Bounds on new physics from $B \to V_1 V_2$ decays

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We consider the possibility that physics beyond the standard model contributes to the decays $B \to V_1 V_2$, where V_1 and V_2 are vector mesons. We show that a time-dependent angular analysis of $B \to V_1 V_2$ decays provides many tests for this new physics (NP). Furthermore, although one cannot solve for the NP parameters, we show that this angular analysis allows one to put bounds on these parameters. This can be useful in estimating the scale of NP, and can tell us whether any NP found directly at future high-energy colliders can be responsible for effects seen in $B \to V_1 V_2$ decays.

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I. INTRODUCTION

Within the standard model (SM), a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is responsible for *CP* violation [1]. By studying *CP*-violating processes in the *B* system, one can test this explanation. If any discrepancy with the SM predictions is found, this would be evidence for physics beyond the SM.

There are a great many tests for the presence of new physics (NP) in *B* decays [2]. Should a signal for NP be found, there are basically two ways to proceed. One can examine various models of physics beyond the SM to see whether a particular model can account for the experimental results. Alternatively, one can perform a model-independent analysis to learn about general properties of the NP responsible for the signal. Most theoretical work has concentrated on the first approach.

For example, within the SM, the *CP*-violating asymmetries in $B_d^0(t) \to J/\Psi K_s$ and $B_d^0(t) \to \phi K_s$ both measure the *CP* phase β, to a good approximation [3]. However, although the BaBar measurement of the *CP* asymmetry in $B_d^0(t) \to \phi K_s$ agrees with that found in $B_d^0(t) \to J/\psi K_s$ (within errors), the Belle measurement disagrees at the level of 3.5σ [4]. This suggests that physics beyond the SM—specifically new decay amplitudes in $B \to \phi K$—may be present. In light of this, many papers have been written to show how particular models of NP can account for this discrepancy [5]. On the other hand, only two papers contain a model-independent analysis of $B_d^0(t) \to \phi K_s$ [6] (and even here some theoretical numerical input is required).

In this paper, we show how model-independent information about new physics can be obtained from an angular analysis of $B \to V_1 V_2$ decays, where V_1 and V_2 are vector mesons. This method is applicable to those $B \to V_1 V_2$ decays in which (i) $\overline{V}_1 \overline{V}_2 = V_1 V_2$, so that this final state is accessible to both B^0 and $\overline{B}^0$, and (ii) a single decay amplitude dominates in the SM. The only theoretical assumption we make is that there is only a single NP amplitude, with a different weak phase from that of the SM amplitude, contributing to these decays. In the event that a signal for NP is found, we demonstrate that one can place *lower* bounds on the NP parameters [7].

If physics beyond the SM contributes to $B_d^0(t) \to \phi K_s$, there should also be NP signals in the corresponding $B \to V_1 V_2$ decay, $B_d^0(t) \to \phi K^{*0}$. Our method can be used in this situation to get information about the NP. It can also be applied to $B_d^0(t) \to J/\psi K^{*0}$, $B_d^0(t) \to K^{*0}\overline{K}^{*0}$, $B_s^0(t) \to J/\psi\phi$, etc., should NP signals be found in these decays.[1]

Any new-physics effects in *B* decays are necessarily virtual. On the other hand, future experiments at the CERN Large Hadron Collider (LHC) and at a linear e^+e^- collider (GLC) will make direct searches for such NP. Should NP be found in both $B \to V_1 V_2$ decays and at the LHC/GLC, the bounds from the angular analysis can tell us whether the NP seen at LHC/GLC can be responsible for the effects in $B \to V_1 V_2$ decays.

We begin in Sec. II by describing the theoretical framework of our method. Signals of new physics are examined in Sec. III. The main results—how to place bounds on the theoretical NP parameters—are presented in Sec. IV. We discuss and summarize these results in Sec. V.

II. THEORETICAL FRAMEWORK

Consider a $B \to V_1 V_2$ decay which is dominated by a single weak decay amplitude within the SM. This holds for processes which are described by the quark-level decays $\overline{b} \to \overline{c}\overline{c}\overline{s}$, $\overline{b} \to \overline{s}\overline{s}\overline{s}$, or $\overline{b} \to \overline{s}\overline{d}\overline{d}$. In all cases, the weak phase of the SM amplitude is zero in the standard parametrization [1]. Suppose now that there is a single new-physics amplitude, with a different weak phase, that contributes to the decay.

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[1]Our analysis treats only the situation where there are additional NP decays amplitudes; it does not apply to the case where the NP appears only in B^0-$\overline{B}^0$ mixing.

The decay amplitude for each of the three possible helicity states may be written as

$$A_\lambda \equiv \mathrm{amp}(B\to V_1V_2)_\lambda = a_\lambda e^{i\delta_\lambda^a} + b_\lambda e^{i\phi} e^{i\delta_\lambda^b},$$
$$\overline{A}_\lambda \equiv \mathrm{amp}(\overline{B}\to (V_1V_2)_\lambda = a_\lambda e^{i\delta_\lambda^a} + b_\lambda e^{-i\phi} e^{i\delta_\lambda^b}, \tag{1}$$

where a_λ and b_λ represent the SM and NP amplitudes, respectively, ϕ is the new-physics weak phase, the $\delta_\lambda^{a,b}$ are the strong phases, and the helicity index λ takes the values $\{0,\|,\perp\}$. Using *CPT* invariance, the full decay amplitudes can be written as

$$\mathcal{A} = \mathrm{amp}(B\to V_1V_2) = A_0 g_0 + A_\| g_\| + iA_\perp g_\perp ,$$
$$\overline{\mathcal{A}} = \mathrm{amp}(\overline{B}\to V_1V_2) = \overline{A}_0 g_0 + \overline{A}_\| g_\| - i\overline{A}_\perp g_\perp , \tag{2}$$

where the g_λ are the coefficients of the helicity amplitudes written in the linear polarization basis. The g_λ depend only on the angles describing the kinematics [8].

Note that it is not a strong assumption to consider a single NP amplitude. First, the new physics is expected to be heavy, so that all strong phases δ_λ should be small. In this case, since the δ_λ are all of similar size, our parametrization above is adequate. Second, if it happens that this is not the case, and there are several different contributing NP amplitudes, our analysis pertains to the dominant signal. Finally, if all the NP amplitudes are of the same size, our approach provides an order-of-magnitude estimate for the size of new physics.

Equations (1) and (2) enable us to write the time-dependent decay rates as

$$\Gamma\left(\overset{(-)}{B}(t)\to V_1V_2\right) = e^{-\Gamma t}\sum_{\lambda\le\sigma}\left[\Lambda_{\lambda\sigma}\pm\Sigma_{\lambda\sigma}\cos(\Delta M t) \mp \rho_{\lambda\sigma}\sin(\Delta M t)\right] g_\lambda g_\sigma . \tag{3}$$

Thus, by performing a time-dependent angular analysis of the decay $B(t)\to V_1V_2$, one can measure 18 observables. These are

$$\Lambda_{\lambda\lambda} = \frac{1}{2}\left(|A_\lambda|^2 + |\overline{A}_\lambda|^2\right), \quad \Sigma_{\lambda\lambda} = \frac{1}{2}\left(|A_\lambda|^2 - |\overline{A}_\lambda|^2\right),$$
$$\Lambda_{\perp i} = -\mathrm{Im}\left(A_\perp A_i^* - \overline{A}_\perp \overline{A}_i^*\right), \quad \Lambda_{\|0} = \mathrm{Re}\left(A_\| A_0^* + \overline{A}_\| \overline{A}_0^*\right),$$
$$\Sigma_{\perp i} = -\mathrm{Im}\left(A_\perp A_i^* + \overline{A}_\perp \overline{A}_i^*\right), \quad \Sigma_{\|0} = \mathrm{Re}\left(A_\| A_0^* - \overline{A}_\| \overline{A}_0^*\right),$$
$$\rho_{\perp i} = \mathrm{Re}\left(\frac{q}{p}\left[A_\perp^* \overline{A}_i + A_i^* \overline{A}_\perp\right]\right), \quad \rho_{\perp\perp} = \mathrm{Im}\left(\frac{q}{p} A_\perp^* \overline{A}_\perp\right),$$
$$\rho_{\|0} = -\mathrm{Im}\left(\frac{q}{p}\left[A_\|^* \overline{A}_0 + A_0^* \overline{A}_\|\right]\right), \quad \rho_{ii} = -\mathrm{Im}\left(\frac{q}{p} A_i^* \overline{A}_i\right), \tag{4}$$

where $i=\{0,\|\}$. In the above, q/p is the weak phase factor associated with B-$\overline{B}$ mixing. For B_d^0 mesons, $q/p = \exp(-2i\beta)$, while $q/p = 1$ for B_s^0 mesons. Henceforth we concentrate on the decays of B_d^0 mesons, though our results can easily be adapted to B_s^0 decays. Note that β may include NP effects in B_d^0-$\overline{B}_d^0$ mixing. Note also that the signs of the various $\rho_{\lambda\lambda}$ terms depend on the *CP* parity of the various helicity states. We have chosen the sign of ρ_{00} and $\rho_{\|\|}$ to be -1, which corresponds to the final state ϕK^*.

Not all of the 18 observables are independent. There are a total of six amplitudes describing $B\to V_1V_2$ and $\overline{B}\to V_1V_2$ decays [Eq. (1)]. Thus, at best one can measure the magnitudes and relative phases of these six amplitudes, giving 11 independent measurements.

The 18 observables given above can be written in terms of 13 theoretical parameters: three a_λ's, three b_λ's, β, ϕ, and five strong phase differences defined by $\delta_\lambda \equiv \delta_\lambda^b - \delta_\lambda^a$, $\Delta_i \equiv \delta_\perp^a - \delta_i^a$. The explicit expressions for the observables are as follows:

$$\Lambda_{\lambda\lambda} = a_\lambda^2 + b_\lambda^2 + 2a_\lambda b_\lambda \cos\delta_\lambda \cos\phi,$$

$$\Sigma_{\lambda\lambda} = -2a_\lambda b_\lambda \sin\delta_\lambda \sin\phi,$$

$$\Lambda_{\perp i} = 2\left[a_\perp b_i \cos(\Delta_i - \delta_i) - a_i b_\perp \cos(\Delta_i + \delta_\perp)\right]\sin\phi,$$

$$\begin{aligned}\Lambda_{\|0} = 2\big[&a_\| a_0 \cos(\Delta_0 - \Delta_\|) \\ &+ a_\| b_0 \cos(\Delta_0 - \Delta_\| - \delta_0)\cos\phi + a_0 b_\| \cos(\Delta_0 - \Delta_\| \\ &+ \delta_\|)\cos\phi + b_\| b_0 \cos(\Delta_0 - \Delta_\| + \delta_\| - \delta_0)\big],\end{aligned}$$

$$\begin{aligned}\Sigma_{\perp i} = -2\big[&a_\perp a_i \sin\Delta_i + a_\perp b_i \sin(\Delta_i - \delta_i)\cos\phi \\ &+ a_i b_\perp \sin(\Delta_i + \delta_\perp)\cos\phi \\ &+ b_\perp b_i \sin(\Delta_i + \delta_\perp - \delta_i)\big],\end{aligned}$$

$$\begin{aligned}\Sigma_{\|0} = 2\big[&a_\| b_0 \sin(\Delta_0 - \Delta_\| - \delta_0) \\ &- a_0 b_\| \sin(\Delta_0 - \Delta_\| + \delta_\|)\big]\sin\phi,\end{aligned}$$

$$\begin{aligned}\rho_{ii} = a_i^2 \sin 2\beta &+ 2a_i b_i \cos\delta_i \sin(2\beta + \phi) \\ &+ b_i^2 \sin(2\beta + 2\phi),\end{aligned}$$

$$\begin{aligned}\rho_{\perp\perp} = -a_\perp^2 \sin 2\beta &- 2a_\perp b_\perp \cos\delta_\perp \sin(2\beta + \phi) \\ &- b_\perp^2 \sin(2\beta + 2\phi),\end{aligned}$$

$$\begin{aligned}\rho_{\perp i} = 2\big[&a_i a_\perp \cos\Delta_i \cos 2\beta \\ &+ a_\perp b_i \cos(\Delta_i - \delta_i)\cos(2\beta + \phi) \\ &+ a_i b_\perp \cos(\Delta_i + \delta_\perp)\cos(2\beta + \phi) \\ &+ b_i b_\perp \cos(\Delta_i + \delta_\perp - \delta_i)\cos(2\beta + 2\phi)\big],\end{aligned}$$

$$\begin{aligned}\rho_{\|0} = 2\big[&a_0 a_\| \cos(\Delta_0 - \Delta_\|)\sin 2\beta \\ &+ a_\| b_0 \cos(\Delta_0 - \Delta_\| - \delta_0)\sin(2\beta + \phi) \\ &+ a_0 b_\| \cos(\Delta_0 - \Delta_\| + \delta_\|)\sin(2\beta + \phi) \\ &+ b_0 b_\| \cos(\Delta_0 - \Delta_\| + \delta_\| - \delta_0)\sin(2\beta + 2\phi)\big].\end{aligned} \tag{5}$$

In subsequent sections, we will work extensively with these expressions.

It is straightforward to see that, in the presence of new physics, one cannot extract the phase β. There are 11 independent observables, but 13 theoretical parameters. Since the number of measurements is fewer than the number of parameters, one cannot express any of the theoretical unknowns purely in terms of observables. In particular, it is impossible to extract β cleanly. Nevertheless, we will show that the angular analysis does allow one to obtain significant *lower bounds* on the NP parameters, as well as on the deviation of β from its measured value.

In our analysis, we usually assume that β has not been measured independently, so that there are indeed 13 unknown theoretical parameters. However, this might not be the case. For example, the decay $B_d^0(t)\to J/\psi K_s$ [or $B_d^0(t)\to J/\psi K^{*0}$] is dominated by the tree contribution. Even if there is new physics in the $\bar{b}\to\bar{c}c\bar{s}$ penguin amplitude, its effect will probably be very small. If it is found experimentally that this is so (e.g., using the NP signals discussed in the next section), the measurement of the *CP* asymmetry in this mode gives the true (SM) value of β. This can then be used as an input for other modes, such as $B_d^0(t)\to\phi K^{*0}$. In this case there are only 12 theoretical parameters, and the analysis simplifies. We will comment on this possibility in Sec. IV F.

III. SIGNALS OF NEW PHYSICS

As mentioned in the Introduction, lower bounds on new-physics parameters are possible only if there is a signal of physics beyond the SM. In this section, we discuss the possible new-physics signals in $B\to V_1V_2$ decays.

In the absence of NP, the b_λ are zero in Eq. (1). The number of parameters is then reduced from 13 to 6: three a_λ's, two strong phase differences (Δ_i), and β. It is straightforward to show that all six parameters can be determined cleanly in terms of observables [Eq. (5)]. However, there are a total of 18 observables. Thus, there must exist 12 relations among the observables in the absence of NP. These are

$$\Sigma_{\lambda\lambda}=\Lambda_{\perp i}=\Sigma_{\parallel 0}=0,$$

$$\frac{\rho_{ii}}{\Lambda_{ii}}=-\frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}}=\frac{\rho_{\parallel 0}}{\Lambda_{\parallel 0}},$$

$$\Lambda_{\parallel 0}=\frac{1}{2\Lambda_{\perp\perp}}\left(\frac{\Lambda_{\lambda\lambda}^2\rho_{\perp 0}\rho_{\perp\parallel}+\Sigma_{\perp 0}\Sigma_{\perp\parallel}(\Lambda_{\lambda\lambda}^2-\rho_{\lambda\lambda}^2)}{\Lambda_{\lambda\lambda}^2-\rho_{\lambda\lambda}^2}\right),$$

$$\frac{\rho_{\perp i}^2}{4\Lambda_{\perp\perp}\Lambda_{ii}-\Sigma_{\perp i}^2}=\frac{\Lambda_{\perp\perp}^2-\rho_{\perp\perp}^2}{\Lambda_{\perp\perp}^2}. \qquad (6)$$

The key point is the following: *the violation of any of the above relations will be a smoking-gun signal of NP*. We therefore see that the angular analysis of $B\to V_1V_2$ decays provides numerous tests for the presence of NP.[2]

Since there are 11 independent observables and six parameters in the SM, one might expect that only five tests are needed to verify the presence of NP. However, since the equations in Eq. (5) are nonlinear, this logic can fail: if the SM parameters take certain special values, more tests are needed. For example, suppose that $b_\parallel=b_\perp=0$ and $\delta_0=0$. Since $b_0\neq 0$, NP is present. We have $\Sigma_{\lambda\lambda}=\Lambda_{\perp\parallel}=0$. If Δ_0 takes the value $\pi/2$, we will also find that $\Lambda_{\perp 0}=0$. Thus, despite the presence of NP, 5 of the 12 tests above agree with the SM. In this case, further tests are needed to confirm the fact that NP is present. In the most general case, *all* 12 tests above are needed to search for NP. (In any event, because it is not known *a priori* which observables will be measured, it is important to have a list of all NP tests.)

We should stress here that the list of NP signals is independent of the parametrization of new physics. That is, even if there are several contributing amplitudes, the NP can still be discovered through the tests in Eq. (6). Furthermore, even in this general case, it is necessary to perform all 12 tests in order to show that NP is not present.

The observable $\Lambda_{\perp i}$ deserves special attention. It is the coefficient of the *T*-odd "triple product" in $B\to V_1V_2$ decays, $\vec{q}\cdot(\vec{\varepsilon}_1\times\vec{\varepsilon}_2)$, where $\vec{q}$ is the momentum of one of the final vector mesons in the rest frame of the B, and $\vec{\varepsilon}_{1,2}$ are the polarizations of V_1 and V_2 [9]. From Eq. (5), one sees that even if the strong phase differences vanish, $\Lambda_{\perp i}$ is nonzero in the presence of new physics ($\phi\neq 0$), in contrast to the direct *CP* asymmetries (proportional to $\Sigma_{\lambda\lambda}$). This is due to the fact that the $\perp$ helicity is *CP*-odd, while the 0 and $\parallel$ helicities are *CP*-even. Thus, $\perp$-0 and $\perp$-$\parallel$ interferences include an additional factor of "i" in the full decay amplitudes [Eq. (2)], which leads to the cosine dependence on the strong phases.

Although the reconstruction of the full $B_d^0(t)$ and $\overline{B}_d^0(t)$ decay rates in Eq. (3) requires both tagging and time-dependent measurements, the $\Lambda_{\lambda\sigma}$ terms remain even if the two rates for $B_d^0(t)$ and $\overline{B}_d^0(t)$ decays are added together. Note also that these terms are time-independent. Therefore, *no tagging or time-dependent measurements are needed to extract* $\Lambda_{\perp i}$. It is only necessary to perform an angular analysis of the final state V_1V_2. Thus, this measurement can even be made at a symmetric B factory.

The decays of charged B mesons to vector-vector final states are even simpler to analyze since no mixing is involved. One can in principle combine charged and neutral B

[2]Note that, despite the many tests, it is still possible for the NP to remain hidden. If the three strong phase differences δ_λ vanish, and the ratio $r_\lambda\equiv b_\lambda/a_\lambda$ is the same for all helicities, i.e., $r_0=r_\parallel=r_\perp$, then it is easy to show that the relations in Eq. (6) are all satisfied. Thus, if these very special conditions happen to hold, the angular analysis of $B\to V_1V_2$ would show no signal for NP even if it is present, and the measured value of β would not correspond to its true (SM) value. Still, we should stress that it is highly unlikely that the NP parameters would respect such a singular situation.

decays to increase the sensitivity to new physics. For example, for $B\to J/\psi K^*$ decays, one simply performs an angular analysis on all decays in which a J/ψ is produced accompanied by a charged or neutral K^*. A nonzero value of $\Lambda_{\perp i}$ would be a clear signal for new physics [10].

IV. BOUNDS ON THE THEORETICAL PARAMETERS

In this section, we explore the constraints on the size of new physics, assuming that a NP signal is observed in $B\to V_1V_2$. As we have shown, the amplitudes are written in terms of 13 theoretical parameters (including β), but there are only 11 independent observables. Since the number of unknowns is greater than the number of observables, naively one would think that it is not possible to obtain any information about the NP parameters. However, since the expressions for the observables in terms of the theoretical parameters are nonlinear [Eq. (5)], it is in fact possible to obtain *bounds* on the NP parameters. One can even put a lower bound on the difference between the measured value of β (which is affected by the presence of NP) and its true (SM) value.

The first step is to reduce the number of unknowns in the expressions for the observables. That is, even though one cannot solve for the theoretical parameters in terms of observables, one can obtain a partial solution, in which observables are written in terms of a smaller number of parameters plus other observables.

For $B\to V_1V_2$ decays, the analogue of the usual direct *CP* asymmetry $a_{\rm dir}^{CP}$ is $a_\lambda^{\rm dir}\equiv\Sigma_{\lambda\lambda}/\Lambda_{\lambda\lambda}$, which is helicity-dependent. We define the related quantity

$$y_\lambda\equiv\sqrt{1-\Sigma_{\lambda\lambda}^2/\Lambda_{\lambda\lambda}^2}. \tag{7}$$

The measured value of $\sin 2\beta$ can also depend on the helicity of the final state: $\rho_{\lambda\lambda}$ can be recast in terms of a measured weak phase $2\beta_\lambda^{\rm meas}$, defined as

$$\sin 2\beta_\lambda^{\rm meas}\equiv\frac{\pm\rho_{\lambda\lambda}}{\sqrt{\Lambda_{\lambda\lambda}^2-\Sigma_{\lambda\lambda}^2}}, \tag{8}$$

where the $+$ $(-)$ sign corresponds to $\lambda=0,\|$ $(\perp)$.

It is possible to express the nine theoretical parameters a_λ, b_λ, and δ_λ in terms of the nine observables $\Lambda_{\lambda\lambda}$, $\Sigma_{\lambda\lambda}$, and $\rho_{\lambda\lambda}$ and the parameters β and ϕ. The other observables can in turn be expressed in terms of $\Lambda_{\lambda\lambda}$, $\Sigma_{\lambda\lambda}$, and $\rho_{\lambda\lambda}$, along with the three theoretical parameters $\beta+\phi$ and Δ_i. Using the expressions for $\Lambda_{\lambda\lambda}$, $\Sigma_{\lambda\lambda}$, and $\beta_\lambda^{\rm meas}$ above, one can express a_λ and b_λ as follows:

$$2a_\lambda^2\sin^2\phi=\Lambda_{\lambda\lambda}[1-y_\lambda\cos(2\beta_\lambda^{\rm meas}-2\beta-2\phi)], \tag{9}$$

$$2b_\lambda^2\sin^2\phi=\Lambda_{\lambda\lambda}[1-y_\lambda\cos(2\beta_\lambda^{\rm meas}-2\beta)]. \tag{10}$$

These expressions will play a critical role in the derivation of bounds on the NP parameters.

The seemingly impossible task of eliminating 10 combinations of the theoretical parameters a_λ, b_λ, δ_λ, β, and ϕ in terms of the observables $\Lambda_{\lambda\lambda}$, $\Sigma_{\lambda\lambda}$, and $\rho_{\lambda\lambda}$ and variable $\beta+\phi$ becomes possible by using the following relation:

$$\begin{aligned}\frac{b_\lambda}{a_\lambda}\cos\delta_\lambda\cos\phi&=\frac{-2\Lambda_{\lambda\lambda}\cos^2\phi+y_\lambda\Lambda_{\lambda\lambda}[\cos(2\beta_\lambda^{\rm meas}-2\beta-2\phi)+\cos(2\beta_\lambda^{\rm meas}-2\beta)]}{2\Lambda_{\lambda\lambda}[1-y_\lambda\cos(2\beta_\lambda^{\rm meas}-2\beta-2\phi)]}\\&=-\cos^2\phi\left(1+\frac{y_\lambda\sin(2\beta_\lambda^{\rm meas}-2\beta-2\phi)\tan\phi}{1-y_\lambda\cos(2\beta_\lambda^{\rm meas}-2\beta-2\phi)}\right),\end{aligned} \tag{11}$$

where we have used the expression for $\Lambda_{\lambda\lambda}$ given in Eq. (5). We introduce a compact notation to express Eq. (11) by defining

$$P_\lambda^2\equiv\Lambda_{\lambda\lambda}[1-y_\lambda\cos(2\beta_\lambda^{\rm meas}-2\beta-2\phi)], \tag{12}$$

$$\xi_\lambda\equiv\frac{\Lambda_{\lambda\lambda}y_\lambda\sin(2\beta_\lambda^{\rm meas}-2\beta-2\phi)}{P_\lambda^2}. \tag{13}$$

This results in

$$\frac{b_\lambda}{a_\lambda}\cos\delta_\lambda\cos\phi=-\cos^2\phi-\xi_\lambda\cos\phi\sin\phi. \tag{14}$$

Similarly, we define

$$\sigma_\lambda\equiv\frac{\Sigma_{\lambda\lambda}}{P_\lambda^2}, \tag{15}$$

which allows us to write

$$\frac{b_\lambda}{a_\lambda}\sin\delta_\lambda\sin\phi=-\sigma_\lambda\sin^2\phi. \tag{16}$$

We can now express the remaining nine observables in terms of Δ_i, $\beta+\phi$, and the newly defined parameters P_λ, ξ_λ, and σ_λ as follows:

$$\begin{aligned}\Sigma_{\perp i}=P_iP_\perp[&(\xi_\perp\sigma_i-\xi_i\sigma_\perp)\cos\Delta_i\\&-(1+\xi_i\xi_\perp+\sigma_i\sigma_\perp)\sin\Delta_i],\end{aligned} \tag{17}$$

$$\Lambda_{\perp i}=P_iP_\perp[(\xi_\perp-\xi_i)\cos\Delta_i-(\sigma_i+\sigma_\perp)\sin\Delta_i], \quad (18)$$

$$\begin{aligned}\rho_{\perp i}=P_iP_\perp\{&[(-1+\xi_i\xi_\perp+\sigma_i\sigma_\perp)\cos(2\beta+2\phi)\\&-(\xi_i+\xi_\perp)\sin(2\beta+2\phi)]\cos\Delta_i\\&+[(-\xi_i\sigma_\perp+\xi_\perp\sigma_i)\cos(2\beta+2\phi)\\&-(\sigma_i-\sigma_\perp)\sin(2\beta+2\phi)]\sin\Delta_i\},\end{aligned} \quad (19)$$

$$\begin{aligned}\Sigma_{\parallel 0}=P_\parallel P_0[&(\xi_\parallel-\xi_0)\sin(\Delta_0-\Delta_\parallel)\\&+(\sigma_\parallel+\sigma_0)\cos(\Delta_0-\Delta_\parallel)],\end{aligned} \quad (20)$$

$$\begin{aligned}\Lambda_{\parallel 0}=P_\parallel P_0[&(\xi_0\sigma_\parallel-\sigma_0\xi_\parallel)\sin(\Delta_0-\Delta_\parallel)\\&+(1+\xi_0\xi_\parallel+\sigma_\parallel\sigma_0)\cos(\Delta_0-\Delta_\parallel)],\end{aligned} \quad (21)$$

$$\begin{aligned}\rho_{\parallel 0}=P_\parallel P_0\{&[(-1+\xi_\parallel\xi_0+\sigma_\parallel\sigma_0)\sin(2\beta+2\phi)\\&+(\xi_\parallel+\xi_0)\cos(2\beta+2\phi)]\cos(\Delta_0-\Delta_\parallel)\\&+[(\xi_\parallel\sigma_0-\xi_0\sigma_\parallel)\sin(2\beta+2\phi)+(\sigma_0\\&-\sigma_\parallel)\cos(2\beta+2\phi)]\sin(\Delta_0-\Delta_\parallel)\}.\end{aligned} \quad (22)$$

The notable achievement of the above relations is the expression of observables involving the interference of helicities in terms of only three theoretical parameters $(\Delta_i,\beta+\phi)$, thereby reducing the complexity of the extremization problem. The above relations are extremely important in obtaining bounds on NP parameters.

We now turn to the issue of new-physics signals. The presence of NP is indicated by the violation of at least one of the relations given in Eq. (6). This in turn implies that $b_\lambda\neq 0$ and $|\beta_\lambda^{\rm meas}-\beta|\neq 0$ for at least one helicity λ. Clearly, any bounds on NP parameters will depend on the specific signal of NP. We therefore examine several different NP signals and explore the restrictions they place on NP parameter space.

Note that we do not present an exhaustive study of new-physics signals. The main point of the present paper is to show that it is possible to obtain bounds on the NP parameters, even though there are more unknowns than observables. Furthermore, the relations for the observables are sufficiently complicated that it is not possible to derive analytic bounds for every signal of NP. Whenever possible, we present analytic bounds on the NP parameters. However, for certain NP signals, we can only obtain numerical bounds. In all cases, the bounds are found without any approximations. This demonstrates the power of angular analysis and its usefulness in constraining NP parameters.

We will see that, while b_λ and b_λ/a_λ can be constrained with just one signal of NP, obtaining a bound on $|\beta_\lambda^{\rm meas}-\beta|$ requires at least two NP signals. Also, because the equations are nonlinear, there are often discrete ambiguities in the bounds. These can be reduced, leading to stronger bounds on NP, if a larger set of observables is used.

In the subsections below, we present bounds for several different signals of NP.

A. $\Sigma_{\lambda\lambda}\neq 0$

Suppose first that one observes direct *CP* violation in at least one helicity, i.e., $\Sigma_{\lambda\lambda}\neq 0$. The minimum value of b_λ^2 can be obtained by minimizing b_λ^2 [Eq. (10)] with respect to β and ϕ,

$$b_\lambda^2\geq\tfrac{1}{2}[\Lambda_{\lambda\lambda}-\sqrt{\Lambda_{\lambda\lambda}^2-\Sigma_{\lambda\lambda}^2}]. \quad (23)$$

Thus, if direct *CP* violation is observed, one can place a lower bound on the new-physics amplitude b_λ.

On the other hand, it follows from Eq. (10) that no upper bound can ever be placed on b_λ^2. One can always take $b_\lambda\to\infty$, as long as $\phi\to 0$ with $b_\lambda\sin\phi$ held constant. For the same reason, one can never determine the NP weak phase ϕ, or place a lower bound on it. (This no longer holds if the true value of β is known. We discuss this possibility in Sec. IV F.)

It is possible, however, to place lower bounds on other NP quantities. Using Eqs. (9) and (10), it is straightforward to obtain the constraints

$$\begin{aligned}&\tfrac{1}{2}\Lambda_{\lambda\lambda}(1-y_\lambda)\leq b_\lambda^2\sin^2\phi\leq\tfrac{1}{2}\Lambda_{\lambda\lambda}(1+y_\lambda),\\&\frac{1-y_\lambda}{1+y_\lambda}\leq r_\lambda^2\leq\frac{1+y_\lambda}{1-y_\lambda},\end{aligned} \quad (24)$$

where

$$r_\lambda\equiv\frac{b_\lambda}{a_\lambda}. \quad (25)$$

If $\Sigma_{\lambda\lambda}\neq 0$, these give nontrivial lower bounds. The lower bound on r_λ is very useful in estimating the magnitude of NP amplitudes or the scale of NP.

One interesting observation can be made regarding bounds on b_λ^2. Saying that new physics is present implies that the NP amplitude b_λ must be nonzero for at least one helicity; the other two helicities could have vanishing NP amplitudes. A nonzero direct asymmetry $a_{\rm dir}^{CP}\neq 0$ (i.e., $\Sigma_{\lambda\lambda}\neq 0$) implies a nonzero NP amplitude with a lower bound given by Eq. (23). Other NP signals [Eq. (6)] do not bound the NP amplitude b_λ^2 for a single helicity, but can bound combinations $(b_\lambda^2\pm b_\sigma^2)$. This is perhaps surprising but may be understood as follows. Consider, for example, the NP signal $\Lambda_{\perp i}\neq 0$. Even in the presence of such a signal, it is possible that one of either b_i or $b_\perp$ is zero, but not both [see Eq. (5)]. Thus, one can only obtain a lower bound when simultaneously bounding b_i^2 and $b_\perp^2$. Hence, for $\Lambda_{\perp i}\neq 0$, we must consider bounds on sums and differences of the NP amplitudes, $b_i^2\pm b_\perp^2$. A similar argument applies to all signals of NP in Eq. (6) involving two helicities. We will encounter such lower bounds in subsequent subsections.

B. $\beta_\lambda^{\rm meas} \neq \beta_\sigma^{\rm meas}$

Another signal of NP is if the measured value of β is different in two helicities, i.e., $\beta_\lambda^{\rm meas} \neq \beta_\sigma^{\rm meas}$. We define

$$2\omega_{\sigma\lambda} \equiv 2\beta_\sigma^{\rm meas} - 2\beta_\lambda^{\rm meas}, \quad \eta_\lambda \equiv 2(\beta_\lambda^{\rm meas} - \beta). \tag{26}$$

Using Eq. (10), we have

$$(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi = \frac{\Lambda_{\lambda\lambda} \pm \Lambda_{\sigma\sigma}}{2} - \frac{y_\lambda \Lambda_{\lambda\lambda}\cos\eta_\lambda \pm y_\sigma \Lambda_{\sigma\sigma}\cos(2\omega_{\sigma\lambda} + \eta_\lambda)}{2}. \tag{27}$$

Extremizing this expression with respect to η_λ, we obtain a solution for η_λ,

$$\sin\eta_\lambda = \pm \frac{y_\sigma \Lambda_{\sigma\sigma}\sin 2\omega_{\sigma\lambda}}{\sqrt{y_\lambda^2\Lambda_{\lambda\lambda}^2 + y_\sigma^2\Lambda_{\sigma\sigma}^2 - 2y_\lambda y_\sigma \Lambda_{\lambda\lambda}\Lambda_{\sigma\sigma}\cos 2\omega_{\sigma\lambda}}}. \tag{28}$$

Taking into account the sign of the second derivative, we get the bounds

$$(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi \geqslant \frac{\Lambda_{\lambda\lambda} \pm \Lambda_{\sigma\sigma}}{2} - \frac{\sqrt{y_\lambda^2\Lambda_{\lambda\lambda}^2 + y_\sigma^2\Lambda_{\sigma\sigma}^2 \pm 2y_\lambda y_\sigma \Lambda_{\lambda\lambda}\Lambda_{\sigma\sigma}\cos 2\omega_{\sigma\lambda}}}{2}, \tag{29}$$

$$(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi \leqslant \frac{\Lambda_{\lambda\lambda} \pm \Lambda_{\sigma\sigma}}{2} + \frac{\sqrt{y_\lambda^2\Lambda_{\lambda\lambda}^2 + y_\sigma^2\Lambda_{\sigma\sigma}^2 \pm 2y_\lambda y_\sigma \Lambda_{\lambda\lambda}\Lambda_{\sigma\sigma}\cos 2\omega_{\sigma\lambda}}}{2}. \tag{30}$$

Extremizing with respect to ϕ as well, one obtains the bounds

$$(b_\lambda^2 \pm b_\sigma^2) \geqslant \frac{\Lambda_{\lambda\lambda} \pm \Lambda_{\sigma\sigma}}{2} - \frac{|y_\lambda \Lambda_{\lambda\lambda} \pm y_\sigma \Lambda_{\sigma\sigma} e^{2i\omega_{\sigma\lambda}}|}{2}, \tag{31}$$

where it has been assumed that $\Lambda_{\lambda\lambda} > \Lambda_{\sigma\sigma}$, and that the right-hand side of the inequality is positive. [Note that an upper bound on $(b_\lambda^2 \pm b_\sigma^2)$ cannot be obtained.] We will see below that Eq. (31) plays a central role in deriving bounds for other signals of NP.

We emphasize that all of the above bounds are exact—no approximations or limits have been used. From the constraints on $(b_\lambda^2 \pm b_\sigma^2)$ one can obtain lower bounds on b_λ^2 and b_σ^2 individually.

Even without extremization, careful examination of Eq. (27) implies minimum and maximum possible values for $(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi$. These can also be derived from Eq. (24) and are given by

$$(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi \geqslant \frac{\Lambda_{\lambda\lambda} \pm \Lambda_{\sigma\sigma}}{2} - \frac{y_\lambda \Lambda_{\lambda\lambda} + y_\sigma \Lambda_{\sigma\sigma}}{2},$$

$$(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi \leqslant \frac{\Lambda_{\lambda\lambda} \pm \Lambda_{\sigma\sigma}}{2} + \frac{y_\lambda \Lambda_{\lambda\lambda} + y_\sigma \Lambda_{\sigma\sigma}}{2}. \tag{32}$$

Note that if $2\omega_{\sigma\lambda} = 0$, Eqs. (29) and (30) reproduce the bounds of Eq. (32) for $(b_\lambda^2 + b_\sigma^2)\sin^2\phi$; if $2\omega_{\sigma\lambda} = \pi$, one reproduces the bounds on $(b_\lambda^2 - b_\sigma^2)\sin^2\phi$. If one uses other NP signals to constrain the NP parameters, then unless these other signals result in constraining the value of $2\omega_{\sigma\lambda}$ to be other than 0 or π, one cannot obtain better bounds than those of Eq. (32). Note also that, while $2\omega_{\sigma\lambda}$ can be measured directly up to discrete ambiguities, additional measurements will result in the reduction of such ambiguities and lead to tighter bounds.

C. $\Lambda_{\perp i} \neq 0$ with $\Sigma_{\lambda\lambda} = 0$

We now turn to the NP signal $\Lambda_{\perp i} \neq 0$. Here we assume that the phase of B_d^0-$\bar{B}_d^0$ mixing has not been measured in any helicity, i.e., the parameter $\omega_{\perp i}$ is unknown. This situation is plausible: as discussed above, $\Lambda_{\perp i}$ can be obtained without tagging or time dependence, while the measurement of $\omega_{\perp i}$ requires both.

In order to obtain analytic bounds which depend on $\Lambda_{\perp i}$, it is simplest to consider the limit in which all direct *CP*-violating asymmetries vanish $(\Sigma_{\lambda\lambda} = 0)$. In this limit, with a little algebra Eq. (18) reduces to

$$\frac{\Lambda_{\perp i}}{2\sqrt{\Lambda_{ii}\Lambda_{\perp\perp}}} = -\sin\omega_{\perp i}\cos\Delta_i, \tag{33}$$

where $\omega_{\perp i} \equiv \beta_\perp^{\rm meas} - \beta_i^{\rm meas}$. We solve the above for $\sin\omega_{\perp i}$ and substitute it into the expressions for $(b_i^2 \pm b_\perp^2)\sin^2\phi$ [Eq. (27)]. The resulting expressions are minimized straightforwardly with respect to $\cos\Delta_i$ and η_i to obtain new bounds. The extrema with respect to Δ_i for both $(b_i^2 \pm b_\perp^2)$ occur at

$$\cos^2\Delta_i = \left\{ 1, \frac{\Lambda_{\perp i}^2}{4\Lambda_{ii}^2\Lambda_{\perp\perp}^2\cos^2(\eta_i/2)}, \frac{\Lambda_{\perp i}^2}{4\Lambda_{ii}^2\Lambda_{\perp\perp}^2\sin^2(\eta_i/2)} \right\}, \tag{34}$$

while that with respect to η_i depends on $\Lambda_{\perp i}$, and occurs for both $(b_i^2 \pm b_\perp^2)$ at

$$\sin\eta_i = \pm \frac{2R\sqrt{1-R^2}\Lambda_{\perp\perp}}{\sqrt{\Lambda_{ii}^2 \pm 2(1-2R^2)\Lambda_{ii}\Lambda_{\perp\perp}+\Lambda_{\perp\perp}^2}}, \quad (35)$$

where

$$R = \frac{\Lambda_{\perp i}}{2\sqrt{\Lambda_{ii}\Lambda_{\perp\perp}}}. \quad (36)$$

These extrema yield new lower limits on $(b_i^2 \pm b_\perp^2)$,

$$2(b_i^2 \pm b_\perp^2) \geqslant \Lambda_{ii} \pm \Lambda_{\perp\perp} - \sqrt{(\Lambda_{ii} \pm \Lambda_{\perp\perp})^2 \mp \Lambda_{\perp i}^2}. \quad (37)$$

Interference terms such as $\Lambda_{\perp i}$ also allow us to obtain bounds for η_λ. Using Eqs. (27) and (37), one can easily derive the bound

$$(\Lambda_{ii} + \Lambda_{\perp\perp}\cos 2\omega_{\perp i})\cos\eta_i + \Lambda_{\perp\perp}\sin 2\omega_{\perp i}\sin\eta_i \leqslant \sqrt{(\Lambda_{ii}+\Lambda_{\perp\perp})^2 - \Lambda_{\perp i}^2}, \quad (38)$$

which can be rewritten as

$$\Lambda_{ii}\cos\eta_i + \Lambda_{\perp\perp}\cos\eta_\perp \leqslant \sqrt{(\Lambda_{ii}+\Lambda_{\perp\perp})^2 - \Lambda_{\perp i}^2}. \quad (39)$$

Thus, if $\Lambda_{\perp i} \neq 0$, one cannot have $\eta_i = \eta_\perp = 0$. These constraints therefore place a lower bound on $|\beta_i^{\rm meas} - \beta|$ and/or $|\beta_\perp^{\rm meas} - \beta|$.

This procedure can also be applied to $\Sigma_{\parallel 0}$, and different lower bounds on $(b_\parallel^2 \pm b_0^2)$ and on $\eta_\parallel$, η_0 can be derived.

Analytic bounds on r_λ are not easy to derive, hence only numerical bounds are obtained. We describe this in the next subsection.

D. $\Lambda_{\perp i} \neq 0$ with $\Sigma_{\lambda\lambda} \neq 0$

We now assume that both $\Lambda_{\perp i} \neq 0$ and $\Sigma_{\lambda\lambda} \neq 0$, but no measurement has been made of the parameter $\omega_{\perp i}$. In this case the procedure outlined in the previous subsection cannot be used to obtain analytic bounds on $(b_i^2 \pm b_\perp^2)$. The reason is that one does not find a simple solution for $\omega_{\perp i}$ such as that given in Eq. (33). In this case, we are forced to turn to numerical solutions. We use the same method as in the previous subsection—we solve Eq. (18) for $\omega_{\perp i}$ and substitute it into Eq. (27)—except that now the minimization is performed numerically with respect to the variables η_i, ϕ, and Δ_i using the computer program MINUIT [11].

We assume the new-physics signal $\Lambda_{\perp 0} \neq 0$. In order to perform numerical minimization, we must choose values for the observables. Here and in the next subsection, we take $\Lambda_{00} = 0.6$, $\Lambda_{\perp\perp} = 0.16$, $y_0 = 0.60$, and $y_\perp = 0.74$.

In Fig. 1, we present the lower and upper bounds on $(b_0^2 + b_\perp^2)\sin^2\phi$ as a function of $\Lambda_{\perp 0}$. As in the previous subsection, these bounds are obtained by minimizing with respect to the variables Δ_i and η_i. Since the minimum value of $(b_0^2 + b_\perp^2)$ can be obtained from that of $(b_0^2 + b_\perp^2)\sin^2\phi$ by setting $\sin\phi = 1$ (its maximum value), the lower bound on

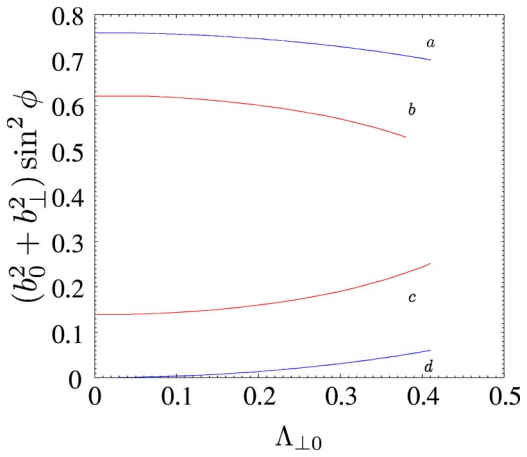

FIG. 1. The lower and upper bounds on $(b_0^2 + b_\perp^2)\sin^2\phi$ as a function of $\Lambda_{\perp 0}$. For curves *b* and *c* we have assumed the following values for the observables: $\Lambda_{00} = 0.6$, $\Lambda_{\perp\perp} = 0.16$, $y_0 = 0.60$, and $y_\perp = 0.74$. Curves *a* and *d* represent the corresponding case with no direct *CP* asymmetry (i.e., $y_0 = y_\perp = 1.0$).

$(b_0^2 + b_\perp^2)$ is identical to that of $(b_0^2 + b_\perp^2)\sin^2\phi$. However, upper bounds can only be derived for $(b_0^2 + b_\perp^2)\sin^2\phi$. For comparison, we include the bounds for the case of vanishing direct *CP* asymmetry, i.e., $\Sigma_{00} = \Sigma_{\perp\perp} = 0$ [Eq. (37)]. It is clear that the bounds are stronger if there are more signals of NP.

As in the previous subsection, the constraints on $(b_0^2 + b_\perp^2)\sin^2\phi$ imply certain allowed regions for η_0 and $\eta_\perp$ [see Eq. (39) and the surrounding discussion]. These are shown in Fig. 2. Recall that $\eta_\lambda \equiv 2(\beta_\lambda^{\rm meas} - \beta)$. Since it is not possible to simultaneously have $\eta_0 = \eta_\perp = 0$ (or π), this is a clear sign of NP (as is $\Lambda_{\perp 0} \neq 0$). However, since neither η_0 nor $\eta_\perp$ is constrained to lie within a certain range, no bounds on β can be derived.

One can perform a similar numerical extremization for $(b_0^2 - b_\perp^2)\sin^2\phi$. However, for this particular data set, we

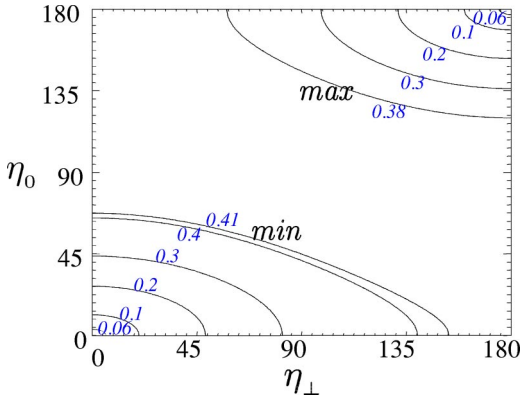

FIG. 2. Contours showing the (correlated) lower and upper bounds on η_0 and $\eta_\perp$, corresponding to the different values of $\Lambda_{\perp 0}$ shown in the figure. We have assumed the following values for the observables: $\Lambda_{00} = 0.6$, $\Lambda_{\perp\perp} = 0.16$, $y_0 = 0.60$, and $y_\perp = 0.74$. Values of η_0 and $\eta_\perp$ above (below) and to the right (left) of the minimum (maximum) contours are allowed.

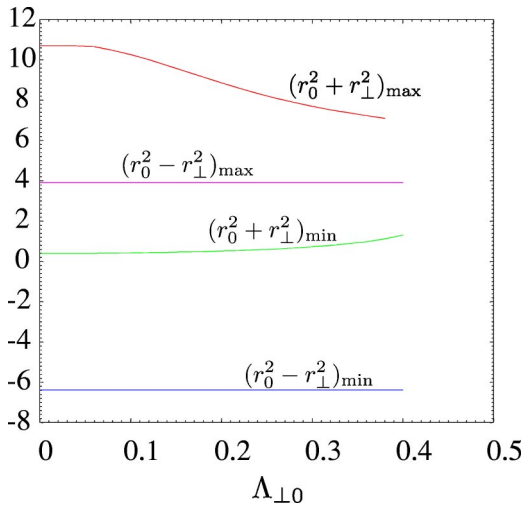

FIG. 3. Upper and lower bounds on $r_0^2 \pm r_\perp^2$ as a function of $\Lambda_{\perp 0}$. We have assumed the following values for the observables: $\Lambda_{00}=0.6$, $\Lambda_{\perp\perp}=0.16$, $y_0=0.60$, and $y_\perp=0.74$.

simply reproduce the bounds of Eq. (32): $-0.02 \leqslant (b_0^2 - b_\perp^2)\sin^2\phi \leqslant 0.46$. Since this bound is independent of $\Lambda_{\perp 0}$, we have not plotted it.

The easiest way to see whether the numerical extremization of $(b_0^2 \pm b_\perp^2)\sin^2\phi$ depends on $\Lambda_{\perp 0}$ or not is as follows. We refer to Eq. (27), and note that $2\omega_{\perp 0} + \eta_0 = \eta_\perp$. The minimal (maximal) value of $(b_0^2 + b_\perp^2)\sin^2\phi$ occurs at the point $(\eta_0, \eta_\perp) = (0,0)$ $[(\pi, \pi)]$. Thus, the minimal (maximal) value of $(b_0^2 + b_\perp^2)\sin^2\phi$ depends on $\Lambda_{\perp 0}$ only if the point (0, 0) $[(\pi, \pi)]$ is excluded. Similarly, the minimal (maximal) value of $(b_0^2 - b_\perp^2)\sin^2\phi$ depends on $\Lambda_{\perp 0}$ only if the point (0, π) $[(\pi, 0)]$ is excluded. Referring to Fig. 2, we note that the points $(\eta_0, \eta_\perp) = (0,0)$, (π, π) are excluded. Thus, the minimal and maximal values of $(b_0^2 + b_\perp^2)\sin^2\phi$ depend on $\Lambda_{\perp 0}$, as in Fig. 1. On the other hand, the points $(\eta_0, \eta_\perp) = (0, \pi)$ and $(\pi, 0)$ are allowed, so the minimal and maximal values of $(b_0^2 - b_\perp^2)\sin^2\phi$ are independent of $\Lambda_{\perp 0}$, as described above.

As noted previously, the minimal values for $(b_0^2 \pm b_\perp^2)$ are equal to those for $(b_0^2 \pm b_\perp^2)\sin^2\phi$. These values can then be combined to give individual minima on b_0^2 and $b_\perp^2$.

It is also possible to obtain numerical bounds on the combinations of ratios $r_0^2 \pm r_\perp^2$ [Eq. (25)]. The procedure is very similar to that used to obtain bounds on $(b_0^2 \pm b_\perp^2)\sin^2\phi$. The bounds on $r_0^2 \pm r_\perp^2$ are shown in Fig. 3. As was the case for $(b_0^2 - b_\perp^2)\sin^2\phi$, the bounds on $r_0^2 - r_\perp^2$ are independent of $\Lambda_{\perp i}$ and follow directly from Eq. (24): $-6.44 \leqslant r_0^2 - r_\perp^2 \leqslant 3.85$. However, unlike $b_0^2 \pm b_\perp^2$, upper bounds on $r_0^2 \pm r_\perp^2$ can also be obtained. The upper and lower bounds on $r_0^2 \pm r_\perp^2$ can then be used to bound r_0^2 and $r_\perp^2$ individually. This constrains the scale of new physics, and so is very significant.

E. Observation of $\Lambda_{\perp 0}$ and $\Sigma_{\perp 0}$ with $\Sigma_{00} \neq 0, \Sigma_{\perp\perp} \neq 0$

In this subsection we assume that, in addition to $\Lambda_{\perp 0}$, $\Sigma_{\perp 0}$ is also known ($\omega_{\perp 0}$ is still assumed not to have been measured). We then see, from Eqs. (17) and (18), that both $\cos(\Delta_0)$ and $\sin(\Delta_0)$ can be determined in terms of these two observables. Thus, Δ_0 can be obtained without ambiguity. Furthermore, using the relation $\cos^2(\Delta_0) + \sin^2(\Delta_0) = 1$, we can *solve* for $\omega_{\perp 0}$, up to an eightfold discrete ambiguity (i.e., a fourfold ambiguity in $2\omega_{\perp 0}$).[3] This is shown explicitly in the Appendix. Thus, $\omega_{\perp 0}$ does not take a range of values, as in the previous subsections, but instead takes specific values. (In fact, one can solve for $\omega_{\perp 0}$, up to discrete ambiguities, whenever two observables are known which involve the interference of two helicity amplitudes.)

The expressions and values for Δ_0 and $\omega_{\perp 0}$ are then substituted into Eq. (27), and we use MINUIT to numerically minimize the resulting expression with respect to n_i and ϕ. As before, we take $\Lambda_{00}=0.6$, $\Lambda_{\perp\perp}=0.16$, $y_0=0.60$, and $y_\perp = 0.74$.

The numerical constraints on $(b_0^2 \pm b_\perp^2)\sin^2\phi$ and $(r_0^2 \pm r_\perp^2)$ are shown in Fig. 4. In these figures, we have only presented results for positive values of $\Lambda_{\perp 0}$. A point on a plot with a negative value of $\Lambda_{\perp 0}$ is equivalent to that with a positive $\Lambda_{\perp 0}$ and negative $\Sigma_{\perp 0}$. This interchange reverses the signs of $\cos(\Delta_0)$ and $\sin(\Delta_0)$, but does not change the value of $\omega_{\perp 0}$.

As noted above, the knowledge of both $\Lambda_{\perp 0}$ and $\Sigma_{\perp 0}$ allows us to fix the value of $\omega_{\perp 0}$, up to an eightfold discrete ambiguity. In this case, we can use Eqs. (29), (30), and (31) to directly bound $(b_\lambda^2 \pm b_\sigma^2)\sin^2\phi$. This is illustrated in Fig. 5 for $\Lambda_{\perp 0}=0.2$ and $\Sigma_{\perp 0}=0.2$.

Of course, it is also possible to measure $2\omega_{\perp 0}$ directly [Eq. (8)], up to a fourfold discrete ambiguity. As we show in the Appendix, in general these four values only partially overlap with the four values obtained from the derivation of $2\omega_{\perp 0}$ from measurements of $\Lambda_{\perp 0}$ and $\Sigma_{\perp 0}$—the discrete ambiguity in $2\omega_{\perp 0}$ is reduced to twofold. Thus, by combining the two ways of obtaining $2\omega_{\perp 0}$, the discrete ambiguity can be reduced. This will in turn improve the bounds on the NP parameters.

As in the previous subsection, one can also place (correlated) constraints on η_0 and $\eta_\perp$. In itself, this does not lead to a bound on β. However, if in addition $2\beta_\lambda^{\rm meas}$ is measured directly [Eq. (8)], then β can be constrained.

F. Measurement of β

Finally, suppose that an angular analysis of $B_d^0(t) \to J/\psi K^{*0}$ is done, and no new physics is found. This implies that the true B_d^0-$\overline{B}_d^0$ mixing phase β can be extracted from measurements of *CP* violation in this decay. Now suppose that some NP signal is found in $B_d^0(t) \to \phi K^{*0}$. The

[3]It is to be expected that we can solve for $\omega_{\perp 0}$ in this case. If the theoretical parameters β and ϕ did not vanish from the equation $\cos^2(\Delta_0) + \sin^2(\Delta_0) = 1$, then we would have a relation between the *independent* parameters β and ϕ, which is impossible. β and ϕ are eliminated because this equation depends on $(2\beta_\perp^{\rm meas} - 2\beta - 2\phi) - (2\beta_0^{\rm meas} - 2\beta - 2\phi) = 2\omega_{\perp 0}$.

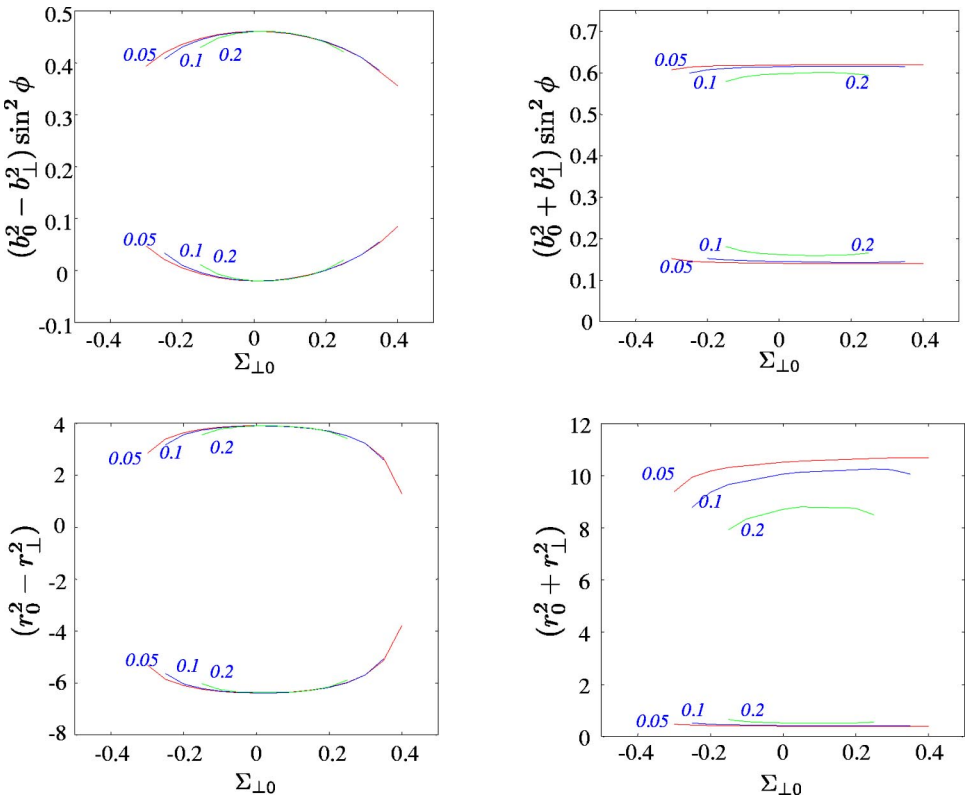

FIG. 4. The lower and upper bounds on $(b_0^2 \pm b_\perp^2)\sin^2\phi$ and $(r_0^2 \pm r_\perp^2)$ as a function of $\Sigma_{\perp 0}$. Each curve corresponds to a specific value of $\Lambda_{\perp 0}$, shown in the figure. We have assumed the following values for the observables: $\Lambda_{00}=0.6$, $\Lambda_{\perp\perp}=0.16$, $y_0=0.60$, and $y_\perp=0.74$.

analysis described in the previous sections can now be applied, except that in this case we *know* the value of β. In addition to improving bounds on b_λ^2 and r_λ^2 using previous techniques, we can now constrain the NP phase ϕ.

For example, assuming that β is known, one can use Eq. (10) to improve the bound on b_λ^2,

$$b_\lambda^2 \geqslant \frac{\Lambda_{\lambda\lambda}}{2}[1 - y_\lambda \cos(2\beta_\lambda^{\rm meas} - 2\beta)]. \tag{40}$$

$2\beta_\lambda^{\rm meas}$ and 2β can each be obtained with a twofold ambiguity. Their combination leads to a twofold ambiguity in the bound for b_λ^2. Obviously, to be conservative, we take the weaker of the two bounds.

To obtain a meaningful bound on ϕ, we require the use of r_λ^2. In previous subsections, we have derived bounds on $(r_0^2 \pm r_\perp^2)$ (Figs. 3 and 4). Bounds on $(r_\parallel^2 \pm r_\perp^2)$ and $(r_0^2 \pm r_\parallel^2)$ can also be obtained. These can all be combined to yield upper and lower bounds on r_λ^2. Together, Eqs. (9) and (10) provide a constraint on ϕ,

$$(r_\lambda^2)_{\rm min} \leqslant \frac{1 - y_0 \cos(2\beta_0^{\rm meas} - 2\beta)}{1 - y_0 \cos(2\beta_0^{\rm meas} - 2\beta - 2\phi)} \leqslant (r_\lambda^2)_{\rm max}. \tag{41}$$

In this case, there is an eightfold ambiguity on the bounds on $\sin 2\phi$.

V. DISCUSSION AND SUMMARY

In this paper we consider $B \to V_1 V_2$ decays in which $\overline{V}_1 \overline{V}_2 = V_1 V_2$, so that both B^0 and $\overline{B}^0$ can decay to the final state $V_1 V_2$. If a time-dependent angular analysis of $B^0(t) \to V_1 V_2$ can be performed, it is possible to extract 18 observ-

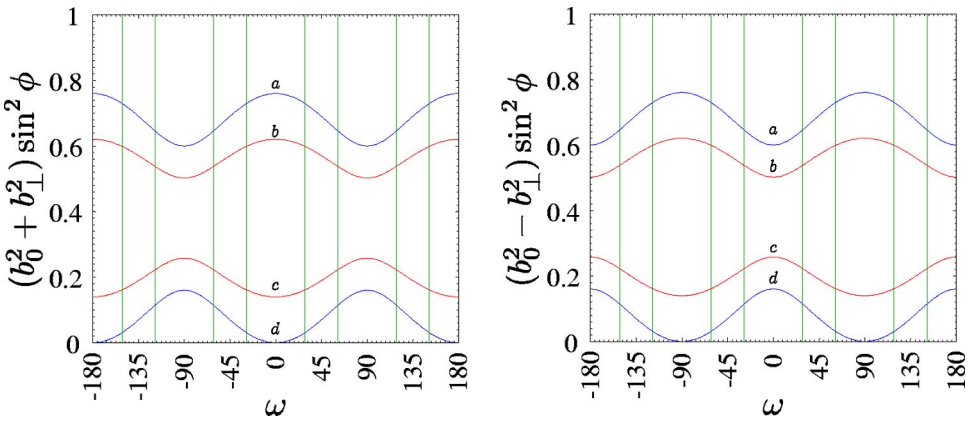

FIG. 5. The lower and upper bounds on $(b_0^2 \pm b_\perp^2)\sin^2\phi$ as a function of $\omega_{\perp 0}$. For curves *b* and *c* we have assumed the following values for the observables: $\Lambda_{00}=0.6$, $\Lambda_{\perp\perp}=0.16$, $y_0=0.60$, and $y_\perp=0.74$. Curves *a* and *d* represent the corresponding case with no direct *CP* asymmetry (i.e., $y_0=y_\perp=1.0$). The solutions for $\omega_{\perp 0}$ for $\Lambda_{\perp 0}=0.2$ and $\Sigma_{\perp 0}=0.2$ are shown as vertical lines.

ables [Eq. (4)]. However, there are only six helicity amplitudes describing the decays $B\to V_1V_2$ and $\overline{B}\to V_1V_2$. There are therefore only 11 independent observables (equivalent to the magnitudes and relative phases of the six helicity amplitudes).

We assume that the $B\to V_1V_2$ decays are dominated by a single decay amplitude in the standard model (SM). The SM parametrization of such decays contains six theoretical parameters: three helicity amplitudes a_λ, two relative strong phases, and the weak phase β (the phase of B_d^0-$\overline{B_d^0}$ mixing). Because there are 18 observables, one has a total of 12 relations to test for the presence of new physics (NP) [Eq. (6)]. With 11 independent observables and six SM parameters, one might expect that only five tests are necessary to search for NP. However, because the equations relating the observables to the theoretical parameters are nonlinear [Eq. (5)], for certain (fine-tuned) values of the SM parameters, some tests can agree with the SM predictions, even in the presence of NP. To take this possibility into account, all 12 NP tests are needed to perform a complete search for NP.

In this paper we assume that a single NP amplitude contributes to $B\to V_1V_2$ decays. In this case one finds a total of 13 theoretical parameters: in addition to the six SM parameters, there are three NP helicity amplitudes b_λ, three additional relative strong phases, and one NP weak phase ϕ. Suppose now that a NP signal is seen. With only 11 independent observables, it is clear that one cannot extract any of the NP parameters. However, precisely because the equations in Eq. (5) are nonlinear, one can place *lower bounds* on the theoretical parameters. This is the main point of the paper.

In the previous section we presented several such constraints, which we summarize here. The form of the constraints depends on which observables have been measured. In some cases, it is possible to obtain analytic results; in other cases only numerical bounds are possible.

For example, three distinct NP signals are $\Sigma_{\lambda\lambda}\neq 0$, $\beta_\lambda^{\rm meas}\neq\beta_\sigma^{\rm meas}$, and $\Lambda_{\perp i}\neq 0$ (with $\Sigma_{\lambda\lambda}=0$). In all three cases one can derive analytic lower bounds on the size of b_λ,

$$
\begin{aligned}
b_\lambda^2 &\geqslant \frac{1}{2}[\Lambda_{\lambda\lambda}-\sqrt{\Lambda_{\lambda\lambda}^2-\Sigma_{\lambda\lambda}^2}],\\
(b_\lambda^2\pm b_\sigma^2) &\geqslant \frac{\Lambda_{\lambda\lambda}\pm\Lambda_{\sigma\sigma}}{2}-\frac{|y_\lambda\Lambda_{\lambda\lambda}\pm y_\sigma\Lambda_{\sigma\sigma}e^{2i\omega_{\sigma\lambda}}|}{2},\\
2(b_i^2\pm b_\perp^2) &\geqslant \Lambda_{ii}\pm\Lambda_{\perp\perp}-\sqrt{(\Lambda_{ii}\pm\Lambda_{\perp\perp})^2\mp\Lambda_{\perp i}^2},
\end{aligned}
\tag{42}
$$

where $y_\lambda\equiv\sqrt{1-\Sigma_{\lambda\lambda}^2/\Lambda_{\lambda\lambda}^2}$ and $2\omega_{\sigma\lambda}\equiv 2\beta_\sigma^{\rm meas}-2\beta_\lambda^{\rm meas}$. *A priori*, one does not know which of the above constraints will be strongest—this will depend on the measured values of the observables and/or which NP signals are seen.

Constraints on other theoretical parameters are possible. For example, if one measures $\Lambda_{\perp i}\neq 0$ (with $\Sigma_{\lambda\lambda}=0$), one finds

$$
\Lambda_{ii}\cos\eta_i+\Lambda_{\perp\perp}\cos\eta_\perp\leqslant\sqrt{(\Lambda_{ii}+\Lambda_{\perp\perp})^2-\Lambda_{\perp i}^2}, \tag{43}
$$

where $\eta_\lambda\equiv 2(\beta_\lambda^{\rm meas}-\beta)$. Thus, if $\Lambda_{\perp i}\neq 0$, one obtains correlated lower bounds on $|\beta_i^{\rm meas}-\beta|$ and $|\beta_\perp^{\rm meas}-\beta|$.

If more observables or NP signals are measured, then it is not possible to obtain analytic constraints—one must perform a numerical analysis. In Sec. IV D we presented numerical results for $\Lambda_{\perp 0}\neq 0$ with $\Sigma_{00}\neq 0$ and $\Sigma_{\perp\perp}\neq 0$. In Sec. IV E we assumed that in addition $\Sigma_{\perp 0}$ was measured. In both cases we were able to put lower bounds on $(b_0^2\pm b_\perp^2)$. [Upper bounds are possible only for $(b_0^2+b_\perp^2)\sin^2\phi$.] We also obtained bounds on $r_0^2\pm r_\perp^2$ ($r_\lambda\equiv b_\lambda/a_\lambda$).

The bounds improve as more NP signals are included in the fits. This is logical. For a particular NP signal, the bounds are weakest if that signal is zero. (Indeed, the bounds vanish if all NP signals are zero.) If a nonzero value for that signal is found, the bound will improve. Similarly, the bounds generally improve if additional observables are measured, even if they are not signals of NP. This is simply because additional measurements imply additional constraints, which can only tighten bounds on the theoretical parameters.

This behavior is seen most clearly in Secs. IV C–IV E. Consider the lower bound on $(b_0^2+b_\perp^2)\sin^2\phi$ as a function of $\Lambda_{\perp 0}$. In Sec. IV C, it is assumed that the NP signal $\Sigma_{\lambda\lambda}=0$. In Fig. 1, we see that the bound is strengthened, varying from 0 ($\Lambda_{\perp 0}=0$) to about 0.05 ($\Lambda_{\perp 0}=0.4$). In Sec. IV D, the values $y_0=0.60$ and $y_\perp=0.74$ are taken, i.e., it is assumed that both NP signals Σ_{00} and $\Sigma_{\perp\perp}$ are nonzero. In this case Fig. 1 shows that the lower bound varies from 0.14 ($\Lambda_{\perp 0}=0$) to 0.24 ($\Lambda_{\perp 0}=0.4$). For $\Lambda_{\perp 0}=0.2$, the bound is 0.16. In Sec. IV E, the measurement of $\Sigma_{\perp 0}$ (not a NP signal) is added. Now the lower bound on $(b_0^2+b_\perp^2)\sin^2\phi$ depends on the values of both $\Lambda_{\perp 0}$ and $\Sigma_{\perp 0}$. From Fig. 4, we see that it takes the value 0.18 for $\Lambda_{\perp 0}=0.2$ and $\Sigma_{\perp 0}=-0.15$.

In addition to the bounds on the b_λ and r_λ, it is possible to find correlated numerical constraints on the η_λ, as in Fig. 2. If these are combined with a measurement of $2\beta_\lambda^{\rm meas}$, one can then obtain a bound on β, even though NP is present.

Even if $2\omega_{\sigma\lambda}$ is not measured directly, one can obtain its value (up to a fourfold ambiguity) through measurements of two observables involving the interference of two helicity amplitudes (as well as the $\Lambda_{\lambda\lambda}$ and $\Sigma_{\lambda\lambda}$). These can be converted into bounds on the other NP parameters. If $2\omega_{\sigma\lambda}$ is measured directly, this reduces the discrete ambiguity to twofold, and improves the bounds.

Finally, all of the above bounds assume that the true (SM) value of β is not known. However, it is possible that no NP is seen in $B_d^0(t)\to J/\psi K^{*0}$, in which case measurements of *CP* violation in this decay allow one to extract the true value of β. This value of β can then be used as an *input* to the analysis of other decays, such as $B_d^0(t)\to\phi K^{*0}$, in which NP signals might be found. If β is assumed to be known, then the bounds on b_λ^2 and r_λ^2 described above are tightened, in general. In addition, it is possible to place bounds on the NP weak phase ϕ.

We stress that we have not presented a complete list of constraints on the NP parameters—the main aim of this paper was simply to show that such bounds exist. Our results have assumed that only a subset of all observables has been measured, and the bounds vary depending on the NP signal

found. In practice, the constraints will be obtained by performing a numerical fit using all measurements. If it is possible to measure all observables, one will obtain the strongest constraints possible.

As a specific application, we have noted the apparent discrepancy in the value of $\sin 2\beta$ as obtained from measurements of $B_d^0(t)\to J/\psi K_s$ and $B_d^0(t)\to\phi K_s$. In this case, the angular analyses of $B_d^0(t)\to J/\psi K^*$ and $B_d^0(t)\to\phi K^*$ would allow one to determine if new physics is indeed present. If NP is confirmed, the method described in this paper would allow one to put constraints on the NP parameters. If NP is subsequently discovered in direct searches at the LHC or GLC, these bounds would indicate whether this NP could be responsible for that seen in B decays.

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APPENDIX

Assume that, in addition to Λ_{00}, $\Lambda_{\perp\perp}$, Σ_{00}, and $\Sigma_{\perp\perp}$, $\Lambda_{\perp 0}$ and $\Sigma_{\perp 0}$ are also known. The expressions for these last two quantities are (repeated for convenience)

$$\Sigma_{\perp 0}=P_0P_\perp[(\xi_\perp\sigma_0-\xi_0\sigma_\perp)\cos\Delta_0 - (1+\xi_0\xi_\perp+\sigma_0\sigma_\perp)\sin\Delta_0], \tag{A1}$$

$$\Lambda_{\perp 0}=P_0P_\perp[(\xi_\perp-\xi_0)\cos\Delta_0-(\sigma_0+\sigma_\perp)\sin\Delta_0], \tag{A2}$$

where

$$P_\lambda^2\equiv\Lambda_{\lambda\lambda}[1-y_\lambda\cos(2\beta_\lambda^{\rm meas}-2\beta-2\phi)],$$

$$\xi_\lambda\equiv\frac{\Lambda_{\lambda\lambda}y_\lambda\sin(2\beta_\lambda^{\rm meas}-2\beta-2\phi)}{P_\lambda^2},$$

$$\sigma_\lambda\equiv\frac{\Sigma_{\lambda\lambda}}{P_\lambda^2}, \tag{A3}$$

with

$$y_\perp\equiv\left(1-\frac{\Sigma_{\lambda\lambda}^2}{\Lambda_{\lambda\lambda}^2}\right)^{1/2}. \tag{A4}$$

Equations (A1) and (A2) can be solved for $\cos\Delta_i$ and $\sin\Delta_i$. Writing

$$\Lambda_{\perp 0}=A\cos\Delta_0+B\sin\Delta_0,$$

$$\Sigma_{\perp 0}=A'\cos\Delta_0+B'\sin\Delta_0, \tag{A5}$$

where

$$A\equiv P_0P_\perp(\xi_\perp-\xi_0),\quad B\equiv-P_0P_\perp(\sigma_0+\sigma_\perp),$$

$$A'\equiv P_0P_\perp(\xi_\perp\sigma_0-\xi_0\sigma_\perp),$$

$$B'\equiv-P_0P_\perp(1+\xi_0\xi_\perp+\sigma_0\sigma_\perp), \tag{A6}$$

we get

$$\cos\Delta_0=\frac{B'\Lambda_{\perp 0}-B\Sigma_{\perp 0}}{AB'-BA'},$$

$$\sin\Delta_0=\frac{A'\Lambda_{\perp 0}-A\Sigma_{\perp 0}}{A'B-B'A}. \tag{A7}$$

Then the relation $\cos^2\Delta_0+\sin^2\Delta_0=1$ results in

$$1=\frac{(A'^2+B'^2)\Lambda_{\perp 0}^2+(A^2+B^2)\Sigma_{\perp 0}^2-2(AA'+BB')\Lambda_{\perp 0}\Sigma_{\perp 0}}{(AB'-BA')^2}. \tag{A8}$$

The point is that each of the four combinations (A^2+B^2), $(A'^2+B'^2)$, $(AA'+BB')$, and $(AB'-BA')$ is independent of β and ϕ.

In order to show this, the following relations are useful:

$$\xi_\lambda^2=-\sigma_\lambda^2+2\frac{\Lambda_{\lambda\lambda}}{P_\lambda^2}-1,$$

$$\xi_0\xi_\perp=\frac{\Lambda_{00}\Lambda_{\perp\perp}y_0y_\perp}{P_0^2P_\perp^2}\cos 2\omega_{\perp 0}-1 - \frac{\Lambda_{00}\Lambda_{\perp\perp}}{P_0^2P_\perp^2}+\frac{\Lambda_{00}}{P_0^2}+\frac{\Lambda_{\perp\perp}}{P_\perp^2},$$

$$P_\lambda^4=-\Lambda_{\lambda\lambda}^2+2\Lambda_{\lambda\lambda}P_\lambda^2 + \Lambda_{\lambda\lambda}^2y_\lambda^2\cos^2(2\beta_\lambda^{\rm meas}-2\beta-2\phi), \tag{A9}$$

where

$$2\omega_{\perp 0}\equiv 2\beta_\perp^{\rm meas}-2\beta_0^{\rm meas}. \tag{A10}$$

With these one can show that

$$(A^2+B^2)=2\Lambda_{00}\Lambda_{\perp\perp}+2\Sigma_{00}\Sigma_{\perp\perp} - 2\Lambda_{00}\Lambda_{\perp\perp}y_0y_\perp\cos 2\omega_{\perp 0},$$

$$(A'^2+B'^2)=2\Lambda_{00}\Lambda_{\perp\perp}+2\Sigma_{00}\Sigma_{\perp\perp}+2\Lambda_{00}\Lambda_{\perp\perp}y_0y_\perp\cos 2\omega_{\perp 0},$$
$$(AA'+BB')=2(\Lambda_{\perp\perp}\Sigma_{00}+\Lambda_{00}\Sigma_{\perp\perp}),$$
$$(AB'-BA')^2=4(\Lambda_{00}^2-\Sigma_{00}^2)(\Lambda_{\perp\perp}^2-\Sigma_{\perp\perp}^2)\sin^2 2\omega_{\perp 0}. \quad \text{(A11)}$$

Thus, the relation $\cos^2\Delta_0+\sin^2\Delta_0=1$ gives a quadratic equation in $\cos 2\omega_{\perp 0}$,

$$N_1(1-\cos^2 2\omega_{\perp 0})=(N_2+M_2\cos 2\omega_{\perp 0})+(N_3+M_3\cos 2\omega_{\perp 0})+N_4, \quad \text{(A12)}$$

where

$$N_1=4(\Lambda_{00}^2-\Sigma_{00}^2)(\Lambda_{\perp\perp}^2-\Sigma_{\perp\perp}^2),$$
$$N_2=2(\Lambda_{00}\Lambda_{\perp\perp}+2\Sigma_{00}\Sigma_{\perp\perp})\Lambda_{\perp 0}^2,$$
$$M_2=2\Lambda_{00}\Lambda_{\perp\perp}y_0y_\perp\Lambda_{\perp 0}^2,$$
$$N_3=2(\Lambda_{00}\Lambda_{\perp\perp}+2\Sigma_{00}\Sigma_{\perp\perp})\Sigma_{\perp 0}^2,$$
$$M_3=-2\Lambda_0\Lambda_{\perp\perp}y_0y_\perp\Sigma_{\perp 0}^2,$$
$$N_4=-2(\Lambda_{\perp\perp}\Sigma_{00}+\Lambda_{00}\Sigma_{\perp\perp})\Lambda_{\perp 0}\Sigma_{\perp 0}. \quad \text{(A13)}$$

The solution for $\cos 2\omega_{\perp 0}$ is

$$\cos 2\omega_{\perp 0}=\frac{-(M_2+M_2)\pm\sqrt{(M_2+M_3)^2-4N_1(N_2+N_3+N_4-N_1)}}{2N_1}. \quad \text{(A14)}$$

Thus, we obtain $2\omega_{\perp 0}$ with a fourfold discrete ambiguity (or, equivalently, $\omega_{\perp 0}$ with an eightfold ambiguity).

It is also possible to obtain $2\omega_{\perp 0}$ from direct measurements of ρ_{00} and $\rho_{\perp\perp}$ [Eq. (8)]. However, it is $\sin 2\beta_\lambda^{\rm meas}$ which is measured, so that one extracts two values,

$$2\beta_\lambda^{\rm meas}, \quad \pi-2\beta_\lambda^{\rm meas}. \quad \text{(A15)}$$

This leads to a fourfold discrete ambiguity in $2\omega_{\perp 0}$,

$$2\omega_\pm=\pm(2\beta_\perp^{\rm meas}-2\beta_0^{\rm meas}), \pm(2\beta_\perp^{\rm meas}+2\beta_0^{\rm meas}-\pi). \quad \text{(A16)}$$

Of these four values, in general only two will be found among those obtained by deriving $2\omega_{\perp 0}$ from measurements of Λ_{00}, $\Lambda_{\perp\perp}$, Σ_{00}, $\Sigma_{\perp\perp}$, $\Lambda_{\perp 0}$, and $\Sigma_{\perp 0}$. Thus, be extracting $2\omega_{\perp 0}$ in these two different ways, one can reduce the discrete ambiguity to twofold.

Note that this can only be done if new physics is found. If no NP signal is observed, then $2\omega_{\perp 0}=0$, and discrete ambiguities are irrelevant.

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