account for this difference. It remains to be seen whether the effect of the hadronic energy cut used in Ref. [13] is large enough to explain the difference [22].

VI. THEORETICAL ESTIMATE OF THE SHIFT IN $\sin^2\theta_W$

The approximate results for the shifts in R^ν and $R^{\bar{\nu}}$ obtained in the previous section should in principle allow an estimate of the corrections to the value of $\sin^2\theta_W$ determined by the NuTeV Collaboration. In practice, however, there are several elements in the NuTeV analysis that make a theoretical estimate somewhat problematic. Here we point out a few complications.

A. Relation between $\sin^2\theta_W$ and R^ν, $R^{\bar{\nu}}$

The NuTeV analysis includes a phenomenological description of the so-called longitudinal structure function, which changes the relation between the $\mathcal{F}_1$ and $\mathcal{F}_2$ structure functions. Effectively, this procedure approximately accounts for the QCD corrections to $\mathcal{F}_1$. We will therefore consider only the impact of the QCD corrections to $\mathcal{F}_3$, which lead to the values for δR_3^ν and $\delta R_3^{\bar{\nu}}$ given in Eqs. (5.14) and (5.15).

Naively, the shift in $\sin^2\theta_W$ due to a shift in the predicted value of R^ν can be derived immediately from the expression of R_0^ν given in Eq. (5.1):

$$\delta \sin^2\theta_W \approx \frac{\delta R_3^\nu}{1-(10/9)(1+r)\sin^2\theta_W} \approx 0.7\times 10^{-3}. \qquad (6.1)$$

However, various effects change this relation. These include a "cross-talk" between the charged- and neutral-current events, experimental cuts, and the corrections to the structure functions listed in Eq. (2.3). The NuTeV analysis has computed these effects using a Monte Carlo simulation. Note that δR_3^ν and $\delta R_3^{\bar\nu}$ can be viewed as approximate shifts in the results for R^ν and $R^{\bar\nu}$ given by the Monte Carlo simulation used by NuTeV. The relation between these shifts and the shift in $\sin^2\theta_W$ is given in Sec. 8 of Ref. [17]:

$$\delta \sin^2\theta_W = \frac{1}{b}(\delta R_3^\nu - a\,\delta R_3^{\bar\nu}). \qquad (6.2)$$

For the fit reported in the NuTeV result [1], where the charm mass is constrained, $a=0.249$ and $b=0.617$, giving $\delta\sin^2\theta_W \approx 1.1\times 10^{-3}$, which is an increase of about 0.7σ. For the fit without constraints, $a=0.453$ and $b=0.612$, and the increase in $\sin^2\theta_W$ is close to 1σ. Thus, the inclusion of the corrections to $\mathcal{F}_3$ alone tend to increase the deviation from the standard model.

B. QCD corrections to the parton distributions

The Q^2 dependence of the PDF's is an effect of order $\alpha_s(\overline{Q^2})/\pi \ln(Q^2/\overline{Q^2})$, where $\overline{Q^2}$ is an average value for Q^2. The NuTeV Collaboration has approximated the Q^2 dependence by the Buras-Gaemers evolution [23]. Using the exact QCD evolution could modify the values derived from the Chicago-Columbia-Fermilab-Rochester (CCFR) data of the PDF's at our reference point of $Q^2=20$ GeV^2. We will not attempt here to estimate this effect. We only mention that this leads to a correction to $\sin^2\theta_W$ that is independent of the one given in Eq. (6.2). Only at order $g_R^2\alpha_s^2(\overline{Q^2})/\pi^2\ln(Q^2/\overline{Q^2})$ do the two corrections become correlated.

The PDF's used by the NuTeV Collaboration are extracted from a fit of the differential cross sections to the ν and $\bar\nu$ charged-current CCFR data. The inclusion of order α_s terms in the cross sections changes the fit. The shifts in the quark PDF's lead to corrections of order α_s to the r ratio that enters in the expression for R_0^ν given in Eq. (5.1). Therefore, we expect additional corrections to $\sin^2\theta_W$ of order $g_R^2\alpha_s/\pi$ that may change the result by a factor of order unity and unknown sign.

VII. CONCLUSIONS

We have presented an analysis of the $O(\alpha_s)$ radiative corrections to the ratios of neutral- and charged-current cross sections, R^ν and $R^{\bar\nu}$. We have shown that these effects are smaller than the $O(\alpha_s/\pi)$ one might expect *a priori*, because of a suppression factor of $\sin^4\theta_W$ in the dominant contribution. On the other hand, the effects turn out to be of the same order as the 1-σ error in the experimental results of NuTeV.

Our results indicate the importance of a full NLO analysis of the NuTeV data, which would include the NLO QCD corrections to the cross sections [see Eqs. (3.1) and (3.3)] as well as the QCD evolution of the PDF's, in both the Monte Carlo simulation used for determining $\sin^2\theta_W$ and the fit to the charged-current data used for extracting the PDF's. In addition, our results will provide a simple check when such an analysis is performed.

It is important to keep in mind that the NLO QCD corrections discussed here are independent at this order of the corrections discussed in Refs. [18–20], which require a refit of the data that allows *both* a strange asymmetry and a violation of isospin symmetry.

Note added. After we submitted this paper, Kretzer and Reno added a note to Ref. [13] regarding the sign difference between their numerical result for δR^ν and our approximate analytical result. They state that the hadronic energy cut does not flip the sign of δR^ν, but the inclusion of the evolution of the PDF's used for computing the NLO QCD correction does in fact flip the sign. Although not expected based on the parametric estimate of the effect, such a sign flip is possible in view of the fact that δR^ν is given by the sum of two comparable contributions of opposite signs, δR_1^ν and δR_3^ν [see Eq. (5.14)]. In any case, our main conclusion, which refers to the order of magnitude of the NLO QCD correction, remains valid.

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