Supergravity analysis of the hybrid inflation model from a D3-D7 system

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(Received 17 December 2003; published 7 May 2004)

Slow-roll inflation is a beautiful paradigm, yet the inflaton potential can hardly be sufficiently flat when unknown gravitational effects are taken into account. However, the hybrid inflation models constructed in $D=4$, $\mathcal{N}=1$ supergravity can be consistent with $\mathcal{N}=2$ supersymmetry, and can be naturally embedded into string theory. This article discusses the gravitational effects carefully in the string model, using a $D=4$ supergravity description. We adopt the D3-D7 system of type IIB string theory compactified on a $K3\times T^2/\mathbb{Z}_2$ orientifold for definiteness. It turns out that the slow-roll parameter can be sufficiently small despite the nonminimal Kähler potential of the model. The conditions for this to happen are given in terms of string vacua. We also find that the geometry obtained by blowing up the singularity, which is necessary for the positive vacuum energy, is stabilized by introducing certain three-form fluxes.

DOI: 10.1103/PhysRevD.69.106001 PACS number(s): 11.25.Mj, 04.65.+e, 98.80.Cq

I. INTRODUCTION

Slow-roll inflation is a beautiful paradigm, in which not only the flatness and homogeneity of the Universe but also the origin of the scale-invariant density perturbation is understood. However, it is not easy to obtain a scalar potential V that satisfies the slow-roll conditions [1]

$$\eta \equiv \frac{M_{\rm Pl}^2 V''}{V} \ll 1, \quad \epsilon \equiv \frac{1}{2}\left(\frac{M_{\rm Pl} V'}{V}\right)^2 \ll 1, \tag{1}$$

where V' and V'' are the first and second derivatives of V with respect to the inflaton, and $M_{\rm Pl}$ is the Planck scale $\simeq 2.4\times 10^{18}$ GeV. Suppose that there is a vacuum energy v_0^4, and then one can see that even gravitational corrections to the potential

$$V(\sigma) = v_0^4\left[1 + c\left(\frac{\sigma}{M_{\rm Pl}}\right) + c'\left(\frac{\sigma}{M_{\rm Pl}}\right)^2 + \cdots\right] \tag{2}$$

are not allowed by the slow-roll conditions if the coefficients c, c' are of the order of unity. Thus, slow-roll inflation is sensitive even to physics at the Planck scale, and can be a good probe in uncovering the fundamental laws of physics.

The hybrid inflation model [2] is realized by quite simple models of $D=4$, $\mathcal{N}=1$ supergravity (SUGRA) [3–6]. Thus, the inflaton potential is protected from radiative corrections. However, $D=4$, $\mathcal{N}=1$ SUGRA is not enough to control the gravitational corrections. In SUGRA as an effective-field-theory approach, no assumption except symmetry is imposed on ultraviolet physics. Thus, higher-order terms are expected in the Kähler potential with $O(1)$ coefficients:

$$K = X^\dagger X + k\frac{(X^\dagger X)^2}{M_{\rm Pl}^2} + \cdots, \tag{3}$$

where X is a chiral multiplet containing the inflaton σ. The second term contributes to the slow-roll parameter η, unless the vacuum energy is carried only by the D term. Thus, the inflaton potential is not expected to be sufficiently flat. This is called the η problem.

It is remarkable that the hybrid inflation model in $\mathcal{N}=1$ supersymmetry (SUSY) is consistent with $D=4$, $\mathcal{N}=2$ rigid SUSY [7]. The inflaton belongs to a vector multiplet of $\mathcal{N}=2$ SUSY, and its interactions, including the Kähler potential, are highly constrained. Thus, it was argued in [7] that the $\mathcal{N}=2$ SUSY might ease the η problem. However, it was far from clear how $\mathcal{N}=2$ SUSY can coexist with chiral quarks and leptons in $D=4$ theories.

Superstring theory is a promising candidate for the quantum theory of gravity. One can work out what the gravitational corrections look like, once a vacuum configuration is fixed. Thus, it is quite important in its own right to consider whether it can realize slow-roll inflation. Moreover, extended SUSY and higher-dimensional spacetime are generic ingredients of string theory, and hence it is a plausible framework in accommodating the hybrid inflation model with $\mathcal{N}=2$ SUSY; enhanced $\mathcal{N}=2$ SUSY can coexist with other $\mathcal{N}=1$ supersymmetric sectors owing to the internal spacetime.

It was shown in [8] that the hybrid inflation model with $\mathcal{N}=2$ SUSY is realized by a D3-D7 system placed on a local geometry ALE$\times$**C**. Thus, this framework of type IIB string theory enables us to examine if the inflaton potential can really be flat even when the internal dimensions are compactified and gravitational effects are taken into account. Note that an analysis at the level of rigid SUSY, where $M_{\rm Pl}$-suppressed corrections are neglected, is not sufficient to see the flatness of the inflaton potential.

This article is organized as follows. In Sec. II, we describe how the hybrid inflation model can be embedded in a local part of a realistic Calabi-Yau compactification of type IIB string theory. After that, we show that short-distance effects in the inflaton potential are not harmful, partly because of a translational invariance of the local geometry ALE$\times$**C**, and partly because of a property specific to string theory. In Sec. III, we adopt $K3\times T^2$ as a toy model of a Calabi-Yau threefold, and show that in the $D=4$ SUGRA description the potential is flat in the presence of dynamical gravity, consistent with the intuitive picture obtained in string theory. The special form of the Kähler potential and interactions derived from string theory play a crucial role there. In Sec. IV, an explicit model that stabilizes nonzero Fayet-Iliopoulos parameters is given in Sec. IV A. The slow-roll parameter η is evaluated for the model, and we obtain a condition that leads to slow-roll inflation in Sec. IV B.

We note that an article [9] was submitted to the e-print archive when we were completing this article. It has some overlap with this article in subjects discussed.

There was an error (in identification of closed-string zero modes with fields in SUGRA) in the first e-print version of this article, which was pointed out in [10].[1] It is corrected in this version, yet the main stream of logic (related to inflation) has not been changed from the first version.

II. STRING THEORY SETUP AND SHORT-DISTANCE EFFECTS IN INFLATON POTENTIAL

The low-energy spectrum consists of an $\mathcal{N}=2$ SUSY vector multiplet (X,V) when a space-filling fractional D3-brane is moving in ALE$\times$**C**. The fractional D3-brane is regarded as a D5-brane wrapped on a two-cycle of the ALE space [11], and hence is trapped at a tip of the ALE space. When a space-filling D7-brane is further introduced and stretched in the ALE direction, $\mathcal{N}=2$ SUSY is preserved, and one massless hypermultiplet $(Q,\overline{Q})$ arises from strings connecting the D3 and D7 branes. The D7-D7 open string and closed string are not dynamical degrees of freedom because of the infinite volume of ALE$\times$**C**. The superpotential is given by

$$W=\sqrt{2}g(\overline{Q}XQ-\zeta^2X), \tag{4}$$

and there may be a Fayet-Iliopoulos D term $\mathcal{L}=-\xi^2D$. The inflaton is X, which corresponds to the distance between the D3- and D7-branes in the **C** direction. When the D3-brane comes close enough to the D7-brane, i.e., $X\lesssim|\zeta|,\xi$, the D3-D7 open-string modes $(Q,\overline{Q})$ become tachyonic and begin to condense, a D3-D7 bound state is formed, the vacuum energy $g^2/2\times(|2\zeta^2|^2+\xi^4)$ disappears, and the inflation comes to an end. There is no massless moduli in this vacuum, and this is the reason why the fractional brane is adopted. The Fayet-Iliopoulos parameters $(-2\,\mathrm{Im}\,\zeta^2,2\,\mathrm{Re}\,\zeta^2,\xi^2)$ are nonzero when a singularity $\mathbf{C}^2/\mathbb{Z}_M$ is blown up to be a smooth ALE space [12].[2]

Type IIB string theory has to be compactified on a Calabi-Yau threefold in order to obtain dynamical gravity. The D7-brane should be wrapped on a homomorphic four-cycle so that the $D=4$, $\mathcal{N}=1$ SUSY is preserved [14]. We consider that there is a point on the four-cycle around which the local geometry of the Calabi-Yau threefold is ALE$\times$**C**. The fractional D3-brane is trapped at the tip of the ALE space and is able to move along the **C** direction. On the other hand, the $\mathcal{N}=1$ vector multiplet is usually the only Kaluza-Klein zero mode from the D7-D7 open string, and, in particular, the coordinate of the D7-brane in the **C** direction is fixed. Other particles such as quarks and leptons can be realized by the local construction of D-branes at another place in the Calabi-Yau threefold, as in [15]. Thus, the noncompact model above can be embedded as a local model of a realistic Calabi-Yau compactification.

The world-sheet amplitude of string theory is expanded in powers of the string coupling g_s. The expansion begins with the sphere amplitude, which is proportional to g_s^{-2}. In particular, M_{Pl}^2 is proportional to g_s^{-2}.

The disk amplitude comes at the next-to-leading order g_s^{-1}. It is calculated by restricting the boundary of the world sheet to the fractional D3-brane. The kinetic term of the inflaton arises at this level, and hence its coefficient is proportional to g_s^{-1}. The kinetic term of the U(1) vector field, the $\mathcal{N}=2$ SUSY partner of the inflaton, also has a coefficient proportional to g_s^{-1}. Thus, the U(1) gauge coupling constant g is related to g_s via $g^2\sim g_s$. The vacuum energy also arises at this level. Therefore, the vacuum energy is proportional to $g_s^1\sim g^2$ when $M_{\mathrm{Pl}}^2\sim g_s^{-2}$ is factored out from the scalar potential (see also the discussion at the end of this section).

We are interested only in the disk amplitude whose boundary is on the D3-brane. The D7-brane is irrelevant, and only the local background geometry around the D3-brane, ALE$\times$**C**, is relevant to the disk amplitude. Since ALE$\times$**C** has translational invariance in the **C** direction, the translational invariance is respected in the disk amplitude. Thus, the amplitude does not depend on the position of the D3-brane. Therefore, the disk amplitude does not induce the inflaton potential.

The cylinder amplitude is at the next order, g_s^0. The one-loop amplitude of the open string and the amplitude exchanging closed string at the tree level are contained here. The inflaton potential comes from a cylinder with one end on the D3-brane and the other on the D7-branes. The amplitude contains a potential logarithmic in the distance r between the two D-branes. This potential corresponds to the one-loop radiative correction in [4]. There are also terms damping exponentially in r. They are interpreted as the forces between the two D-branes induced by exchanging stringy excited states at the tree level. These terms are suppressed very much when the D-branes are separated by a distance longer than the string length $\sim\sqrt{\alpha'}$. Finally, there is also a term quadratic in the inflaton r. This potential is induced by exchanging massless twisted sector fields; both the fractional D3-branes and the D7-brane carry twisted Ramond-Ramond charges.

Putting all the above together, we have obtained

$$\mathcal{L}\sim(g_s^{-2}\sim M_{\mathrm{Pl}}^2)[R+g_s(\partial r)^2 \\ -g_s(1+g_s\ln r-g_se^{-r}+g_sr^2+\cdots)], \tag{5}$$

where α' is set to unity and r is the distance between the two

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[1] We are grateful to the authors of Ref. [10].

[2] See also [13], where the vacuum energy is given by the vacuum expectation value of the B field.

D-branes. Let us now rescale the inflaton r so that the kinetic term is canonical; $\sigma \equiv \sqrt{g_s} r M_{\rm Pl}$. Then the scalar potential is given by

$$V \propto g_s M_{\rm Pl}^2 \left\{ 1 + g_s \ln\left(\frac{\sigma}{M_{\rm Pl}} \right) - g_s e^{-\sigma/(\sqrt{g_s} M_{\rm Pl})} + \left(\frac{\sigma}{M_{\rm Pl}} \right)^2 + O(g_s) \left(\frac{\sigma}{M_{\rm Pl}} \right)^2 + O\left[\left(\frac{\sigma}{M_{\rm Pl}} \right)^3 \right] \right\}. \quad (6)$$

The correct mass dimension of the scalar potential is restored by multiplying quantities that have been set to unity, including α' and the volume of the compactified manifold. Note that the short-distance effects appear only as the exponentially damping potential. This is partly because the local translational invariance of the internal space dimensions forbids the potential from the disk amplitude. This is also because the cylinder amplitude is interpreted as the Yukawa potential induced by heavy states, and hence the short-distance (ultraviolet) effects is irrelevant unless the D3-brane is in a short distance from the D7-brane in the internal space dimensions. This kind of picture is hardly obtained without assuming string theory. The logarithmic correction is not harmful when the coupling is sufficiently small, just as in field theoretical models [4]. The quadratic potential induced by the twisted-sector exchange, which can be the only harmful effect, is suppressed in certain string vacua as shown in Sec. IV. Although the volume of the Calabi-Yau threefold has not been treated carefully, it is also shown in Secs. III and IV that this parameter is irrelevant to the flatness of the inflaton potential.

III. $D=4$ SUGRA ANALYSIS OF THE INFLATON POTENTIAL

Both the Planck scale and the Kaluza-Klein scale are finite, as well as the string scale, when the internal dimensions are compactified. We show in this section that the inflaton potential still reflects the translational invariance of the local geometry, and is sufficiently flat, even in the low-energy effective $D=4$ SUGRA description obtained after the compactification. In particular, the inflaton potential does not grow exponentially for large field value, even when the vacuum energy is carried by the F term. It is another purpose of this section and of Sec. IV to examine the volume-parameter (in)dependence of the potential, which was neglected in the previous section.

We adopt $K3 \times T^2$ as the model of a Calabi-Yau threefold. It surely contains ALE$\times$**C** as a local geometry, but it also preserves extended SUSY. Thus, the analysis based on $K3 \times T^2$ has a limited meaning. However, this toy model has another virtue that we can analyze more precisely owing to the extended SUSY. Furthermore, a related discussion is found at the end of this section.

The scalar potential of the $D=4$, $\mathcal{N}=2$ SUGRA is given by [16]

$$V = 4 h_{uv} k_\Lambda^u k_\Sigma^v L^\Lambda L^{*\Sigma} + (g^{ij^*} f_i^\Lambda f_{j^*}^\Sigma - 3 L^{*\Lambda} L^\Sigma) P_\Lambda^x P_\Sigma^x . \quad (7)$$

P_Λ^x are momentum maps, which roughly correspond to the D term (Killing potential) and the F-term potential, k_Λ^u are Killing vectors, L^Λ is roughly the scalar partner of the Λth vector field, f_i^Λ its covariant derivative with respect to the ith scalar of the vector multiplets, and g_{ij*} and h_{uv} the metric of the vector multiplets and hypermultiplets, respectively. See [16] for more details.

Let us define

$$L^\Lambda \equiv e^{K_V/2} X^\Lambda, \quad (8)$$

$$W_0 \equiv X^\Lambda (P^1 + i P^2)_\Lambda , \quad (9)$$

where K_V is the Kähler potential of vector multiplets. Then the first term of Eq. (7) becomes $e^{K_V} |\partial W_0|^2$ for hypermultiplets, and the second contains $e^{K_V} |\partial W_0|^2$ for $\mathcal{N}=1$ chiral components of $\mathcal{N}=2$ vector multiplets. The last term contains $-3 e^{K_V} |W_0|^2$. Thus, the $\mathcal{N}=2$ SUGRA scalar potential is not completely different from that of $\mathcal{N}=1$ SUGRA. See [17] for more details about the relation between $\mathcal{N}=2$ SUGRA and $\mathcal{N}=1$ SUGRA. We come back to this issue at the end of this section.

The $\mathcal{N}=1$ chiral multiplet X in Sec. II, identified with the inflaton, belongs to an $\mathcal{N}=2$ vector multiplet. Thus, one of the X^Λ's is approximately X. The $\mathcal{N}=2$ hypermultiplet $(Q,\overline{Q})$ in Sec. II is in the momentum maps as

$$P_\Lambda^3 = \langle e^3 \rangle + |Q|^2 - |\overline{Q}|^2 + \cdots , \quad (10)$$

$$i (P^1 + i P^2)_\Lambda = i \langle e^1 + i e^2 \rangle + 2 Q \overline{Q} + \cdots . \quad (11)$$

The Fayet-Iliopoulos parameters are now obtained as the vacuum expectation values (VEVs) $\langle e^m \rangle$ $(m=1,2,3)$ of massless fields in the closed-string sector; $i \langle e^1 + i e^2 \rangle = -2\zeta^2$ and $\langle e^3 \rangle = \xi^2$. The first term of Eq. (7) contains

$$g^2 (|XQ|^2 + |X\overline{Q}|^2), \quad (12)$$

which prevents the D3-D7 open-string modes $(Q,\overline{Q})$ from condensing during the inflation because $\langle X \rangle$ is large. The vacuum energy (and the inflaton potential) during the inflation is (are) provided by the last two terms

$$(g^{ij^*} f_i^\Lambda f_{j^*}^\Sigma - 3 L^{*\Lambda} L^\Sigma) \langle P_\Lambda^x P_\Sigma^x \rangle , \quad (13)$$

as we see explicitly in this section. Although the first term also contributes to the inflaton potential, we show in Sec. IV that this contribution is negligible in certain string vacua.

Let us suppose that the inflaton potential comes dominantly from Eq. (13). Then we only have to know the special geometry, which determines $g^{ij^*} f_i^\Lambda f_{j^*}^\Sigma - 3 L^{*\Lambda} L^\Sigma$, to see whether the inflaton potential is flat. Therefore, we just assume in this section that the positive $\langle P_\Lambda^x P_\Sigma^x \rangle$ is realized, and postpone discussing how the momentum maps are determined until Sec. IV. Section IV A discusses how to stabilize nonzero $\langle e^m \rangle$'s in Eqs. (10) and (11) by examining the quaternionic geometry of hypermultiplets. Section IV B explains when the first term in Eq. (7), which contains the quadratic term in Eq. (6), is not harmful to the slow-roll condition.

Special geometry of the vector multiplets and Calabi-Visentini basis

We begin by determining the Kähler metric of the moduli space of vector multiplets (special geometry). After that, a symplectic vector (X^Λ, F_Σ) is chosen suitably and $(g^{ij*} f_i^\Lambda f_{j*}^\Sigma - 3L^{*\Lambda} L^\Sigma)$ in the potential (13) is calculated.

As we see later, one cannot capture the essential reason of the flatness in this SUGRA analysis without considering carefully the interaction of the inflaton with other vector multiplets arising from the closed-string sector. There are three $\mathcal{N}=2$ vector multiplets in the low-energy effective theory when type IIB theory is compactified on $K3\times T^2/\mathbb{Z}_2$. Here, $\mathbb{Z}_2$ is generated by $\Omega(-1)^{F_L} R_{T^2}$, where R_{T^2} reflects the coordinates of T^2. The three complex scalars in these multiplets are denoted by S, T, and U; $S = C_{(0)} + i g_s^{-1}$, $\mathrm{Im}\, T \propto g_s^{-1} \mathrm{vol}(K3)$, and U is the complex structure of T^2. We adopt a convention in which imaginary parts of all S, T, and U are positive.

The kinetic terms of these fields are determined from [18], since a model T dual to ours (type I theory compactified on $K3\times T^2$) is discussed there. We take the T duality transformation from [18], and find that the kinetic term is given by

$$\frac{\partial_\mu S \partial_\mu \overline{S}}{(S-\overline{S})^2} + \frac{\partial_\mu T \partial_\mu \overline{T}}{(T-\overline{T})^2} + \frac{\partial_\mu U \partial_\mu \overline{U}}{(U-\overline{U})^2} \tag{14}$$

after Kaluza-Klein reduction and Weyl rescaling. All the scalar fields are chosen to be dimensionless, and these terms become a part of the $D=4$ Lagrangian when multiplied by M_{Pl}^2. This metric of the special geometry, which is the target space of the nonlinear σ model of the scalar components, is obtained from a Kähler potential

$$K_V = -\log[i(S-\overline{S})(T-\overline{T})(U-\overline{U})], \tag{15}$$

which can be derived from a prepotential

$$\mathcal{F} = -STU. \tag{16}$$

Let us now introduce D3-branes to this system. The coordinates of the D3-branes on T^2 are denoted by $(x_i, y_i) \sim (x_i + 1, y_i) \sim (x_i, y_i + 1)$. We introduce a complex scalar $Z_i = x_i + U y_i$. The twisted Ramond-Ramond (RR) charge does not vanish when there is only one fractional D3-brane. But the RR charge can be canceled in a system where D7-branes and other fractional D3-branes are introduced. They will be scattered at different points in T^2. We are interested in only one[3] of the fractional D3-branes $Z = Z_1$, which corresponds to X in Sec. II.

[3]Since we are interested only in the disk-level potential in this section, other D-branes are irrelevant to the inflaton potential.

The kinetic terms of the bulk particles and the D3-brane are given by [18]

$$\frac{\partial_\mu S \partial_\mu \overline{S}}{(S-\overline{S})^2} + \frac{|\partial_\mu T + (x\partial_\mu y - y\partial_\mu x)/2|^2}{(T-\overline{T})^2} + \frac{\partial_\mu U \partial_\mu \overline{U}}{(U-\overline{U})^2} + \frac{(\partial_\mu x + U\partial_\mu y)(\partial_\mu x + \overline{U}\partial_\mu y)}{(U-\overline{U})(T-\overline{T})} \tag{17}$$

after Kaluza-Klein reduction.[4] The cross term in the kinetic term of T has its origin in the Wess-Zumino term on the D-branes

$$\int_{D3} C^{(4)}_{\mu\nu xy} (\partial_\rho x)(\partial_\sigma y) dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma = -\frac{1}{2}\int d^4x (\partial_\rho C^{(4)}_{\mu\nu xy}) \epsilon^{\mu\nu\rho\sigma} (x\partial_\sigma y - y\partial_\sigma x). \tag{18}$$

Now, a new coordinate

$$\tilde{T} = T + \frac{1}{2} y_i Z_i \tag{19}$$

is introduced, and $\tilde{T}$ is regarded as one of the special coordinates; T is no longer a special coordinate. The Kähler potential for the metric (17) is given by

$$K_V = -\log(i(S-\overline{S})((\tilde{T}-\overline{\tilde{T}})(U-\overline{U}) - (Z-\overline{Z})^2/2)) \tag{20}$$

$$= -\log(i(S-\overline{S})(T-\overline{T})(U-\overline{U})), \tag{21}$$

and this Kähler potential is derived from a prepotential

$$\mathcal{F} = -S\tilde{T}U + SZ^2/2. \tag{22}$$

Thus, the newly introduced $\tilde{T}$ is in the correct set of special coordinates, along with S, U, and Z. Note that the complexified coupling of the gauge field on the D3-brane is S, as desired. The special geometry obtained here turns out to be

$$\frac{SU(1,1)}{SO(2)} \times \frac{SO(2,3)}{SO(2)\times SO(3)}. \tag{23}$$

One of the special coordinates S, which factorizes in Eq. (20), parametrizes $SU(1,1)/SO(2)$.

The symplectic section $\Omega = (X^\Lambda, F_\Lambda)$ of the special manifold is given by

$$X^\Lambda = (1, S, \tilde{T}, U, Z), \tag{24}$$

[4]The relative normalization between the bulk particles (S,T,U) and the D-brane Z is not precise. It turns out, however, that the slow-roll parameter η is independent of the normalization. Thus, we do not pay attention to the numerical coefficients, say, of the last term, very much in this article.

$$F_\Lambda=(S\tilde{T}U-SZ^2/2,-\tilde{T}U+Z^2/2,-SU,-S\tilde{T},SZ). \tag{25}$$

Although the symplectic transformation of Ω does not change the Kähler potential, a different choice of basis leads to different coupling with hypermultiplets [19]. We choose a base in which the bidoublet representation of $SU(1,1)$ acting on S and $SU(1,1)\subset SO(2,2)\subset SO(2,3)$ acting on U is realized in the coordinates X^Λ. This is for the same reason as in [20,10]. Choosing a suitable symplectic transformation, one finds that

$$X^\Lambda=\left(\frac{1-SU}{\sqrt{2}},-\frac{S+U}{\sqrt{2}},\frac{-1-SU}{\sqrt{2}},\frac{-S+U}{\sqrt{2}},Z\right), \tag{26}$$

$$F_\Lambda=\left(-\frac{\tilde{T}(1-SU)+SZ^2/2}{\sqrt{2}},-\frac{\tilde{T}(-S-U)+Z^2/2}{\sqrt{2}},\right.$$
$$\left.-\frac{\tilde{T}(1+SU)-SZ^2/2}{\sqrt{2}},-\frac{\tilde{T}(S-U)+Z^2/2}{\sqrt{2}},SZ\right). \tag{27}$$

This is the so-called Calabi-Visentini basis.

Now that we have holomorphic symplectic section $\Omega=(X^\Lambda,F_\Sigma)$ in a suitable basis, it is straightforward to calculate the potential (13). One finds that

$$\begin{aligned}(g^{ij^*}f_i^\Lambda f_{j^*}^\Sigma-3e^{K_V}X^{*\Lambda}X^\Sigma)P_\Lambda^xP_\Sigma^x\\
=-\eta^{\Lambda\Sigma}\frac{1}{2\,\mathrm{Im}\,T}P_\Lambda^xP_\Sigma^x|_{\Lambda,\Sigma=0,\ldots,3}+\frac{1}{2\,\mathrm{Im}\,S}P_{\Lambda=4}^xP_{\Lambda=4}^x\\
-\frac{x}{\sqrt{2}\,\mathrm{Im}\,T}(P_{\Lambda=0}^x+P_{\Lambda=2}^x)P_{\Lambda=4}^x+\frac{y}{\sqrt{2}\,\mathrm{Im}\,T}\\
\times(P_{\Lambda=1}^x+P_{\Lambda=3}^x)P_{\Lambda=4}^x,\end{aligned} \tag{28}$$

where $\eta^{\Lambda\Sigma}=\mathrm{diag}(1,1,-1,-1)$. The Kähler potential (20) is far from minimal, and the holomorphic symplectic section Ω in Eqs. (26),(27) exhibits an intricate mixture of the special coordinates. However, the inflaton potential (28) is completely independent of the inflaton field Z, when $\langle P_4P_4\rangle$ is nonzero. This result shows that the flat inflaton potential is not lifted when the internal dimensions are compactified and the Planck scale (as well as the string scale) becomes finite. See also Sec. IV for a discussion related to the second line, which depends linearly on the inflaton.

The translational symmetry in the **C** direction, or in the T^2 direction, is preserved in the kinetic term of the bosons (17), where a scalar Re T from the Ramond-Ramond four-form potential is also shifted:

$$x\to x+\epsilon,\quad \mathrm{Re}\,T\to\mathrm{Re}\,T-\epsilon y/2, \tag{29}$$

$$y\to y+\epsilon',\quad \mathrm{Re}\,T\to\mathrm{Re}\,T+\epsilon' x/2, \tag{30}$$

or in terms of the special coordinates

$$Z\to Z+\epsilon,\quad (\tilde{T}U-Z^2/2)\to(\tilde{T}U-Z^2/2)-\epsilon Z, \tag{31}$$

$$Z\to Z+\epsilon' U,\quad \tilde{T}\to\tilde{T}+\epsilon' Z. \tag{32}$$

The translational symmetry of T^2 is now part of $SO(2,3)$ isometry along with $SO(2,2)\simeq SL_2\mathbb{R}\times SL_2\mathbb{R}$.

There is another interesting feature in Eq. (28). Notice that the F-term and D-term scalar potentials are completely different in $D=4$, $\mathcal{N}=1$ SUGRA, namely,

$$V_F=e^{K_V+K_H}(g^{ij^*}\mathcal{D}_iW_1\mathcal{D}_{j^*}W_1^*-3|W_1|^2),\quad V_D=g^2|D|^2. \tag{33}$$

However, the $\mathcal{N}=2$ scalar potential (28) "becomes"[5]

$$\begin{aligned}V_{x=1,2}&=e^{K_V}(g^{ij^*}\mathcal{D}_iW_0\mathcal{D}_{j^*}W_0^*-3|W_0|^2)\\
&\text{“=”}V_F\ \text{ in Eq. (33)},\end{aligned} \tag{34}$$

where

$$W_1\text{“=”}e^{-K_H/2}W_0, \tag{35}$$

while

$$V_{x=3}=g_s|P^3|^2=V_D\ \text{ in Eq. (33)}, \tag{36}$$

when the relation (28) holds. Thus, the flat potential obtained in Eq. (28) may still be expected when the internal manifold is not $K3\times T^2$ but a Calabi-Yau threefold with local ALE$\times$**C** geometry. Then, an important consequence is that the inflaton potential is not growing exponentially at large field value, no matter how much the vacuum energy is carried by the F term in realistic models.

IV. MODULI STABILIZATION AND SLOW-ROLL CONDITIONS

In the previous section, we assumed that $\langle P_{\Lambda=4}^xP_{\Sigma=4}^x\rangle$ is nonzero. It is, however, realized as VEVs of dynamical fields, and would have vanished if those fields were not stabilized. Thus, we need to ensure that the nonzero VEVs of the dynamical fields are stabilized.

It has been clarified [22,23] that most of the moduli are stabilized by introducing three-form fluxes. Moduli that are not stabilized by the three-form fluxes can also be stabilized by nonperturbative effects. Thus, it is not the main focus of our attention whether moduli are stabilized or not. Rather, the question is whether the stabilized Fayet-Iliopoulos parameter can be nonzero.

Another important aspect of the moduli stabilization in models of inflation is that an extra inflaton potential is generically generated when stabilized heavy moduli are integrated out. Since even Planck-suppressed corrections are harmful to the flatness of the inflaton potential, extra contri-

[5]Here we keep quotation marks because there is a subtlety in defining the Kähler potential K_H for quaternionic geometry. See [21] for more details.

TABLE I. (Some of) the moduli particles are classified in terms of two different $\mathcal{N}=2$ SUSYs. Particles in the right column are odd under the orientifold projection $\mathbb{Z}_2$ and are projected out. Σ stands for a two-cycle in $K3$ and γ for a one-cycle of T^2. Ω is the global holomorphic three-form of CY_3.

	$\mathcal{N}=2$ hypermultiplet of $K3\times T^2/\mathbb{Z}_2$	$\mathcal{N}=2$ vector multiplet of $K3\times T^2/\mathbb{Z}_2$
$\mathcal{N}=2$ hypermultiplet of CY_3	$\int_\Sigma C^{(4)}_{\Sigma\mu\nu}$, $e^3=\int_\Sigma \omega_{K3}$	$\int_\Sigma B, \int_\Sigma C^{(2)}$
$\mathcal{N}=2$ vector multiplet of CY_3	$e^1+ie^2=\int_{\Sigma\gamma}\Omega$	$\int_{\Sigma\gamma} C^{(4)}$

butions to the potential are also harmful when they are suppressed by masses of moduli. It also happens that the stabilizing potential sometimes constrains moduli as functions of the inflaton. Thus, the VEVs of moduli can change during the inflation, and the dynamics of the inflation can be different from the ordinary one. Therefore, the moduli stabilization is an important ingredient of the inflation model in string theory [24].

One can analyze the effects of introducing the fluxes in terms of $D=4$ gauged SUGRA [23]. We adopt $K3\times T^2$ as a toy model of the Calabi-Yau threefold in this section (except in Sec. IV C), to see explicitly how the nonzero Fayet-Iliopoulos parameters are stabilized and how the inflaton is mixed with other moduli.

The kinetic term of the Ramond-Ramond four-form potential and the Chern-Simons term are

$$\int d^{10}x \frac{1}{2}\left| dC^{(4)} - \frac{1}{2}C^{(2)}\wedge dB + \frac{1}{2}B\wedge dC^{(2)}\right|^2 + \int C^{(4)}\wedge dB \wedge dC^{(2)} \tag{37}$$

in the $D=10$ action of type IIB theory. When the type IIB theory is compactified on $K3\times T^2/\mathbb{Z}_2$, the dimensional reduction of this action contains

$$\int d^4x \frac{1}{2}\left| \partial_\mu \int_{\Sigma\gamma\gamma'} C^{(4)} + \frac{1}{2}\int_{\Sigma\gamma'}\langle dB\rangle \int_\gamma C^{(2)}_{\gamma\mu} - \frac{1}{2}\int_{\Sigma\gamma'}\langle dC^{(2)}\rangle\int_\gamma B_{\gamma\mu}\right|^2, \tag{38}$$

where Σ denote two-cycles of the $K3$ manifold and γ,γ' one-cycles of T^2. The quantities $\int_{\Sigma\gamma'}\langle dB\rangle$ and $\int_{\Sigma\gamma'}\langle dC^{(2)}\rangle$ are the numbers of flux quanta penetrating the three-cycles $\Sigma\times\gamma'$ and are nonzero. Thus, the Killing vectors of the vector fields (in $D=4$ effective theory) $\int_\gamma C_{\gamma\mu}$ and $\int_\gamma B_{\gamma\mu}$ act nontrivially in the direction of the scalar $\int_{\Sigma\gamma\gamma'}C^{(4)}$. The introduction of fluxes turns on gauge coupling of the vector fields originating in the closed string sector.

The Ramond-Ramond scalars $\int_{\Sigma\gamma\gamma'}C^{(4)}$ are absorbed by the vector fields $\int_\gamma B_{\gamma\mu}$ through the Higgs mechanism in Eq. (38). The Fayet-Iliopoulos D-term parameters $e^3=\int_\Sigma \omega_{K3}$ are scalar $\mathcal{N}=1$ SUSY partners of the Ramond-Ramond scalars $\int_{\Sigma\gamma\gamma'}C^{(4)}$ (see Table I), and hence the D-term parameters are also stabilized by the fluxes as long as the $\mathcal{N}=1$ SUSY is preserved. The Fayet-Iliopoulos F-term parameters $e^1+ie^2=\int_\Sigma \Omega_{K3}$ are also stabilized when the $\mathcal{N}=2$ SUSY is preserved. They are stabilized by the scalar potential (28), where they are contained in the momentum maps P^x_Λ. The scalar partners of the vector fields, which are certain linear combinations of X^Λ's ($\Lambda=0,1,2,3$) in Eq. (26), also become massive when the $\mathcal{N}=2$ SUSY is preserved. Their mass term arises from the first term of Eq. (7), because the k^u_Λ's are nonzero for $u=\int_{\Sigma\gamma\gamma'}C^{(4)}$'s.

We introduce the fluxes so that the Killing vectors for $\Lambda=2,3$ are turned on. This is because we do not want vacuum instability that arises due to the positive sign of $\eta^{\Lambda\Sigma}$ in Eq. (28). The Killing vectors we introduce later (and corresponding fluxes) preserve $\mathcal{N}=2$ SUSY. Thus, all the moduli mentioned above acquire masses.

Other moduli, including the volumes of $K3$ and T^2, are not stabilized in the toy model discussed in Secs. IV A and IV B. However, those moduli can be stabilized in the general framework of $\mathcal{N}=1$ supersymmetric vacua, and we just assume that they are stabilized at finite values and do not cause extra problems. A related discussion is found in Secs. IV B and IV C.

In Sec. IV A, we discuss in detail the potential stabilizing the Fayet-Iliopoulos parameters (blow-up modes) $e^m (m=1,2,3)$. The potential is roughly given by

$$V\sim \frac{1}{2\,\mathrm{Im}\,T}\left[|P^x_{\Lambda=2}(\text{function of } e^m\text{'s})|^2 + |P^x_{\Lambda=3}(\text{function of } e^m\text{'s})|^2\right] + \frac{1}{2\,\mathrm{Im}\,S}|P^x_{\Lambda=4=\text{inflaton}}(e^x + \text{function of } (Q,\overline{Q}) + \cdots)|^2, \tag{39}$$

where the first two terms arise from turning on nontrivial Killing vectors for the bulk gauge fields, and the last term is for the gauge field on the fractional D3-brane. The first two terms fix the vacuum of e^m's so that P^x_2 and P^x_3 vanish. The second line of Eq. (28), which is omitted here, also vanishes. On the other hand, the effective Fayet-Iliopoulos parameters $P^x_4|_{Q,\overline{Q}=0}$ do not vanish, because the function of e^m's can be different for $P_{2,3}$ and for P_4, as we show explicitly in Sec. IV A. In particular, the positive vacuum energy for the inflation is stabilized (when the volumes of both $K3$ and T^2 are finite). The purpose of Sec. IV A is to show explicitly that $P_{2,3}$ and P_4 can be different functions of the blow-up parameters.

In Sec. IV B, we discuss the mixing of the inflaton with moduli S and U that is caused by the moduli stabilization. It turns out that there is no extra mass term generated by this mixing. Although the inflaton mass does not vanish, we see that there is a flux configuration where the inflaton mass is sufficiently small.

A. Quaternionic geometry of the hypermultiplets and stabilization of the positive vacuum energy

The Fayet-Iliopoulos parameters are realized by VEVs of a hypermultiplet. There are 20 hypermultiplets coming from the closed string sector, when the type IIB theory is compactified on $K3\times T^2/\mathbb{Z}_2$. The 80 scalars consist of the moduli of $K3$ metric e^{ma} ($m=1,2,3$, $a=1,\ldots,19$) [25], $3+19=22$ scalars $c^m (m=1,2,3)$ and c^a ($a=1,\ldots,19$) from the Ramond-Ramond four-form, and $e^{-2\phi}$, which is the volume of T^2. There are 19 anti-self-dual two-cycles in the $K3$ manifold, and each of them has a triplet moduli e^{ma} ($m=1,2,3$) describing the blow-up of the cycle. The Fayet-Iliopoulos parameters we are interested in are e^{ma} ($m=1,2,3$) for one of these cycles (one of $a\in\{1,\ldots,19\}$).

In order to stabilize nonzero Fayet-Iliopoulos parameters, one has to know the quaternionic geometry for a wider range of the moduli space, not just around the orbifold limit. The global geometry of the quaternionic manifold is $SO(4,20)/SO(4)\times SO(20)$ [26]. The global parametrization of this manifold, where the coordinates are (e^{ma},c^m,c^a,ϕ), is explicitly described in [20].

Massless modes from the D3-D7 open string are also hypermultiplets, and thus the total quaternionic geometry is spanned by 80 coordinates of the bulk modes and extra coordinates of the open-string modes. The metric of the total quaternionic space is not known. However, the D3-D7 open string is given a large mass via Eq. (12) and its VEV is zero during the inflation. Therefore, it is sufficient to know the geometry of the submanifold where the VEVs of open string modes are zero, as long as we are concerned about the stabilization of the positive vacuum energy during the inflation.

We introduce the following Killing vectors:

$$k_{\Lambda=2}=g_1\partial_{c^{m=1}}+g_2\partial_{c^{a=1}} \quad (g_1<g_2), \tag{40}$$

$$k_{\Lambda=3}=g_1\partial_{c^{m=2}}+g_2\partial_{c^{a=2}}. \tag{41}$$

The Killing vectors above are constant shifts in the c^m and c^a directions, and it is easy to see that they are isometry; the metric of the quaternionic geometry is as follows:

$$\begin{aligned} ds^2=&d\phi^2+\sum_m e^{2\phi}\left(\sqrt{1+e\cdot e^t}^{\,mn}dc^n+e^{ma}dc^a\right)^2\\ &+\sum_a e^{2\phi}\left(dc^m e^{ma}+dc^b\sqrt{1+e^t\cdot e}^{\,ba}\right)^2\\ &+\sum_{a,m}\left(\sqrt{1+e\cdot e^t}^{\,mn}de^{na}-e^{mb}d\sqrt{1+e^t\cdot e}^{\,ba}\right)^2, \end{aligned} \tag{42}$$

which does not depend on c^m and c^a. This isometry is the remnant of the gauge symmetry adding an exact four-form to $C^{(4)}$. The $\mathcal{N}=2$ SUSY is preserved when the Killing vectors are chosen as in Eqs. (40),(41).

The introduction of the Killing vectors (40) and (41) corresponds to introducing three-form fluxes in the $D=10$ picture. One can determine the fluxes in the $D=10$ picture through Eq. (38), but we do not pursue this issue further in this article. The Killing vectors are sufficient information for later purposes.

The Killing vectors are given, and now the momentum maps are obtained by [23,27]

$$P^x_\Lambda=\omega^x_u k^u=\omega^x_{c^m}k^{c^m}_\Lambda+\omega^x_{c^a}k^{c^a}_\Lambda. \tag{43}$$

Here, ω^x is the $su(2)_R$ connection associated with the quaternionic manifold, which is given by

$$\begin{aligned}\omega^x=&\omega^x_{c^m}dc^m+\omega^x_{c^a}dc^a+\cdots\\ =&e^{\phi}\left(\sqrt{1+e\cdot e^t}^{\,xm}dc^m-e^{xa}dc^a\right)+\cdots \quad (x=1,2,3).\end{aligned} \tag{44}$$

The ellipses stand for one-form de^{ma} and $d\phi$. Thus, the momentum maps are obtained:

$$P^x_{\Lambda=2}=e^{\phi}\left(g_1\sqrt{1+e\cdot e^t}^{\,x1}-g_2e^{x1}\right), \tag{45}$$

$$P^x_{\Lambda=3}=e^{\phi}\left(g_1\sqrt{1+e\cdot e^t}^{\,x2}-g_2e^{x2}\right). \tag{46}$$

All e^{m1}'s and e^{m2}'s are stabilized and their VEVs are determined by requiring the potential $(P^x_2)^2+(P^x_3)^2$ to be minimized. Their VEVs are

$$e^{11}=e^{22}=\frac{g_1}{\sqrt{g_2^2-g_1^2}}, \tag{47}$$

$$\sqrt{1+(e^{11})^2}=\sqrt{1+(e^{22})^2}=\frac{g_2}{\sqrt{g_2^2-g_1^2}}, \tag{48}$$

$$e^{21}=e^{31}=e^{12}=e^{32}=0, \tag{49}$$

and in particular, we see that the Fayet-Iliopoulos parameters can really be nonzero at the stabilized vacuum.

The Killing vector associated with $\Lambda=4$, i.e., the inflaton, is given by

$$k_{\Lambda=4}=g_3\partial_{c^{a=2}}+i(Q\partial_Q-\bar{Q}\partial_{\bar{Q}})+\text{H.c.}, \tag{50}$$

and the momentum maps are roughly given by Eqs. (10) and (11), where the e^m's in Eqs. (10) and (11) are replaced by $e^{\phi}g_3e^{m2}$. Thus, the positive vacuum energy $\langle P^x_{\Lambda=4}P^x_{\Lambda=4}\rangle$ is stable during the inflation (here, T and $e^{-\phi}$ are assumed to be stabilized). The vacuum energy is given by

$$\rho_{\cos}=\frac{1}{\text{Im}\,S}(e^{\phi}g_3e^{22})^2=\frac{e^{2\phi}}{\text{Im}\,S}\frac{g_1^2}{g_2^2-g_1^2}g_3^2. \tag{51}$$

We have minimized $|P^x_{\Lambda=2}|^2$ and $|P^x_{\Lambda=3}|^2$ without considering the potential from $P_{\Lambda=4}$, and evaluated the potential from $P_{\Lambda=4}$ at the vacuum determined by $P_{\Lambda=2}$ and $P_{\Lambda=3}$.

This treatment is justified when the mass of the moduli e^{ma} (denoted by m_e) is sufficiently larger than the Hubble parameter of the inflation $H\equiv\sqrt{\rho_{cos}}/(\sqrt{3}M_{\rm Pl})\simeq\sqrt{\rho_{cos}}$, i.e.,

$$\frac{H^2}{m_e^2}\sim\frac{g_3^2\langle e\rangle^2/\mathrm{Im}\,S}{[(g_2^2-g_1^2)/g_2]^2/\mathrm{Im}\,T/\langle e\rangle^2}\sim\frac{g_3^2\,\mathrm{vol}(K3)}{(g_2^2-g_1^2)^4/(g_1^4g_2^2)}\lesssim 1. \tag{52}$$

This is also a necessary condition for an inflation model. Otherwise the value of the Fayet-Iliopoulos parameters would change considerably along the evolution of the inflation.

B. Inflaton-moduli mixing and slow-roll conditions

We have assumed so far that the first term in Eq. (7) does not play an important role. This term, however, contains a potential corresponding to the quadratic term in Eq. (6) and hence can be harmful to the evolution of the inflaton. Therefore, let us now turn our attention to this term and determine in what circumstances it is not harmful.

The vacuum of the hypermultiplets is determined from the potential (28) in the previous subsection. Now it turns out that $\langle h_{uv}k^u_\Lambda k^v_\Sigma\rangle$ does not vanish. Thus, this term generates mass terms to the scalar particles in the vector multiplets. The mass term is given by

$$\frac{e^{2\phi}}{\mathrm{Im}\,S\;\mathrm{Im}\,T\;\mathrm{Im}\,U}\Bigg[((g_1X^2)^\dagger,(g_2X^2)^\dagger)\begin{pmatrix}c^2+s^2 & 2sc\\ 2sc & c^2+s^2\end{pmatrix}\times\begin{pmatrix}g_1X^2\\ g_2X^2\end{pmatrix}+((g_1X^3)^\dagger,(g_2X^3+g_3X^4)^\dagger)\begin{pmatrix}c^2+s^2 & 2sc\\ 2sc & c^2+s^2\end{pmatrix}\times\begin{pmatrix}g_1X^3\\ g_2X^3+g_3X^4\end{pmatrix}\Bigg], \tag{53}$$

where $X^{\Lambda=2,3,4}$ are those in Eq. (26) and abbreviated notations $c^2\equiv g_2^2/(g_2^2-g_1^2)$ and $s^2\equiv g_1^2/(g_2^2-g_1^2)$ are introduced. Here, the metric (42), the Killing vectors (40), (41), (50), and the Kähler potential (21) are used along with Eq. (8). The first line of the above potential leads $X^{\Lambda=2}$ to zero. The mass matrix in the second line is diagonalized:

$$\text{eigenvalue:}\quad (c+s)^2=\frac{g_2+g_1}{g_2-g_1},$$

$$\text{eigenstate:}\quad \frac{1}{\sqrt{2}}[(g_2+g_1)X^3+g_3X^4], \tag{54}$$

$$\text{eigenvalue:}\quad (c-s)^2=\frac{g_2-g_1}{g_2+g_1},$$

$$\text{eigenstate:}\quad \frac{1}{\sqrt{2}}[(g_2-g_1)X^3+g_3X^4]. \tag{55}$$

The former eigenstate is dominantly $X^{\Lambda=3}$ since $g_3\ll g_1,g_2$, and the other has the (mass)^2 eigenvalue suppressed by $(g_2-g_1)/(g_2+g_1)$.

The (mass)^2 of X^2 and $[X^3+g_3/(g_2+g_1)X^4]$ are not smaller than the squared Hubble parameter because

$$m^2\gtrsim\frac{e^{2\phi}}{\mathrm{Im}\,S}g_1^2\frac{g_2+g_1}{g_2-g_1}\gtrsim\frac{e^{2\phi}}{\mathrm{Im}\,S}g_3^2\frac{g_1^2}{g_2^2-g_1^2}=H^2. \tag{56}$$

Thus, the moduli S and U are determined by

$$X^2=0\quad\text{and}\quad X^3=-\frac{g_3}{g_2+g_1}X^4 \tag{57}$$

as functions of the inflaton $X^{\Lambda=4}=Z$. In particular,

$$\mathrm{Im}\,S=1-\left(\frac{g_3}{2(g_1+g_2)}Z\right)^2+\cdots, \tag{58}$$

where Z is assumed to be real for simplicity.

The moduli S and U are integrated out, i.e., the relations (57) are substituted into the potential (53)+(51). The net effect of integrating out heavy moduli is to replace $\mathrm{Im}\,S$ with Eq. (58) in Eq. (51) and the original inflaton $X^4=Z$ with a linear combination of X^3 and X^4 in Eq. (53). The inflaton Z is canonically normalized, and now we finally obtain the total effective action relevant to the inflation.

$$\mathcal{L}\simeq M_{\rm Pl}^2\Bigg[|\partial\tilde{Z}|^2-\left(1+\frac{g_3^2\,\mathrm{vol}(K3)}{4(g_1+g_2)^2}\tilde{Z}^2\right)\times e^{2\phi}\left(\frac{1}{2}\frac{g_2-g_1}{g_2+g_1}\left|\frac{2g_1}{g_2+g_1}g_3\tilde{Z}\right|^2+\frac{g_1^2}{g_2^2-g_1^2}g_3^2\right)\Bigg], \tag{59}$$

where $\tilde{Z}$ is the canonically normalized inflaton. Thus, the slow-roll condition (1) implies that

$$\eta\simeq\frac{1}{2}\left(\frac{g_2-g_1}{(g_2+g_1)/2}\right)^2+\frac{g_3^2\,\mathrm{vol}(K3)}{4(g_1+g_2)^2}\ll 1. \tag{60}$$

The first term is sufficiently small when the two flux quanta g_1 and g_2 are degenerate by 10%. Under this condition, the blow-up parameters of the $K3$ manifold are

$$e^{11}=e^{22}=\sqrt{\frac{g}{g_2-g_1}}\gtrsim(2-3), \tag{61}$$

where $g_1\sim g_2\sim g$. The second term, which comes from the inflaton dependence of the string coupling, is sufficiently small when the above condition and Eq. (52) are satisfied.

It is also easy to see that the other slow-roll parameter ϵ is also sufficiently small under the above condition. The second line in Eq. (28), which has linear dependence on the inflaton, contributes to ϵ by

$$\epsilon \sim \frac{H^2}{m_e^2}\frac{\mathrm{Im}\,S}{\langle e\rangle^4} \lesssim \eta. \tag{62}$$

It has been assumed so far that the volume of the torus $e^{-2\phi}$ does not have Z dependence. If it were stabilized as a function of the inflaton Z, the inflaton potential in Eq. (59) would be no longer flat. Therefore, the conditions for the slow-roll inflation are (i) the T^2 volume is stabilized independently from Z, (ii) the flux quanta g_1 and g_2 are degenerate by 10%, and (iii) the flux quanta $g_1 \sim g_2$ are sufficiently large so that the moduli mass is larger than the Hubble parameter.

The volume of the torus $e^{-2\phi}$ is irrelevant to the slow-roll condition. Thus, it can be arbitrary (from the viewpoint of phenomenology), and, in particular, can be moderately large so that the exponential terms in Eq. (6) are sufficiently suppressed.

The two moduli S and U change through Eq. (57) as the value of the inflaton Z changes. However, the slow-roll condition implies that the resulting changes of S and U are not significant.

Finally, one remark is in order here. The coordinate of the D3-brane $Z = X^4$ explicitly appears in the scalar potential, and it looks as if the origin of the torus has a physical meaning. This is actually an artifact of our treatment, where we focused on only one fractional D3-brane. When all the D-branes relevant to the twisted RR-charge cancellation are introduced, we expect that the potential will be a function only of the distance between those D-branes. We consider that the "Z" we used in this article is an approximation, in some sense, to the distance between the fractional D3-brane and one of those D7-branes.

C. Moduli stabilization in generic Calabi-Yau manifold

Some of the results obtained in Secs. IV A and IV B are specific to the choice of $K3 \times T^2$ as the Calabi-Yau threefold. Thus, we go back to the most generic setup described at the beginning of Sec. II, where the Calabi-Yau threefold is required only to have local geometry ALE$\times\mathbf{C}$, and discuss issues relevant to the moduli stabilization again.

Let us start with the type IIB theory compactified on a Calabi-Yau threefold without space-filling D-branes. Moduli particles are classified into $\mathcal{N}=2$ SUSY multiplets.[6] There are $h^{2,1}$ vector multiplets $(\int_A \Omega, \int_A C^{(4)}_{A\mu})$ and $h^{1,1}$ hypermultiplets $((\int_\Sigma C^{(4)}_{\Sigma\mu\nu}, \int_\Sigma \omega), (\int_\Sigma B, \int_\Sigma C^{(2)}))$, where A and Σ denote three-cycles and two-cycles of the Calabi-Yau threefold, respectively. There is another hypermultiplet $((S \equiv C^{(0)} + ie^{-\phi}), (B_{4D}, C^{(2)}_{4D}))$. When three-form fluxes and O3-planes are introduced, only $\mathcal{N}=1$ SUSY can be preserved, and $\mathcal{N}=1$ multiplets $\int_A C^{(4)}_{A\mu}$, $(\int_\Sigma B, \int_\Sigma C^{(2)})$ and $(B_{4D}, C^{(2)}_{4D})$ are projected out. The $\mathcal{N}=1$ chiral multiplets $\int_A \Omega$ are stabilized by the effective superpotential induced by fluxes [28]

$$W = \int_{CY_3} \Omega \wedge G = \int_A \Omega \left\langle \int_B G \right\rangle - \int_B \Omega \left\langle \int_A G \right\rangle, \tag{63}$$

$$G \equiv dC^{(2)} - S dB, \tag{64}$$

where the $\int_B \Omega$'s are written as functions of the $\int_A \Omega$'s. Thus, in particular, the Fayet-Iliopoulos F term $e^1 + ie^2 = \int_\Sigma \Omega_{K3} = \int_{\Sigma\gamma} \Omega_{CY_3}$ and the chiral multiplet S are stabilized by this superpotential. The stable minimum of $\int_A \Omega_{CY_3}$ depends on the fluxes introduced and can be nonzero. On the other hand, the Fayet-Iliopoulos D-term parameter $e^3 = \int_\Sigma \omega_{K3} = \int_\Sigma \omega_{CY_3}$ is not stabilized through this superpotential (64). But nonperturbative effects of gauge theories might help in stabilizing these moduli.

It is surely possible that all the moduli are stabilized and that the effective Fayet-Iliopoulos parameters are nonzero. However, this is not enough for the model of inflation. Let us suppose that the moduli stabilization in Eq. (64) is effectively described by the following superpotential:

$$W_{\mathrm{moduli}} = M_0 + M_2(\Xi - \zeta^2)^2 + O((\Xi - \zeta^2)^3), \tag{65}$$

where Ξ denotes a modulus chiral multiplet whose VEV provides the Fayet-Iliopoulos parameter, and M_0, M_2, and ζ are numerical parameters. Then, the total system is governed by

$$W = \sqrt{2} g X (Q\bar{Q} - \Xi) + W_{\mathrm{moduli}}, \tag{66}$$

and the effective superpotential obtained after the modulus Ξ is integrated out contains a mass term of the inflaton X. Thus, the inflaton potential is no longer flat.

This is not the case when the effective model of the moduli stabilization (65) is replaced by

$$W_{\mathrm{moduli}} = X' \times \mathrm{function}(\Xi), \tag{67}$$

where X' is another modulus. One linear combination of X and X' is integrated out, while the other combination remains light and plays the role of the inflaton. The toy model of the moduli stabilization given in Secs. IV A and IV B is partly described by this superpotential; $X^{\Lambda=3}$ plays the role of X'.

One of the remarkable features of the hybrid inflation model [3–6] is that there is a (discrete) R symmetry, under which X carries R charge 2 [4,7]. Thus, if there is a moduli stabilization that preserves such a (discrete) R symmetry, as in the superpotential (67), the effective superpotential of the inflaton is still constrained by the R symmetry even after the moduli are integrated out, and the inflaton potential remains flat. Therefore, the string realization of the R-invariant moduli stabilization deserves further investigation.

[6]Note that the eight SUSY charges of this $\mathcal{N}=2$ SUSY are not the same subset of the 32 SUSY charges of the type IIB theory as those of the $\mathcal{N}=2$ SUSY in Sec. III. Only four SUSY charges ($\mathcal{N}=1$ SUSY) belong to both. See Table I.

ACKNOWLEDGMENTS

The authors thank S. Yamaguchi and T. Yanagida for discussion. This work is supported in part (T.W.) by the Japan Society for the Promotion of Science, the Miller Institute for Basic Research of Science, and the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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