BPS preons, generalized holonomies, and $D=11$ supergravities

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We develop the BPS (Bogomol'nyi-Prasad-Sommerfield) preon conjecture to analyze the supersymmetric solutions of $D=11$ supergravity. By relating the notions of Killing spinors and BPS preons, we develop a moving G-frame method $[G=GL(32,\mathbb{R}),\ SL(32,\mathbb{R})$ or $Sp(32,\mathbb{R})]$ to analyze their associated generalized holonomies. As a first application we derive here the equations determining the generalized holonomies of $\nu=k/32$ supersymmetric solutions and, in particular, those solving the necessary conditions for the existence of BPS preonic ($\nu=31/32$) solutions of the standard $D=11$ supergravity. We also show that there exist elementary preonic solutions, i.e., solutions preserving 31 out of 32 supersymmetries in a Chern-Simons type supergravity. We present as well a family of worldvolume actions describing the motion of pointlike and extended BPS preons in the background of a D'Auria-Fré type $OSp(1|32)$-related supergravity model. We discuss the possible implications for M theory.

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I. INTRODUCTION

A complete, algebraic classification of M-theory Bogomol'nyi-Prasad-Sommerfield (BPS) states, based on the number k of supersymmetries preserved by a given BPS state, has been given in [1]. BPS states preserving k out of 32 supersymmetries are denoted $\nu=k/32$ states; $k=32$ corresponds to fully supersymmetric vacua. The observation [1] (see also Sec. II) that BPS states that break $32-k$ supersymmetries can be treated as composites of those preserving all but one supersymmetry suggests that the $k=31$ states might be considered as fundamental constituents of M theory. These $\nu=31/32$ BPS states were accordingly named *BPS preons* [1].

Interestingly enough, the notion of BPS preons may be extended to arbitrary dimensions, including $D=4$. In this case, a pointlike BPS preon may be identified with a tower of massless higher spin fields of all possible helicities (see [2–4] and references therein).

The actual existence of preonic, $\nu=31/32$ supersymmetric, BPS states as solitonic solutions of the standard Cremmer-Julia-Scherk (CJS) $D=11$ supergravity [5] has been the subject of recent studies [6,7]. Although no obstructions for their existence have been found by geometric considerations based on the notion of generalized holonomy [6] (see also [8–11]), no $\nu=31/32$ solutions were found either.

In this paper we develop the notion of BPS preons to analyze the various supersymmetric solutions of the supergravity equations. A $\nu=k/32$ supersymmetric solution corresponds to a BPS state composed of $n=(32-k)$ BPS preons. The corresponding $(32-k)$ bosonic spinors $\lambda_\alpha{}^r$ ($r=1,\ldots,n$) are orthogonal to the k Killing spinors $\epsilon_J{}^\alpha$ characterizing the $\nu=k/32$ solution,

$$\epsilon_J{}^\alpha\lambda_\alpha{}^r=0, \tag{1}$$

$$\alpha=1,\ldots,32, \quad J=1,\ldots,k,$$

$$r=1,\ldots,(32-k).$$

Thus, BPS preonic spinors and Killing spinors provide an alternative (dual) characterization of a ν-supersymmetric solution; either one can be used and, for solutions with supernumerary supersymmetries [12,13], the characterization provided by BPS preons is a more economic one. Moreover, the use of both BPS preonic ($\lambda_\alpha{}^r$) and Killing ($\epsilon_J{}^\alpha$) spinors allows us to develop (in Sec. II) a *moving G-frame* method, which may be useful in the search for new supersymmetric solutions of CJS supergravity.

We apply this moving G-frame method [in Sec. III, $G=SL(32,\mathbb{R})$] to studying the generalized holonomies of CJS supergravity and discuss the basic equations characterizing the still hypothetical BPS preonic solutions [Sec. IV B, $G=GL(32,\mathbb{R}),\ SL(32,\mathbb{R})$ or $Sp(32,\mathbb{R})$]. Although no definite answer to the question of the existence of BPS preonic solutions for the standard CJS supergravity is given here, we do show (in Sec. IV A) that $\nu=31/32$ supersymmetric preonic configurations solve the equations in the Chern-Simons (CS) supergravity case [14] (for a review, see [15]) i.e., that CS supergravity does have preonic solutions.

Using the recent results [16,17] on a gauge-fixed form of the action for dynamical supergravity interacting with dynamical superbrane sources (see Sec. IV C), we also propose (in Sec. IV D) a $D=11$ worldvolume action for BPS preons in the background of a D'Auria-Fré $OSp(1|32)$-related supergravity [18] (see also [19]), a model allowing for an economic "embedding" of the standard $D=11$ CJS supergravity.

In this paper we use a "mostly minus" metric, $\eta_{ab}=(+,-,\ldots,-)$; the exterior derivative d acts from the right, $d\Omega_q=(1/q!)dx^{\mu_q}\wedge\cdots\wedge dx^{\mu_1}\wedge dx^\nu\partial_\nu\Omega_{\mu_1\cdots\mu_q}$.

A. Equations of $D=11$ supergravity

The purely bosonic limit of the "free" CJS supergravity equations is given by

$$E_{ab}:=R_{ab}-\frac{1}{3}F_{a[3]}F_b{}^{[3]}+\frac{1}{36}\eta_{ab}F_{[4]}F^{[4]}=0, \qquad (2)$$

$$\mathcal{G}_8:=d*F_4-F_4\wedge F_4=0, \qquad (3)$$

$$dF_4\equiv 0 \quad \Leftrightarrow \quad F_4=dA_3, \qquad (4)$$

which include the Einstein equations with a contribution from the energy-momentum tensor of the antisymmetric gauge field $A_{\mu\nu\rho}(x)$ only, Eq. (2), as well as the free equations (3) and Bianchi identities (4) for the gauge field in curved $D=11$ spacetime. Here $a,b,c=0,\ldots,9,10=\mu,\nu,\rho$,

$$A_3=\frac{1}{3!}dx^\rho\wedge dx^\nu\wedge dx^\mu A_{\mu\nu\rho}(x),$$

$$F_4=\frac{1}{4!}e^d\wedge e^c\wedge e^b\wedge e^a F_{abcd}(x), \qquad (5)$$

$e^a=dx^\mu e_\mu{}^a(x)$, $F_{a[3]}F^{b[3]}\equiv F_{ac_1c_2c_3}F^{bc_1c_2c_3}$, etc. It is also assumed that the torsion and the gravitino vanish,

$$T^a:=De^a=de^a-e^b\wedge\omega_{L\,b}{}^a=0, \qquad (6)$$

$$\psi^\alpha=dx^\mu\psi_\mu^\alpha=0, \quad \alpha=1,\ldots,32, \qquad (7)$$

where $\omega_{L\,b}{}^a$ is the Lorentz connection. Such equations possess *nonsingular* pp-wave solutions with supernumerary supersymmetries [12,13].

As the supergravity multiplet is the only one without higher spin fields in $D=11$, no usual field-theoretical (spacetime) matter contribution to the right-hand sides (r.h.s.'s) of Eqs. (2), (3), (4) may appear. However, these equations might be modified by higher order corrections in curvature [20,21] (a counterpart of the string α' corrections [22] in $D=10$, see also [23]), or/and by the presence of sources from p-branes.

The $\nu=16/32$ supersymmetric M2-brane solution of $D=11$ supergravity (see [24,25] and references therein) possesses a singularity on the $(2+1)$-dimensional worldvolume surface $W^{2+1}\subset M^{11}$, $x^\mu=\hat{x}^\mu(\xi)\equiv\hat{x}^\mu(\tau,\sigma^1,\sigma^2)$ (a caret indicates a function of the worldvolume coordinates). In other words, it solves the Einstein field equation $E_{ab}=\mathcal{T}_{ab}-\frac{1}{9}\eta_{ab}\mathcal{T}_c{}^c$ with a singular energy-momentum tensor density $\mathcal{T}_{ab}\propto\delta^3(x-\hat{x}(\xi))$ [42]. The gauge field equation also possesses a singular contribution J_8 in the r.h.s., $\mathcal{G}_8=J_8$, similar to that of the electric current to the r.h.s. of Maxwell equations. In this sense, the M2-brane carries a supergravity counterpart of the electric charge in Maxwell electrodynamics (see [26] for a discussion).

The other basic $\nu=16/32$ supersymmetric solution of the CJS supergravity, the M5-brane (see [24,25,27] and Refs. therein), is a counterpart of the Dirac monopole, i.e. of the magnetically charged particle. It is characterized by a modification of the Bianchi identities [Eq. (4)] with the analogue of a magnetic current on the r.h.s., $dF_4=\mathcal{J}_5$.

The coincident M-brane solutions still possess $\nu=16/32$ supersymmetry, while the intersecting brane solutions correspond to $\nu<16/32$. Thus, as supernumerary supersymmetric solutions ($\nu>1/2$) are known only for "free" CJS, it is reasonable to consider first the "free" bosonic CJS equations (2)–(4),(6),(7) in the search for a hypothetical BPS preonic, $\nu=31/32$ solution.

Notice, however, that one should not exclude the possible existence of a brane solution [i.e. solutions of $E_{ab}=\mathcal{T}_{ab}-\frac{1}{9}\eta_{ab}\mathcal{T}_c{}^c$ with a $\mathcal{T}_{ab}\propto\delta^{p+1}(x-\hat{x}(\xi))$] with supernumerary supersymmetries, although certainly these solutions would describe quite unusual branes. The reason why the "standard" brane solutions (like M-waves, M2 and M5-branes in $D=11$) always break 1/2 of the supersymmetry is that their κ-symmetry projector (the bosonic part of which is identical to the projector defining the preserved supersymmetries [28,29]) has the form $(1-\bar{\Gamma})$ with $\mathrm{tr}\bar{\Gamma}=0$, $\bar{\Gamma}^2=I$. However, worldvolume actions for branes with a different form for the κ-symmetry projector are known [30,31,4] although in an enlarged superspace (see [32]). A question arises, whether such actions may be written in usual spacetime or superspace.

As a partial answer to this question, we present here a $D=11$ spacetime action for a BPS preon in the background of a D'Auria-Fré type supergravity [18]. The experience provided by the usual $D=11$ M-branes and $D=10$ D-branes together with the analysis [16,17] of the partial preservation of local supersymmetry by the purely bosonic limit of the super-p-brane action suggests that the existence of such a preonic action implies that $\nu=31/32$ solitons should exist in a D'Auria-Fré type model.

B. Killing spinors, generalized connection and generalized holonomy

A bosonic solution of the CJS supergravity equations preserving k out of 32 supersymmetries can be characterized by k independent bosonic spinors (Killing spinors), $\epsilon_J{}^\alpha(x)$, $J=1,\ldots,k$, obeying the Killing spinor equation

$$\mathcal{D}\epsilon_J{}^\alpha=d\epsilon_J{}^\alpha-\epsilon_J{}^\beta\omega_\beta{}^\alpha=0. \qquad (8)$$

The *generalized connection* $\omega_\beta{}^\alpha$ in Eq. (8) includes, besides the Lorentz (spin) connection $\omega_{L\,\beta}{}^\alpha=1/4\omega_L^{ab}\Gamma_{ab\,\beta}{}^\alpha$, a tensorial part $t_\beta{}^\alpha=\omega_\beta{}^\alpha-\omega_{L\,\beta}{}^\alpha$ constructed from the field strength F_{abcd}, Eqs. (4), (5),

$$\omega_\beta{}^\alpha=\frac{1}{4}\omega_L^{ab}\Gamma_{ab\,\beta}{}^\alpha+\frac{i}{18}e^aF_{ab_1b_2b_3}\Gamma^{b_1b_2b_3}{}_\beta{}^\alpha+\frac{i}{144}e^a\Gamma_{ab_1b_2b_3b_4\,\beta}{}^\alpha F^{b_1b_2b_3b_4}. \qquad (9)$$

The Killing spinor equation (8) comes from the requirement of invariance under supersymmetry of the purely bosonic solution [Eq. (7)] that requires $\delta_\varepsilon \psi_\mu^\alpha = \mathcal{D}_\mu \varepsilon^\alpha = 0$. In $OSp(1|32)$-related supergravity models, including CS-type supergravities [14], $\mathcal{D}_\mu$ and, hence, the Killing spinor equation involve an $sp(32)$-valued $\omega_\beta{}^\alpha$ connection ($\omega^{\beta\alpha} := C^{\beta\gamma}\omega_\gamma{}^\alpha = \omega^{(\beta\alpha)}$) which is a true connection, i.e., it is associated with the actual gauge symmetry of the model.

In CJS supergravity, as well as in type IIB $D=10$ supergravity, the gauge symmetry is restricted to $SO(1,10)$, and ω in Eq. (9) is not a true connection. However, the self-consistency (integrability) condition for Eq. (8), $\mathcal{D}\mathcal{D}\epsilon_J{}^\alpha = 0$ has the suggestive form [6,8]

$$\epsilon_J{}^\beta \mathcal{R}_\beta{}^\alpha = 0, \tag{10}$$

in terms of the *generalized curvature*

$$\mathcal{R}_\beta{}^\alpha = d\omega_\beta{}^\alpha - \omega_\beta{}^\gamma \wedge \omega_\gamma{}^\alpha. \tag{11}$$

Equations (8),(10) *formally* possess a $GL(32,\mathbb{R})$ gauge invariance with ω transforming as a $GL(32,\mathbb{R})$ connection. However, $GL(32,\mathbb{R})$ is not a gauge invariance of CJS supergravity, hence the name "generalized" connection and curvature for ω and $\mathcal{R}$.

Notice that, in contrast, a rigid $GL(k,\mathbb{R})$ transformation acting on the index I results in a redefinition of the Killing spinors $\epsilon_I{}^\alpha$, i.e., in replacing the Killing spinors by independent linear combinations with constant coefficients. This is clearly allowed and $GL(k,\mathbb{R})$ can be treated as a rigid symmetry of the system of k Killing spinors characterizing the $\nu = k/32$ supersymmetric solution of any model.

A true connection takes values in the Lie algebra $\mathcal{G}$ of the *structure group* G of the principal fiber bundle. The curvature may take values in a smaller subalgebra $\mathcal{H} \subset \mathcal{G}$ which is associated with a proper subgroup $H \subset G$ of the structure group, the *holonomy group*. For generalized connections ω one may accordingly introduce the notion of *generalized holonomy group* $H \subset G$ [6] (see also [7,8,10]), such that $\mathcal{R}_\beta{}^\alpha \in \mathcal{H}$ (while $\omega_\beta{}^\alpha \in \mathcal{G}$). In this light, the necessary condition for the existence of k Killing spinors, Eq. (10), can be treated as a restriction on the generalized holonomy group H [6,7,10,11,13].

It has been shown that for both CJS $D=11$ supergravity [7] and for type IIB supergravity [11], the generalized holonomy group H is a subgroup of $SL(32,\mathbb{R})$, $H \subset SL(32,\mathbb{R})$. As the generalized connections are clearly traceless in both cases, one also has $G \subset SL(32,\mathbb{R})$.

For $OSp(1|32)$-related models including CS supergravities [14] the (true) structure group is $G \subset Sp(32,\mathbb{R})$ and the holonomy group is $H \subset Sp(32,\mathbb{R})$.

A full expression for the generalized curvature $\mathcal{R}_\alpha{}^\beta$ corresponding to purely bosonic solutions of CJS supergravity may be found e.g., in [9,33] (see Appendix B of [9] and references therein; there, η_{ab} and F correspond to $-\eta_{ab}$, $-2F$). For our purposes here it is sufficient to note that this $\mathcal{R}_\alpha{}^\beta$ obeys

$$i_a \mathcal{R}_\alpha{}^\gamma \Gamma^a{}_\gamma{}^\beta = -\frac{1}{4} e^b R_{b[c_1c_2c_3]} \Gamma^{c_1c_2c_3} + \frac{1}{2} e^a E_{ab} \Gamma^b_\alpha{}^\beta + \frac{i}{36} e^a (\Gamma_a{}^{b_1b_2b_3} + 6\delta_a^{[b_1}\Gamma^{b_2b_3]})_\alpha{}^\beta [{}^*\mathcal{G}_8]_{b_1b_2b_3}$$
$$+ \frac{i}{720} e^a [dF_4]_{b_1\ldots b_5} (\Gamma_a{}^{b_1\ldots b_5} + 10\delta_a^{[b_1}\Gamma^{b_2\ldots b_5]})_\alpha{}^\beta, \tag{12}$$

where $E_{ab}, \mathcal{G}_8$ are the r.h.s's of the Einstein and the gauge field equations as defined in Eqs. (2),(3) and i_a is defined by $i_a e^b = \delta_a^b$ so that i.e., for $\Omega_p = (1/p!) e^{a_p} \wedge \ldots \wedge e^{a_1} \Omega_{a_1\ldots a_p}$,

$$i_a \Omega_p = \frac{1}{(p-1)!} e^{a_p} \wedge \ldots \wedge e^{a_2} \Omega_{aa_2\ldots a_p}; \tag{13}$$

in particular $i_a \mathcal{R}_\alpha{}^\beta = e^b \mathcal{R}_{ab\alpha}{}^\beta$. The equality (12) implies that the set of the *free bosonic* [Eq. (7)] equations for the CJS supergravity, Eqs. (2), (3), (4), is equivalent to the following simple equation for the generalized curvature of Eq. (11), $e^b \mathcal{R}_{ab\alpha}{}^\gamma \Gamma^a{}_\gamma{}^\beta = 0$ or

$$i_a \mathcal{R}_\alpha{}^\gamma \Gamma^a{}_\gamma{}^\beta = 0, \tag{14}$$

since the r.h.s. of Eq. (12) is zero on account of the equations of motion (2), (3), the Bianchi identity (4) and that $R_{b[c_1c_2c_3]} = 0$ [since $R_{abcd} = R_{cdab}$ and $DT^a = 0$ by Eq. (6)].

II. KILLING SPINORS, PREONS AND GENERALIZED *G*-FRAME

A. Killing spinors and BPS states

A BPS state $|BPS,k\rangle$ described by a solitonic solution preserving k supersymmetries is, schematically, one satisfying

$$\epsilon_J{}^\alpha Q_\alpha |BPS,k\rangle = 0, \quad J = 1,\ldots,k, \quad k \leqslant 31, \tag{15}$$

where Q_α are the supersymmetry generators obeying

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}, \quad [Q_\alpha, P_{\beta\gamma}] = 0 \tag{16}$$

$$\alpha,\beta,\gamma = 1,2,\ldots,32,$$

so that $P_{\alpha\beta} = P_{\beta\alpha}$. The generalized momentum $P_{\alpha\beta}$ may be decomposed e.g. in the basis of $D=11$ $Spin(1,10)$ (32×32) Dirac matrices,

$$P_{\alpha\beta}=P_{\mu}\Gamma^{\mu}_{\alpha\beta}+Z_{\mu\nu}\Gamma^{\mu\nu}_{\alpha\beta}+Z_{\mu_1\dots\mu_5}\Gamma^{\mu_1\dots\mu_5}_{\alpha\beta}, \tag{17}$$

giving then rise to the standard $D=11$ momentum P_μ and to the tensorial "central" charge generators $Z_{\mu\nu}, Z_{\mu_1\dots\mu_5}$ of the M-algebra $\{Q_\alpha, Q_\beta\}=P_{\alpha\beta}$. These generators may be identified as topological charges [34] related to the M2- and M5-branes (as well as to the M9-brane and KK7-brane of M theory [35]; see [29] for the role of the worldvolume gauge fields of the M5-brane, and [32,36] for D-branes).

B. BPS preons as constituents

A BPS preon [1] state $|BPS,31\rangle\equiv|\lambda\rangle$ is a state characterized by a single bosonic spinor λ_α,

$$P_{\alpha\beta}|\lambda\rangle=\lambda_\alpha\lambda_\beta|\lambda\rangle \tag{18}$$

(hence the notation $|\lambda\rangle$) or as a state preserving all supersymmetries but one (hence the notation $|BPS,31\rangle$) [1]. The bosonic spinor parameters $\epsilon_I{}^\alpha$ corresponding to the supersymmetries preserved by a BPS preon $|\lambda\rangle$,

$$\epsilon_I{}^\alpha Q_\alpha|\lambda\rangle=0, \quad I=1,\dots,31, \tag{19}$$

are "orthogonal" to the bosonic spinor λ_α that labels it,

$$\epsilon_I{}^\alpha\lambda_\alpha=0, \quad I=1,\dots,31 \tag{20}$$

(see below). Identifying $\epsilon_I{}^\alpha$ with 31 Killing spinors satisfying Eq. (8), one finds that Eq. (20) expresses the fact that these Killing spinors are orthogonal to the *single bosonic spinor* λ_α *characterizing a hypothetical BPS preonic solution.*

For BPS states $|BPS,k\rangle$ preserving $k\leqslant 30$ supersymmetries (15) one may introduce $n=32-k$ bosonic spinors $\lambda_\alpha{}^r$ corresponding to the n broken supersymmetries. They may be treated as characterizing n BPS preons $|\lambda^r\rangle$, $r=1,\dots,n$, out of which the corresponding $k/32$ BPS state is composed [1].

To make this transparent, let us consider the eigenvalue matrix $p_{\alpha\beta}$ of the generalized momentum operator $P_{\alpha\beta}$ corresponding to the BPS state $|BPS,k\rangle$ (which is usually assumed to be an eigenstate of the generalized momentum, i.e. having definite energy and definite brane charges), $P_{\alpha\beta}|BPS,k\rangle=p^{(k)}_{\alpha\beta}|BPS,k\rangle$. This is a symmetric matrix of $rank(p^{(k)}_{\alpha\beta})=n=32-k$ (a relation justified below). Hence $p^{(k)}_{\alpha\beta}$ may be diagonalized by a general linear transformation i.e.,

$$p^{(k)}_{\alpha\beta}=g_\alpha{}^{(\gamma)}p_{(\gamma)(\delta)}g_\beta{}^{(\delta)} \tag{21}$$

with $p_{(\gamma)(\delta)}=\mathrm{diag}(\dots)$. Moreover, as $g_\alpha{}^{(\gamma)}\in GL(n,\mathbb{R})$, this diagonal matrix can be put in the form

$$p_{(\gamma)(\delta)}=\mathrm{diag}(\underbrace{1,\dots,1,-1,\dots,-1}_{n=32-k},\underbrace{0,\dots,0}_{k}), \tag{22}$$

where the number of nonvanishing elements, all $+1$ or -1, is equal to $n=rank(p^{(k)}_{\alpha\beta})$. However, the usual assumptions for the supersymmetric quantum mechanics describing BPS states do not allow for negative eigenvalues of $P_{\alpha\beta}=\{Q_\alpha,Q_\beta\}$ [$p_{11}=-1$, e.g., would imply $(Q_1)^2|BPS,k\rangle=-|BPS,k\rangle$, contradicting positivity]. Thus, only positive eigenvalues are allowed and

$$p_{(\gamma)(\delta)}=\mathrm{diag}(\underbrace{1,\dots,1}_{n=32-k},\underbrace{0,\dots,0}_{k}). \tag{23}$$

Substituting Eq. (23) into Eq. (21), one arrives at

$$p^{(k)}_{\alpha\beta}=g_\alpha{}^{(\gamma)}\begin{pmatrix}1 & & & & & \\ & \ddots & & & 0 & \\ & & 1 & & & \\ & & & 0 & & \\ & 0 & & & \ddots & \\ & & & & & 0\end{pmatrix}_{(\gamma)(\delta)} g_\beta{}^{(\delta)}, \tag{24}$$

or, equivalently, denoting $g_\alpha{}^1=\lambda_\alpha{}^1, \dots, g_\alpha{}^n=\lambda_\alpha{}^n$,

$$P_{\alpha\beta}|BPS,k\rangle=\sum_{r=1}^{n=32-k}\lambda_\alpha{}^r\lambda_\beta{}^r|BPS,k\rangle \equiv(\lambda_\alpha{}^1\lambda_\beta{}^1+\dots+\lambda_\alpha{}^n\lambda_\beta{}^n)|BPS,k\rangle. \tag{25}$$

One sees using Eq. (16) that, if the preserved supersymmetries correspond to the generators $\epsilon_J{}^\alpha Q_\alpha$, $J=1,\dots,k$, Eq. (15), then

$$\sum_{r=1}^{n=32-k}\epsilon_{(J}{}^\alpha\lambda_\alpha{}^r\epsilon_{K)}{}^\beta\lambda_\beta{}^r=0, \tag{26}$$

which immediately implies

$$\epsilon_J{}^\alpha\lambda_\alpha{}^r=0, \tag{27}$$

$$J=1,\dots,k, \quad r=1,\dots,n,$$

making clear that $k=32-n$. This explains the relation $n=32-k$ between the number of preons $n=rank(p_{\alpha\beta})$ and the number of preserved supersymmetries k. The $k=31$ case $(n=1)$ is Eq. (20) for BPS preons.

Equation (25) may be looked at as a manifestation of the *composite structure* of the $\nu=k/32$ BPS state $|BPS,k\rangle$,

$$|BPS,k\rangle=|\lambda^1\rangle\otimes\dots\otimes|\lambda^{(32-k)}\rangle, \tag{28}$$

where $|\lambda^1\rangle, \dots, |\lambda^n\rangle$ are BPS elementary, preonic states characterized by the spinors $\lambda_\alpha{}^1,\dots,\lambda_\alpha{}^n$, respectively.

C. Moving *G*-frame

When a BPS state $|k\rangle$ is realized as a solitonic solution of supergravity, it is characterized by k Killing spinors $\epsilon_J{}^{\beta}(x)$ or by the $n=32-k$ bosonic spinors $\lambda_\alpha{}^r(x)$ associated with the n BPS preonic components of the state $|BPS,k\rangle$. The Killing spinors and the preonic spinors are orthogonal,

$$\epsilon_J{}^{\alpha}\lambda_\alpha{}^r=0,$$
$$J=1,\dots,k, \quad r=1,\dots,n=32-k \tag{29}$$

and, hence, may be completed to obtain bases in the spaces of spinors with upper and with lower indices by introducing $n=32-k$ spinors $w_r{}^{\alpha}$ and k spinors $u_\alpha{}^L$ satisfying

$$w_s{}^{\alpha}\lambda_\alpha{}^r=\delta_s^r, \quad w_s{}^{\alpha}u_\alpha{}^J=0, \quad \epsilon_J{}^{\alpha}u_\alpha{}^K=\delta_J{}^K. \tag{30}$$

Either of these two dual bases defines a *generalized moving G-frame* described by the nondegenerate matrices

$$g_\alpha{}^{(\beta)}=(\lambda_\alpha{}^s,u_\alpha{}^J), \quad g^{-1}{}_{(\beta)}{}^{\alpha}=\begin{pmatrix} w_s{}^{\alpha}\\ \epsilon_J{}^{\alpha}\end{pmatrix}, \tag{31}$$

where $(\alpha)=(s,J)=(1,\dots,32-k;J=1,\dots,k)$. Indeed, $g^{-1}{}_{(\beta)}{}^{\gamma}g_\gamma{}^{(\alpha)}=\delta_{(\beta)}{}^{(\alpha)}$ is equivalent to Eqs. (30) and (29), while

$$\delta_\alpha{}^{\beta}=g_\alpha{}^{(\gamma)}g^{-1}{}_{(\gamma)}{}^{\beta}\equiv\lambda_\alpha{}^r w_r{}^{\beta}+u_\alpha{}^J\epsilon_J{}^{\beta} \tag{32}$$

provides the unity decomposition or completeness relation in terms of these dual bases.

One may consider the dual basis $g^{-1}{}_{(\beta)}{}^{\alpha}$ to be constructed from the bosonic spinors in $g_\alpha{}^{(\beta)}$ by solving Eq. (32) or $g^{-1}g=I$ [Eqs. (30) and (29)]. Alternatively, one may think of $w_r{}^{\alpha}$ and $u_\alpha{}^J$ as being constructed from $\epsilon_J{}^{\alpha}$ and $\lambda_\alpha{}^r$ through a solution of the same constraints. In this sense *the generalized moving G-frame* (31) *is constructed from k Killing spinors* $\epsilon_J{}^{\alpha}$ *characterizing the supersymmetries preserved by a BPS state (realized as a solution of the supergravity equations) and from the* $n=32-k$ *bosonic spinors* $\lambda_\alpha{}^r$ *characterizing the BPS preons from which the BPS state is composed.*

Although many of the considerations below are general, we shall be mainly interested here in the cases $G=SL(32,\mathbb{R})$ and $G=Sp(32,\mathbb{R})$.

Clearly, in $D=11$, the charge conjugation matrix $C^{\alpha\beta}=-C^{\beta\alpha}$ allows us to express the dual basis g^{-1} in terms of the original one g or *vice versa*. In particular, in the preonic $k=31$ case one finds that, as $\lambda_\alpha C^{\alpha\beta}\lambda_\beta\equiv 0$, $\lambda^{\alpha}=C^{\alpha\beta}\lambda_\beta$ has to be expressed as $\lambda^{\alpha}=\lambda^I\epsilon_I{}^{\alpha}$, for some coefficients λ^I, $I=1,\dots,31$. In general (as e.g., in CJS supergravity with nonvanishing F_4), the charge conjugation matrix is not "covariantly constant," $\mathcal{D}C^{\alpha\beta}=-2\omega^{[\alpha\beta]}\neq 0$. This relates the coefficients $\lambda^I=\lambda^{\alpha}u_\alpha{}^I$ to the antisymmetric (nonsymplectic) part of the generalized connection, $\omega^{[\alpha\beta]}=C^{[\alpha\gamma}\omega_\gamma{}^{\beta]}$ by $d\lambda^I-A\lambda^I=2\lambda_\alpha\omega^{[\alpha\beta]}u_\beta{}^I$ [43]. In $OSp(1|32)$-related models $\omega^{[\alpha\beta]}=0$ and $A=0$, hence λ^I is constant and we may set $\lambda^I=\delta_{31}^I$ using the *global* transformations of $GL(31,\mathbb{R})$, which is a rigid symmetry of the system of Killing spinors. This allows us to identify λ^{α} itself with one of the Killing spinors

$$G=Sp(32,\mathbb{R}): \quad \epsilon_I{}^{\alpha}=(\epsilon_i{}^{\alpha},\lambda^{\alpha}), \quad \lambda^{\alpha}:=C^{\alpha\beta}\lambda_\beta, \tag{33}$$
$$i=1,\dots,30.$$

Without specifying a solution of the constraints (32) (or $g^{-1}g=I$), the moving frame possesses a $G=GL(32,\mathbb{R})$ symmetry. One may impose as additional constraints $\det(g)=1$ or $\det(g^{-1})=1$ reducing G to $SL(32,\mathbb{R})$,

$$G=SL(32,\mathbb{R}): \ \det(g_\beta^{(\alpha)})=1=\det(g^{-1}_{(\alpha)}{}^{\beta}). \tag{34}$$

For instance, in the preonic case $k=31$ this would imply $w^{\alpha}=1/(31)!\,\varepsilon^{\alpha\beta_1\cdots\beta_{31}}u_{\beta_1}{}^1\dots u_{\beta_{31}}{}^{31}$. Such a frame is most convenient to study the bosonic solutions of CJS supergravity.

III. GEOMETRY OF BPS PREONS AND $\nu=k/32$-SUPERSYMMETRIC SOLUTIONS

A. Generalized connection and moving *G*-frame

The Killing equation (8) for a $\nu=k/32$ supersymmetric solution,

$$\mathcal{D}\epsilon_J{}^{\alpha}=d\epsilon_J{}^{\alpha}-\epsilon_J{}^{\beta}\omega_\beta{}^{\alpha}=0, \quad J=1,\dots,k, \tag{35}$$

implies the following equations for the other components of the moving G-frame:

$$\mathcal{D}\lambda_\alpha{}^r:=d\lambda_\alpha{}^r+\omega_\alpha{}^{\beta}\lambda_\beta{}^r=\lambda_\alpha{}^sA_s{}^r, \tag{36}$$

$$\mathcal{D}u_\alpha{}^J:=du_\alpha{}^J+\omega_\alpha{}^{\beta}u_\beta{}^J=\lambda_\alpha{}^rB_r^J, \tag{37}$$

$$\mathcal{D}w_r{}^{\alpha}:=dw_r{}^{\alpha}-w_r{}^{\beta}\omega_\beta{}^{\alpha}=-A_r{}^sw_s{}^{\alpha}-B_r^J\epsilon_J{}^{\alpha}, \tag{38}$$

$$\alpha,\beta=1,\dots,32;$$
$$J=1,\dots,k; \quad r,s=1,\dots,(32-k),$$

where $A_s{}^r$ and $B_r{}^I$ are $(32-k)\times(32-k)$ and $(32-k)\times k$ arbitrary one-form matrices.

To obtain Eqs. (36), (37), (38) one can take firstly the derivative $\mathcal{D}$ of the orthogonality relations (29), (30). After using Eq. (35), this results in

$$\epsilon_I{}^{\alpha}\mathcal{D}\lambda_\alpha{}^r=0, \quad \epsilon_I{}^{\alpha}\mathcal{D}u_\alpha{}^J=0, \tag{39}$$

$$w_s{}^{\alpha}\mathcal{D}\lambda_\alpha{}^r=-\mathcal{D}w_s{}^{\alpha}\lambda_\alpha{}^r, \quad w_s{}^{\alpha}\mathcal{D}u_\alpha{}^J=-\mathcal{D}w_s{}^{\alpha}u_\alpha{}^J. \tag{40}$$

Then, for instance, to derive Eq. (36), one uses the unity decomposition (32) to express $\mathcal{D}\lambda_\alpha{}^r$ through the contractions $w_s{}^{\alpha}\mathcal{D}\lambda_\alpha{}^r$ and $\epsilon_I{}^{\alpha}\mathcal{D}\lambda_\alpha{}^r$: $\mathcal{D}\lambda_\alpha{}^r\equiv\lambda_\alpha{}^sw_s{}^{\beta}\mathcal{D}\lambda_\beta{}^r+u_\alpha{}^I\epsilon_I{}^{\beta}\mathcal{D}\lambda_\beta{}^r$. The second term vanishes due to Eq. (39), while the first one is not restricted by the consequences of the Killing spinor equations and may be written as in Eq. (36) in terms of an arbitrary form $A_s{}^r\equiv w_s{}^{\alpha}\mathcal{D}\lambda_\alpha{}^r$.

Notice that, using the unity decomposition (32), one may also solve formally Eqs. (35), (36), (37), (38) with respect to the generalized connection $\omega_\alpha{}^\beta$,

$$\omega_\alpha{}^\beta = A_r{}^s \lambda_\alpha{}^r w_s{}^\beta + B_r{}^J \lambda_\alpha{}^r \epsilon_J{}^\beta - (dgg^{-1})_\alpha{}^\beta, \tag{41}$$

where $g_\alpha^{(\beta)}$ and $g_{(\beta)}^{-1\,\alpha}$ are defined in Eq. (31) and, hence,

$$(dgg^{-1})_\alpha{}^\beta = d\lambda_\alpha{}^r w_r{}^\beta + du_\alpha{}^I \epsilon_I{}^\beta. \tag{42}$$

For a BPS $\nu = 31/32$, preonic configuration Eqs. (36), (37),(38) read

$$\mathcal{D}\lambda_\alpha := d\lambda_\alpha + \omega_\alpha{}^\beta \lambda_\beta = A\lambda_\alpha, \tag{43}$$

$$\mathcal{D}u_\alpha{}^I := du_\alpha{}^I + \omega_\alpha{}^\beta u_\beta{}^I = B^I\lambda_\alpha, \tag{44}$$

$$\mathcal{D}w^\alpha := dw^\alpha - w^\beta \omega_\beta{}^\alpha = -Aw^\alpha - B^I \epsilon_I{}^\alpha \tag{45}$$

and contain $1+31=32$ arbitrary one-forms A and B^I.

For $G = SL(32,\mathbb{R})$ one may choose $\det(g)=1$, Eq. (34), which implies $\mathrm{tr}(dgg^{-1}) := (dgg^{-1})_\alpha{}^\alpha = 0$. Then the $sl(32,\mathbb{R})$-valued generalized connection $\omega_\alpha{}^\beta$ ($\omega_\alpha{}^\alpha = 0$) allowing for a $\nu = k/32$ supersymmetric configuration is determined by Eq. (41) with $A_r{}^r = 0$,

$$G = SL(32,\mathbb{R}): \quad A_r{}^r = 0. \tag{46}$$

In particular, the $sl(32,\mathbb{R})$-valued generalized connection allowing for a BPS preonic, $\nu = 31/32$, configuration, should have the form

$$G = SL(32,\mathbb{R}), \quad \nu = 31/32:$$

$$\omega_\alpha{}^\beta = B^I \lambda_\alpha \epsilon_I{}^\beta - (dgg^{-1})_\alpha{}^\beta \tag{47}$$

in terms of 31 arbitrary one-forms B^I, $I = 1, \ldots, 31$.

Assuming a definite form of the generalized connections, e.g. the one characterizing bosonic solutions of the "free" CJS supergravity equations (9), one finds that Eqs. (41) become differential equations for k Killing spinors $\epsilon_J{}^\alpha$ *and* $n = 32-k$ *BPS preonic spinors* $\lambda_\alpha{}^r$ once $(dgg^{-1}) = d\lambda_\alpha{}^r w_r{}^\beta - u_\alpha{}^I d\epsilon_I{}^\beta$ [Eq. (42)] is taken into account.

On the other hand, one might reverse the argument and ask for the structure of a theory allowing for $\nu = k/32$ supersymmetric solutions. This question is especially interesting for the case of BPS preonic and $\nu = 30/32$ solutions as, for the moment, such solutions are unknown in the standard $D = 11$ CJS and $D = 10$ type II supergravities.

B. Generalized holonomy for BPS preons and for $\nu = k/32$ supersymmetric solutions

The simplest application of the moving G-frame construction above is to find an explicit form for the general solution of Eq. (10), which expresses the necessary conditions for the existence of k Killing spinors. As the Killing spinor equation (35) implies Eqs. (36),(37), one may solve instead the self-consistency conditions for these equations,

$$\mathcal{D}\mathcal{D}\lambda_\alpha{}^r = \mathcal{R}_\alpha{}^\beta \lambda_\beta{}^r = \lambda_\alpha{}^s (dA - A \wedge A)_s{}^r, \tag{48}$$

$$\mathcal{D}\mathcal{D}u_\alpha{}^I = \mathcal{R}_\alpha{}^\beta u_\beta{}^I = \lambda_\alpha{}^r (dB_r^I + B_s^I \wedge A_r{}^s). \tag{49}$$

Using the unity decomposition (32), which implies $\mathcal{R}_\alpha{}^\beta = \mathcal{R}_\alpha{}^\gamma \lambda_\gamma{}^r w_r{}^\beta + \mathcal{R}_\alpha{}^\gamma u_\gamma{}^I \epsilon_I{}^\beta$, one finds the following expression for the generalized curvature:

$$\mathcal{R}_\alpha{}^\beta = G_r{}^s \lambda_\alpha{}^r w_s{}^\beta + \nabla B_r^I \lambda_\alpha{}^r \epsilon_I{}^\beta, \tag{50}$$

where

$$G_r{}^s := (dA - A \wedge A)_r{}^s, \tag{51}$$

$$\nabla B_r^I := dB_r^I - A_r{}^s \wedge B_s^I. \tag{52}$$

For $k = 31$, corresponding to the case of a BPS preon, Eq. (50) simplifies to

$$\mathcal{R}_\alpha{}^\beta = dA\lambda_\alpha w^\beta + (dB^I + B^I \wedge A)\lambda_\alpha \epsilon_I{}^\beta. \tag{53}$$

Equations (50) and (53) imply $\mathcal{R}_\alpha{}^\beta = \lambda_\alpha{}^r (\cdots)_r{}^\beta$ and, thus, clearly solve Eq. (10), $\epsilon_I{}^\beta \mathcal{R}_\beta{}^\alpha = 0$.

The conditions $G \subset SL(32,\mathbb{R})$ and hence $H \subset SL(32,\mathbb{R})$, $\mathcal{R}_\alpha{}^\alpha = 0$ (which is always the case for bosonic solutions of "free" CJS and type IIB supergravities [7,11]), imply $A_r{}^r = 0$ in Eq. (50) [see Eq. (46)], while for $k = 31$ Eq. (53) simplifies to

$$H \subset SL(32,\mathbb{R}), \quad k = 31: \quad \mathcal{R}_\alpha{}^\beta = dB^I \lambda_\alpha \epsilon_I{}^\beta. \tag{54}$$

Finally, for $G \subset Sp(32,\mathbb{R})$ $\omega^{[\alpha\beta]} = 0$, the holonomy group $H \subset Sp(32,\mathbb{R})$, $\mathcal{R}^{\alpha\beta} := C^{\alpha\gamma}\mathcal{R}_\gamma{}^\beta = \mathcal{R}^{(\alpha\beta)}$, and Eq. (54) reduces to

$$H \subset Sp(32,\mathbb{R}), \quad k = 31: \ \mathcal{R}_\alpha{}^\beta = dB\lambda_\alpha \lambda^\beta, \tag{55}$$

where only one arbitrary one-form B appears [to obtain Eq. (55) one has to keep in mind that $\epsilon_I{}^\alpha = (\epsilon_i{}^\alpha, C^{\alpha\beta}\lambda_\beta)$, $I = (i, 31)$, Eq. (33)]. *Equations (54),(55) solve Eq. (10) for preons when* $G = SL(32,\mathbb{R}), Sp(32,\mathbb{R})$, *respectively*.

Equation (50) with $A_r{}^r = 0$ [Eq. (46), and, hence, $(dA - A \wedge A)_r{}^r = 0$] provides an explicit expression for the results of [7,11] on generalized holonomies of k-supersymmetric solutions of $D = 11$ and of $D = 10$ type IIB supergravity, namely $H \subset SL(32-k,\mathbb{R}) \ltimes \mathbb{R}^{k(32-k)}$. For a BPS preon $k = 31$, and $H \subset \mathbb{R}^{31}$ as expressed by Eq. (54). However, our explicit expressions for the $[sl(32-k,\mathbb{R}) \oplus \mathbb{R}^{k(32-k)}]$-valued generalized curvatures $\mathcal{R}_\alpha{}^\beta$, Eqs. (50),(54), given in terms of the Killing spinors $\epsilon_I{}^\beta$ and bosonic spinors $\lambda_\alpha{}^r$ characterizing the BPS preon contents of a $\nu = k/32$ BPS state, may be useful in searching for new supersymmetric solutions, including preonic $\nu = 31/32$ ones. Some steps in this direction are taken in the next section.

IV. ON BPS PREONS IN $D=11$ CJS SUPERGRAVITY AND BEYOND

A. BPS preons in Chern-Simons supergravity

The first observation is that the generalized curvature allowing for a BPS preonic ($k=31$ supersymmetric) configuration for the case of $H\subset SL(32,\mathbb{R})$ holonomy, Eq. (54), is nilpotent,

$$\mathcal{R}_{\alpha}{}^{\gamma}\wedge\mathcal{R}_{\gamma}{}^{\beta}=0 \quad \text{for} \quad H\subset SL(32,\mathbb{R}), \quad k=31. \tag{56}$$

As a result it solves the purely bosonic equations of a Chern-Simons supergravity (see [14,15], although our statement may be related to a more general version of a hypothetical Chern-Simons-like supergravity),

$$\mathcal{R}_{\alpha}{}^{\gamma_1}\wedge\mathcal{R}_{\gamma_1}{}^{\gamma_2}\wedge\mathcal{R}_{\gamma_2}{}^{\gamma_3}\wedge\mathcal{R}_{\gamma_3}{}^{\gamma_4}\wedge\mathcal{R}_{\gamma_4}{}^{\beta}=0. \tag{57}$$

Clearly, the same is true for $H\subset Sp(32,\mathbb{R})\subset SL(32,\mathbb{R})$, where $\mathcal{R}$ is given by Eq. (55). Thus, there exist BPS preonic solutions in CS supergravity theories, including $OSp(1|32)$-type ones.

Note that Eq. (56) follows in general for a preonic configuration only. For the configurations preserving $k\leqslant 30$ of the 32 supersymmetries, the bosonic equations of a CS supergravity, Eqs. (57) reduce to [see Eqs. (51),(52)]

$$G_s{}^{s_2}\wedge G_{s_2}{}^{s_3}\wedge G_{s_3}{}^{s_4}\wedge G_{s_4}{}^{s_5}\wedge G_{s_5}{}^{r}=0,$$
$$G_s{}^{s_2}\wedge G_{s_2}{}^{s_3}\wedge G_{s_3}{}^{s_4}\wedge G_{s_4}{}^{r}\wedge \nabla B_r{}^{I}=0, \tag{58}$$

which are not satisfied identically for $G_r{}^r=0$. Equations (58) are satisfied e.g., by configurations with $G_s{}^r=0$, for which the generalized holonomy group is reduced down to $H\subset\mathbb{R}^{\otimes k(32-k)}$, $\mathcal{R}_{\beta}{}^{\alpha}=\nabla B_r^I\lambda_{\beta}{}^{r}\epsilon_I{}^{\alpha}$.

Thus, *only* the preonic, $\nu=31/32$, configurations *always* solve the Chern-Simons supergravity equations (57).

B. Searching for preonic solutions of "free" bosonic CJS equations

We go back now to the question of whether BPS $\nu=31/32$ (preonic) solutions exist for the standard CJS supergravity [5]. As it was noted in Sec. I A, this problem can be addressed step by step, beginning by studying the existence of preonic solutions for "free" bosonic CJS equations. To this aim it is useful to observe [9] that these equations may be collected in a compact expression for the generalized curvature, Eq. (14). The generalized curvature of a BPS preonic configuration satisfies Eq. (54), and thus it solves the "free" bosonic CJS supergravity equations (14) if

$$i_a dB^I\epsilon_I{}^{\alpha}\Gamma^a{}_{\alpha}{}^{\beta}=0. \tag{59}$$

Actually, Eq. (54) in Eq. (14) gives $\lambda_\alpha i_a dB^I\epsilon_I{}^{\gamma}\Gamma^a{}_{\gamma}{}^{\beta}=0$. However, as $\lambda_\alpha\neq 0$, this is equivalent to Eq. (59).

Equation (59) contains a summed $I=1,\dots,31$ index and, as a result, it is not easy to handle. It would be much easier to deal with the expression $\Gamma^a{}_{\alpha}{}^{\gamma}i_a\mathcal{R}_{\gamma}{}^{\beta}$ which, with Eq. (54) is equal to $\Gamma^a{}_{\alpha}{}^{\gamma}\lambda_\gamma i_a dB^J\epsilon_J{}^{\beta}$. Indeed, $(\Gamma^a\lambda)_\alpha i_a dB^J\epsilon_J{}^{\beta}=0$, for instance, would imply $(\Gamma^a\lambda)_\alpha i_a dB^J=0$ which may be shown to have only trivial solutions. However, $\Gamma^a{}_{\alpha}{}^{\gamma}i_a\mathcal{R}_{\gamma}{}^{\beta}\neq 0$ in general *for a solution of the "free" bosonic CJS equations* [Eq. (14)],

$$\Gamma^a{}_{\alpha}{}^{\gamma}i_a\mathcal{R}_{\gamma}{}^{\beta}=-\frac{i}{12}[D\hat{F}_{\alpha}{}^{\beta}+\mathcal{O}(FF)], \tag{60}$$

where $D=e^aD_a$ is the Lorentz covariant derivative [not to be confused with $\mathcal{D}$ defined in Eqs. (8),(9)],

$$\hat{F}_{\alpha}{}^{\beta}=F_{a_1a_2a_3a_4}(\Gamma^{a_1a_2a_3a_4})_{\alpha}{}^{\beta}, \tag{61}$$

and $\mathcal{O}(FF)$ denotes the terms of second order in $F_{c_1c_2c_3c_4}$,

$$\mathcal{O}(FF)=\frac{1}{(3!)^2 4!}e^a(\Gamma_a{}^{b_1b_2b_3}+2\delta_a^{[b_1}\Gamma^{b_2b_3]})\epsilon_{b_1b_2b_3[4][4']}F^{[4]}F^{[4']}+\frac{2i}{3}e^a(\Gamma_a{}^{b_1b_2b_3b_4}+3\delta_a^{[b_1}\Gamma^{b_2b_3b_4]})F_{cdb_1b_2}F^{cd}{}_{b_3b_4}$$
$$+\frac{8i}{9}e^aF_{abb_1b_2}F^{b}{}_{b_3b_4b_5}\Gamma^{b_1b_2b_3b_4b_5}. \tag{62}$$

Equation (54) then implies that for a hypothetical preonic solution of the "free" bosonic CJS equations, the gauge field strength F_{abcd} should be nonvanishing (otherwise $dB^J=0$ and $\mathcal{R}_{\alpha}{}^{\beta}=0$, see above) and satisfy

$$\Gamma^a{}_{\alpha}{}^{\gamma}\lambda_\gamma i_a dB^J\epsilon_J{}^{\beta}=-\frac{i}{12}[D\hat{F}_{\alpha}{}^{\beta}+\mathcal{O}(FF)]. \tag{63}$$

Using Eq. (30), Eqs. (63) split into a set of restrictions for F_{abcd},

$$[D\hat{F}+\mathcal{O}(FF)]_{\alpha}{}^{\beta}\lambda_\beta=0, \tag{64}$$

and equations for dB^I,

$$\Gamma^a{}_{\alpha}{}^{\gamma}\lambda_\gamma i_a dB^I=-\frac{i}{12}[D\hat{F}+\mathcal{O}(FF)]_{\alpha}{}^{\beta}u_{\beta}{}^{I}. \tag{65}$$

Equation (63) or, equivalently, Eqs. (64),(65) are the equations to be satisfied by a CJS preonic configuration. Note that if a nontrivial solution of the above equations with some

$F_{abcd}\neq 0$ and some $dB^I\neq 0$ is found, one would have then to check in particular that such a solution satisfies $ddB^I=0$ and $D_{[e}F_{abcd]}=0$.

On the other hand, if the general solution of the above equation turned out to be trivial, $dB^I=0$, this would imply $\mathcal{R}_\alpha{}^\beta=0$ and, thus, a trivial generalized holonomy group, $H=1$. However, this is the necessary condition for fully supersymmetric, $k=32$, solutions [8]. Hence a trivial solution for Eqs. (64),(65) would indicate that a solution preserving 31 supersymmetries possesses all 32 (thus corresponding to a fully supersymmetric vacuum) and, hence, that there are no preonic, $\nu=31/32$ solutions of the *free bosonic* CJS supergravity equations (2), (3), (4), (6), and (7).

If this happened to be the case, one would have to study the existence of preonic solutions for the CJS supergravity equations with nontrivial right hand sides. These could be produced by corrections of higher order in curvature [20,21,23] (a counterpart of the string α' corrections in $D=10$ [22]) and by the presence of sources (from some exotic p-branes).

C. On brane solutions and worldvolume actions

As far as supersymmetric p-brane solutions of supergravity equations are concerned, one notices that for most of the known $\nu=1/2$ supersymmetric solutions ($\nu=16/32$ in $D=11$ and $D=10$ type II cases) there also exist worldvolume actions in the corresponding ($D=11$ or $D=10$ type II) superspaces possessing 16 κ-symmetries, exactly the number of supersymmetries preserved by the supergravity solitonic solutions. The *κ-symmetry-preserved supersymmetry* correspondence was further discussed and extended for the case of $\nu<1/2$ multi-brane solutions in [28,29].

In this perspective one may expect that if preonic $\nu=31/32$ supersymmetric solutions of the CJS equations with a source do exist, a worldvolume action possessing 31 κ-symmetries should also exist in a curved $D=11$ superspace. For a moment no such actions are known in the *standard* $D=11$ superspace, but they are known in a superspace enlarged with additional tensorial ‘‘central’’ charge coordinates [30,31]. One might expect that the role of these additional tensorial coordinates could be taken over by the tensorial fields of $D=11$ supergravity. But this would imply that the corresponding action does not exist in the flat standard $D=11$ superspace as it would require a contribution from the above additional field degrees of freedom (replacing the tensorial coordinate ones in the spirit of [32]). This lack of a clear flat *standard* superspace limit hampers the way towards a hypothetical worldvolume action for a BPS preon in the usual curved $D=11$ superspace.

Nevertheless, a shortcut in the search for such an action may be provided by the recent observation [17] that the superfield description of the dynamical supergravity-superbrane interacting system, described by the sum of the *superfield* action for supergravity (still unknown for $D=10,11$) and the super-p-brane action, is gauge equivalent to the much simpler dynamical system described by the sum of the spacetime, component action for supergravity and the action *for the purely bosonic limit* of the super-p-brane. This bosonic p-brane action carries the memory of being the bosonic limit of a super-p-brane by still possessing 1/2 of the spacetime local supersymmetries [16]; this preservation of local supersymmetry reflects the κ-symmetry of the original super-p-brane action.

Thus the κ-symmetric worldvolume actions for super-p-branes have a clear spacetime counterpart: the purely bosonic actions in spacetime possessing a part of local spacetime supersymmetry of a ‘‘free’’ supergravity theory.

This fact, although explicitly discussed for the standard, $\nu=1/2$ superbranes in [17], is general since it follows from symmetry considerations only and thus it applies to any superbrane, including a hypothetical preonic one. The number of supersymmetries possessed by this bosonic brane action coincides with the number of κ-symmetries of the parent super-p-brane action. Moreover, these supersymmetries are extracted by a projector which may be identified with the bosonic limit of the κ-symmetry projector for the superbrane. With this guideline in mind one may simplify, in a first stage, the search for a worldvolume action for a BPS preon in standard supergravity (or in a model minimally extending the standard supergravity) by discussing the bosonic limit that such a hypothetical action should have.

D. BPS preons in D'Auria-Fré supergravity

Let us consider a symmetric spin-tensor one-form $e^{\alpha\beta}=e^{\beta\alpha}=dx^\mu e_\mu^{\alpha\beta}(x)$ transforming under local supersymmetry by

$$\delta_\varepsilon e^{\alpha\beta}=-2i\psi^{(\alpha}\varepsilon^{\beta)}, \tag{66}$$

where ψ^α is a fermionic one-form,

$$\psi^\alpha=dx^\mu\psi_\mu^\alpha(x), \tag{67}$$

which we may identify with the gravitino. Let us consider for simplicity the worldline action (cf. [30])

$$\begin{aligned} S&=\int_{W^1}\lambda_\alpha(\tau)\lambda_\beta(\tau)\hat{e}^{\alpha\beta}\\ &=\int d\tau\lambda_\alpha(\tau)\lambda_\beta(\tau)e_\mu^{\alpha\beta}(\hat{x}(\tau))\partial_\tau\hat{x}^\mu(\tau), \end{aligned}\tag{68}$$

where τ parametrizes the worldline W^1 in $D=11$ spacetime, $\hat{e}^{\alpha\beta}:=d\tau\partial_\tau\hat{x}^\mu(\tau)e_\mu^{\alpha\beta}(\hat{x}(\tau))$ and $\lambda_\alpha(\tau)$ is an auxiliary spinor field on the worldline W^1. The extended ($p\geqslant 1$) object counterpart of this worldline action is the following action for tensionless p-branes (cf. [31]):

$$\begin{aligned} S_{p+1}&=\int_{W^{p+1}}\lambda_\alpha\lambda_\beta\hat{\rho}\wedge\hat{e}^{\alpha\beta}\\ &=\int_{W^{p+1}}d^{p+1}\xi\rho^k\lambda_\alpha\lambda_\beta\hat{e}_\mu^{\alpha\beta}\partial_k\hat{x}^\mu, \end{aligned}\tag{69}$$

where $\hat{\rho}(\xi)$ is a p-form auxiliary field, and $\rho^k(\xi)$ is the worldvolume vector density (see [38]) related to $\hat{\rho}(\xi)$ by $\hat{\rho}(\xi)=(1/p!)d\xi^{j_p}\wedge\ldots\wedge d\xi^{j_1}\rho_{j_1\ldots j_p}(\xi)=(1/p!)d\xi^{j_p}\wedge\ldots\wedge d\xi^{j_1}\epsilon_{j_1\ldots j_p k}\rho^k(\xi)$.

One easily finds that the action (68) possesses all but one of the local spacetime supersymmetries [44], Eq. (66), 31 for $\alpha,\beta=1,\ldots,32$ corresponding to $D=11$. Indeed, performing a supersymmetric variation δ_ε of Eq. (68) assuming $\delta_\varepsilon\lambda_\alpha(\tau)=0$, one finds

$$\delta_\varepsilon S=-2i\int_{W^1}\hat{\psi}^\alpha\lambda_\alpha(\tau)\hat{\varepsilon}^\beta\lambda_\beta(\tau). \tag{70}$$

Thus, one sees that $\delta_\varepsilon S=0$ for the supersymmetry parameters on W^1 that obey [cf. Eq. (20)]

$$\hat{\varepsilon}^\beta\lambda_\beta(\tau)=0 \quad [\hat{\varepsilon}^\beta:=\varepsilon^\beta(\hat{x}(\tau))]. \tag{71}$$

Clearly Eq. (71) possesses 31 solutions, which may be expressed through worldvolume spinors $\hat{\epsilon}_I{}^\alpha(\tau)$ (the worldline counterparts of the Killing spinors) orthogonal to $\lambda_\alpha(\tau)$, $\hat{\epsilon}_I{}^\alpha(\tau)\lambda_\alpha(\tau)=0$, as

$$\hat{\varepsilon}^\beta=\varepsilon^I(\tau)\hat{\epsilon}_I{}^\beta, \quad I=1,\ldots,31, \tag{72}$$

for some arbitrary $\varepsilon^I(\tau)$. The same is true for the tensionless p-branes described by the action (69).

Thus, the actions (68), (69) possess 31 of the 32 local spacetime supersymmetries (66) and, in light of the discussion of the previous section, can be considered as the spacetime counterparts of a superspace BPS-preonic action (hypothetical in the standard superspace but known [2,30,31] in flat maximally enlarged or tensorial superspaces).

The question that remains to be settled is the meaning of the symmetric spin-tensor one-form $e^{\alpha\beta}$ with the local supersymmetry transformation rule (66) in $D=11$ supergravity. The contraction of $e^{\alpha\beta}$ with Γ_a,

$$e^a=e^{\alpha\beta}\Gamma^a_{\alpha\beta}, \tag{73}$$

may be identified with the $D=11$ vielbein. Decomposing $e^{\alpha\beta}$ in the basis of the $D=11$ $Spin(1,10)$ gamma-matrices,

$$\begin{aligned} e^{\alpha\beta}&=e^{\beta\alpha} \\ &=\frac{1}{32}e^a\Gamma_a^{\alpha\beta}-\frac{1}{2!32}B_1^{ab}\Gamma_{ab}^{\alpha\beta} \\ &\quad+\frac{1}{5!32}B_1^{a_1\ldots a_5}\Gamma^{\alpha\beta}_{a_1\ldots a_5}, \end{aligned} \tag{74}$$

one finds that $e^{\alpha\beta}$ also contains the antisymmetric tensor one-forms $B_1^{ab}(x)=dx^\mu B_\mu^{ab}(x)$ and $B_1^{a_1\ldots a_5}(x)=dx^\mu B_\mu^{a_1\ldots a_5}(x)$. Such fields, moreover, with exactly the same supersymmetry transformation rules, are involved in the $D=11$ supergravity model by D'Auria and Fré [18].

Thus the action (68) can be teated as a worldline action for a BPS preon in the background of the D'Auria-Fré, $OSp(1|32)$-related "gauge" supergravity model. This might be regarded as an indirect indication of the existence of BPS preonic $\nu=31/32$ solutions in D'Auria-Fré $D=11$ supergravity.

The possibility of having preonic actions in the standard CJS supergravity requires additional study.

V. DISCUSSION AND OUTLOOK

In this paper we have studied the role of the BPS preon notion [1] in the analysis of supersymmetric solutions of $D=11$ supergravity. This notion suggests the moving G-frame method, which we propose as a new tool in the search for supersymmetric solutions of $D=11$ and $D=10$ supergravity. We used this method here to make a step towards answering whether the standard CJS supergravity [5] possesses a solution preserving 31 supersymmetries, a solution that would correspond to a BPS preon state. Although this question has not been settled for the CJS supergravity case, we have shown in our framework that preonic, $\nu=31/32$ solutions do exist in a Chern-Simons type $D=11$ supergravity [14].

Although the main search for preonic solutions concerns the "free" bosonic CJS supergravity equations, one should not exclude other possibilities, both inside and outside the CJS standard supergravity framework. When, e.g., super-p-brane sources are included, the Einstein equation (2), and possibly the gauge field equations (3) and even the Bianchi identities (4), acquire r.h.s.'s. In this case [see Eq. (12)], a r.h.s. also appears in Eq. (14) and the situation would have to be reconsidered. Another source of modification of the CJS supergravity equations might be due to "radiative" corrections of higher order in curvature. Such modified equations might also allow for preonic solutions not present in the unmodified ones. If it were found that only the inclusion of these higher-order curvature terms allows for preonic BPS solutions, this would indicate that BPS preons cannot be seen in a classical low energy approximation of M theory and, hence, that they are intrinsically quantum objects.

The special role of BPS preons in the algebraic classification of all the M-theory BPS states [1] allows us to conjecture that they are elementary (quarklike) necessary ingredients of any model providing a more complete description of M theory. In such a framework, if the standard supergravity did not contain $\nu=31/32$ solutions, neither in its "free" form, nor in the presence of a super-p-brane source, this might just indicate the need for a wider framework for an effective description of M theory. Such an approach could include Chern-Simons supergravities [14] and/or the use of larger, extended superspaces (see [32,37] and references therein), in particular with additional tensorial coordinates (also relevant in the description of massless higher-spin theories [2,3]). In this perspective our observation that the BPS preonic configurations do solve the bosonic equations of Chern-Simons supergravity models looks interesting.

Note added. We mention that it might be interesting to look at the role of vectors and higher order tensors that may be constructed from the preonic spinors $\lambda_\alpha{}^r$, in analogy with the use of the Killing vectors $K^a_{IJ}=\epsilon_I\Gamma^a\epsilon_J$ and higher order bilinears $\epsilon_I\Gamma^{a_1\cdots a_s}\epsilon_J$ made in Refs. [9,39–41].

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[42] Notice that the tensor E_{ab} in Eq. (2) is related to the l.h.s. of the true Einstein equation of CJS supergravity, $\tilde{E}_{ab}$, by $E_{ab}=\tilde{E}_{ab}-[1/(D-2)]\eta_{ab}\tilde{E}_c{}^c$.

[43] To see this, one calculates $d\lambda^I=\mathcal{D}\lambda^I=(\mathcal{D}C^{\alpha\beta})\lambda_\beta u_\alpha{}^I+C^{\alpha\beta}(\mathcal{D}\lambda_\beta)u_\alpha{}^I+C^{\alpha\beta}\lambda_\beta\mathcal{D}u_\alpha{}^I$ and uses Eqs. (43),(44) to find $d\lambda^I=A\lambda^I+2\lambda_\alpha\omega^{[\alpha\beta]}u_\beta{}^I$.

[44] Notice that when a brane action is considered in a supergravity *background*, the local spacetime supersymmetry is not a gauge symmetry of that action but rather a transformation of the background; it becomes a gauge symmetry only when a supergravity action is added to the brane one so that supergravity is dynamical.