Uncertainty in Newton's constant and precision predictions of the primordial helium abundance

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The current uncertainty in Newton's constant G_N is of the order of 0.15%. For values of the baryon to photon ratio consistent with both cosmic microwave background observations and the primordial deuterium abundance, this uncertainty in G_N corresponds to an uncertainty in the primordial ^{4}He mass fraction Y_P of $\pm 1.3\times 10^{-4}$. This uncertainty in Y_P is comparable to the effect from the current uncertainty in the neutron lifetime τ_n, which is often treated as the dominant uncertainty in calculations of Y_P. Recent measurements of G_N seem to be converging within a smaller range; a reduction in the estimated error on G_N by a factor of 10 would essentially eliminate it as a source of uncertainty in the calculation of the primordial ^{4}He abundance.

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Big bang nucleosynthesis (BBN) represents one of the key successes of modern cosmology [1,2]. In recent years, the BBN production of deuterium has emerged as the most useful constraint on the baryon density in the Universe, both because of observations of deuterium in (presumably unprocessed) high-redshift quasistellar object absorption line systems (see Ref. [3] and references therein), and because the predicted BBN yields of deuterium are highly sensitive to the baryon density. These arguments give a baryon density parameter $\Omega_b h^2$ of [3]

$$\Omega_b h^2 = 0.0194\text{–}0.0234, \tag{1}$$

which is in excellent agreement with the baryon density derived by the Wilkinson Microwave Anisotropy Probe team from recent cosmic microwave background observations [4]:

$$\Omega_b h^2 = 0.022\text{–}0.024. \tag{2}$$

Given these limits on the baryon density, BBN predicts the primordial abundances of ^{4}He and ^{7}Li. Because the ^{4}He abundance is particularly sensitive to new physics beyond the standard model, a comparison between the predicted and observed abundances of ^{4}He can be used to constrain, for example, neutrino degeneracy or extra relativistic degrees of freedom (see, e.g., Refs. [5,6]).

For this reason, it is useful to obtain the most accurate possible theoretical predictions for the primordial ^{4}He mass fraction Y_P. The primordial production of ^{4}He is controlled by the competition between the rates for the processes that govern the interconversion of neutrons and protons,

$$\begin{aligned} n+\nu_e &\leftrightarrow p+e^-, \\ n+e^+ &\leftrightarrow p+\overline{\nu}_e, \\ n &\leftrightarrow p+e^-+\overline{\nu}_e, \end{aligned} \tag{3}$$

and the expansion rate of the Universe, given by

$$\frac{\dot{R}}{R} = \left(\frac{8}{3}\pi G_N \rho\right)^{1/2}. \tag{4}$$

In BBN calculations, the weak interaction rates are scaled by the inverse of the neutron lifetime τ_n. When these rates are faster than the expansion rate, the neutron to proton ratio (n/p) tracks its equilibrium value. As the Universe expands and cools, the expansion rate comes to dominate and n/p essentially freezes out. Nearly all the neutrons that survive this freeze-out are bound into ^{4}He when deuterium becomes stable against photodisintegration. Following the initial calculations of Wagoner, Fowler, and Hoyle [7], numerous groups examined higher-order corrections to the ^{4}He production. The first such systematic attempt was undertaken by Dicus *et al.* [8], who examined the effects of Coulomb and radiative corrections to the weak rates, finite-temperature QED effects, and incomplete neutrino coupling. Later investigations included more detailed examination of Coulomb and radiative corrections to the weak rates [9–12], finite-temperature QED effects [13], and incomplete neutrino decoupling [14], as well as an examination of the effects of finite nuclear mass [15]. These effects were systematized by Lopez and Turner [16] (see also Ref. [17]), who argued that all theoretical corrections larger than the effect of the uncertainty in the neutron lifetime had been accounted for, yielding a total theoretical uncertainty $\Delta Y_P < 0.0002$. Assuming an uncertainty of ± 2 sec in the neutron lifetime, the corresponding experimental uncertainty in Y_P is $\Delta Y_P = \pm 0.0004$. Similar results were obtained in Ref. [17].

Two relevant changes have occurred since the publication of Refs. [16,17]. First, the estimated uncertainty in the neutron lifetime has decreased, with the current value being [18]

$$\tau_n = 885.7 \pm 0.8 \text{ sec}. \tag{5}$$

Second, the estimated uncertainty in the value of G_N has *increased*. The value currently recommended by CODATA (the Committee on Data for Science and Technology) is [19]

$$G_N = 6.673 \pm 0.010 \times 10^{-8} \ \text{cm}^3\,\text{gm}^{-1}\,\text{sec}^{-2}. \tag{6}$$

This represents a factor of 12 increase over the previous recommended uncertainty [20] and is primarily due to an anomalously high value for G determined by Michaelis, Haars, and Augustin [21]. (See Table I.)

TABLE I. Recent experimental values of Newton's constant. Digits in parentheses are the 1σ uncertainties in the last digits of the given value.

$10^8 G_N$ ($\mathrm{cm}^3\,\mathrm{g}^{-1}\,\mathrm{sec}^{-2}$)	Reference
6.67407(22)	Schlamminger *et al.* (2002) [29]
6.67559(27)	Quinn *et al.* (2001) [30]
6.674215(92)	Gundlach and Merkowitz (2000) [31]
6.6699(7)	Luo *et al.* (1999) [32]
6.6742(7)	Fitzgerald and Armstrong (1999) [33]
6.6873(94)	Schwarz *et al.* (1999) [34]
6.6749(14)	Nolting *et al.* (1999) [35]
6.6735(29)	Kleinevoss *et al.* (1999) [36]
6.673(10)	CODATA (1998) [19]
6.67259(85)	CODATA (1986) [20]

It is easy to calculate the effect of both of these uncertainties on the primordial ^{4}He abundance. For the range of values of $\Omega_b h^2$ given in Eqs. (1) and (2), we find, numerically,

$$\Delta Y_P = 0.088(\Delta G_N/G_N), \tag{7}$$

$$\Delta Y_P = 0.18(\Delta \tau_n/\tau_n). \tag{8}$$

The coefficient in Eq. (8) is twice that in Eq. (7). This factor of 2 comes from the fact that the expansion rate [Eq. (4)] scales as $G_N^{1/2}$, while the weak interaction rates scale as τ_n^{-1}, and the abundance of ^{4}He is essentially unchanged if the ratio of the weak interaction rates to the expansion rate is held constant.

Then the current uncertainties in G_N and τ_n given in Eqs. (5) and (6) yield, for the 1σ uncertainties in Y_P,

$$\Delta Y_P = \pm 1.6\times 10^{-4} \tag{9}$$

from the uncertainty in τ_n, and

$$\Delta Y_P = \pm 1.3\times 10^{-4} \tag{10}$$

from the uncertainty in G_N.

These two uncertainties are roughly comparable. This is significant because the uncertainty in τ_n is often taken to be the dominant uncertainty in calculations of Y_P. Of course, both of these effects are exceedingly small, and well below the dispersion in the estimates of the primordial ^{4}He abundance from observations of low-metallicity systems [2]. (The effect of the uncertainty in G_N is comparable to the corrections to Y_P due to QED plasma effects and residual neutrino heating [16].) A further source of uncertainty in theoretical calculations of Y_P is the uncertainty in the nuclear reaction rates. A recent exhaustive study of this effect has been undertaken by Cyburt [22], who concluded that the effect on Y_P of the uncertainties in the nuclear reaction rates is currently subdominant. (See also earlier work in Refs. [23,24].)

The uncertainty in G_N has consequences in other astrophysical settings. Lopes and Silk [25] investigated the effect on the sound speed in the sun. They argued that helioseismology (in combination with improved solar neutrino measurements) might eventually provide an independent constraint on G_N, although this claim has been disputed by Ricci and Villante [26].

In principle, the uncertainty in G_N also affects cosmic microwave background (CMB) measurements. The change in the observed CMB fluctuation spectrum due to a fixed change in G_N was investigated by Zahn and Zaldarriaga [27]. Even under the most optimistic conditions for future observations, the smallest change in G_N which is, in principle, detectable in CMB measurements is $\Delta G_N/G_N \sim 0.006$, well above the current CODATA uncertainty.

It is likely, of course, that current and future measurements will lead to a reduction in the uncertainty in G_N. A set of the most recent measurements of G_N is displayed in Table I. (For a survey of measurements of G_N over a longer timeline, see Ref. [28].) The three most recent measurements all yield a value of G_N within a very narrow range. Reduction in the uncertainty in G_N by, for example, a factor of 10 (e.g., back to the 1986 CODATA level of uncertainty) would essentially eliminate any significant effect on BBN calculations.

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