Regge analysis of pion-pion (and pion-kaon) scattering for energy $s^{1/2} > 1.4$ GeV

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We perform a detailed Regge analysis of *NN*, πN, *KN*, $\pi\pi$, and πK scattering. From it, we find expressions that represent the $\pi\pi$ scattering amplitudes with an accuracy of a few percent for exchange of isospin zero and $\sim$15% for exchange of isospin 1, and this for energies $s^{1/2} > 1.4$ GeV and for momentum transfers $|t|^{1/2} \leq 0.4$ GeV. These Regge formulas are perfectly compatible with the low energy ($s^{1/2} \sim 1.4$ GeV) scattering amplitudes deduced from $\pi\pi$ phase shift analyses as well as with higher energy ($s^{1/2} \gtrsim 1.4$ GeV) experimental $\pi\pi$ cross sections. They are also compatible with *NN*, *KN*, and πN experimental cross sections using factorization, a property that we check with precision. This contrasts with results from current phase shift analyses of the $\pi\pi$ scattering amplitude, which bear little resemblance to reality in the region $1.4 < s^{1/2} < 2$ GeV, as they are not well defined and increasingly violate a number of physical requirements when the energy grows. πK scattering is also considered, and we present a Regge analysis for these processes valid for energies $s^{1/2} > 1.7$ GeV. As a by-product of our analysis, we obtain also a fit of *NN*, πN, and *KN* cross sections valid from c.m. kinetic energy $E_{\rm kin} \simeq 1$ GeV to multi-TeV energies.

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I. INTRODUCTION

A precise and reliable knowledge of the $\pi\pi$ scattering amplitude has become increasingly important in the last years. This is so, in particular, because $\pi\pi$ scattering is one of the few places where one has more observables than unknown constants in a chiral perturbation theory analysis, so it provides a window to higher order terms. Moreover, an accurate determination of the *S*-wave scattering lengths and of the phase shifts at $s^{1/2} = m_K$ provides essential information for two subjects under intensive experimental investigation at present: viz., pionic atom decays and *CP* violation in the kaonic system. In recent papers, Ananthanarayan, Colangelo, Gasser, and Leutwyler (ACGL) [1] Colangelo, Gasser, and Leutwyler [2], Descotes *et al.* [3], and Kamiński, Leśniak, and Loiseau [3] have used experimental information, analyticity, and unitarity (in the form of the Roy equations) and, in Ref. [2], chiral perturbation theory to construct the $\pi\pi$ scattering amplitude at low energy $s^{1/2} \leq 0.8$ GeV. For these analyses one needs as input the imaginary part of the $\pi\pi$ amplitudes above the energy at which the Roy analysis stops; in particular, one needs the scattering amplitudes for $s^{1/2}$ above 1.4 GeV, which will be the subject of the present paper.

Unfortunately, the authors in Refs. [2,3] take their $\pi\pi$ scattering amplitude in this energy region from ACGL [1], which presents a number of serious drawabacks.[1] First of all, the input scattering amplitude at energy $s^{1/2} \gtrsim 2$ GeV which these authors use (following Pennington [5]) is not physically acceptable, as it contradicts known properties of standard Regge theory and, moreover, is quite incompatible with *experimental*[2] $\pi\pi$ total cross sections [7], and this in spite of the large errors assumed by ACGL. Second, the scattering amplitude for 1.4 GeV $\leq s^{1/2} \leq$ 1.9 GeV that ACGL (and, following them, the authors in Refs. [2,3]) use is obtained from phase shift analyses, specifically the Cern-Munich set of analyses [8], which are subject to large uncertainties and which, indeed, can be shown to contradict a number of physical requirements. [Although we will not discuss this here (see Ref. [9]), it is also clear that the errors ACGL and the authors in Ref. [2] take for some of their lower energy experimental input data are excessively optimistic and, moreover, certain of their chiral parameters are likely to be biased [10].] One should imagine that the use of incorrect high energy input should lead to inconsistent low energy output. In fact, this occurs in the work by Colangelo, Gasser, and Leutwyler [2], where the central values are probably displaced and the errors claimed are excessively optimistic and lead to several mismatches, as shown in Refs. [9,11].

In the present paper we will not concern ourselves with the reliability or otherwise of the *low energy* consequences of faulty high energy input, but will concentrate our efforts in ascertaining what a *correct* high energy input should be. To do this, we will perform a detailed Regge analysis and show

[1] In Ref. [4], the Regge parameters of ACGL are also used for πK scattering; perhaps this is the reason why they are not able to get a satisfactory description of this process.

[2] It should be noted that Pennington has publicly stated (in the Conversano workshop, 2003) that his analysis, tenable in 1974, is superseded by more recent developments, both experimental and theoretical. In fact, already by 1977 it was clear to experts that standard Regge behavior also holds for $\pi\pi$ scattering; see, e.g., Froggatt and Petersen [6], who use the correct Regge behavior in their dispersive analysis of $\pi\pi$ scattering.

that it is compatible with *experimental* data for all values of $s^{1/2} \gtrsim 1.4$ GeV (for some $\pi\pi$ processes, even down to $s^{1/2} \sim 1$ GeV). The resulting $\pi\pi$ amplitudes, summarized in Eqs. (5), (6), (7), (17), (18), and (27) and Table II below, should provide a correct and accurate input for dispersive studies of $\pi\pi$ scattering.

Our analysis will be an improvement on standard ones not only for $\pi\pi$ and πK, but even for πN, KN, and NN in that we will be able to give an accurate description of the amplitudes for energies ranging from a kinetic energy in the center of mass $E_{\rm kin} \simeq 1$ GeV to the TeV region. This accuracy reaches the level of a few percent for zero isospin exchange, and it is less precise for the isospin-1 exchange amplitude, for which the errors may go up to ~15% at low energy.

An analysis of high energy πK scattering is possible by a straightforward extension of the methods here; it is given in Sec. III, where we present precise Regge formulas for zero isospin exchange, valid for energies $s^{1/2} > 1.7$ GeV.

The analysis of $\pi\pi$ and πK scattering up to (relatively) low energies, ~14 GeV, is described in Secs. II and III; in Sec. IV, we extend it to multi-TeV energies. As a by-product of our analysis, we present also a parametrization of NN, πN, and KN total cross sections compatible with the Froissart bound and valid from $E_{\rm kin} \simeq 1$ GeV to ~30 TeV. In particular, we predict the total pp cross section at the LHC to be

$$\sigma_{pp} = \begin{cases} 104 \pm 4 \ \text{mb} & \text{(B)}, \\ 113 \pm 4 \ \text{mb} & \text{(C)}, \end{cases}$$

where B and C refer to the fits in Table II.[3]

Our results are summarized in Sec. V, where a brief discussion is also presented.

II. REGGE ANALYSIS OF $\pi\pi$ SCATTERING ($s^{1/2} \geqslant 1.4$ GeV)

We normalize scattering amplitudes to

$$\sigma_{AB} = \frac{4\pi^2}{\lambda^{1/2}(s, m_A^2, m_B^2)} \operatorname{Im} F_{A+B\to A+B}(s,0),$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc.$$

σ_{AB} is the total $A+B$ cross section; for NN ($\bar{p}p, pp$) and πN scattering, we understand that the cross sections are spin averaged. According to Regge theory, the imaginary part of a scattering amplitude with fixed isospin in the t channel, $\operatorname{Im} F^{(I_t)}_{A+B\to A+B}(s,t)$, *factorizes*[4] as a product: for each Regge pole R, we can write

$$\operatorname{Im} F^{(I_t)}_{A+B\to A+B}(s,t) \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\simeq} f_A^{(R)}(t) f_B^{(R)}(t) (s/\hat{s})^{\alpha_R(t)}. \quad (1)$$

Here $\hat{s}$ is a constant, usually taken to be 1 GeV^2; we will do so here. A similar formula holds for the real parts:

$$\operatorname{Re} F^{(I_t)}_{A+B\to A+B}(s,t) \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\simeq} \operatorname{Re} \xi(R) f_A^{(R)}(t) f_B^{(R)}(t) (s/\hat{s})^{\alpha_R(t)}. \quad (2)$$

$\xi(R)$, with $\operatorname{Im} \xi(R) = 1$, is known as the *signature* factor; for the Pomeron (P), P', and rho Regge poles one has

$$\operatorname{Re} \xi(R) = -\frac{1 + \cos \pi\alpha_R}{\sin \pi\alpha_R}, \quad R = P, P'$$

$$\operatorname{Re} \xi(\rho) = \frac{1 - \cos \pi\alpha_\rho}{\sin \pi\alpha_\rho}. \quad (3)$$

The residue functions $f_i^{(R)}(t)$ depend on the quantum numbers of the Regge pole exchanged, on the particles that couple to it, and if we had external currents, also on their virtuality, but the power $(s/\hat{s})^{\alpha_R(t)}$ is universal and depends only on the Regge pole exchanged in channel t. The exponent $\alpha_R(t)$ is the Regge trajectory associated with the quantum numbers in channel t. For the Pomeron, which is rather flat, we will take it linear; for the rho, a more precise quadratic formula may be used. We thus write, for small t,

$$\alpha_P(t) \underset{t\sim 0}{\simeq} \alpha_P(0) + \alpha_P' t, \quad \alpha_\rho(t) \underset{t\sim 0}{\simeq} \alpha_\rho(0) + \alpha_\rho' t + \tfrac{1}{2}\alpha_\rho'' t^2. \quad (4)$$

For the ρ and Pomeron pole, fits to high energy πN and NN processes give

$$\alpha_\rho(0) = 0.52 \pm 0.02, \quad \alpha_\rho' = 0.90 \ \text{GeV}^{-2},$$

$$\alpha_\rho'' = -0.3 \ \text{GeV}^{-4},$$

$$\alpha_P(0) = 1, \quad \alpha_P' = 0.2 \pm 0.1 \ \text{GeV}^{-2}.$$

The Regge parameters taken here are based on those in the global fit of Rarita *et al.* [14], which are still the best available as there are few modern data for the *slopes* in the relevant energy range. There are a few differences, however. For $\alpha_\rho(0)$, we take the value 0.52 ± 0.02 instead of 0.58. This is more consistent with determinations based on deep inelastic scattering (see, e.g., the paper of Adel *et al.* [13]) as well as with fits to πN cross sections; see Sec. IV here. Moreover, for $\alpha_\rho(t)$ we use a quadratic formula that agrees with the average slope of Ref. [14] for small, negative t, and which fulfills the condition $\alpha_\rho(M_\rho^2) = 1$. Finally, for α_P', Rarita *et al.* give 0.11, Froggatt and Petersen [6] give 0.3, and the shrinking of the diffraction peak at the Tevatron suggests 0.26. Our choice here encompasses these three values. These are minor improvements as, in fact, for our fits in the present paper we only need the values of the $\alpha_R(0)$; the slopes only intervene in sum rules.

[3]This number agrees with the one obtained in Ref. [12]. We thank Professor Nicolescu for pointing this out to us.

[4]In potential theory factorization can be proved rigorously; in relativistic theory, it follows from extended unitarity or, in QCD, from the DGLAP formalism [13].

Let us now turn to the functions $f_i(t)$. With respect to them we have two quite separate questions. First of all, we have the question of their normalization—that is to say, the values $f_i(0)$. These can be obtained with little ambiguity and small errors by fitting experimental NN, πN, and $\pi\pi$ total cross section data; we will do precisely that below. A different matter is the dependence of the $f_i(t)$ on t—i.e., the ratios $f_i(t)/f_i(0)$—which is important in particular for Roy equations or sum rules like the ones at the end of the present section. These are obtained from fits to the slopes of $NN, \pi N$ differential cross sections. Unfortunately, these fits are not unique, because both the background and the functional forms assumed for the $f_i(t)$ have a non-negligible influence on the results and because for the differential cross sections also the *real* part of the scattering amplitudes intervene. Moreover, the parameters of these fits were obtained before QCD emerged as the theory of strong interactions; these fits were extended to large values of t where, as we now know, Regge theory must fail and one has instead the Brodsky-Farrar behavior [15]. They are thus forced fits.

The situation, however, is not hopeless; the difference between the *numerical* results of various fits is small, for small values of $|t|$. For example, the numerical difference for the ratios $f_P(t)/f_P(0)$ between Refs. [9] and [17] is below the 10% level for $|t|^{1/2} \leqslant 0.4$ GeV, which covers the values of t in which we are interested here. In the present paper we have chosen the t dependence of Ref. [14], which was obtained in a detailed fit to many data.

Before writing explicit formulas for the various processes $(NN, \pi N, \pi\pi)$ we have to decide in which variable we assume Regge behavior to hold, which is important for us since we are going down to rather low energies. In Eqs. (1), (2) we have taken the c.m. energy squared, $s=(p_1+p_2)^2$, with p_i the momenta of the incoming particles. Other possibilities are the s-u crossing symmetric variable $\nu=2p_1\cdot p_2$, and $E_{\rm kin}^2$, so we could assume behaviors like ν^{α_P} or $E_{\rm kin}^{2\alpha_P}$ instead of s^{α_P}, etc. We have, in our fits, tried all three possibilities; the fits using s, as in Eqs. (1), (2), all have substantially better $\chi^2/N_{\rm DOF}$ than those using $\nu=2p_1\cdot p_2$ or $E_{\rm kin}^2$. Therefore, we stick to Regge behavior in the variable s, as in Eqs. (1), (2).

Regge formulas for $\pi\pi$, πN, and NN scattering. We start with $\pi\pi$ scattering. For exchange of isospin $I_t=0$ in the t channel, containing the Pomeron and P' pole [the second associated with the $f_2(1270)$ resonance], we have

$$\mathrm{Im}\,F_{\pi\pi}^{(I_t=0)}(s,t) \underset{\substack{s\to\infty \\ t\ \text{fixed}}}{\simeq} P(s,t)+P'(s,t),$$

$$P(s,t)=\beta_P\alpha_P(t)\,\frac{1+\alpha_P(t)}{2}\,e^{bt}(s/\hat{s})^{\alpha_P(t)},$$

$$P'(s,t)=\beta_{P'}\frac{\alpha_{P'}(t)[1+\alpha_{P'}(t)]}{\alpha_{P'}(0)[1+\alpha_{P'}(0)]}\,e^{bt}(s/\hat{s})^{\alpha_{P'}(t)},$$

$$\alpha_{P'}(t)=\alpha_\rho(t),$$

$$b=(2.4\pm 0.2)\ \ \mathrm{GeV}^{-2}. \tag{5}$$

Here $\beta_P=[f_\pi^{(P)}]^2$, $\beta_{P'}=[f_\pi^{(P')}]^2$.

The expression (5) is like its counterpart in Ref. [14], except for the P' pole parameters. In fact, the subleading contribution of the P' pole, which is necessary at the lowest energy range, is added somewhat empirically; its parameters are not well known, and we start by assuming the corresponding trajectory to be degenerate with the one of the rho, as is suggested by a number of theoretical developments (in particular the QCD theory of Regge trajectories [13]) and as is done in Ref. [6]: $\alpha_{P'}(t)=\alpha_\rho(t)$. In Ref. [14], a larger value (0.7 instead of 0.52) was given for the intercept of the P' pole and a smaller number was taken for its residue. In Sec. IV we will present global fits to data, leaving, in particular, $\alpha_{P'}(0)$ as a free parameter. The results for it are in reasonable agreement with other modern determinations and altogether vary from 0.68 to 0.54, not far from the degeneracy assumption value of $\alpha_{P'}(0)=0.52\pm 0.02$.

It should perhaps also be remarked that Eq. (5), in what respects the Pomeron, is of limited validity (up to 10–15 GeV) since, at higher energies, total cross sections are known to rise. A modification of $P(s,t)$ in Eq. (5) that will make the parametrization valid up to multi-TeV energies will be given in Sec. IV.

For $I_t=1$, we also take the parametrization of Ref. [14]. We write

$$\mathrm{Im}\,F_{\pi\pi}^{(I_t=1)}(s,t) \underset{\substack{s\to\infty \\ t\ \text{fixed}}}{\simeq} \rho(s,t),$$

$$\rho(s,t)=\beta_\rho[(1.5+1)e^{bt}-1.5]\,\frac{1+\rho_\rho(t)}{1+\alpha_\rho(0)}(s/\hat{s})^{\alpha_\rho(t)}. \tag{6}$$

b is as before and $\beta_\rho=[f_\pi^{(\rho)}]^2$. The universal value of the slope of the diffractive factor, e^{bt}, for all three trajectories rho, P, and P', is what was found in Ref. [14] from fit to actual NN and πN data; it can nowadays be understood physically as a consequence of the universality of the Regge mechanism in QCD. We note that Froggatt and Petersen [6], who fit $\pi^+\pi^-$ data, find a value for b similar to ours for the Pomeron, but somewhat different ones for rho and P'. This last fact is not very meaningful as, in the fits to $\pi^+\pi^-$, the ρ, P' Regge poles are subleading and easily hidden by the Pomeron. We also remark that, in Ref. [11], we had added a small background to $\mathrm{Im}\,F_{\pi\pi}^{(I_t=1)}$ to join smoothly the asymptotic formulas to the experimental cross section at $s^{1/2}\sim 1.4$ GeV. With the value of the parameter β_ρ found in the present section, such a background is unnecessary.

For $\pi\pi$ scattering we have to add an amplitude for exchange of isospin 2, corresponding to double rho exchange, which we do by writing

$$\mathrm{Im}\,F_{\pi\pi}^{(I_t=2)}(s,t) \underset{\substack{s\to\infty \\ t\ \text{fixed}}}{\simeq} R_2(s,t)\equiv\beta_2 e^{bt}(s/\hat{s})^{\alpha_\rho(t)+\alpha_\rho(0)-1}. \tag{7}$$

We will discuss this quantity $R_2(s,t)$ later on; in particular, we will determine the quantity β_2, which is small. We will start by putting $\beta_2=0$ and correct for this afterwards.

The important parameters are β_P, $\beta_{P'}$, β_ρ. We can obtain them fitting NN (pp plus $\bar{p}p$) and πN cross sections (including the forward differential cross section for the charge exchange reaction $\pi^- p \to \pi^0 n$), from $\pi\pi$ cross sections or from a global fit to the two sets. We write

$$\frac{\sigma_{pp}+\sigma_{\tilde{p}p}}{2} \underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s,m_p^2,m_p^2)} \frac{1}{2} f_{N/\pi}^2 [P(s,0)+(1+\epsilon)P'(s,0)],$$

$$\sigma_{\pi^\pm p} \underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s,m_\pi^2,m_p^2)} f_{N/\pi} \times \left\{ \frac{1}{\sqrt{6}}[P(s,0)+P'(s,0)] \mp \frac{1}{2}\overline{\rho}(s,0) \right\},$$

$$\left.\frac{d\sigma(\pi^- p \to \pi^0 n)}{dt}\right|_{t=0} \underset{s \text{ large}}{\simeq} f_{N/\pi}^2 \frac{1-\cos\pi\alpha_\rho}{\sin^2\pi\alpha_\rho} \frac{\pi^3}{\lambda(s,m_\pi^2,m_p^2)} |\overline{\rho}(s,0)|^2. \quad (8)$$

Here $f_{N/\pi} \equiv f_N^{(P)}/f_\pi^{(P)}$, and we have defined

$$\overline{\rho}(s,t) = \beta_\rho^{(N\pi)}[(1.5+1)e^{bt}-1.5] \frac{1+\alpha_\rho(t)}{1+\alpha_\rho(0)} (s/\hat{s})^{\alpha_\rho(t)}, \quad (9)$$

with

$$\beta_\rho^{(N\pi)} = [f_\pi^{(P)} f_N^{(\rho)}/f_\pi^{(\rho)} f_N^{(P)}]\beta_\rho. \quad (10)$$

In Eq. (8), ϵ measures the admixture of the a_2 trajectory, which couples to nucleons (and, to a lesser extent, to kaons), but not to pions. In this equation we have put the same values of $f_{N/\pi}$ for Pomeron and P'. In Sec. IV we will discuss fits, allowing for different $f_{N/\pi}^{(P)}$, $f_{N/\pi}^{(P')}$; their central values will be somewhat displaced, but the improvement in the χ^2/N_{DOF} obtained by so doing is not significative.

We will, in this section, assume that the contribution of the a_2 trajectory to NN scattering is negligible—that is to say, that $\epsilon=0$. Current fits give a small value for this quantity; in Sec. IV, we will repeat the fits, leaving ϵ free.

Fits. We will not fit data for scattering off neutrons which would not improve the precision while, because the neutrons are necessarily bound, they could distort the fits. We will also not include the difference of cross sections $\sigma_{\bar{p}p}-\sigma_{pp}$ in the fits, as this would involve the contribution of at least three Regge poles (ω, ϕ, and π) which do not contribute to $\pi\pi$. One could include the reaction $\bar{p}p \to \bar{n}n$, which only involves exchange of the rho, but the data for it are few and with (comparatively) large errors, so it would add little to the analysis. For the charge-exchange reaction $\pi^- p \to \pi^0 n$, only data in the forward direction are included. This reaction is interesting in that, although it has much larger errors than the others, it receives contribution from the real part of the corresponding Regge pole, so it represents a completely independent test of the Regge formulas.

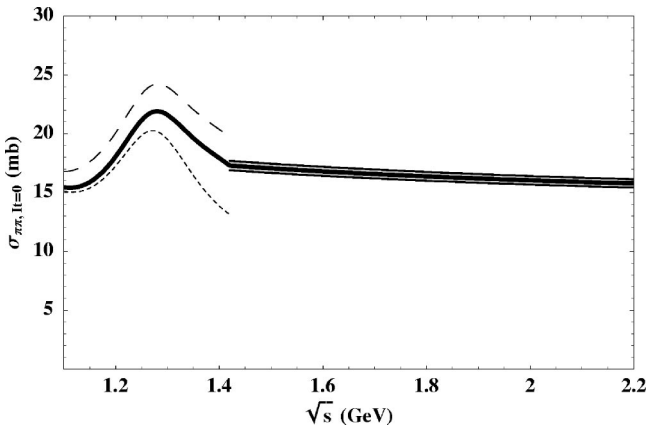

FIG. 1. The average cross section $\frac{1}{3}[2\sigma_{\pi^0\pi^+}+\sigma_{\pi^0\pi^0}]$, which is pure $I_t=0$. Solid lines, for $s^{1/2}>1.4$ GeV: Regge formula. The lines cover the errors in the values of the Regge residues. Solid lines, up to $s^{1/2}=1.4$ GeV: experimental cross section (from the fits in Ref. [11], actually with a slightly improved $D2$ wave). The dotted and dashed lines are representative of the experimental errors in the cross section.

Before going on to the actual fits, a few words have to be said on the energy regions in which one may expect Regge behavior (and, in particular, factorization) to hold. Generally speaking, we expect this to occur when one is past the region of elastic resonances and one also has $E_{\text{kin}}^2 \gg \Lambda^2$ ($\Lambda \simeq 0.4$ GeV is the QCD parameter), which means for $E_{\text{kin}} \gtrsim 1$ GeV, but the precise details vary for different reactions. Thus, for $pp, \bar{p}p$ scattering, there are no resonances and hence Regge behavior is expected to occur precociously: here we will actually fit from $E_{\text{kin}}=0.98$ GeV.

For $\pi\pi$ scattering it is difficult to tell when exactly one may use Regge formulas since data, particularly for $\pi^-\pi^-$, are not very good. For the cross section $\sigma^{(I_t=0)} \equiv \frac{1}{3}[2\sigma_{\pi^0\pi^+}+\sigma_{\pi^0\pi^0}]$, Eqs. (5), (6) provide a good representation for energies as low as $E_{\text{kin}}=1$ GeV, as shown in Fig. 1, but when resonances are more important, Regge behavior is a good approximation only at slightly higher energies. Another matter is that, at low energies ($s^{1/2} \sim 1.5$ GeV), the $\pi\pi$ data are of poor quality. Because of this, we will consider two extreme possibilities for actual fits. The first, which we will call *no-cut*, consists in including all $\pi\pi$ data for $E_{\text{kin}} > 1.1$ GeV ($s^{1/2} \geqslant 1.38$ GeV). The second possibility, which we call *cut*, consists in cutting out all data for energies below $s^{1/2}=2$ GeV. The difference in results between the two fits will be an indication of the *systematic* errors in our calculation.

For πN the formulas (8) fit well data down to $E_{\text{kin}} \sim 1.3$ GeV, but for the sum $\sigma_{\pi^+ p}+\sigma_{\pi^- p}$, one can go to $E_{\text{kin}} \sim 1$ GeV. For the difference $\sigma_{\pi^+ p}-\sigma_{\pi^- p}$ and for the charge-exchange reaction $\pi^- p \to \pi^0 n$, resonances somewhat spoil *local* agreement, but Eq. (8) provides a good *average* representation even down to 1 GeV, as has been known for a long time (see, e.g., Ref. [16]) and as can be seen in the lower energy region in our fit to $\pi^+ p$ data in Fig. 2. We will

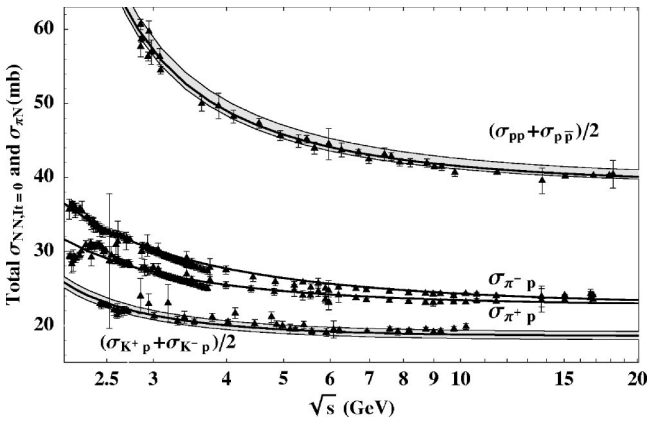

FIG. 2. The total cross sections $\sigma_{\pi^{\pm}p}$, $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$ and $\frac{1}{2}(\sigma_{K^+p}+\sigma_{K^-p})$. Black dots, triangles, and squares: experimental points. Solid lines: Regge formulas, with parameters as in our best fit. For $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$ and $\frac{1}{2}(\sigma_{K^+p}+\sigma_{K^-p})$, the grey bands cover the errors in the values of the Regge residues. For πN the theoretical error is of the order of that for $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$. Note that the thick line in the low energy experimental cross sections for πN is merely due to the accumulation of closely spaced data.

here start from $E_{\rm kin}=1.08$ GeV.

Another question is how high one goes in energy. In the present section we fit experimental data for c.m. kinetic energies $E_{\rm kin}\lesssim 16.5$ GeV: this is what is required for applications to $\pi\pi$ Roy equations, dispersion relations, and sum rules, since here the importance of the very high energy region is negligible. Nevertheless, and as stated before, parametrizations and fits valid up to multi-TeV energies will be given in Sec. IV.

The data on $\pi^- p\to\pi^0 n$ are from the compilation in Ref. [16]. For *NN* and πN we will take the data from the COMPAS Group compilations, as given in the Particle Data Tables [17]. For those data where systematic errors are not given, we have included a common systematic error of 0.5% for *pp*, 1% for $\bar{p}p$, and 1.5% for πp, which are like the standard systematic errors in other data. Another possibility is to take a common systematic error of 1.5% for all data: the difference of the results with the two will indicate the systematic errors of our fit. Since we are only interested in $\sigma^{pp}+\sigma^{\bar{p}p}$, we have also made a selection of *NN* data, as follows. We take only data at energies at which there are results for both *pp* and $\bar{p}p$, and, when there are, at a given energy, data from various experiments, we have taken only the most recent. This is designed to thin out the data to a number comparable in order of magnitude to that of $\pi\pi$, so that $\pi\pi$ data have a non-negligible weight in the joint fits. For $\pi\pi$ scattering we have taken the errors as given by the various experimental groups except for those of Abramowicz *et al.* [7], who only give statistical errors, much smaller than those of the other groups, and for which we have added a common systematic error of 1.5 mb to all points; even with this, the error, though comparable, is smaller than what other groups find.

We could fit separately the $NN,\pi N$ data and the $\pi\pi$ data of Ref. [7] or make a global fit. The results of these fits, in which we have put $\beta_2=0$ and fixed $\alpha_\rho(0)=0.52$, are given in Table I, where the errors correspond to one standard deviation. The best values are average values, with errors enlarged to overlap other results. A graphical representation of this best fit may be seen, compared with experimental $NN,\pi N$ cross sections in Fig. 2 and, for $\pi\pi$ data, in Fig. 3. We note that, in Fig. 3, for $\pi\pi$, we have used the values of β_ρ and β_2 from Eqs. (17), (18) below.

A few features of our results worth noting are the following. First, the equality of $f_{N/\pi}$ and $\beta_P,\beta_{P'}$, for fits with and without $\pi\pi$ data is a very satisfactory test of factorization. Another interesting point is the stability and accuracy of the parameters $f_{N/\pi}$, $\beta_\rho^{(N\pi)}$, β_P. The parameter $\beta_{P'}$ is less well determined, and β_ρ is not fixed with precision by fits to data alone; we will improve its accuracy in a moment using sum rules. Second, the matching between the low energy ($s^{1/2}\leqslant 1.42$ GeV) results for cross sections from phase shift analyses and the high energy ($s^{1/2}\geqslant 1.42$ GeV) Regge representations is excellent for $\pi^0\pi^-$, $\pi^-\pi^-$, and $\sigma^{(I_t=0)}$. It is less good for $\pi^+\pi^-$, where matching occurs only at the 1.5σ level, no doubt due to the coinciding tails of the $f_2(1270)$ and $f_0(1370)$ resonances. And, third, the fact that, for *NN* and πN, the $\chi^2/N_{\rm DOF}$ is somewhat larger than unity is due to the following effects. First, we use only two poles for vacuum exchange and one for charge exchange: we are thus missing the contributions of other poles, likely small, but not negligible at the lower energy range. Second, at the very low energy range, the experimental cross sections oscillate a little around the Regge formulas, as is seen very clearly for the $\pi^+ p$ cross section in Fig. 2. Third, we have neglected the a_2 contribution for *NN* scattering [ϵ in Eq. (8)]. Finally, we have that, to cover well the upper part of the energy range, we need more sophisticated expressions: see Sec. IV.

Besides this, we have a few technical points to make in connection with the fits including $\pi\pi$ data. As is clear from Fig. 3, the low energy ($s^{1/2}<2.5$ GeV) results for $\pi^-\pi^-$ cross sections of various experiments are quite incompatible with one another, which is the reason for the large $\chi^2/N_{\rm DOF}$ in *no-cut* fits. There is certainly a bias in the experimental $\pi^-\pi^-$ cross sections of Biswas *et al.*, and Robertson, Walker, and Davis [7] in the lower energy range. This is probably due to incorrect treatment of final state interactions, which, at these lower energies, are influenced by the Δ_{33} and other resonances. At higher energies the influence of this resonance seems to become negligible as, indeed, the $\pi^-\pi^-$ cross sections found by Robertson, Walker, and Davis overlap those of Abramowicz *et al.* [7] and both tend to the $\pi^+\pi^-$ one, as Regge theory and the Pomeranchuk theorem imply. We consider that this problem is solved by considering our two types of fits, *cut* or *no-cut*, for $\pi\pi$ scattering.

We next discuss the isospin-2 exchange piece $R_2(s,t)$. We have three methods to get the quantity β_2. First, we fix the values of β_P and $\beta_{P'}$ to their best values, as given in Table I, and fit the $\pi\pi$ data using Eqs. (5), (6), (7). Note that one cannot leave the parameters $\beta_P,\beta_{P'}$ free in these fits because one would get spureous minima, since the data are not precise enough. We find $\beta_\rho=1.07$ and a very small $\beta_2\sim -2\times 10^{-8}$. Alternatively, we could obtain β_2 by fitting $\sigma_{\pi^0\pi^0}-\sigma_{\pi^0\pi^+}$ at $s^{1/2}=1.42$ GeV, as was done in Ref. [11]. This gives $\beta_2=0.55\pm 0.2$. Finally, we can use the first cross-

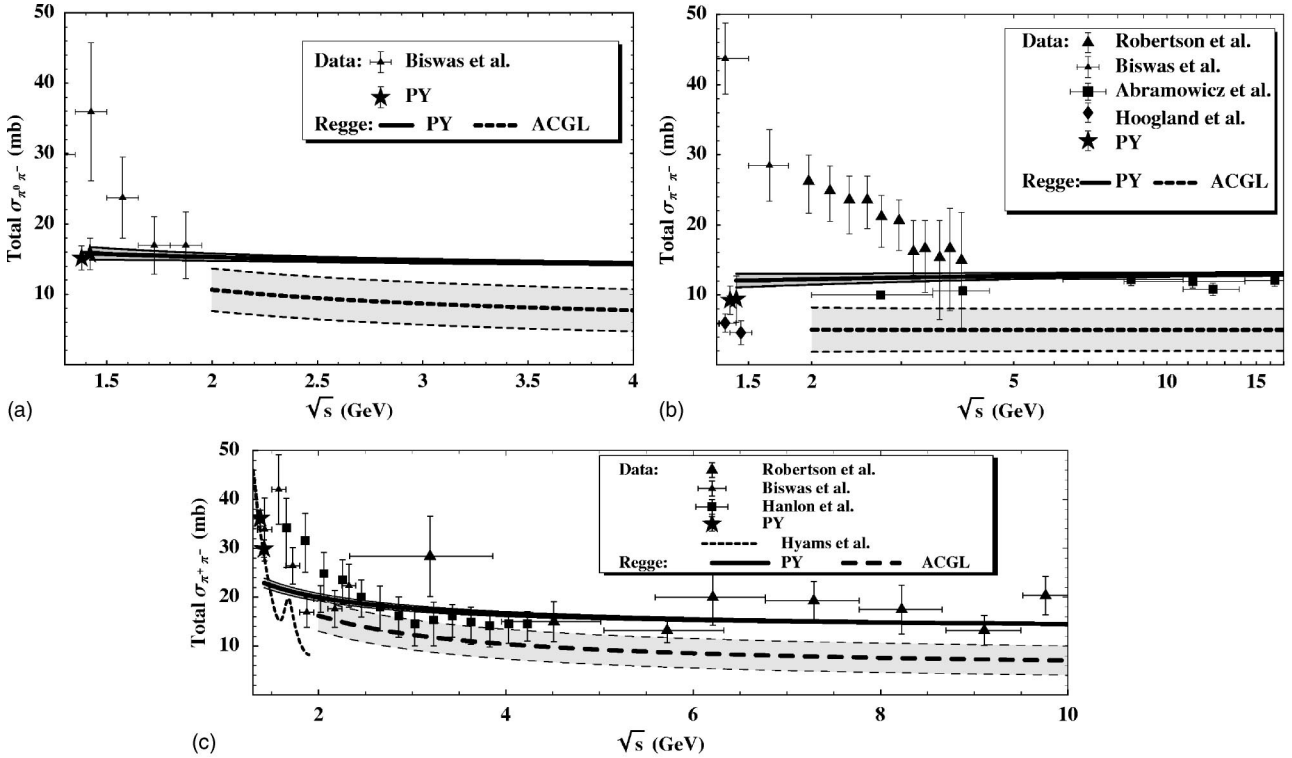

FIG. 3. Total cross sections $\sigma_{\pi^0\pi^-}$, $\sigma_{\pi^-\pi^-}$, and $\sigma_{\pi^+\pi^-}$. Black dots, triangles, and squares: experimental points from Ref. [7]. The stars at 1.38 and 1.42 GeV [Peláez and Yndurain (PY)] are from the phase shift analysis of experimental data given in Ref. [11], slightly improved for the $D2$ wave. Solid lines, from 1.42 GeV (PY): Regge formula, with parameters as in our best fit (the three lines per fit cover the error in the theoretical values of the Regge residues). Dashed lines, above 2 GeV: the cross sections following from ACGL [1]; the gray band covers their error band. Below 2 GeV, the dotted line corresponds to the $\pi^+\pi^-$ cross section from the Cern-Munich analysis; cf. Fig. 7 in the paper of Hyams *et al.* [8]

ing sum rule in the Appendix to Ref. [11] [identical to Eq. (B7) in ACGL], which would give a β_2 compatible with zero. We take as a compromise the number

$$\beta_2 = 0.2 \pm 0.2. \tag{11}$$

However, we should note that the t dependence of $R_2(s,t)$ is little more than guesswork.

Sum rules. We now say a few words on the sum rules discussed in Ref. [11]. Because these sum rules were verified with Regge expressions slightly different from what we have now found, one may wonder what happens to them. Since the formulas in Eqs. (5), (6), (7), with parameters as in Table I, agree with those of Ref. [11] within $\lesssim 2\,\sigma$ and the decrease of β_P is (partially) compensated by the increase in $\beta_{P'}$, it can be expected that the various sum rules would still be satisfied within errors, as indeed happens. Our numbers here leave the agreement of the Olsson sum rule and the value of the P-wave scattering length and effective range still within 1σ. We have already discussed the first crossing sum rule in

TABLE I. Parameters of the fits using Eqs. (8).

	NN, πN [enlarged error[a]]	Only $\pi\pi$ [cut[b,c]]	NN, πN, $\pi\pi$ [cut[b]]	Best values
$f_{N/\pi}$	1.407 ± 0.001 [1.409 ± 0.001]		1.407 ± 0.003 [1.407 ± 0.003]	1.407 ± 0.004
$\beta_\rho^{(N\pi)}$	0.377 ± 0.007 [0.380 ± 0.007]		0.377 ± 0.007 [0.377 ± 0.007]	0.377 ± 0.008
β_ρ		1.30 ± 0.13 [0.59 ± 0.27]	1.33 ± 0.13 [0.59 ± 0.25]	1.0 ± 0.3[d]
β_P	2.545 ± 0.002 [2.538 ± 0.002]	2.50 ± 0.08 [2.55, fix]	2.545 ± 0.007 [2.545 ± 0.007]	2.54 ± 0.03
$\beta_{P'}$	1.05 ± 0.01 [1.06 ± 0.01]	1.46 ± 0.17 [1.04, fix]	1.05 ± 0.02 [1.05 ± 0.02]	1.05 ± 0.05
$\frac{\chi^2}{N_{\rm DOF}}$	$\frac{460}{354-4}\left[\frac{436}{354-4}\right]$	$\frac{109}{58-3}\left[\frac{45}{39-1}\right]$	$\frac{573}{412-5}\left[\frac{505}{393-5}\right]$	

[a]We here endow all πN numbers with a minimum systematic error of 1.5%.
[b]By "cut" we mean that $\pi\pi$ data for $s^{1/2} < 2$ GeV are removed from the fit.
[c]We here fix β_P, $\beta_{P'}$ as given by NN, πN, to avoid spureous minima.
[d]The error in this quantity will be improved using crossing sum rules; see Eq. (17) below.

the Appendix in Ref. [11] in connection with β_2, so we turn to the second crossing sum rule. It reads

$$\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\, F^{(I_t=1)}(s,4m_\pi^2) - \mathrm{Im}\, F^{(I_t=1)}(s,0)}{s^2} = \int_{4m_\pi^2}^{\infty} ds \frac{8m_\pi^2[s-2m_\pi^2]}{s^2(s-4m_\pi^2)^2} \mathrm{Im}\, F^{(I_s=1)}(s,0). \quad (12)$$

The interest of this sum rule lies in that its high energy ($s^{1/2} \geqslant 1.42$ GeV) is dominated by $\rho(s,t)$, while the low energy piece ($s^{1/2} \leqslant 1.42$ GeV) is such that the contributions of the S waves cancel, so it is dominated by the P wave, which is very well known. Thus it provides an independent, reliable way of fixing the parameter β_ρ. We find Eq. (12) satisfied provided one has

$$\beta_\rho = 0.82 \pm 0.12. \quad (13)$$

Since this is compatible with the independent determinations in Table I, we may include fulfillment of Eq. (12) in the fits. If we do so for the fit with *cut* $\pi\pi$ data, we get the value

$$\beta_\rho = 0.78 \pm 0.11. \quad (14)$$

If we include Eq. (12) in the fit with all $\pi\pi$ data (*no-cut*), we find, instead,

$$\beta_\rho = 1.07 \pm 0.09. \quad (15)$$

Combining Eqs. (14), (15) we can then take

$$\beta_\rho = 0.94 \pm 0.10(\mathrm{stat}) \pm 0.10(\mathrm{syst}). \quad (16)$$

Best values. We can now present our best values and compare them with the values given in Ref. [11] (PY), obtained basically from those by Rarita *et al.* [14], or those of Refs. [1,5] (ACGL):

	[our best values]	[PY]	[ACGL]
β_ρ	0.94 ± 0.14	0.84 ± 0.10	1.48 ± 0.25
β_P	2.54 ± 0.03	3.0 ± 0.3	1.0 ± 0.6
$\beta_{P'}$	1.05 ± 0.05	0.72 ± 0.07	2.22 ± 0.38
β_2	0.2 ± 0.2	0.55 ± 0.20	0

(17)

Besides these, we have also

$$f_{N/\pi} = 1.407 \pm 0.04, \quad \beta_\rho^{(N\pi)} = 0.377 \pm 0.008. \quad (18)$$

Our present results are compatible with those in Refs. [6, 11,14]. We note, however, that our fits include much more information on the total cross sections than those in Refs. [6,14]. The first only includes $\pi^+\pi^-$ data while the more complete fit of Rarita *et al.* [14] includes 24 total cross section data for NN (we have 34) and 28 for πN (we have 141); the energy range we cover is also wider, by a factor 6 in the variable s. We also have 58 $\pi\pi$ data points (none in Ref. [14]). Of course, the situation is different for the t dependence of the residue functions $f_i(t)$ for which the fit of Rarita *et al.* [14] cannot be really improved.

The results in Eqs. (17) and (18) may be compared with some theoretical models. The value $f_{N/\pi} \simeq 1.4$ is similar to what one gets in the naive quark model [18] with additive quark-quark cross sections, which gives $f_{N/\pi} = 3/2$. (It is, however, not clear why the naive quark model works, as its mechanism is very different from the orthodox QCD one.) Likewise, the value of $\beta_\rho = 0.94 \pm 0.14$ is similar to what one has in the Veneziano model [19] ($\beta_\rho \simeq 0.95$). β_ρ also agrees with the rho dominance model, in which one couples the rho universally to pions and nucleons according to

$$g\bar{N}\vec{t}\gamma^\mu N\vec{\rho}_\mu \quad g(\vec{\pi} \times \overleftrightarrow{\partial}_\mu \vec{\pi})\vec{\rho}^\mu, \quad (19)$$

with $\vec{t} = \vec{\sigma}/2$, $\vec{\sigma}$ the Pauli matrices, which gives $\beta_\rho = \sqrt{\frac{8}{3}} f_{N/\pi} \beta_\rho^{(N\pi)} \simeq 0.84$.

III. πK SCATTERING

The analysis of πK scattering follows similar lines. For exchange of isospin zero we have

$$\mathrm{Im}\, F_{\pi K}^{(I_t=0)}(s,t) \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\simeq} f_{K/\pi}[P(s,t) + rP'(s,t)],$$

$$f_{K/\pi} = f_K^{(P)}(0)/f_\pi^{(P)}(0). \quad (20)$$

P, P' are as above, and r is related to the branching ratio for the $\bar{K}K$ decay of the resonances[5] $f_2(1270)$, $a_2(1320)$, which is $r \sim \mathrm{BR} \simeq 5 \times 10^{-2}$. For isospin-1 exchange,

$$\mathrm{Im}\, F_{\pi K}^{(I_t=1)}(s,t) \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\simeq} g_{K/\pi}\rho(s,t),$$

$$g_{K/\pi} = f_K^{(\rho)}(0)/f_\pi^{(\rho)}(0); \quad (21)$$

$\rho(s,t)$ is as before. To find the desired representations for the πK amplitude we have to determine the ratios $f_{K/\pi}$, $g_{K/\pi}$. For the first, this is done taking the $f_{N/\pi}$ from $NN, \pi N$ scattering, as in the previous sections and with the help of the even combination of cross sections for KN scattering:

$$\sigma_{K^+p} + \sigma_{K^-p} \underset{s \text{ large}}{\simeq} \frac{4\pi^2}{\lambda^{1/2}(s,m_K^2,m_p^2)} f_{N/\pi} f_{K/\pi}[P(s,0) + rP'(s,0)]. \quad (22)$$

The parameter r measures the projection of a_2, f_2 trajectories on KN scattering. For $g_{K/\pi}$, unfortunately, we cannot use the charge exchange reaction $K^-p \to K^0n$ because there are two trajectories of comparable importance—ρ and that corre-

[5]Since the P' pole couples so weakly to kaons, one may consider the importance of other Regge poles for the subleading contribution to kaon scattering. For KK scattering, the Regge pole associated with the $f_2(1525)$ resonance gives a substantial contribution, but for KN or πK scattering, this trajectory contributes very little since it is almost uncoupled to pions and nucleons and its intercept is small, $\alpha_{f_2(1525)} \simeq -0.3$. For KN and πK, the amplitude for exchange of zero isospin is almost pure Pomeron.

sponding to $a_2(1320)$ exchange—that contribute; for a discussion, cf., for instance, the text of Barger and Cline [13]. The difference of cross sections K^+p and K^-p also contains extra contributions $(\omega,\phi,...)$.

For the KN cross sections we will take data in the region $E_{\rm kin}>1$ GeV and go up to $E_{\rm kin}=10$ GeV. At higher energies the logarithmic increase of the total cross section for K^+p scattering is noticeable, and we would need more complicated Regge formulas (that we will give in Sec. IV), while, as occurs for the $\pi\pi$ case, the importance of the very high energy region is negligible in most applications to πK scattering. For πK scattering we thus expect the ensuing Regge expressions to be accurately valid for a corresponding energy range—say, for 1.7 GeV$<s^{1/2}<$11 GeV.

The $K^{\pm}p$ data we take also from the COMPAS Group compilations; see the Particle Data Tables [17]. For those data where systematic errors are not given, we have included a common systematic error of 0.3 mb, as we did for the πN case. We take only data at energies at which there are results for both K^+p and K^-p. In the fits we use the very precise values of the parameters $f_{N/\pi},\beta_P$ obtained before, and we set $r=0$, since it is very small and not very well known; in Sec. IV, we will make fits, leaving r free. We find

$$f_{K/\pi}=0.67\pm0.01 \quad [\text{from } K^+p+K^-p,$$
$$\chi^2/N_{\rm DOF}=50/(43-1)],$$
$$g_{K/\pi}=1.1\pm0.1. \tag{23}$$

The results for $(\sigma_{K^+p}+\sigma_{K^-p})/2$ are shown in Fig. 2. The value of $g_{K/\pi}$ is taken from the classical analysis of Ref. [20], which takes into account the $a_2(1320)$ exchange. The value of $f_{K/\pi}$ is within 20% of its SU(3) value $\sqrt{\frac{2}{3}}\simeq0.82$.

IV. GLOBAL FIT VALID UP TO MULTI-TeV ENERGIES

A simple parametrization of scattering amplitudes which fits data at energies $s^{1/2}>12$ GeV (with a $\chi^2/N_{\rm DOF}=1.2-1.8$, depending on the process) may be found in in Refs. [21,22]. Here the Pomeron is allowed an intercept larger than unity, $\alpha_P(0)\sim1.095$, and the intercept of the P' is given as $\alpha_{P'}(0)=0.66$. This parametrization, which we will call ''power Pomeron'' parametrization, is purely phenomenological, as explicitly mentioned in Refs. [21,22]. Only data with energy larger than ~10 GeV are used in the fits which, if extended to energies below 5 GeV, miss widely the data. These parametrizations also must fail at very large energies since they are incompatible with unitarity in that they violate the Froissart bound. As a matter of fact, in Ref. [23] the inadequacy of such a parametrization is remarked on and a parametrization verifying the Froissart bound [i.e., with a term in $(\text{const})\times\log^2 s/s_0+\text{const}$] is substituted in place of the ''power Pomeron.'' This improves substantially the $\chi^2/N_{\rm DOF}$ of the fit and gives an intercept $\alpha_{P'}(0)=0.54\pm0.02$, perfectly compatible with our choice 0.52 ± 0.02. The corresponding parametrization holds down to $s^{1/2}=5$ GeV.

It is possible to write a parametrization, similar to that of Ref. [23], obtained by a modification of the Pomeron in Eq. (5), which fits data for kinetic energies from 1 GeV to the multi-TeV region and which, moreover, is compatible with unitarity, by adding a slightly more complicated logarithmic term. We do this as follows: we note that one can improve the Froissart bound to a bound of the form [24]

$$\sigma_{\rm tot}\leqslant a\log^2\frac{s}{s_1\log^{7/2}s/s_2}, \tag{24}$$

which is maximal in the sense that one cannot increase the power of the logarithm in the denominator to more than $\frac{7}{2}$. For the *bound* for $\pi\pi$ scattering, one can evaluate the constants a,s_1,s_2 in terms of the pion mass and low energy parameters for the D wave, with $a=\pi/4m_\pi^2\simeq15$ mb^2, $s_1=m_\pi^2$ if we assume the cross section to be mostly inelastic. What this suggests is that we add a term like Eq. (24) to the Pomeron given in Eq. (5), but leaving a,s_1,s_2 as free parameters. Thus we replace,

$$P(s,t)=\beta_P\alpha_P(t)\frac{1+\alpha_P(t)}{2}e^{bt}(s/\hat{s})^{\alpha_P(t)}\to P_F(s,t),$$
$$P_F(s,t)=\left\{\tilde{\beta}_P+A\log^2\frac{s}{s_1\log^{7/2}s/s_2}\right\}\times\alpha_P(t)\frac{1+\alpha_P(t)}{2}e^{bt}(s/\hat{s})^{\alpha_P(t)}. \tag{25}$$

This replacement should also be made in Eqs. (8), (20), and (22). The logarithmic term has an appealing physical interpretation as the contribution of the Regge cuts which, as Mandelstam showed long ago [25], should accompany the Pomeron. The parameter β_P that we used before is to be viewed as an effective parameter, the sum of $\tilde{\beta}_P$ and the average value, for low energy ($s^{1/2}\lesssim15$ GeV), of the logarithmic piece in Eqs. (25).

With Eqs. (25) we fit data for $\pi^{\pm}p$, K^+p+K^-p, $\pi\pi$, and $pp+\bar{p}p$ cross sections[6] up to the highest energies attained experimentally, 30 TeV in cosmic ray experiments [26].

Because we have so many experimental data, covering such a wide energy range, we may fit all hadronic data (i.e., including NN, all πN data, KN and $\pi\pi$ data) leaving all parameters free; in particular, this will test the quality of the assumption of degenerate rho and f_2 trajectories, the value of $\alpha_\rho(0)$, the equality of $f_{N/\pi}^{(P)},f_{N/\pi}^{(P')}$, and the smallness of the parameters r and ϵ. We find

$$f_{N/\pi}^{(P)}=1.348\pm0.004, \quad f_{N/\pi}^{(P')}=1.26\pm0.03,$$
$$f_{K/\pi}=0.746\pm0.003,$$
$$\alpha_{P'}(0)=0.68\pm0.01, \quad \alpha_\rho(0)=0.52\pm0.02,$$

[6]Above 30 GeV we approximate $\sigma_{\bar{p}p}-\sigma_{pp}=(66.7\text{ mb})(s/\hat{s})^{-0.55}$, where this difference comes from the phenomenological fit of Ref. [17], since we do not have data at coinciding energies. For $\pi\pi$ only data above 2 GeV are included in these fits.

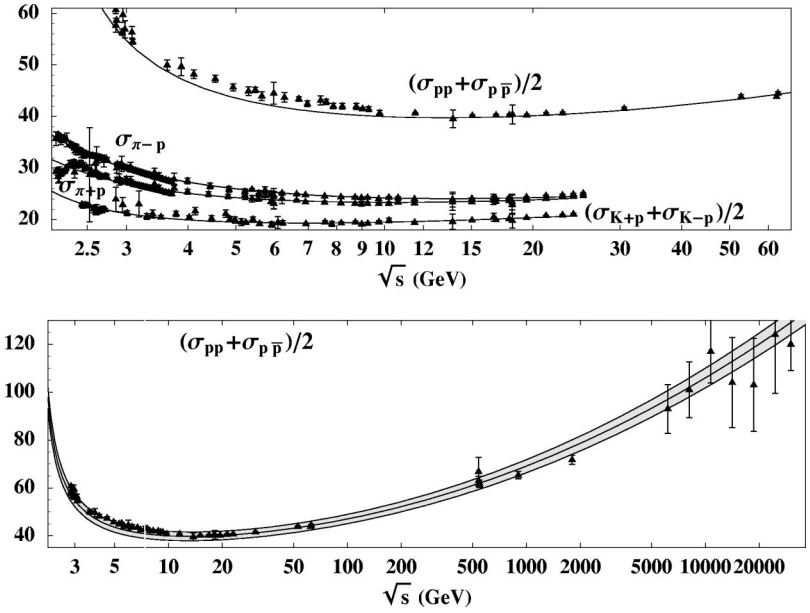

FIG. 4. The total cross sections $\sigma_{\pi^{\pm}p}$, $\frac{1}{2}(\sigma_{K^+p}+\sigma_{K^-p})$ and $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$ up to 30–60 GeV (upper graph) and $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$ up to 30 TeV (lower graph). Black dots, triangles, and squares: experimental points. For energies above 30 GeV, we have depicted the experimental values of $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$ as if they equalled $\sigma_{\bar{p}p}$ or σ_{pp}. Solid lines: Regge formulas, with parameters as in Eqs. (27). In the lower figure we have given the error bands for $\frac{1}{2}(\sigma_{\bar{p}p}+\sigma_{pp})$ that follow from (27).

$$\epsilon = 0.11 \pm 0.03, \quad \tilde{\beta}_P = 2.13 \pm 0.01,$$

$$\beta_{P'} = 1.84 \pm 0.03, \quad \beta_\rho^{(N\pi)} = 0.39 \pm 0.02,$$

$$\beta_\rho = 0.94 \pm 0.14, \quad r = 0.18 \pm 0.01,$$

$$A = 0.0200 \pm 0.0005,$$

$$s_1 = (0.54 \pm 0.04) \times 10^{-4} \ \text{GeV}^2,$$

$$s_2 = (0.27 \pm 0.06) \times 10^{-7} \ \text{GeV}^2,$$

$$\chi^2/(N_{\text{DOF}}) = 559/(497-13) \simeq 1.15. \tag{26}$$

The value of β_ρ given here is that found before, Eq. (16); since there are no $\pi\pi$ data at very high energy, the value of this quantity essentially decouples from the very high energy analysis.[7]

What is interesting about Eqs. (26) is that $f_{N/\pi}^{(P)}$ and $f_{N/\pi}^{(P')}$ are not far from each other, as required by (strong) factorization. In fact, this had already been noticed in Ref. [23]: in a fit with a formula compatible with theory (the Froissart bound), the results respect other theoretical constraints reasonably well.

The problem with the fit in Eqs. (26) is that there is, unfortunately, a very strong correlation among $\tilde{\beta}_P$, $\beta_{P'}$, $\alpha_{P'}(0)$, s_1, and s_2 and, if we leave all of them free as we did in getting Eqs. (26), there exist a large number of equally significant minima: the parameters are not well determined. In fact, s_1, s_2, $\beta_{P'}$, and $\alpha_{P'}(0)$ can one mock the effects of each other. In particular, a set of fits with quality essentially unchanged may be obtained by varying simultaneously s_1 and s_2. In view of this, we require $f_{N/\pi}^{(P)} = f_{N/\pi}^{(P')}$, $\alpha_\rho(0) = 0.52 \pm 0.02$, and to fix the parameters, choose $s_1 = 0.01$ GeV^2 and repeat the fit with all other parameters free. We now get the results

$$f_{N/\pi} = 1.359 \pm 0.004, \quad f_{K/\pi} = 0.723 \pm 0.004,$$

$$\alpha_{P'}(0) = 0.59 \pm 0.03, \quad r = 0 \pm 0.007,$$

$$\tilde{\beta}_P = 2.32 \pm 0.04, \quad \beta_{P'} = 1.41 \pm 0.03,$$

$$\beta_\rho^{(N\pi)} = 0.392 \pm 0.008, \quad \beta_\rho = 0.94 \pm 0.14 [\text{fix}],$$

$$A = 0.033 \pm 0.001, \quad s_1 = 0.01 \ \text{GeV}^2 \ [\text{fix}],$$

$$s_2 = 0.15 \pm 0.05 \ \text{GeV}^2, \quad \epsilon = 0.24 \pm 0.03,$$

$$\chi^2/(N_{\text{DOF}}) = 584/(497-10) \simeq 1.20. \tag{27}$$

We note that, although the χ^2/N_{DOF} is slightly worse than that in Eqs. (26), we consider the fit in Eqs. (27) to be equally satisfactory physically. The values of the parameters s_1, s_2 in Eqs. (26) were too small for comfort, and one should not force too good a fit at the expense of physical considerations (like factorization or degeneracy), particularly since we are fitting with formulas that, at the lowest energies, should be corrected by including other Regge poles (or cuts). Equations (27) have the nice properties that degeneracy $[\alpha_\rho(0) = \alpha_{P'}(0)]$ is reasonably verified and that $f_{K/\pi}$ agrees better with its SU(3) value.

At the lower energies (below 15 GeV) Eqs. (25) plus Eqs. (26) or (27) overlap with the previous fits, using Eqs. (5) for the Pomeron and P' for vacuum exchange. In fact, for Kp or πN, the corresponding curves could not be distinguished from those obtained using Eqs. (5) in Fig. 2; see Fig. 4. For $\bar{p}p + pp$, the result of the fits with the two types of formulas

[7] If we had fitted also β_ρ, including the sum rule (12), its value would depend on whether we had included all $\pi\pi$ data above 1.4 GeV (in which case we would have got 1.05 ± 0.009) or only data for $s^{1/2} \geqslant 2$ GeV, which gives 0.80 ± 0.11: essentially the same numbers as in the fits in Sec. II, Eqs. (14), (15).

TABLE II. Parameters of the fits using Eqs. (8) (column A) and Eqs. (25) (columns B,C).

	(A) $E_{\rm kin}\lesssim 15$ GeV	(B) 1 GeV$\lesssim E_{\rm kin}\lesssim$30 TeV all parameters free	(C) 1 GeV$\lesssim E_{\rm kin}\lesssim$30 TeV $s_1=0.01$, $f^{(P)}_{N/\pi}=f^{(P')}_{N/\pi}$
$f^{(P)}_{N/\pi}$	1.407 ± 0.004	1.384 ± 0.002	1.359 ± 0.004
$f^{(P')}_{N/\pi}$	$\equiv f^{(P)}_{N/\pi}$ [fix]	1.26 ± 0.03	$\equiv f^{(P)}_{N/\pi}$ [fix]
$f_{K/\pi}$	0.67 ± 0.01	0.746 ± 0.003	0.723 ± 0.004
r	0 [fix]	0.18 ± 0.01	0 ± 0.007
$\alpha_\rho(0)$	0.52 ± 0.02 [fix]	0.52 ± 0.02	0.52 ± 0.02 [fix]
$\alpha_{P'}(0)$	0.52 ± 0.02 [fix]	0.68 ± 0.01	0.59 ± 0.03
$\tilde{\beta}_P$	-	2.13 ± 0.01	2.32 ± 0.04
β_P	2.54 ± 0.03	-	-
$\beta_{P'}$	1.05 ± 0.02	1.84 ± 0.03	1.41 ± 0.03
ϵ	0 [fix]	0.11 ± 0.03	0.24 ± 0.03
$\beta^{(N\pi)}_\rho$	0.377 ± 0.008	0.39 ± 0.02	0.392 ± 0.008
ρ_ρ	0.94 ± 0.14	0.94 ± 0.14 [fix]	0.94 ± 0.14 [fix]
A	-	0.0200 ± 0.0005	0.033 ± 0.001
s_1	-	$(0.54\pm0.04)\times10^{-4}$ GeV^2	$\equiv0.01$ GeV^2
s_2	-	$(0.27\pm0.06)\times10^{-7}$ GeV^2	0.15 ± 0.05 GeV^2
$\chi^2/N_{\rm DOF}$	-	1.15	1.20

(5) and (27) are depicted in Fig. 4, where the error bars corresponding to Eqs. (27) are also shown.

The fact that the $\chi^2/N_{\rm DOF}$ of the fits is somewhat larger than unity can be adscribed to the reasons like those described in Sec. II: we have the oscillations of the π^+N cross section around the Regge value[8] (easily seen in Figs. 2 and 4) and the fact that we have not included more Regge trajectories, certainly necessary at the very low energy range. Nevertheless, the quality is comparable to (in fact, slightly better than) that of the fits in Ref. [23], if extended down to $s^{1/2}=3$ GeV—this, in spite of the fact that the fits in Ref. [23] do not cover our range: we go down to 1.4 GeV for $\pi\pi$, 1.7 GeV for KN, 2.2 GeV for πN, and 2.8 GeV for NN scattering.

V. SUMMARY AND A SHORT DISCUSSION

The Regge parameters that ACGL [1] and, following them, the authors in Refs. [2–4], [18] assume not only are unorthodox, but as we have shown, incompatible with experiment. As our Fig. 3 clearly demonstrates, the claimed large errors in ACGL are not large enough to cover the experimental data.

ACGL get these quaint Regge parameters by considering sum rules like Eqs. (12) that link the Regge contributions, which they assume to hold only for $s^{1/2}\geqslant2$ GeV, with the corresponding low energy ($s^{1/2}<2$ GeV) pieces. Unfortunately, the intermediate energy (1.4 GeV$\leqslant s^{1/2}<2$ GeV) that ACGL, again here followed by the authors in Refs. [2–4], take for the $S0$, P, $D0$, and F phases comes basically from the experimental analysis of the Cern-Munich group, whose $\pi^+\pi^-$ cross section is more and more incompatible, as $s^{1/2}$ nears 2 GeV—in fact, as soon as inelasticity becomes important—with the values found by all other experiments [7]: see our Fig. 3. (The interested reader may consult Ref. [9] for a detailed discussion of this and other related issues.) It is thus not surprising that Pennington [5] and Ananthanarayan *et al.* [1] who fix their Regge parameters by balancing them above 2 GeV with phase shifts below 2 GeV, get totally incorrect Regge amplitudes. And given these facts, it also follows that the *low energy* results of Refs. [2,3,4], which borrow their input at energies $s^{1/2}\geqslant1.4$ GeV from ACGL, should be taken with great caution.

Unlike the results of phase shift analyses, the Regge formulas in Eqs. (5), (6), (25) with the parameters as the ''best values'' in Eqs. (17), (18), or (27), and which we summarize in Table II, give a consistent representation for the imaginary part of all the $\pi\pi$ scattering amplitudes, a representation which can be trusted, within the given errors, for $s^{1/2}>1.4$ GeV, provided $|t|^{1/2}<0.4$ GeV. In fact, one has better than that: our Regge formulas give a good representation of those processes in pion-pion scattering where resonances are absent, or are not important, down to lower energies, just as it happens in NN or πN scattering. This occurs, in particular, for $\pi^0\pi^+$ and $\pi^-\pi^-$, for which the Regge formulas reproduce the experimental data down to $s^{1/2}\sim1.1$ GeV. However, by the very nature of things, we are likely to have uncertainties of the order of 15% in the region 1.4 GeV $\leqslant s^{1/2}\leqslant1.8$ GeV when exchange of isospin 1 is important, because the Regge formula probably represents data only in the average there, as occurs for πN scattering. Finally and

[8]In fact, if we excluded from the fit the data on π^+p for $s^{1/2}<3$ GeV, the final $\chi^2/N_{\rm DOF}$ would decrease to 1. However, we have preferred to keep the data below 3 GeV because the difference between the Regge found and the experiment is less than 5%, and the Regge expression gives a very good average representation in that region.

using Eqs. (8), (22) and the formulas in the last columns in Table II, fits B, C, we can fit NN, πN, and KN up to multi-TeV energies, and predict $\pi\pi$ and πK cross sections there.

When performing calculations of $\pi\pi$ scattering in which the lower energy region is dominant (such as Roy equations, dispersion relations, or sum rules) it is irrelevant, within our errors, which form one uses for the Pomeron, Eqs. (5), (26), or (27). The last has better overall fit and (probably) a more realistic value for $\beta_{P'}$, although the first is to be preferred in that it is simpler and fits slightly better the low energy data. The safest procedure is to use all fits A, B, C, and consider their difference as a measure of the influence of the parametrization on the results. We should, however, emphasize that the parameters in the fits are strongly correlated and, even when they are similar, one *cannot* mix parameters from the various columns in Table II; each fit stands on its own.

One may also wonder what happens for values of the momentum transfer larger than $|t|^{1/2}\sim 0.4$ GeV. On general grounds, one expects Regge theory to work when $s\gg\Lambda^2$, $s\gg|t|$, and in fact, as already mentioned, Regge representations for NN or πN become unreliable at large $|t|$. For example, the parametrizations of Rarita *et al.* [14] and Ref. [6] for $f^{(\rho)}(t)$ differ completely from one another already at $-t=0.23$ GeV^2, where the first changes sign. There is unfortunately no sure way out of this problem, and one has to admit that, for $s^{1/2}>1.4$ GeV and values of the momentum transfer $|t|>0.15$ GeV^2, there is no reliable information on the pion-pion scattering amplitude—which, in particular, is an unavoidable cause of uncertainty for Roy equation analyses that require information for values of $|t|$ as large as 0.5 GeV^2.

Note added in proof. Contrary to what is stated in Sec. II, it is also possible to obtain a Regge description of similar quality in terms of the ν variable. The results will be shown in a future publication.

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