Linear and nonlinear perturbations in dark energy models

Luca Amendola
INAF/Osservatorio Astronomico di Roma, Viale Frascati 33, 00040 Monte Porzio Catone (Roma), Italy
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I present linear and second-order perturbation theory in dark energy models with an explicit interaction to matter in view of its application to *N*-body simulations and nonlinear phenomena. Several new or generalized results are obtained: the general equations for the linear perturbation growth, an analytical expression for the bias induced by a species-dependent interaction, the Yukawa correction to the gravitational potential due to dark energy interaction, and the second-order perturbation equations in coupled dark energy and their Newtonian limit. I also show that a density-dependent effective dark energy mass arises if the dark energy coupling is varying.

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I. INTRODUCTION

Dark energy is defined as a fluid distributed almost homogeneously and capable of driving accelerated expansion. As such, it can be observed mainly through large scale effects such as those relating to the cosmic expansion history and its linear fluctuations. Indeed, the most weighty evidence in favor of dark energy comes from the acceleration of the Universe as seen in the Hubble diagram of the supernovae type Ia [1] and on the angular size of the acoustic horizon on the cosmic microwave background [2]. Indications concerning the growth of linear fluctuations, e.g., via the Integrated Sachs-Wolfe (ISW) effect [3,4] are still very tentative, although the prospects from, e.g., weak lensing [5] and Lyman-α clustering [6,7] appear promising.

However, all these observables depend ultimately on dark energy only through the expansion history $H(z)$ and the matter linear growth function $D(z)$, where z is the cosmological redshift. For instance, the luminosity distance in flat space is defined as

$$d_L(z)=(1+z)\int_0^z\frac{dz}{H(z)}, \tag{1}$$

where

$$H(z)=H_0[\Omega_m a^{-3}+\Omega_\phi a^{-3[1+W(a)]}+(1-\Omega_m-\Omega_\phi)a^{-2}]^{1/2}, \tag{2}$$

where $a=(1+z)^{-1}$ is the scale factor, Ω_i denotes the density at the present time of the ith species, and

$$W(\alpha=\log a)=\frac{1}{\alpha}\int_0^\alpha w_\phi(\alpha')d\alpha', \tag{3}$$

$w_\phi(z)$ being the equation of state of dark energy. It is clear that at any given redshift there will be different $w_\phi(z)$'s that give indistinguishable $d_L(z)$'s and that the degree of degeneration will increase with redshift. Similar integrals of $H(z)$ enter the definitions of angular-diameter distance and age, that will therefore be subject to the same ambiguity.

The linear growth function $D(z)$, defined as $D(z)=\delta_m(z)/\delta_m(0)$ if $\delta_m(z)$ denotes the matter density contrast, is a second observable quantity, in general independent of $H(z)$. If the matter density parameter at present is Ω_{m0}, D is given on subhorizon scales by the solution of the perturbation equation

$$D''+\frac{1}{2}\left(1+\frac{\mathcal{H}'}{\mathcal{H}}\right)D'-\frac{3\Omega_{m0}}{2}a^{-3}D=0 \tag{4}$$

(valid for uncoupled dark energy and when radiation is negligible) where the prime denotes derivation with respect to the *e*-folding time $\alpha=\log a$ and where we introduce the conformal Hubble function $\mathcal{H}=d\alpha/d\tau$, τ being the conformal time. The growth function D is therefore an independent probe of dark energy: two models that give an identical $H(z)$ will in general be distinguished by different $D(z)$'s.

In principle, the degeneracy can be broken by a large number of observations at different z's of $H(z)$ and/or $D(z)$. However, this is hardly feasible, since real data is confined to "small" ($z<5$) or very large redshifts ($z\approx1100$). Moreover, in most models the dark energy components become subdominant at $z\gg1$, so that both $H(z)$ and $D(z)$ becomes rapidly insensitive to $w_\phi(z)$ [see Ref. [8] for a detailed discussion on the practical observability of $w_\phi(z)$ at large z].

It would be desirable therefore to add new observables, such as the evolution of the perturbations in the dark energy field itself, the second order growth function or the full nonlinear properties as obtained through *N*-body simulations (see, e.g., Refs. [9–11]). In this paper we derive the general linear and nonlinear perturbation equations in dark energy models in order to provide the basic material for the study of these additional observable quantities. Our dark energy model is quite general: a scalar field with a generic potential and an explicit varying species-dependent coupling to baryons and dark matter. This cover most scalar field models presented in the literature, with the notable exception of models with nonstandard kinetic terms [12]. In a subsequent paper we employ the formalism derived here to evaluate the large scale skewness in coupled dark energy.

II. COUPLED DARK ENERGY

Our dark energy model is characterized by a general potential $V(\phi)$ and general couplings $C_i(\phi)$ to matter. This class of models include those motivated by string theory proposed in Refs. [13–22]. The conservation equations with interacting terms for the field ϕ, cold dark matter (c), and baryons (b) are

$$T^{\mu}_{(c)\nu;\mu} = -C_c(\phi)T_{(c)}\phi_{;\nu},$$
$$T^{\mu}_{(b)\nu;\mu} = -C_b(\phi)T_{(b)}\phi_{;\nu},$$
$$T^{\mu}_{(\phi)\nu;\mu} = [C_c(\phi)T_{(c)} + C_b(\phi)T_{(b)}]\phi_{;\nu}, \tag{5}$$

where the coupling functions $C_{b,c}(\phi)$ depend on the species, as first proposed in Ref. [23]. Radiation (subscript γ) remains uncoupled because it is traceless (models with coupling to the electromagnetic field [24,25] or neutrinos [26] have also been proposed). The standard Einstein equations are assumed to hold. This coupling form is derived, through a conformal transformation, from a Brans-Dicke gravity with species-dependent interaction [23,27]. There are strong limits on the baryon coupling [28] and relatively looser ones on the dark matter coupling from either astrophysics [10,29] or cosmology [30]. However, all the limits have been derived assuming couplings constant in space and time while we wish to derive the equations in all generality: we therefore assume the couplings to be free functions. In a flat-space FRW metric Eq. (5) plus the Friedmann equation reads

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \sqrt{2/3}\kappa(\beta_c\rho_c + \beta_b\rho_b),$$
$$\dot{\rho}_c + 3H\rho_c = -\sqrt{2/3}\kappa\beta_c\rho_c\dot{\phi},$$
$$\dot{\rho}_b + 3H\rho_b = -\sqrt{2/3}\kappa\beta_b\rho_b\dot{\phi}, \tag{6}$$
$$\dot{\rho}_\gamma + 4H\rho_\gamma = 0,$$
$$3H^2 = \kappa^2(\rho_\gamma + \rho_c + \rho_b + \rho_\phi),$$

where $\kappa^2 = 8\pi G$, $\beta_c = C_c\sqrt{3/2\kappa^2}$, $\beta_b = C_b\sqrt{3/2\kappa^2}$, $H = \dot{a}/a$. The matter conservation equations can be integrated out:

$$\rho_{c,b} = \rho(0)_{c,b}a^{-3}\exp\left[-\sqrt{\frac{2}{3}}\kappa\int \beta_{c,b}(\phi)d\phi\right]. \tag{7}$$

This shows one of the basic properties of dark energy interactions: although pressureless, matter density does not scale with the inverse of volume. In other words, matter appears to be nonconserved to observers unaware of dark energy. As far as the potential $V(\phi)$ is concerned, we write in all generality

$$V(\phi) = Ae^{-\kappa\sqrt{2/3}\mu f(\phi)\phi}, \tag{8}$$

where μ is a dimensionless constant (which, of course, could be absorbed in f but we find it convenient to use in order to keep track of the derivative order and to conform to the notation in Ref. [14]). The exponential case studied in Refs. [13,14] therefore corresponds to $f=1$, a constant potential similar to that in [19] to $\mu=0$, and the power law $V\sim\phi^{-n}$ to $f(\phi) = \sqrt{(3/2)}n \log\phi/(\kappa\mu\phi)$. We will also use the definitions

$$\frac{dV}{d\phi} = -\sqrt{\frac{2}{3}}\kappa\mu f_1 V, \tag{9}$$

$$\frac{d^2V}{d\phi^2} = \frac{2}{3}\kappa^2\mu^2 f_2 V, \tag{10}$$

where

$$f_1 = \frac{df}{d\phi}\phi + f, \tag{11}$$

$$f_2 = f_1^2 - \frac{\sqrt{3/2}}{\kappa\mu}\frac{df_1}{d\phi}. \tag{12}$$

The simple exponential case reduces then to $f=f_1=f_2=1$. Later on we will need higher derivatives of V so we give the general rule

$$V^{(n)}_{,\phi} = (-1)^n\left(\frac{2}{3}\right)^{n/2}\kappa^n\mu^n f_n V,$$

where $f_0=1$ and

$$f_n = f_{n-1}f_1 - f'_{n-1}\Big/\left(\sqrt{\frac{2}{3}}\kappa\mu\right). \tag{13}$$

The system (6) is best studied in the new variables [14,31]

$$x = \kappa\frac{\phi'}{\sqrt{6}},\quad y = \frac{\kappa}{H}\sqrt{\frac{V}{3}},\quad z = \frac{\kappa}{H}\sqrt{\frac{\rho_\gamma}{3}},\quad v = \frac{\kappa}{H}\sqrt{\frac{\rho_b}{3}}, \tag{14}$$

and in the e-folding time. Then we obtain

$$x' = \left(\frac{z'}{z} - 1\right)x - \mu f_1 y^2 + \beta_c(1 - x^2 - y^2 - v^2 - z^2) + \beta_b v^2,$$

$$y' = \mu f_1 xy + y\left(2 + \frac{z'}{z}\right),$$

$$z' = -\frac{z}{2}(1 - 3x^2 + 3y^2 - z^2), \tag{15}$$

$$v' = -\frac{v}{2}(3\beta_b x - 3x^2 + 3y^2 - z^2).$$

The CDM energy density parameter is obviously $\Omega_c = 1 - x^2 - y^2 - z^2 - v^2$ while we also have $\Omega_\phi = x^2 + y^2$, $\Omega_\gamma = z^2$, and $\Omega_b = v^2$. The system is subject to the condition

$x^2+y^2+v^2+z^2 \leqslant 1$. To close the system one also needs the relation $f_1(y,H)$ and the Friedmann equation

$$\frac{H'}{H} = -\frac{1}{2}(3+3x^2-3y^2+z^2). \tag{16}$$

In the perturbation calculations we will use the conformal Hubble function $\mathcal{H}=aH$, and we will need the following relation:

$$\frac{\mathcal{H}'}{\mathcal{H}} = 1+\frac{H'}{H}. \tag{17}$$

III. LINEAR PERTURBATIONS

The aim of this section is to write down the linear perturbation equations for a combination of fluid components with general equations of state $p_i=w_i(\rho)\rho_i$ and a scalar field with a general potential $V(\phi)$ and coupling $\beta_i(\phi)$ to the fluids. Several works discussed the linear perturbations for uncoupled dark energy even before the evidence for acceleration [32–35]. A few papers included various kinds of couplings (e.g., in Refs. [19,36] or in the conformally related Jordan frame [37,38]) but not at this level of generality. We choose the longitudinal gauge

$$ds^2 = a^2[-(1+2\Psi)d\tau^2+(1-2\Phi)dx_i dx^i], \tag{18}$$

where τ is the conformal time. In the e-folding, time, our metric is effectively

$$ds^2 = e^{2\alpha}\left[-(1+2\Psi)\frac{d\alpha^2}{\mathcal{H}^2}+(1-2\Phi)dx_i dx^i\right]. \tag{19}$$

It is well known that in the absence of anisotropic stress $\Phi=\Psi$, so we adopt this simplification from the start. We define the perturbation variables

$$\delta=\delta\rho/\rho, \quad \varphi=\kappa\delta\phi/\sqrt{6}, \quad \nabla_i v_i=\theta, \tag{20}$$

where the dimensionless conformal velocity $\mathbf{v}$ of a fluid with equation of motion $x_i(t)$ has components $v_i = a[dx_i(t)/dt]/\mathcal{H}$. Repeated indexes mean summation and all spatial derivatives are with respect to the comoving coordinates x_i. We also define the dark energy mass

$$m_\phi^2 = \frac{d^2V}{d\phi^2}, \tag{21}$$

with its dimensionless version

$$\hat{m}_\phi^2 = \frac{m_\phi^2}{H^2} = 2\mu^2 y^2 f_2. \tag{22}$$

Perturbing the Einstein equations and the conservation equations we obtain the linear perturbations below. We introduce the scale λ: in real space we interpret λ^{-2} as the operator $-\mathcal{H}^{-2}\nabla^2$; in Fourier space, $\lambda=\mathcal{H}/k$. In this way the following equations can be read equivalently in real or Fourier space. Note also that $\beta'=d\beta/d\alpha=\phi' d\beta/d\phi$.

The perturbation equations for a generic equation of state $p=w(\rho)\rho$, which includes unified models such as the Chaplygin gas [40] or phase transition models [41], are as follows. Generic fluid component [equation of state $p=w(\rho)\rho$]:

$$\delta' = -(w+1)\theta+3(1+w)\Phi'-2(1-3w)(\beta\varphi'+\beta'\varphi) -3w_{,\rho}\delta(1-2x\beta), \tag{23}$$

$$\theta' = -\left[(1-3w)(1-2x\beta)-w_{,\rho}A(w)+\frac{\mathcal{H}'}{\mathcal{H}}\right]\theta + \frac{w+w_{,\rho}}{w+1}\lambda^{-2}\delta+2\beta\frac{3w-1}{w+1}\lambda^{-2}\varphi+(1+w)\lambda^{-2}\Psi, \tag{24}$$

where $w_{,\rho}\equiv dw/d\log\rho$ and

$$A(w)=3+2x\beta(1-3w)/(1+w). \tag{25}$$

Note that the sound speed is $c_s^2\equiv dp/d\rho=w+w_{,\rho}$. The equations for the scalar field coupled to several fluids with equations of state $p_i=w_i(\rho_i)\rho_i$ and the metric equations, respectively, are as follows. Scalar field:

$$\varphi''+\left(2+\frac{\mathcal{H}'}{\mathcal{H}}\right)\varphi'+(\lambda^{-2}+\hat{m}_\phi^2)\varphi-4\Phi' x-2y^2\mu f_1\Phi \tag{26}$$

$$= \sum_i \beta_i[1-3w_i-3w_{i,\rho}]\Omega_i\delta_i+2\sum_i \beta_i\Omega_i\Phi + \sum_i (1-3w_i)\frac{\varphi}{x}\beta_i'\Omega_i, \tag{27}$$

metric:

$$\Phi = \frac{-3\lambda^2\left[6x\varphi+2x\varphi'-2y^2\mu f_1\varphi+\sum \Omega_i(\delta_i+3(w_i+1)\lambda^2\theta_i)\right]}{2(1-3\lambda^2x^2)}, \tag{28}$$

$$\Phi' = \frac{1}{2}\left[2(3x\varphi-\Phi)+\lambda^2\sum 3(w_i+1)\theta_i\Omega_i\right]. \tag{29}$$

From these general equations one can derive the following equations for the three perfect fluid components, CDM ($w \approx 0 \approx c_s$), baryons ($w \approx 0$, but c_s^2 non-negligible at small scales), radiation ($w = 1/3$). CDM:

$$\delta_c' = -\theta_c + 3\Phi' - 2\beta_c\varphi' - 2\beta_c'\varphi, \tag{30}$$

$$\theta_c' = -\left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_c x\right)\theta_c + \lambda^{-2}(\Phi - 2\beta_c\varphi). \tag{31}$$

Radiation:

$$\delta_\gamma' = -\frac{4}{3}\theta_\gamma + 4\Phi', \tag{32}$$

$$\theta_\gamma' = -\frac{\mathcal{H}'}{\mathcal{H}}\theta_\gamma + \frac{1}{4}\lambda^{-2}\delta_\gamma + \lambda^{-2}\Phi. \tag{33}$$

Baryons:

$$\delta_b' = -\theta_b + 3\Phi' - 2\beta_b\varphi' - 2\beta_b'\varphi, \tag{34}$$

$$\theta_b' = -\left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_b x\right)\theta_b + c_s^2\lambda^{-2}\delta + \lambda^{-2}(\Phi - 2\beta_b\varphi) \tag{35}$$

(to the equation for θ_b' one should add the standard term describing momentum exchange with photons due to Thomson scattering, see Ref. [39]). Scalar field:

$$\varphi'' + \left(2 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\varphi' + (\lambda^{-2} + \hat{m}_\phi^2)\varphi - 4\Phi' x - 2y^2\mu f_1\Phi = \beta_c\Omega_c(\delta_c + 2\Phi) + \beta_b\Omega_b(\delta_b + 2\Phi) + \frac{\varphi}{x}(\Omega_c\beta_c' + \Omega_b\beta_b'). \tag{36}$$

Let us derive now the Newtonian limit (small scales, $\lambda \ll 1$). The gravitational potential is

$$\Phi = -\frac{3}{2}\lambda^2\left(\sum \Omega_i\delta_i + 6x\varphi + 2x\varphi' - 2y^2\mu f_1\varphi\right), \tag{37}$$

$$\Phi' = 3x\varphi - \Phi. \tag{38}$$

Inserting Eq. (38) in Eq. (36) we obtain

$$\varphi'' + \left(2 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\varphi' + \left(\lambda^{-2} + \hat{m}_\phi^2 - 12x^2 - \frac{1}{x}\sum \Omega_i\beta_i'\right)\varphi + \Phi\left(4x - 2y^2\mu f_1 - 2\sum \Omega_i\beta_i\right) = \sum \beta_i\Omega_i\delta_i, \tag{39}$$

where the sum is on the coupled components (here baryons and dark matter). It is interesting to observe that the terms in β_i' contribute to the equation as effective masses. If, for instance, $\beta_i = \beta_0 e^{\sqrt{2/3}\kappa\beta_1\phi}$, then we can define a "coupling mass"

$$\hat{m}_{\beta_i}^2 \equiv \frac{\Omega_i\beta_i'}{x} = 2\Omega_i\beta_i\beta_1. \tag{40}$$

The Newtonian limit in Eq. (39) allows several simplifications. First, we can neglect the metric potential Φ which is proportional to λ^2. Second, we can also neglect the term $12x\varphi$ since $|x| \leq 1$ is much smaller that λ^{-2}. Finally, we also neglect $\hat{m}_\phi^2$ and $\hat{m}_{\beta_i}^2$ with respect to λ^{-2} since otherwise the dark energy would cluster on astrophysical scales and would reduce to a form of massive dark matter [42]. In Sec. V, however, we remove this approximation. Then we are left with the equation

$$\varphi'' + \left(2 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\varphi' + \lambda^{-2}\varphi = \sum \beta_i\Omega_i\delta_i, \tag{41}$$

whose solution is the sum of the solution $\varphi_{\rm hom}$ of the homogeneous equation and a particular inhomogeneous solution $\varphi_{\rm inh}$. The solution of the homogeneous part is a rapidly ($k/\mathcal{H} \gg 1$) oscillating function with zero average. For the inhomogeneous solution, we assume that its time derivatives $\varphi_{\rm inh}', \varphi_{\rm inh}''$ are much smaller than the remaining terms; this will be proved later on. Finally, we average over the oscillations to obtain $\varphi \approx \varphi_{\rm inh}$

$$\varphi \approx \lambda^2(\beta_c\Omega_c\delta_c + \beta_b\Omega_b\delta_b). \tag{42}$$

Since φ is of order λ^2, Eq. (37) reduces to the usual Poisson equation (hereafter we neglect radiation)

$$\Phi = -\frac{3}{2}\lambda^2(\Omega_b\delta_b + \Omega_c\delta_c). \tag{43}$$

Now, if we substitute in Eq. (31) we can define a new potential acting on dark matter

$$\Phi_c = \Phi - 2\beta_c\varphi = -\frac{3}{2}\lambda^2\Omega_b\delta_b\left(1 + \frac{4}{3}\beta_b\beta_c\right) - \frac{3}{2}\lambda^2\Omega_c\delta_c\left(1 + \frac{4}{3}\beta_c^2\right). \tag{44}$$

In real space, this equation becomes

$$\nabla^2\Phi_c = 4\pi G_{bc}\rho_b\delta_b + 4\pi G_{cc}\rho_c\delta_c, \tag{45}$$

where the gravitational constant is restored and define

$$G_{ij} = G\gamma_{ij}, \quad \gamma_{ij} \equiv 1 + 4\beta_i\beta_j/3, \tag{46}$$

so that $G_{bc} = G(1 + 4\beta_b\beta_c/3)$ and $G_{cc} = G(1 + 4\beta_c^2/3)$. Analogous equations hold for the baryon force equation (35). Therefore the Newtonian linear equations for dark matter and baryons in coupled dark energy are

$$\delta_c' = -\theta_c, \tag{47}$$

$$\theta_c' = -\left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_c x\right)\theta_c - \mathcal{H}^{-2}\nabla^2\Phi_c, \tag{48}$$

$$\delta_b' = -\theta_b, \tag{49}$$

$$\theta_b' = -\left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_b x\right)\theta_b - \mathcal{H}^{-2}\nabla^2\Phi_b, \tag{50}$$

$$\nabla^2\Phi_c = 4\pi G_{bc}\rho_b\delta_b + 4\pi G_{cc}\rho_c\delta_c, \tag{51}$$

$$\nabla^2\Phi_b = 4\pi G_{bb}\rho_b\delta_b + 4\pi G_{bc}\rho_c\delta_c. \tag{52}$$

The $\beta'\varphi$ terms in the δ' equations have been dropped because φ is of order λ^2. Deriving the δ_c' equations we obtain

$$\delta_c'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_c x\right)\delta_c' - \frac{3}{2}(\gamma_{cc}\delta_c\Omega_c + \gamma_{bc}\delta_b\Omega_b) = 0, \tag{53}$$

and similarly for δ_b'

$$\delta_b'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_b x\right)\delta_b' - \frac{3}{2}(\gamma_{bc}\delta_c\Omega_c + \gamma_{bb}\delta_b\Omega_b) = 0. \tag{54}$$

These equations generalize previous results [36] because they are also valid for nonconstant β (provided $\hat{m}_\beta^2 \ll \lambda^{-2}$).

It is clear that since baryons and dark matter obey different equations, they will develop a bias already at the linear level. A simple result can be obtained in the case in which one component dominates. Assuming $\Omega_b \ll \Omega_c$, in fact, the baryon solution will be forced by the dominating CDM to follow asymptotically its evolution. Putting then $\delta_c \sim e^{\int m(\alpha)d\alpha}$ and $\delta_b = b\delta_c$ with $b = \text{const}$ we obtain the coupled equations for the growth exponent $m(\alpha) \equiv d\log\delta_c/d\alpha$ (not to be confused with the scalar field mass)

$$m' + m^2 + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_c x\right)m - \frac{3}{2}\gamma_{cc}\Omega_c = 0,$$

$$m' + m^2 + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta_b x\right)m - \frac{3}{2b}\gamma_{bc}\Omega_c = 0, \tag{55}$$

from which by subtraction

$$b = \frac{3\gamma_{bc}\Omega_c}{3\gamma_{cc}\Omega_c + 4(\beta_c - \beta_b)xm}. \tag{56}$$

Notice that all terms on the right hand side are, in general, functions of time. This shows that a linear bias of gravitational nature develops whenever $\beta_c \neq \beta_b$. This bias extends to all Newtonian scales and therefore is distinguishable from the hydrodynamical or nonlinear bias that takes place in collapsed objects.

Further insight can be gained when these equations have constant coefficients, i.e., when w_ϕ and $\Omega_{c,b}$ are constant (in Ref. [14] we denoted these cases as stationary solutions). As shown in Ref. [14] this is realized on the critical points of a field governed by an exponential potential and a constant coupling β. In this case is convenient to define a total equation of state that in absence of radiation is

$$w_e = p_{\text{tot}}/\rho_{\text{tot}} = x^2 - y^2, \tag{57}$$

instead of $w_\phi = p_\phi/\rho_\phi$. Neglecting the baryons the relation is simply

$$w_e = (1 - \Omega_c)w_\phi,$$

so that now

$$\frac{\mathcal{H}'}{\mathcal{H}} = -\frac{1}{2}[1 + 3w_e]. \tag{58}$$

The scale factor in this case grows as $a \sim \tau^{p/(1-p)} \sim t^p$, where $p = 2/[3(w_e+1)]$. The solutions are $\delta_c = a^{m_\pm}$ and $\delta_b = b_\pm a^{m_\pm}$ where $b_\pm$ is as in Eq. (56) and, again neglecting the baryons, i.e., for $\Omega_b \ll \Omega_c$ [36]

$$m_\pm = \frac{1}{4}(-1 + 3w_e + 4\beta_c x \pm \Delta), \tag{59}$$

where $\Delta^2 = 24\gamma_{cc}\Omega_c + (-1 + 3w_e + 4\beta_c x)^2$. In this case then m and b are constant.

The scalar field solution is

$$\varphi \approx \lambda^2\beta_c\delta_c\Omega_c = H_0^2 k^{-2} a^{2(p-1)/p}\beta_c\delta_c\Omega_c. \tag{60}$$

The derivative φ' is

$$\varphi' = \varphi\left[\frac{2(p-1)}{p} + m + \frac{\Omega_c'}{\Omega_c}\right],$$

which is much smaller than $\lambda^{-2}\varphi$ in Eq. (41) for realistic values of p, m, and Ω_c. The same applies to φ'', which completes our derivation of Eq. (42). For small wavelengths φ (which here is proportional to $\delta\rho_\phi/\rho_\phi$) is always much smaller than δ_c, δ_b at the present time, unless, of course, β_c is exceedingly large. It is interesting to observe that φ could outgrow the matter perturbations in the future in an accelerated epoch, i.e., if $p > 1$ and if Ω_c does not vanish.

IV. SYNCHRONOUS GAUGE

Since most Boltzmann codes in CMB are implemented via the synchronous gauge we give here the relevant equations in this gauge.

Generic fluid component [equation of state $p = w(\rho)\rho$]:

$$\delta' = -(w+1)\theta - \frac{1}{2}(w+1)h' - 2(1-3w)(\beta\varphi' + \beta'\varphi) - 3w_{,\rho}\delta(1 - 2x\beta), \tag{61}$$

$$\theta' = -\left[(1-3w)(1-2\beta x) - w_{,\rho}A(w) + \frac{\mathcal{H}'}{\mathcal{H}}\right]\theta + \frac{w+w_{,\rho}}{w+1}\lambda^{-2}\delta + 2\beta\frac{3w-1}{w+1}\lambda^{-2}\varphi. \quad (62)$$

Scalar field:

$$\varphi'' + \left(2+\frac{H'}{H}\right)\varphi' + \lambda^{-2}\varphi + \frac{1}{2}h'x + 2\mu^2 y^2 f_2\varphi = \sum \beta_i[1-3w_i-3w_{i,\rho}]\Omega_i\delta_i + \sum \Omega_i(1-3w_i)\frac{\beta_i'}{x}\varphi. \quad (63)$$

Metric:

$$h' = 2\lambda^{-2}\eta + 3\sum \delta_i\Omega_i + 6\varphi' x - 6\mu f_1 y^2\varphi,$$

$$\eta' = \frac{3}{2}\lambda^2\sum (w_i+1)\Omega_i\theta_i + 3\varphi x, \quad (64)$$

$$h'' = -\left(1+\frac{H'}{H}\right)h' - 2[12\varphi' x + 6\mu f_1 y^2\varphi] - 3\sum (1+3w+3w_{,\rho})\delta_i\Omega_i.$$

Again in view of CMB applications, it is useful to detail the adiabatic initial conditions. The condition of zero entropy perturbations is

$$\delta S = \frac{\delta_i}{1+w_i} - \frac{\delta_j}{1+w_j} = 0, \quad (65)$$

$$\delta S' = \left(\frac{\delta_i}{1+w_i}\right)' - \left(\frac{\delta_j}{1+w_j}\right)' = 0. \quad (66)$$

For the scalar field

$$\frac{\delta_\phi}{1+w_\phi} = \frac{\varphi'\phi' + \varphi a^2\mathcal{H}^{-2}V_{,\phi}}{\phi'^2} = \frac{x\varphi' - \varphi y^2\mu f_1}{x^2}, \quad (67)$$

so that applying Eqs. (65), (66) to the scalar field and the other components we obtain the initial conditions as

$$\varphi = -\frac{x^2(8y^2\mu f_1\delta_c + 2xh' + 4x\delta_c')}{4[x^2 + y^2\mu f_1\lambda^2(6x-\beta\Omega_c)]}, \quad (68)$$

$$\varphi' = -\frac{x[4x^2\delta_c - y^2\mu f_1\lambda^2(-24x\delta_c + 8\mu f_1 y^2\delta_c + 4\beta\delta_c\Omega_c + 2xh' + 4x\delta_c')]}{4[x^2 + y^2\mu f_1\lambda^2(6x-\beta\Omega_c)]}. \quad (69)$$

In a radiation dominated era in which $\Omega_c \to 0$ and on super horizon scales $(\lambda \gg 1)$, these become

$$\varphi = -\frac{x(4y^2\mu f_1\delta_c + xh' + 2x\delta_c')}{12y^2\mu f_1}, \quad (70)$$

$$\varphi' = \frac{-12x\delta_c + 4\mu f_1 y^2\delta_c + xh' + 2x\delta_c'}{12}. \quad (71)$$

Inserting δ_c' from Eq. (61) and putting initially $\theta_c = 0$, we can further simplify

$$\varphi = -\frac{x(y^2\mu f_1 + \beta x^2)}{y^2\mu f_1}\delta_c, \quad (72)$$

$$\varphi' = \frac{-3x + \mu f_1 y^2}{3+\beta x}\delta_c. \quad (73)$$

V. A MASSIVE DARK ENERGY FIELD

Here we take a digression to consider the two effective masses of the dark energy field, previously neglected. In this section we assume the dark energy is coupled to a single matter component, subscript m, or, equivalently, that has a universal coupling to all fields. If λ^{-2} is not much larger than $\hat{m}^2 = \hat{m}_\phi^2 + \hat{m}_\beta^2$ (but still is larger than the coefficients of Φ), Eq. (42) in Fourier space becomes

$$\varphi \approx Y(k)\lambda^2\beta\Omega_m\delta_m, \quad (74)$$

where

$$Y(k) = \frac{k^2}{k^2 + a^2m^2}, \quad (75)$$

where $m = \hat{m}H$. If we substitute in Eq. (31) we see that the effective potential is (neglecting the baryons)

$$\hat{\Phi} = -\frac{3}{2}\lambda^2\Omega_m\delta_m\left[1 + \frac{4}{3}\beta^2 Y(k)\right]. \quad (76)$$

Now, let us write down the density contrast for a particle of mass M_0 located at the origin in empty space

$$\Omega_m \delta_m = \frac{\rho_M - \rho_m}{\rho_{\rm crit}} = \frac{\kappa^2 M(\phi)}{3\mathcal{H}^2 a}, \tag{77}$$

where

$$M(\phi) = M_0 e^{-\sqrt{2\kappa^2/3}\int \beta d\phi}, \tag{78}$$

where we used Eq. (7). It turns out then that the potential originated by a dark matter particle is

$$\hat{\Phi} = -\frac{3}{2}\Omega_m \delta_m \lambda^2 \left[1 + \frac{4}{3}\beta^2 Y(k)\right]$$

$$= -4\pi G M(\phi)\left(\frac{1}{k^2} + \frac{4}{3}\beta^2 \frac{1}{k^2 + a^2 m^2}\right)\frac{1}{a}, \tag{79}$$

which, upon inverse Fourier transform

$$\hat{\Phi}(r) = \frac{1}{(2\pi)^3}\int e^{i\mathbf{k}\cdot\mathbf{x}}\hat{\Phi} d^3k \tag{80}$$

becomes the Yukawa potential

$$\hat{\Phi}(r) = -\frac{GM(\phi)}{r}\left(1 + \frac{4\beta^2}{3}e^{-mr}\right), \tag{81}$$

where $\mathbf{r} = a\mathbf{x}$ is the physical coordinate.

It is useful, in view of application to N-body simulations, to write down the acceleration on particles explicitly. Taking Eq. (31) and using the definition of peculiar velocity $\mathbf{v}_p = a d\mathbf{x}/dt$ in terms of the velocity used in Eq. (20)

$$v_p = \mathcal{H} v,$$

we can write the acceleration equation in ordinary time $dt = a d\tau$ as

$$\dot{v}_{p,i} = -(1-2\beta x)H v_{p,i} - \frac{d\hat{\Phi}}{dr_i}. \tag{82}$$

If we define

$$G_Y(r) = G\left[1 + \frac{4\beta^2}{3}(1+mr)e^{-mr}\right], \tag{83}$$

we obtain the force on a dark matter particle

$$\frac{d\hat{\Phi}}{dr_i} = \frac{G_Y(r)M(\phi)}{r^2}. \tag{84}$$

In Eq. (82) the three effects of the coupling appear clearly: the mass M depends on the time evolution of ϕ; the gravitational potential acquires the Yukawa correction; and the mass variation of the test particle induces an extra friction $-2\beta x$.

It is straightforward to generalize to the acceleration of a test particle of type s' due by a distribution of several particles of species s at distances $r_{ss'}$ (dropping the p subscript):

$$\dot{\mathbf{v}}_{s'} = -(1-2\beta_{s'}x)H\mathbf{v}_{s'} - \sum_s \frac{G_{Yss'}(r_{ss'})M_s(\beta_s,\phi)}{r^3_{ss'}}\mathbf{r}_{ss'}, \tag{85}$$

where

$$G_{Yss'}(r) = G\left[1 + \frac{4\beta_s\beta_{s'}}{3}(1+mr)e^{-mr}\right]. \tag{86}$$

In Ref. [10] these equations have been applied to N-body simulations in the limit of constant β and $m \rightarrow 0$.

In practice, in any dark energy model one expects the mass scale to be much larger than the galaxy cluster scale ≈ 1 Mpc, in order to prevent clustering, so $r \ll 1/m$ for all astrophysical scales. However, a nonvanishing mass m can have some interesting effects.

First, it is to be observed that the background dynamics depends on the potential and its first derivative only, while the mass depends on the second derivative. It is then possible to build viable dark energy models that accelerate the expansion but whose mass scale $1/m$ is between a few Mpc and $H_0^{-1} = 3000$ Mpc/h; in this case the perturbations of the scalar field would be directly observable through, e.g., weak lensing.

Second, while the mass m_ϕ depends exclusively on the potential $V(\phi)$, the coupling mass

$$m_\beta^2 \equiv x^{-1}\Omega_m \beta_m' H^2 = \sqrt{\frac{6}{\kappa^2}}\Omega_m \beta_{,\phi} H^2 \tag{87}$$

depends on the matter content Ω_m. In a inhomogeneous background, we can expect that m_β^2 will be proportional to ρ_m. This raises an interesting question, recently posed in the context of α-varying models [25] and in the "chameleon" model of Ref. [43]: is it possible to have a scalar field with a large mass near a massive body such as the Earth and a very low one in space? This would open the possibility that scalar gravity escapes detection in Earth laboratories, where the Yukawa term would be exponentially suppressed if $1/m$ is on the submillimetric scale, even if β were of order unity (here we neglect any possible upper bound from cosmology). In Ref. [43] it has been hypothesized that a model with a constant β does in fact contain a density-dependent mass but our calculations clearly show that $\beta' \neq 0$ is a necessary condition. This possibility will be discussed in another paper.

VI. SECOND ORDER EQUATIONS

Here we extend the previous calculations to second order in the Newtonian regime and in the nonrelativistic limit. This will allow us to evaluate higher order moments of the gravitational clustering. Higher order perturbation equations in a varying dark matter mass scenario and in scalar-tensor theories which may be reduced to particular cases of the present model were studied in Ref. [38].

Here for simplification we consider a single matter fluid with $w=0$. In the nonrelativistic limit the quadrivelocity $\mathbf{u}$ remains first order, with components

$$u^{\alpha}=\frac{dx^{\alpha}}{ds}\approx\frac{dx^{\alpha}}{\sqrt{g_{00}}d\tau}. \tag{88}$$

The general conservation equations at second order are then ($v^2=v_iv^i$)

$$\begin{aligned}\delta'+\nabla_i(1+\delta)v_i&=3(1+\delta)\Phi'-2\beta\varphi'(1+\delta)+6\Phi\Phi'\\&\quad-v_i\nabla_i(\Psi-3\Phi)-H^2v^2(1-2\beta x)\\&\quad-2\beta'\varphi\left(1+\delta+\frac{\varphi'}{x}\right)-\frac{\varphi^2}{x}\left(\beta''-\beta'\frac{\phi''}{\phi'}\right),\end{aligned}$$

$$\begin{aligned}&(1+\delta-2\Psi-2\Phi)\left[v_i'+\left(1+\frac{\mathcal{H}'}{\mathcal{H}}-2\beta x\right)v_i\right]\\&=-\frac{1+\delta}{\mathcal{H}^2}\nabla_i(\Phi-2\beta\varphi)+2\Psi\frac{\nabla_i\Psi}{\mathcal{H}^2}-v_j\nabla_jv_i\\&\quad+v_i(2\Phi'+\Psi')+2\beta'\frac{\varphi}{x\mathcal{H}^2}\nabla_i\varphi,\end{aligned} \tag{89}$$

where the second line of each equation contains the terms from the variation of $\beta(\phi)$. The scalar field equation is

$$\begin{aligned}&\left[\varphi''+\left(2+\frac{\mathcal{H}'}{\mathcal{H}}\right)\varphi'-2y^2\mu f_1\Psi\right](1-2\Psi)\\&\quad+2y^2\mu^2\varphi(f_2+\mu f_3\varphi)+\varphi'(3\Phi'+\Psi')\\&\quad-\Phi'x(3+6\Phi-6\Psi)-\Psi'x(1-4\Psi)\\&\quad-(1+2\Phi)\frac{\nabla^2}{\mathcal{H}^2}\varphi+\frac{1}{\mathcal{H}^2}(\nabla_i\varphi)\nabla_i(\Phi-\Psi)\\&=\beta\Omega_m[2\Psi(1-2\Psi)+\delta-\mathcal{H}^2v^2]\\&\quad+\frac{\varphi}{x}\Omega_m\beta'(1+\delta_c)+\frac{\varphi^2}{2x^2}\Omega_m\left(\beta''-\beta'\frac{\phi''}{\phi'}\right),\end{aligned} \tag{90}$$

where f_3 is defined in Eq. (13).

Let us now derive the Newtonian limit. We can use the metric equations at first order, in particular the relation $\Phi=\Psi$ since, as before, Φ and φ are of order λ^2 (with respect to δ). Again as before, we neglect the time derivatives of φ. We obtain from Eq. (89)

$$\delta'+\nabla_i(1+\delta)v_i=2v_i\nabla_i\Phi-H^2v^2(1-2\beta x),$$

$$\begin{aligned}v_i'+\left(1+\frac{\mathcal{H}'}{\mathcal{H}}-2\beta x\right)v_i&=-\frac{1}{\mathcal{H}^2}\nabla_i(\Phi-2\beta\varphi)\\&\quad-\frac{1}{1+\delta}v_j\nabla_jv_i.\end{aligned} \tag{91}$$

Furthermore, in the first equation we can neglect the term $H^2v^2(1-2\beta x)$ because of the nonrelativistic approximation and the term $2v_i\nabla_i\Phi$ because both v_i and $\nabla_i\Phi$ are of order λ at first order. Finally, in the second equation we can approximate $1+\delta\approx1$ at the denominator in the last term, since it gives a third order correction. Therefore we are left with

$$\delta'+\nabla_i(1+\delta)v_i=0, \tag{92}$$

$$\begin{aligned}v_i'+\left(1+\frac{\mathcal{H}'}{\mathcal{H}}-2\beta x\right)v_i&=-\frac{1}{\mathcal{H}^2}\nabla_i(\Phi-2\beta\varphi)\\&\quad-v_j\nabla_jv_i.\end{aligned} \tag{93}$$

Applying the same approximations for the scalar field we obtain

$$\begin{aligned}&\varphi''+\left(2+\frac{\mathcal{H}'}{\mathcal{H}}\right)\varphi'-2y^2\mu f_1\Phi-2\beta\Omega_m\Phi-\frac{\nabla^2}{\mathcal{H}^2}\varphi-4\Phi'x\\&\quad+2y^2\mu^2f_2\varphi+2y^2\mu^3f_3\varphi^2\\&\quad=\beta\Omega_m\delta+\frac{\varphi}{x}\Omega_m\beta'(1+\delta)+\frac{\varphi^2}{2x^2}\Omega_m\left(\beta''-\beta'\frac{\phi''}{\phi'}\right),\end{aligned} \tag{94}$$

which neglecting Φ and the time derivatives of φ reduces to the nonlinear Klein-Gordon equation

$$\nabla^2\varphi-m^2\varphi-\sigma_a\varphi^2+\sigma_b\varphi\delta=-\beta\Omega_m\delta\mathcal{H}^2, \tag{95}$$

where we defined the nonlinear correction coefficients

$$\sigma_a=2y^2\mu^3f_3\mathcal{H}^2+\frac{\Omega_m}{2x^2}\left(\beta''-\beta'\frac{\phi''}{\phi'}\right)\mathcal{H}^2, \tag{96}$$

$$\sigma_b=x^{-1}\Omega_m\beta'\mathcal{H}^2. \tag{97}$$

If m and $\sigma_{a,b}$ are negligible with respect to the length scale λ then we see that the nonlinear conservation equations in coupled dark energy coincide with the usual nonlinear Newtonian perturbation equations with an effective potential $\hat{\Phi}$ and a correction in the Euler equation due to the time variation of the dark matter mass

$$\delta'+\nabla_i(1+\delta)v_i=0, \tag{98}$$

$$v_i'+\left(1+\frac{\mathcal{H}'}{\mathcal{H}}-2\beta x\right)v_i+v_j\nabla_jv_i=-\frac{1}{\mathcal{H}^2}\nabla_i\hat{\Phi}, \tag{99}$$

$$\nabla^2\hat{\Phi} = 4\pi G_{mm}\rho_m\delta. \tag{100}$$

If $\sigma_a\varphi^2, \sigma_b\varphi\delta$ are negligible but $m^2\varphi$ is not then one should use the Yukawa correction to G_{mm} [in this case one should also assume that the coefficients of the terms in Φ, Φ' in Eq. (95) are smaller than $\hat{m}^2$]. As has been shown, Eqs. (98)–(100) are also valid when β is a function of ϕ.

VII. CONCLUSIONS

This paper is meant to set up the formalism for future work on nonlinear properties of dark energy, with an emphasis on its coupling to matter. We found several results that we summarize here.

(a) We derived the general equations for the linear perturbation growth for a general dark energy potential and a general species-dependent interaction with matter. This generalizes previous work.

(b) We derived an analytical relation between the bias induced by a species-dependent coupling and the growth exponent of the linear perturbations, as well as their values in term of the fundamental parameters in the case of an exponential potential.

(c) We discussed the Yukawa correction to the gravitational potential due to dark energy interaction and we found that a density-dependent effective dark energy mass arises only if the coupling is nonconstant. The consequence of this effect on equivalence principle experiments will be discussed in another paper.

(d) We derived the second-order perturbation equations in coupled dark energy and their Newtonian limit. We showed that the coupling introduces three corrections to the standard Newtonian fluid equations, one proportional to the velocity and the others which can be absorbed in the gravitational potential. These equations will be used in a subsequent paper to derive the large scale skewness of coupled dark energy.

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