Plausible upper limit on the number of *e*-foldings

Bin Wang*
Department of Physics, Fudan University, Shanghai 200433, People's Republic of China

Elcio Abdalla†
Instituto De Fisica, Universidade De Sao Paulo, C. P. 66.318, CEP 05315-970, Sao Paulo, Brazil

(Received 21 August 2003; published 17 May 2004)

Based solely on the arguments relating the Friedmann equation and the Cardy formula we derive a bound for the number of e-folds during inflation for a standard Friedmann-Robertson-Walker cosmology as well as a nonstandard four-dimensional cosmology induced by a Randall-Sundrum-type model.

DOI: 10.1103/PhysRevD.69.104014 PACS number(s): 04.70.Dy, 98.80.Cq

Motivated by the well-known example of black hole entropy, an influential holographic principle was put forward, suggesting that microscopic degrees of freedom that build up the gravitational dynamics actually reside on the boundary of space-time [1]. This principle developed into Maldacena's conjecture on AdS conformal field theory (CFT) correspondence [2] and further very important consequences, such as Witten's [3] identification of the entropy, energy, and temperature of CFT at high temperatures with the entropy, mass, and Hawking temperature of the AdS black hole [4].

On the other hand, although the standard model of particle physics has been established as the uncontested theory of all interactions down to distances of 10^{-17} m, there are good reasons to believe that there is new physics arising soon at the experimental level, and this fact is going to appear in a clear fashion in the setup of cosmology, where a large number of quite exciting developments is giving rise to a precision cosmology. Thus cosmology may provide an alternative laboratory for string theory. During the 1980s several authors tried to analyze the kinds of cosmology arising from string inspired models, which are essentially general relativity in higher dimensions together with scalar and tensor fields. When we introduce the brane concept also, a consistent picture of the brane universe is achieved, and we can describe the evolution of the universe by means of solutions of the Einstein field equations in higher dimensions, with a four-dimensional membrane. We thus seek a description of the powerful holographic principle in cosmological settings, where its testing is subtle, and the question of holography therein has been considered by several authors [5] who have shown that for flat and open Friedmann-Lemaitre-Robertson-Walker (FLRW) universes the area of the particle horizon should bound the entropy on the backward-looking light cone. In addition to the study of holography in homogeneous cosmologies, attempts to generalize the holographic principle to a generic realistic inhomogeneous cosmological setting were carried out in [6]. Later, a very interesting study of the holographic principle in a FLRW universe filled with CFT with a dual AdS description was done by Verlinde [7], revealing that when a universe-sized black hole can be formed, an interesting and surprising correspondence appears between the entropy of CFT and the Friedmann equation governing radiation dominated closed FLRW universes. Generalizing Verlinde's discussion to a broader class of universes including a cosmological constant [8], we found that the matching of the Friedmann equation to the Cardy formula holds for de Sitter closed and AdS flat universes. However, for the remaining de Sitter and AdS universes, the argument fails due to breaking down of the general philosophy of the holographic principle. In high dimensions, various other aspects of Verlinde's proposal have also been investigated in a number of works [9].

In a recent paper [10], further light on the correspondence between the Friedmann equation and the Cardy formula has been shed from a Randall-Sundrum-type brane-world perspective [11]. Considering the CFT dominated universe as a codimension 1 brane with fine-tuned tension in a background of an AdS black hole, Savonije and Verlinde found the correspondence between the Friedmann equation and the Cardy formula for the entropy of CFT when the brane crosses the black hole horizon. This result was further confirmed by studying a brane universe filled with radiation and stiff matter, quantum-induced brane worlds, and a radially infalling brane [12]. The relation discovered between the Friedmann equation and the Cardy formula for the entropy shed significant light on the meaning of the holographic principle in a cosmological setting. However, the general proof for this correspondence for all CFTs is still difficult at the moment. Other settings have also been considered as in, e.g., [13]. It is worthwhile to further check the validity of the correspondence in broader classes of situations than [7,10].

Our motivation here is to use the correspondence between the CFT gas and the Friedmann equation and to establish an upper bound for the number of *e*-foldings during inflation, using a small number of assumptions. The main point is an upper limit for entropy, a fact that we can derive from the above correspondence. Recently, Banks and Fischler [14] have considered the problem of the number of *e*-foldings in a universe displaying an asymptotic de Sitter phase, like our own. As a result the number of *e*-foldings is not larger than 65/85 depending on the type of matter considered.

Here we reconsider the problem from the point of view of the entropy content of the Universe, and considering the cor-

*Electronic address: binwang@fma.if.usp.br
†Electronic address: eabdalla@fma.if.usp.br

respondence between the Friedmann equation and the Cardy formula in brane universes, as discussed by us [15].

The main points in our argument are the following. First we assume a FRW closed universe with a positive cosmological constant which does not recollapse, as implied by some recent observations. Such an assumption is crucial to our argument. Further on, we assume that there is an upper bound for entropy in the Universe. Such an entropy is obtained from the bulk black hole in the sense of holography and considered to have a bound on its storage to prevent the collapse of the universe. These hypotheses are sufficient conditions for us to arrive at the result. They are nevertheless not necessary, but we think them to be quite natural. Later on, we extend the result for a Randall-Sundrum brane world model. While dynamical details of the AdS/CFT correspondence have been used in the derivation, it is the bound on the entropy of the Universe that is the essential ingredient.

The existence of an upper bound for the number of *e*-foldings before the end of inflation was also studied recently in [16] and [17]. However, in that investigation the number of *e*-foldings is related both to a possible reduction in energy scale during the late stages of inflation and to the complete cosmological evolution, being model dependent. The bound has been obtained in some very simple cosmological settings, while it is still difficult to be obtained in nonstandard models. Using the entropy bound, the consideration of physical details connected with the universe evolution can be avoided. We have obtained the upper bound for the number of *e*-foldings for a standard FRW universe as well as for a nonstandard cosmology based on the brane inspired idea of Randall-Sundrum models.

The starting point is that the scalar factor, in the case of the brane cosmology, is defined by the Darmois-Israel condition [15,18,19]. We consider a bulk metric defined by

$$ds_5^2 = -f dt^2 + f^{-1} da^2 + a^2 d\Sigma_K^2, \tag{1}$$

where $f = k + a^2/L^2 - m/a^2$ and L is the curvature radius of AdS space-time. k takes the values $0, -1, +1$ corresponding to flat, open, and closed geometrics, and $d\Sigma_k^2$ is the corresponding metric on the unit three-dimensional plane, hyperboloid, or sphere. The black hole horizon is located at

$$a_H^2 = \frac{L^2}{2}\left(-k + \sqrt{k^2 + 4m/L^2}\right). \tag{2}$$

The relation between the parameter m and the Arnowitt-Deser-Misner mass of the five dimensional black hole M is [20]

$$M = \frac{3\omega_3}{16\pi G_5} m, \tag{3}$$

where ω_3 is the volume of the unit three-sphere, Vol(S^3), and G_5 is the Newton constant in the bulk. It is related to the Newton constant G_4 on the brane as $G_5 = G_4 L/2$.

Here, the location and the metric on the boundary are time dependent. We can choose the brane time such that $\dot{a}^2 = f^2 \dot{t}^2 - f$, in which case the metric on the brane is given by

$$ds_4^2 = -d\tau^2 + a^2(\tau) d\Sigma_3^2. \tag{4}$$

The conformal field theory lives on the brane, which is the boundary of the AdS hole. The energy for a CFT on a sphere with radius a, of volume $V = a^3\omega_3$, is given by $E = LM/a$. The total energy E is not a constant during the cosmological expansion, but decreases like a^{-1}. This is consistent with the fact that for a CFT energy density we have

$$\rho_{CFT} = E/V = \frac{3mL}{16\pi G_5 a^4} = \frac{3m}{8\pi G_4 a^4}. \tag{5}$$

The entropy of the CFT on the brane is equal to the Bekenstein-Hawking entropy of the AdS black hole [3,21], which is given by the area of the horizon measured in bulk Planckian units, as given by

$$S_{CFT}(4D) = S_{BH}(5D) = \frac{V_H}{4G_5}, \quad V_H = a_H^3 \omega_3. \tag{6}$$

The area of a three-sphere in an AdS space-time equals the volume of the corresponding spatial section for an observer on the brane.

The total entropy S is a constant during the cosmological evolution, but the entropy density of the CFT on the brane is

$$s = \frac{S}{V} = \frac{a_H^3\omega_3}{4G_5}\frac{1}{a^3\omega_3} =_{G_5 = G_4 L/2} \frac{a_H^3}{2G_4 L a^3}. \tag{7}$$

In the brane world interpretation we have to satisfy matching conditions for the gravitational fields due to the immersion of the brane into the bulk (see, e.g., [15,18,19]). From the matching conditions we find now that the cosmological equations in the brane are

$$H^2 = -\frac{k}{a^2} + \frac{m}{a^4} - \frac{1 - (\sigma/\sigma_c)^2}{L^2}, \tag{8}$$

where $\sigma_c = 3/8\pi G_5 L$ is the critical brane tension. Taking $\sigma = \sigma_c$, Eq. (8) reduces to the Friedmann equation of a CFT radiation dominated brane universe without a cosmological constant discussed in [10]. If $\sigma > \sigma_c$ or $\sigma < \sigma_c$, the brane world is a de Sitter universe or AdS universe, respectively. Using Eq. (5) the Friedmann equation can be written in the form

$$H^2 = -\frac{k}{a^2} + \frac{8\pi G_4}{3}\rho_{CFT} + \frac{\lambda}{3}, \tag{9}$$

where λ is the effective positive cosmological constant in four dimensions, in agreement with observations. The arguments and formulas above depend on the holographic properties, which we suppose to be valid in the theory.

The relation between the energy density and the entropy, $\rho = (9mL/16\pi^2 a_H^3 a^4)S$ can be used to rewrite the Friedmann equation as

$$(\dot{a})^2+k-\frac{3G_4 mLS}{2\pi a_H^3 a^2}-\frac{\lambda}{3}a^2=0, \tag{10}$$

which corresponds to the movement of a mechanical nonrelativistic particle in a given potential. This equation is crucial for the developments which follow, being deeply rooted in Eq. (7), which is a direct consequence of holography and the subsequent construction, as given in, e.g., [15]. For a closed universe there is a critical value for which the solution extends to infinity (no big crunch), which is

$$S<\frac{2\pi a_H^3\lambda a^4}{9G_4 mL}\overset{a\approx\lambda^{-1/2}}{\rightarrow}\frac{2\pi a_H^3\lambda^{-1}}{9G_4 mL}. \tag{11}$$

$\lambda^{-1/2}$ is the size of the de Sitter horizon, which is the box holding the maximum amount of entropy. The above equation was obtained by considering the potential obtained from the mechanical problem (10) for a closed universe, namely, $k=1$, which is $V=1-3G_4mLS/2\pi a_H^3 a^2-\lambda a^2/3$. We want a solution that does not collapse to zero but rather develops to infinity. The maximum of the potential occurs at $S_{max}=2\pi a_H^3\lambda a^4/9G_4 mL$ and divides the collapsing region (for a smaller than the one corresponding to the maximum) or an expanding region (for a larger than the one corresponding to the maximum). For the expanding case the first inequality in (11) must be satisfied.

Now, dividing Eq. (7) by Eq. (5) we get $s=(4\pi a_H^3/3mL)a\rho$; thus the entropy in such a universe can be rewritten as

$$S=sV=\frac{4}{3}\pi a^3\frac{4\pi a_H^3 a}{3mL}\rho \tag{12}$$

at the end of inflation. We take ρ to be the energy density during inflation, that is, $\rho\sim\Lambda_I/8\pi G_4$, which for the scale factor at the exit of inflation leads to the value $a\approx\Lambda_I^{-1/2}e^{Ne}$, where $\Lambda_I^{-1/2}$ corresponds to the apparent horizon during inflation, and we obtain

$$N_e=\ln a+\frac{1}{2}\ln\Lambda_I. \tag{13}$$

Using now Eq. (12) in Eq. (11) we get

$$a<(\Lambda_I\lambda)^{-1/4}, \tag{14}$$

from which we arrive at

$$N_e<\frac{1}{4}\ln\frac{\Lambda_I}{\lambda}\approx 64, \tag{15}$$

where we used the usual values $\Lambda_I^{1/4}\approx 10^{16}$ GeV and $\lambda^{1/4}\approx 10^{-3}$ eV. Note that the bound (14) is stricter than the previously used $a\approx\lambda^{-1/2}$. The bound (11) implies a bound for the scale factor such that only a limited amount of entropy can be stored to avoid the big crunch. This is the reason we need (14) in order to get the result (15).

For this standard FRW universe, the bound obtained is in agreement with [16] and [17] as well as with [14] for a universe filled with radiation. The de Sitter closed universe satisfies the correspondence between the Friedmann equation and the Cardy formula, which is the extension of Verlinde's argument (see [8]) showing the spirit of holography. It is doubtful that a similar bound can be obtained along the same lines for open or flat universes. However, we stress the fact that a recent Wilkinson Microwave Anisotropy Probe analysis favors a closed universe, although this is still a result to be further confirmed [22].

Let us consider now very high energy brane corrections to the Friedmann equation. From the Darmois-Israel conditions we find

$$H^2=-\frac{k}{a^2}+\frac{8\pi}{3M_4^2}\rho+\frac{4\pi}{3M_4^2}\frac{\rho^2}{l}+\frac{\lambda}{3}\approx-\frac{k}{a^2}+\frac{4\pi}{3M_4^2}\frac{\rho^2}{l}+\frac{\lambda}{3}, \tag{16}$$

where l is the brane tension and in the very high energy limit the ρ^2 term dominates. M_4 and λ are the four-dimensional Planck scale and the cosmological constant, respectively. Within the high energy regime, the expansion laws corresponding to matter and radiation domination are slower than in the standard cosmology [17]. Slower expansion rates lead to a larger value of the number of *e*-foldings. However, a full calculation has not been obtained due to the lack of knowledge of this high energy regime. Here we study this problem from the point of view of holography.

The energy density of the CFT and the entropy density are related as follows:

$$\rho_{CFT}=\frac{3m}{8\pi G_4 a^4},\qquad s=\frac{a_H^3}{2G_4 La^3},$$

$$\rho=\frac{9mLS}{16\pi^2 a_H^3 a^4}, \tag{17}$$

which can be substituted in the Friedmann equation as before, leading to a bound for the entropy, as well as a bound for the scale factor, as given by

$$S^2=\frac{256\pi^4 a_H^6 a^8\rho^2}{81m^2L^2}<\frac{64\pi^3 l a_H^6}{243G_4 m^2L^2\lambda^3},$$

$$a^8<\frac{3l}{4\pi G_4\rho^2\lambda^3}. \tag{18}$$

We consider the era when the quadratic energy density is important. The brane tension is required to be bounded by $l<(1\ \mathrm{MeV})^4$ [23]. Combining the values of Λ_I and λ we chose before, a bound for N_e is given by

$$N_e<75. \tag{19}$$

The number of *e*-foldings obtained is bigger than the value in standard FRW cosmology, which is consistent with the argument of [17].

In summary, we have derived an upper limit for the number of *e*-foldings based upon the arguments relating the

Friedmann equation and the Cardy formula. For the standard FRW universe our result is in good agreement with [16] and [17] and in the radiation dominated case with [14]. For the brane inspired cosmology in four dimensions we obtained a larger bound. Considering such a high energy context, the expansion laws are slower than in the standard cosmology, and our result can again be considered to be consistent with the argument in [17]. The interesting point here is that by using the holographic point of view we can avoid complicated physics during the universe evolution and give a reasonable value for the upper bound of the number of *e*-foldings. Elsewhere [16,17] the mechanism of ending inflation and the reheating phase are very important. Therefore in those discussions there is a strong model dependence. In the present description using the holographic description we do not refer to those sensitive processes. We thus claim that this discussion is more general.

This work was partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPQ). B.W. would like to acknowledge the support given by NNSF, China, Ministry of Science and Technology of China under Grant No. NKBRSFG19990754 and the Ministry of Education of China. We would like also to thank S. Nojiri for useful correspondence.

[1] G. 't Hooft, gr-qc/9310026; L. Susskind, J. Math. Phys. **36**, 6377 (1995).
[2] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. Gubser, I. Klebanov, and A. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[4] S.W. Hawking and D. Page, Commun. Math. Phys. **87**, 577 (1983).
[5] W. Fischler and L. Susskind, hep-th/9806039; N. Kaloper and A. Linde, Phys. Rev. D **60**, 103509 (1999); R. Easther and D.A. Lowe, Phys. Rev. Lett. **82**, 4967 (1999); G. Veneziano, Phys. Lett. B **454**, 22 (1999); R. Brustein, Phys. Rev. Lett. **84**, 2072 (2000); R. Brustein and G. Veneziano, *ibid.* **84**, 5695 (2000); R. Bousso, J. High Energy Phys. **07**, 004 (1999); **06**, 028 (1999); Class. Quantum Grav. **17**, 997 (2000); B. Wang and E. Abdalla, Phys. Lett. B **466**, 122 (1999); **471**, 346 (2000).
[6] R. Tavakol and G. Ellis, Phys. Lett. B **469**, 37 (1999); B. Wang, E. Abdalla, and T. Osada, Phys. Rev. Lett. **85**, 5507 (2000).
[7] E. Verlinde, hep-th/0008140.
[8] B. Wang, E. Abdalla, and R.K. Su, Phys. Lett. B **503**, 394 (2001).
[9] S. Nojiri and S.D. Odintsov, Int. J. Mod. Phys. A **16**, 3237 (2001); D. Kutasov and F. Larsen, J. High Energy Phys. **01**, 001 (2001); F. Lin, Phys. Lett. B **507**, 270 (2001); R. Brustein, S. Foffa, and G. Veneziano, *ibid.* **507**, 270 (2001); D. Klemm, A.C. Petkou, and G. Siopsis, Nucl. Phys. **B601**, 380 (2001); R.G. Cai, Phys. Rev. D **63**, 124018 (2001); D. Birmingham and S. Mokhtari, Phys. Lett. B **508**, 365 (2001).
[10] I. Savonije and E. Verlinde, Phys. Lett. B **507**, 305 (2001).
[11] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).
[12] A.K. Biswas and S. Mukherji, J. High Energy Phys. **03**, 046 (2001); S. Nojiri and S.D. Odintsov, Class. Quantum Grav. **18**, 5227 (2001); Y.S. Myung, J. Korean Phys. Soc. **43**, 991 (2003).
[13] D. Youm, Phys. Lett. B **515**, 170 (2001); S. Nojiri, O. Obregon, S. Odintsov, H. Quevedo, and M. Ryan, Mod. Phys. Lett. A **16**, 1181 (2001); D. Klemm, A. Petkou, G. Siopsis, and D. Zanon, Nucl. Phys. **B620**, 519 (2002).
[14] T. Banks and W. Fischler, astro-ph/0307459.
[15] Bin Wang, Elcio Abdalla, and Ru-Keng Su, Mod. Phys. Lett. A **17**, 23 (2002).
[16] S. Dodelson and L. Hui, Phys. Rev. Lett. **91**, 131301 (2003).
[17] A.R. Liddle and S.M. Leach, Phys. Rev. D **68**, 103503 (2003).
[18] W. Israel, Nuovo Cimento B **44**, 1 (1966); **48**, 463(E) (1967).
[19] E. Abdalla and B. Cuadros-Melgar, Phys. Rev. D **67**, 084012 (2003).
[20] A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, Phys. Rev. D **60**, 064018 (1999).
[21] J. Garriga and M. Sasaki, Phys. Rev. D **62**, 043523 (2000).
[22] D.N. Spergel *et al.*, Astrophys. J., Suppl. Serv. **148**, 175 (2003); J.-P. Luminet *et al.*, Nature (London) **425**, 593 (2003); N. Cornish, D. Spergel, G. Starkmann, and E. Komatsu, astro-ph/0310233.
[23] R. Maartens, D. Wands, B.A. Bassett, and I.P.C. Heard, Phys. Rev. D **62**, 041301(R) (2000).