Some comments on narrow resonances $D_{s_1}^*(2.46\ \mathrm{GeV}/c^2)$ and $D_{s_0}(2.317\ \mathrm{GeV}/c^2)$

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The newly observed resonances $D_{s_1}^*$ and D_{s_0} are discussed in a potential model. The relationship between the mass difference between the p states $D_{s_1}^*(1^+), D_{s_0}(0^+)$ and the s states $D_s^*(1^-), D_s(0^-)$ is also examined. Some remarks about the states $D_1^*(1^+)$ and $D_0(0^+)$ in the nonstrange sectors are also made.

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Some time back the BaBar Collaboration [1] reported a narrow resonance with a mass near 2.32 GeV/c^2 which decays to $D_s\pi^0$. Recently, the CLEO Collaboration [2] has confirmed the state at 2.317 GeV/c^2 and also reported a new resonance at 2.46 GeV/c^2 which decays to $D_s^*\pi^0$. Since these states decay to $D_s\pi^0$ and $D_s^*\pi^0$, it is natural to assign them the quantum numbers $J^P=0^+$ and 1^+, respectively, as required by angular momentum and parity conservation. In this article we comment on the existence of relatively light $j=1/2$ states in a potential model considered by us in 1993 [3].

In the heavy quark effective theory (HQET) limit, the heavy quark spin is decoupled; it is natural to combine $\vec{j}=\vec{L}+\vec{S}_q$ with $\vec{S}_Q$, i.e., $\vec{J}=\vec{j}+\vec{S}_Q$, where $q=u,d,s$ and $Q=c$ or b [4]. Thus for the p states we get two multiplets, one with $j=3/2$ and the other with $j=1/2$. Hence we have the multiplets for the bound states $Q\bar{q}$ $l=0$ $[D^*(1^-),D(0^-)]$, $l=1$ $[D_2^*(2^+),D_1(1^+)]_{j=3/2}$, and $[D_1^*(1^+),D_0(0^+)]_{j=1/2}$. The degeneracy between $j=3/2$ and $j=1/2$ multiplets is removed by the spin-orbit coupling $\vec{L}\cdot\vec{S}_q$. The hyperfine mass splitting between two members of each multiplet arises from the Fermi term $\vec{S}_q\cdot\vec{S}_Q$, the spin-orbit coupling term $(\vec{S}_Q+\vec{S}_q)\cdot\vec{L}$, and the tensor term [5]

$$S_{12}\equiv\left[\frac{12}{r^2}(\vec{S}_q\cdot\vec{r})(\vec{S}_Q\cdot\vec{r})-4\vec{S}_q\cdot\vec{S}_Q\right].$$

These terms vanish in the heavy quark limit ($m_Q\to\infty$). Based on these considerations the effective Hamiltonian for the $q\bar{Q}$ or $Q\bar{q}$ states can be written [3]

$$\begin{aligned}H=H_0&+\frac{1}{2m_q^2}\vec{S}_q\cdot\vec{L}\left[\frac{1}{r}\frac{dV_1}{dr}\right]+\frac{1}{8m_q^2}\nabla^2V_1\\&+\frac{2}{3m_qm_Q}\vec{S}_q\cdot\vec{S}_Q\nabla^2V_2+\frac{1}{m_qm_Q}(\vec{S}_q+\vec{S}_Q)\cdot\vec{L}\left[\frac{1}{r}\frac{dV_2}{dr}\right]\\&+\frac{1}{12m_qm_Q}S_{12}\left[\frac{1}{r}\frac{dV_2}{dr}-\frac{d^2V_2}{dr^2}\right]\end{aligned}\qquad(1)$$

where

$$H_0=\frac{p^2}{2\mu}+V(r),\qquad \mu=\frac{m_qm_Q}{m_q+m_Q},\qquad \vec{p}=-i\vec{\nabla}.$$

The third term in Eq. (1) is the Darwin term. We take $V(r)$ to be the Cornell potential [6]

$$V(r)=\frac{r}{b^2}-\frac{K}{r}-A\qquad(b=2.34\ \ \mathrm{GeV}^{-1},K=0.52).\qquad(2)$$

This potential is taken to be flavor independent. The constituent quark masses are taken as $m_u=m_d=0.34$ GeV, $m_s=0.48$ GeV, $m_c=1.52$ GeV. We here confine ourselves to the charmed quark only. The potentials V_1 and V_2 which occur in the spin-orbit, the Darwin, the Fermi, and the tensor terms responsible for the fine structure are taken to be one-gluon-induced Coulomb-like potentials:

$$V_1(r)=V_2(r)=-\frac{K'}{r},\qquad K'=0.60.\qquad(3)$$

Based on the potential model outlined above, we discussed the mass spectrum of the $l=1$, charmed resonances in Ref. [3]. In particular, we predicted two $j=1/2$ p states $D_1^*(1^+)$ and $D_0(0^+)$ at 2.29 GeV and 2.19 GeV, respectively. These resonances are only 280 MeV and 321 MeV higher than the corresponding s states $D^*(1^-)$ and $D(0^-)$. But they are above the threshold for their main decay channels $D^*\pi$ and $D\pi$, respectively. Since these decays are s-wave decays in HQET, the decay width will be large; D_1^* and D_0 will appear as broad resonances. These states have now been observed by the Belle Collaboration at somewhat higher mass (see below). Thus, in view of the fact that ($m_s-m_d=0.14$), one would expect the $D_{s_1}^*$ and D_{s_0} states at 2.43 GeV and 2.33 GeV, respectively. The work of Ref. [3] was extended to strange and bottom quarks in Ref. [7]. In Ref. [7], we predicted the masses of $D_{s_1}^{*+}$ and $D_{s_0}^+$ at 2.453 GeV and 2.357 GeV, respectively, about 340 MeV and 389 MeV higher than the corresponding states $D_s^{*+}(1^-)$ and $D_s^+(0^-)$. The former agrees with the resonance $D_{s_1}^*(2.46\ \mathrm{GeV})$ in [2] while the latter is at somewhat higher mass than that observed in [1,2]. Since $D_{s_1}^{*+}$ and $D_{s_0}^+$ are below the threshold of the decay channels D^*K and DK, they will appear as narrow resonances.

We now comment on the mass difference between members of the $j=3/2$ and 1/2 multiplets. In the potential model [7], we find the mass difference $m_{D^*_{s_1}}-m_{D_{s_0}}\sim 100$ MeV, whereas the experimental value for this mass difference [2] is 144 MeV, close to the mass difference $m_{D^*_s}-m_{D_s}=143.8\pm 0.4$ MeV. In order to see whether this deficiency is an artifact of the potential model obtained in the previous paragraph but not of the bound state picture as such, we want to discuss some general features of the model. From Eqs. (1) and (3)

$$m_{D^*_s}-m_{D_s}=\langle V_F\rangle_{\text{triplet}}-\langle V_F\rangle_{\text{singlet}}$$

$$=\frac{2}{3m_cm_s}4\pi K'|\Psi_{1s}(0)|^2\equiv\frac{2}{3}\lambda_s, \tag{4}$$

$$m_{D^*_{s_1}}-m_{D_{s_0}}=\frac{1}{m_cm_s}[(\vec{S}\cdot\vec{L})_{D^*_{s_1}}-(\vec{S}\cdot\vec{L})_{D_{s_0}}]K'\left\langle\frac{1}{r^3}\right\rangle_{1p}+\frac{1}{12m_cm_s}[(S_{12})_{D^*_{s_1}}-(S_{12})_{D_{s_0}}]3K'\left\langle\frac{1}{r^3}\right\rangle_{1p}$$

$$=\frac{8}{3m_cm_s}K'\left\langle\frac{1}{r^3}\right\rangle_{1p}\equiv\frac{8}{3}\lambda'_{1s}, \tag{5}$$

$$m_{D^*_{s_2}}-m_{D_{s_1}}=\frac{1}{m_cm_s}[(\vec{S}\cdot\vec{L})_{D^*_{s_2}}-(\vec{S}\cdot\vec{L})_{D_{s_1}}]K'\left\langle\frac{1}{r^3}\right\rangle_{1p}+\frac{1}{12m_cm_s}[(S_{12})_{D^*_{s_2}}-(S_{12})_{D_{s_1}}]3K'\left\langle\frac{1}{r^3}\right\rangle_{1p}$$

$$=\frac{16}{15}\frac{1}{m_cm_s}K'\left\langle\frac{1}{r^3}\right\rangle_{1p}\equiv\frac{16}{15}\lambda'_{1s}, \tag{6}$$

$$m_{D^*_{s_1}}-m_{D_{s_0}}=\frac{5}{2}(m_{D^*_{s2}}-m_{D_{s1}}). \tag{7}$$

As is well known Eq. (4) is on solid ground. The mass splitting between the $^3S_1$ and $^1S_0$ state is due to the Fermi interaction $\overline{\mu}_q\cdot\overline{\mu}_Q$ and is governed by the short range Coulomb-like potential. For p-wave states, the same interaction induces the tensor term S_{12}. The spin-orbit coupling $\vec{S}\cdot\vec{L}$ is needed to preserve the observed hierarchy in the mass spectrum of heavy p-state mesons. In its absence, we would get $m_{D^*_{s_1}}-m_{D_{s_0}}=-5(m_{D^*_{s_2}}-m_{D_{s_1}})$, in contradiction to the experimentally observed mass heirarchy. The spin orbit coupling $(1/m_q^2)\vec{S}_q\cdot\vec{L}$ gives the mass splitting

$$m_{j=3/2}-m_{j=1/2}=\frac{3}{2}\frac{1}{2m_q^2}\left\langle\frac{1}{r}\frac{dV_1}{dr}\right\rangle=\frac{3}{4}\frac{m_c}{m_q}\lambda_{1q}. \tag{8}$$

With the confining potential $V(r)$ given in Eq. (2), one has the relations [8]

$$4\pi|\Psi_{1s}(0)|^2=2\mu\left\langle\frac{dV}{dr}\right\rangle_{1s}=2\mu\left[\frac{1}{b^2}+K\left\langle\frac{1}{r^2}\right\rangle_{1s}\right], \tag{9}$$

$$\left\langle\frac{1}{r^3}\right\rangle_{1p}=\frac{2\mu}{4}\left\langle\frac{dV}{dr}\right\rangle_{1p}=\frac{2\mu}{4}\left[\frac{1}{b^2}+K\left\langle\frac{1}{r^2}\right\rangle_{1p}\right]. \tag{10}$$

Using these relations we get from Eqs. (4) and (5)

$$m_{D^*_s}-m_{D_s}=\frac{2}{3}K'\frac{2\mu}{m_cm_s}\left[\frac{1}{b^2}+K\left\langle\frac{1}{r^2}\right\rangle_{1s}\right], \tag{11}$$

$$m_{D^*_{s_1}}-m_{D_{s_0}}=\frac{2}{3}K'\frac{2\mu}{m_cm_s}\left[\frac{1}{b^2}+K\left\langle\frac{1}{r^2}\right\rangle_{1p}\right]. \tag{12}$$

Hence we obtain

$$m_{D^*_{s_1}}-m_{D_{s_0}}=m_{D^*_s}-m_{D_s} \tag{13}$$

only if

$$\left\langle\frac{1}{r^2}\right\rangle_{1p}=\left\langle\frac{1}{r^2}\right\rangle_{1s}. \tag{14}$$

It is unlikely that this equality would hold in a potential model. In fact one would expect $\langle 1/r^2\rangle_{1p}$ to be less than $\langle 1/r^2\rangle_{1s}$. This is the reason why we get $(m_{D^*_{s_1}}-m_{D_{s_0}})$ less than $(m_{D^*_s}-m_{D_s})$. In Refs. [3,7], we obtained

$$\left\langle\frac{1}{r^2}\right\rangle_{1s}\approx 0.357\ \text{GeV}^2,$$

$$\left\langle\frac{1}{r^2}\right\rangle_{1p}\approx 0.088\ \text{GeV}^2. \tag{15}$$

These values give

$$m_{D^*_s}-m_{D_s}\approx 147\ \text{MeV},$$

$$m_{D^*_{s_1}}-m_{D_{s_0}}\approx 91\ \text{MeV}. \tag{16}$$

We now wish to comment on recently observed states D_1^* and D_0 in the nonstrange sector by the Belle Collaboration [11]

$$m_{D_1^*}=(2427\pm 26\pm 20\pm 15)\ \text{MeV}/c^2,$$

$$m_{D_0}=(2308\pm 17\pm 15\pm 28)\ \text{MeV}/c^2. \tag{17}$$

The axial vector states mix. The mixing angle is given by [11]

$$\omega = -0.10 \pm 0.03 \pm 0.02 \pm 0.02. \tag{18}$$

Taking these masses at their face values,

$$m_{D_1^*} - m_{D_0} \approx 119 \ \text{MeV}, \quad m_{D_1} - m_{D_1^*} \approx 0. \tag{19}$$

First we wish to show that the observed mass spectra of D mesons imply that the parameters λ_q, λ'_{1q} are independent of the light flavor. Thus

$$m_{D_s^*} - m_{D_s} \approx 144 \ \text{MeV},$$

$$m_{D^*} - m_D \approx 140 \ \text{MeV} \Rightarrow \lambda_s \approx \lambda_d, \tag{20}$$

$$m_{D_{s_2}^*} - m_{D_{s_1}} \approx 40 \ \text{MeV},$$

$$m_{D_2^*} - m_{D_1} \approx 40 \ \text{MeV} \Rightarrow \frac{16}{15}\lambda'_{1s} \approx \frac{16}{15}\lambda'_{1d}, \tag{21}$$

$$\lambda'_{1s} = \lambda'_{1d} = 36 \ \text{MeV}. \tag{22}$$

In view of the definition of λ_{1q} given in Eq. (8), one would expect λ_{1q} to be independent of the light flavor (see below). We note that

$$m_{D_{s_1}} - m_{D_{s_1}^*} = \frac{3}{4}\frac{m_c}{m_s}\lambda_{1s} + \frac{1}{6}\lambda'_{1s}. \tag{23}$$

The spin-orbit potential $V_1(r)$ for the light quark in the HQET as given in Eq. (1) is expected to be of same form as $V_2(r)$. We may further assume $V_1(r)$ to have the same strength as $V_2(r)$ [3,7]. With this assumption

$$\lambda_{1q} = \lambda'_{1q} \approx 36 \ \text{MeV}. \tag{24}$$

Thus we get

$$m_{D_{s_1}} - m_{D_{s_1}^*} \approx 91 \ \text{MeV} \quad (75 \ \text{MeV} \ [2]),$$

$$m_{D_1} - m_{D_1^*} \approx 126 \ \text{MeV},$$

$$m_{D_1^*} = m_{D_1} - 126 \ \text{MeV} = 2.295 \ \text{GeV},$$

$$m_{D_0} \approx 2.195 \ \text{GeV}, \tag{25}$$

compatible with the values of Ref. [7]. However, if we take λ_{1q} slightly less than λ_{1q}, say 30 MeV, i.e., $V_1(r)$ is slightly weaker than $V_2(r)$, then we get

$$m_{D_{s_1}} - m_{D_{s_1}^*} \approx 77 \ \text{MeV},$$

$$m_{D_1} - m_{D_1^*} \approx 106 \ \text{MeV},$$

$$m_{D_1^*} = 2.315 \ \text{GeV},$$

$$m_{D_0} = 2.215 \ \text{GeV}. \tag{26}$$

Finally, the mixing angle between axial vector states D_1 and D_1^* is given by

$$\tan 2\omega = \frac{2m_{D_1-D_1^*}}{m_{D_1} - m_{D_1^*}} = -\frac{(\sqrt{2}/3)\lambda'_{1d}}{(3m_c/4m_s)\lambda_{1d} + \frac{1}{6}\lambda'_{1d}} \approx 0.135, \tag{27}$$

$$\omega = 0.07, \tag{28}$$

to be compared with the value of ω [11] given in Eq. (18). A higher value of ω would imply λ_{1d} less than λ'_{1d}, viz., $V_1(r)$ is weaker than $V_2(r)$. For $\lambda_{1d} \approx 30$ MeV, we get $\omega = 0.08$.

To reinforce the above conclusion, we note from Eqs. (5), (6), and (11) that

$$\frac{5m_{D_{s_2}^*} + 3m_{D_{s_1}}}{8} - \frac{3m_{D_{s_1}^*} + m_{D_{s_0}}}{4} = \frac{3}{2}\left(\frac{m_c}{m_s}\lambda_{1s} + \lambda'_{1s}\right). \tag{29}$$

Using the experimental values for masses [1,2], we get

$$\frac{3}{2}\left(\frac{m_c}{m_s}\lambda_{1s} + \lambda'_{1s}\right) = 133 \ \text{MeV}. \tag{30}$$

Now, using $\lambda'_{1s} \approx 36$ MeV, we get $\lambda_{1s} \approx 33$ MeV compatible with the assumption stated above. The flavor independence of these parameters gives

$$\frac{3m_{D_1^*} + m_{D_0}}{4} = \frac{5m_{D_2^*} + 3m_{D_1}}{5} - 133 \ \text{MeV} \approx 2312 \ \text{MeV}. \tag{31}$$

This value is about 85 MeV, below that of the Belle Collaboration, but only about 30 MeV above the values implied by Eq. (25). However this value is in agreement with that obtained from Eq. (26).

The following comments are in order. In Refs. [9,10], it was suggested that in the heavy quark limit $j = 1/2$ states with $J^P = 1^+$ and 0^+ are chiral partners of 1^- and 0^-. In Ref. [9] a mass difference between parity doublets $(0^-,1^-)$ and $(0^+,1^+)$ arises due to chiral symmetry breaking and is thus expected to be small, a feature which we also get. In particular, they find the mass difference of order 338 MeV. In Ref. [10], they obtained $m_{D_1^*} - m_{D_0} \approx m_{D^*} - m_D$. In the bound state model, we get $m_{D_0} - m_D \approx 326$ MeV. Further, we note that the $j = 1/2$ p-wave multiplet lies below the $j = 3/2$ multiplet, due to spin-orbit coupling of the light quark in the limit of heavy quark spin symmetry reminiscent of the fine structure of the hydrogen atom spectrum.

We now briefly discuss the decays of the resonances $D_{s_1}^*$ and D_{s_0}. The isospin conserving decays

$$D_{s_1}^* \to D^{*0}K^+ (D^{*+}K^0),$$

$$D_{s_0}^+ \to D^0 K^+ (D^+ K^0)$$

are not energetically allowed. The experimentally observed decays $D_s^{*+}\pi^0$ and $D_s^+\pi^0$ violate isospin. One obvious possibility is that these decays proceed via the η meson:

$$D_{s_1}^{*}\rightarrow D_s^{*+}\eta\rightarrow D_s^{*+}\pi^0,$$

$$D_{s_0}^{+}\rightarrow D_s^{+}\eta\rightarrow D_s^{+}\pi^0.$$

In this picture the coupling $g_{D_{s_0}D_s\pi}$ can be expressed in terms of $g_{D_{s_0}D_s\eta}$ as

$$g_{D_{s_0}D_s\pi}=g_{D_{s_0}D_s\eta}\frac{m^2_{\eta-\pi^0}}{m_\eta^2-m_{\pi^0}^2},\tag{32}$$

where [12]

$$m^2_{\eta-\pi^0}=-\frac{1}{\sqrt{3}}[(m_{K^0}^2-m_{K^+}^2)+(m_{\pi^+}^2-m_{\pi^0}^2)].\tag{33}$$

Then using $SU(3)$

$$g_{D_{s_0}^+D_s^+\eta}=-\sqrt{\frac{2}{3}}g_{D_0^+D^+\pi^0},\tag{34}$$

we get

$$\left(\frac{g_{D_{s_0}^+D_s^+\pi^0}}{g_{D_0^+D^+\pi}}\right)^2=\frac{2}{3}\left(\frac{m^2_{\eta-\pi^0}}{m_\eta^2-m_\pi^2}\right)^2\approx 7.7\times10^{-5},\tag{35}$$

$$\frac{\Gamma(D_{s_0}^+\rightarrow D_s^+\pi^0)}{\Gamma(D_0^+\rightarrow D^+\pi^0)}=\frac{|\vec{p}|_{D_s}}{|\vec{p}|_D}\frac{m_{D_0}^2}{m_{D_{s_0}}^2}\left[\frac{(m_{D_{s_0}}-m_{D_s})}{(m_{D_0}-m_D)}\right]^2\times[7.7\times10^{-5}]\tag{36}$$

$$\approx(1.07)(7.7\times10^{-5})=8.2\times10^{-5},\tag{37}$$

where we have defined the dimensionless coupling constant $g_{D_0^+D^+\pi^0}$ as [13]

$$M=\sqrt{4p_0p_0'}\langle D^+(p')|J_\pi|D_0(p)\rangle=i\left(\frac{m_{D_0}^2-m_D^2}{2m_D}\right)g_{D_0^+D^+\pi^0}$$

$$\approx i(m_{D_0}-m_D)g_{D_0^+D^+\pi^0}\tag{38}$$

and have used $m_{D_0}=2.20$ GeV and the experimental values for the other masses. Assuming the decay width

$$\Gamma(D_0^+\rightarrow D^+\pi^0)\sim 200\ \text{MeV},\tag{39}$$

we get

$$\Gamma(D_{s_0}^+\rightarrow D_s^+\pi^0)\sim 16\ \text{keV}.\tag{40}$$

That the decay width $\Gamma(D_0^+\rightarrow D^+\pi^0)$ is of order 200 MeV can be seen as follows. In HQET, the coupling $g_{D^*D\pi}$ is usually parametrized as $g_{D^*D\pi}=\lambda_D m_D/f_\pi$; in the same spirit $g_{D_0D\pi}$ is parameterized as [13]

$$g_{D_0D\pi}=2\lambda_{D_0}\frac{m_D}{f_\pi}\quad(f_\pi=132\ \text{MeV}).\tag{41}$$

Thus we get

$$\Gamma(D_0^+\rightarrow D^+\pi^0)=\frac{1}{8\pi}|M|^2|\vec{p}|\frac{1}{m_{D_0}^2}$$

$$=\frac{1}{8\pi}\frac{1}{m_{D_0}^2}\frac{(4\lambda_{D^0}^2m_D^2)}{f_\pi^2}(m_{D_0}-m_D)^2|\vec{p}|$$

$$\approx 202\lambda_{D^0}^2\ \text{MeV}\lesssim 202\ \text{MeV}.\tag{42}$$

In HQET the decay

$$D_{s_1}^{*+}\rightarrow D_s^{*+}\pi^0$$

is also s wave and is related to

$$D_{s_0}^{+}\rightarrow D_s^{+}\pi^0$$

as follows:

$$\frac{\Gamma(D_{s_1}^{*}\rightarrow D_s^{*+}\pi^0)}{\Gamma(D_{s_0}^{+}\rightarrow D_s^{+}\pi^0)}=\frac{|\vec{p}|_{D_s^*}}{|\vec{p}|_{D_s}}\frac{(m_{D_{s_1}^*}-m_{D_s^*})^2}{(m_{D_{s_0}}-m_{D_s})^2}.\tag{43}$$

Thus, except for phase space, the decay width of $D_{s_1}^{*+}\rightarrow D_s^{*+}\pi^0$ is equal to that of $D_{s_0}^{+}\rightarrow D_s^{+}\pi^0$. However, the mixing between axial vector states D_{s_1} and $D_{s_1}^*$ can contribute to the decay width of $D_{s_1}^*$. The contribution to the decay width of $D_{s_1}^*$ due to mixing is $\omega^2\Gamma_{D_{s_1}}$. Since $\Gamma_{D_{s_1}}<2.3$ MeV we get $\omega^2\Gamma_{D_{s_1}}<23$ keV for the mixing angle $\omega\simeq0.1$ as implied by Eq. (28). Thus the width of $D_{s_1}^*$ is expected to be about twice that of D_{s_0}.

Finally, the resonances $D_{s_1}^{*+}$ and $D_{s_0}^{+}$ can also decay to $D_s^{*+}\gamma$ and $D_s^{+}\gamma$, respectively, by the $E1$ transition [3]. For the decay

$$D_{s_0}^{+}\rightarrow D_s^{+}\gamma,$$

the decay width is given by

$$\Gamma=\frac{4\alpha}{3}|M^+|^2k^3.\tag{44}$$

In the quark model [3]

$$M^+=\mu\left[\frac{2}{3m_c}I_c-\frac{1}{3m_s}I_s\right]\tag{45}$$

where I_c and I_s are overlap integrals. Our estimate for the radiative decay width comes out to be 0.2 keV [7].

To conclude, in a picture in which D mesons are regarded as bound states $c\bar{q}$, the potential model considered by us gives

$$m_{j=1/2} < m_{j=3/2}, \tag{46}$$

$$m_{D^*_{s_1}} - m_{D_{s_0}} = \frac{5}{2}(m_{D^*_{s_2}} - m_{D_{s_1}}), \tag{47}$$

$$m_{D^*_{s_1}} - m_{D_{s_0}} \approx \frac{1}{\sqrt{2}}(m_{D^*_s} - m_{D_s}). \tag{48}$$

Further, using the experimental data of Refs. [1,2], our analysis gives

$$\frac{3m_{D_1^*} + m_{D_0}}{4} \approx 2.312 \text{ GeV}, \quad m_{D_1^*} = 2.35 \text{ GeV},$$

$$m_{D_0} = 2.20 \text{ GeV}, \tag{49}$$

$$\left(\frac{3m_{D_1^*} + m_{D_0}}{4}\right) - \left(\frac{3m_{D^*} + m_D}{4}\right) \approx 337 \text{ MeV}. \tag{50}$$

The p-wave $j=1/2$ multiplet is 337 MeV, above the s-wave multiplet. Except for the mass relation (48) (which is hard to understand in a bound state picture), the general features of the bound state picture are compatible with the experimental data.

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