Cosmology and two-body problem of D-branes

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In this paper, we investigate the dynamics and the evolution of the scale factor of a probe Dp-brane which moves in the background of source Dp-branes. The action of the probe brane is described by the Born-Infeld action and the interaction with the background Ramond-Ramond field. When the probe brane moves away from the source branes, it expands as a power law whose index depends on the dimension of the brane. If the energy density of the gauge field on the brane is subdominant, the expansion decelerates irrespective of the dimension of the brane. On the other hand, when the probe brane is a Nambu-Goto brane, the energy density of the gauge field can be dominant, in which case accelerating expansion occurs for $p \leqslant 4$. The accelerating expansion stops when the brane has expanded sufficiently that the energy density of the gauge field becomes subdominant.

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I. INTRODUCTION

With the discovery of D-branes, not only string theory but also cosmology has been activated significantly. The Randall-Sundrum braneworld model [1–3] is the simplest cosmological model which was induced by the idea of D-branes. In this model, the action of the brane is assumed to be the Nambu-Goto action. Cosmology with the Born-Infeld action has also been investigated in [4–6] and it was found that the behavior of a gauge field confined to the brane is significantly different from that of a gauge field added to the Nambu-Goto brane. Interaction between D-branes by the Ramond-Ramond (RR) charge, which is absent in the Randall-Sundrum model, has been studied by many authors as a potential energy source that inflates the brane [7–14]. For a review of cosmology in the context of string theory, see, for example, [15].

Since D-branes are a fundamental object in superstring theory, their two-body problem is also fundamental. Burgess *et al.* [16] studied the motion of a probe brane in the background spacetime of source branes and found that there exist bound states of a D6-brane and anti-D6-brane, which they called a "branonium." Probe-brane dynamics was also discussed in [17,18]. Recently, cosmology on the probe brane was studied in the context of a bouncing universe [19].

In this paper, we investigate the two-body problem and cosmology of D-branes. The basic approach is the same as [16,19] but we take into account a gauge field confined to the probe brane, which was neglected in [16,19]. The motion of the brane causes the time evolution of the induced metric on it, which is seen as cosmological expansion or contraction by an observer living on the brane. In this sense, our picture is similar to that of "mirage cosmology" [20–23]. Thus, by following the motion of the brane, we can also follow the evolution of the scale factor. We show that the gauge field on the probe brane, which has not been studied rigorously, can affect the behavior of the scale factor.

The rest of this paper is organized as follows. In Sec. II, we review the p-brane solutions in supergravity as the background spacetime of the source D-branes. We consider the motion of a probe brane in this background spacetime in Sec. III and follow the evolution of the scale factor on the probe brane in Sec. IV. In Secs. V and VI, we give a discussion and summary, respectively.

II. BACKGROUND SPACETIME

We consider a system in which a probe D-brane (or anti-D-brane) moves within the background of N parallel source D-branes. In this section, we review the p-brane solutions in supergravity as the background spacetime of the source D-branes. Low-energy effective theories for superstring theories are given by supergravities, among which we consider only types IIA and IIB here for simplicity. The effective actions include the metric, the two-form potential, and the scalar dilaton in the Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector, $(n-1)$-form gauge potentials in the RR sector, and Chern-Simons terms. Here n is even for type IIA and odd for type IIB.

To obtain a tractable system to study, we shall make a *consistent truncation* (see [24] and references therein) of the action down to a simple system comprising only the metric G_{MN}, the scalar dilaton ϕ, and a single $(n-1)$-form gauge potential $A_{[n-1]}$ with corresponding field strength $F_{[n]}$. Then the background spacetime of the source Dp-brane is determined by the following action in the Einstein frame:

$$S=\int D^D x\sqrt{-G}\left[R-\frac{1}{2}\partial^M\phi\partial_M\phi-\frac{1}{2n!}e^{a\phi}F_{[n]}^2\right], \quad (1)$$

where $D=10$ and $a=(5-n)/2$ is the dilaton coupling of the RR field. Assuming asymptotic flatness and spherical symmetry in the transverse directions, flatness of the branes, and an "electric" gauge field, the background spacetime and gauge field for $p \leqslant 6$ are given by

$$ds^2 = h^{-(7-p)/8}\eta_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/8}\delta_{mn}dy^m dy^n, \tag{2}$$

$$e^\phi = h^{(3-p)/4}, \tag{3}$$

$$A_{M_1M_2\cdots M_{p+1}} = \epsilon_{M_1M_2\cdots M_{p+1}}(1-h^{-1}), \tag{4}$$

where $x^\mu(\mu=0,1,\ldots,p)$ and $y^m(m=1,2,\ldots,D-p-1)$ are the coordinates parallel and transverse to the branes, respectively. We define the radial coordinate transverse to the brane as $r^2 \equiv \delta_{mn}y^m y^n$ and thus,

$$h(r) = 1 + \frac{k}{r^{7-p}}. \tag{5}$$

Here k is an integration constant which represent the energy scale of the source branes:

$$k = (2\sqrt{\pi})^{5-p}\Gamma\left(\frac{7-p}{2}\right)g_s l_s^{7-p}N, \tag{6}$$

where g_s is the string coupling constant at infinity, l_s is the string length scale, and N is the number of source branes. It should be noted that these solutions are reliable only for $r \gg l_s$. This is because supergravity is a good approximation of superstring theory only within this region, where the brane interactions are dominated by massless string states.

The asymptotic behaviors of the gravitational field and the gauge field potential can be understood in terms of Gauss's law. Both behave asymptotically like $\sim r^{-(7-p)}$, as expected from the Laplace equation,

$$\nabla^2 f(r) = \left[\frac{d^2}{dr^2} + \frac{8-p}{r}\frac{d}{dr}\right]f(r) = 0. \tag{7}$$

Thus, the potential produced by a D6-brane is, like that of a point particle in ordinary four-dimensional spacetime, $\sim r^{-1}$. For global structures of these solutions, see, for example, [25].

On the other hand, there is no asymptotically flat solution for $p \geq 7$. Hereafter we concentrate on the $p \leq 6$ cases.

III. DYNAMICS OF PROBE BRANE

In this section we consider the motion of a probe brane, which is assumed to be parallel to the source branes, in the background spacetime discussed in the previous section. The dynamics of the probe brane which has "electric" charge is determined by the Born-Infeld action (in the string frame),

$$S_{\mathrm{BI}} = -T_p \int d^{p+1}x e^{-\phi}\sqrt{-\det(g_{\mu\nu} + 2l_s^2 F_{\mu\nu})}, \tag{8}$$

and the interaction with the background gauge field $A_{[p+1]}$,

$$S_{\mathrm{WZ}} = -qT_p \int A_{[p+1]}. \tag{9}$$

Here $g_{\mu\nu}$ is the induced metric on the probe brane, $F_{\mu\nu}$ is the $U(1)$ gauge field strength confined to the brane, and q is the RR charge of the brane, which equals ± 1 for the D-brane and anti-D-brane, respectively. Note that the field strength $F_{\mu\nu}$ should be thermal in nature in order not to break the isotropy of the brane. Therefore, we interpret that $F_{\mu\nu}F^{\mu\nu} \to \langle F_{\mu\nu}F^{\mu\nu}\rangle$, etc. [26]. The induced metric on the brane in the string frame is written as

$$\begin{aligned} d\tilde{s}^2 &= e^{4\phi/(D-2)}ds^2 \\ &= -h^{-1/2}(1-hv^2)dt^2 + h^{-1/2}\delta_{ij}dx^i dx^j, \end{aligned} \tag{10}$$

where we took the static gauge, $t = x^0$, and $i,j = 1,2,\ldots,p$. Here we defined the velocity v of the brane as

$$v^2 \equiv \delta_{mn}\frac{dy^m}{dt}\frac{dy^n}{dt}. \tag{11}$$

Thus the motion of the brane in the dimensions transverse to the brane is described in terms of the radial coordinate r and the velocity v.

Due to the spherical symmetry in the transverse direction, the angular momenta of the brane are conserved. This shows that the motion is confined to the plane that is spanned by the initial position and momentum vectors. We will denote the polar coordinate in this plane by r and θ. Further, due to technical difficulty, we treat the gauge field as a perturbation and consider the leading term. Then the total Lagrangian of the probe brane is

$$L = -mh^{-1}[\sqrt{1-h(\dot{r}^2 + r^2\dot{\theta}^2)}(1 + l_s^4 F_{\mu\nu}F^{\mu\nu}) - q], \tag{12}$$

where we have neglected an additive constant, $m = T_p \int d^p x$ is the "mass" of the brane, and the overdot denotes a derivative with respect to t. The independent variables are r, θ, and the gauge potential A_μ on the brane. The canonical momenta associated with these variables are

$$\begin{aligned} p_r &\equiv m^{-1}\frac{\partial L}{\partial \dot{r}} \\ &= \frac{\dot{r}}{\sqrt{1-h(\dot{r}^2+r^2\dot{\theta}^2)}}(1 + l_s^4 F_{\mu\nu}F^{\mu\nu}), \end{aligned} \tag{13}$$

$$\begin{aligned} l &\equiv m^{-1}\frac{\partial L}{\partial \dot{\theta}} \\ &= \frac{r^2\dot{\theta}}{\sqrt{1-h(\dot{r}^2+r^2\dot{\theta}^2)}}(1 + l_s^4 F_{\mu\nu}F^{\mu\nu}), \end{aligned} \tag{14}$$

$$\begin{aligned} p_A^i &\equiv m^{-1}\frac{\partial L}{\partial \dot{A}_i} \\ &= -\frac{4\sqrt{1-h(\dot{r}^2+r^2\dot{\theta}^2)}}{h}F^{i0}, \end{aligned} \tag{15}$$

where p_A^i is also conserved as we can see from the Euler-Lagrange equation. Thus the "electric field" F^{i0} can be written in terms of the other variables. On the other hand, the "magnetic field" F^{ij} is obtained from the Bianchi identity

$$\partial_\mu F_{\nu\lambda}+\partial_\nu F_{\lambda\mu}+\partial_\lambda F_{\mu\nu}=0 \tag{16}$$

as

$$F_{ij}=C_{ij}=\text{const.} \tag{17}$$

Combining the above results, it follows that

$$\begin{aligned}F_{\mu\nu}F^{\mu\nu}&=\left(\delta^{ik}\delta^{jl}C_{ij}C_{kl}-\frac{\delta_{ij}p_A^i p_A^j}{8}\right)h\\ &\equiv C'h,\end{aligned} \tag{18}$$

where C' is a constant which represents the energy scale of the gauge field.

From Eqs. (13) and (14), we obtain the following useful relation:

$$\dot{r}^2+r^2\dot{\theta}^2=\frac{p_r^2+l^2/r^2}{(1+Ch)^2+h(p_r^2+l^2/r^2)}, \tag{19}$$

where $C\equiv C' l_s^4$ is a dimensionless constant which represents the energy scale of the gauge field in units of l_s^{-1}. Then the Hamiltonian can be written as

$$\begin{aligned}E&\equiv p_r\dot{r}+l\dot{\theta}+p_A^i\dot{A}_i-m^{-1}L\\ &=\frac{\{1+(4D+C)h\}(1+Ch)+h(p_r^2+l^2/r^2)}{h\sqrt{(1+Ch)^2+h(p_r^2+l^2/r^2)}}-\frac{q}{h},\end{aligned} \tag{20}$$

which gives the conserved energy. Here we took the gauge $A_0=0$ and $D\equiv\delta_{ij}p_A^i p_A^j/16$. Note that this agree with (2.22) of [16] in the limit of no gauge field, $C,D\to 0$. Hereafter, we set $D=0$ for simplicity, which means that there is only a magnetic field. From Eq. (20), we expect that the dynamics does not change very much even if there are both electric and magnetic fields.

Following [16], we define the effective potential V_{eff} for the radial motion as

$$\begin{aligned}V_{\mathrm{eff}}(r)&\equiv E(p_r=0)\\ &=h^{-1}\left[\sqrt{(1+Ch)^2+hl^2/r^2}-q\right].\end{aligned} \tag{21}$$

The asymptotic behavior depends on the charge and the dimension of the brane. For $p=6$,

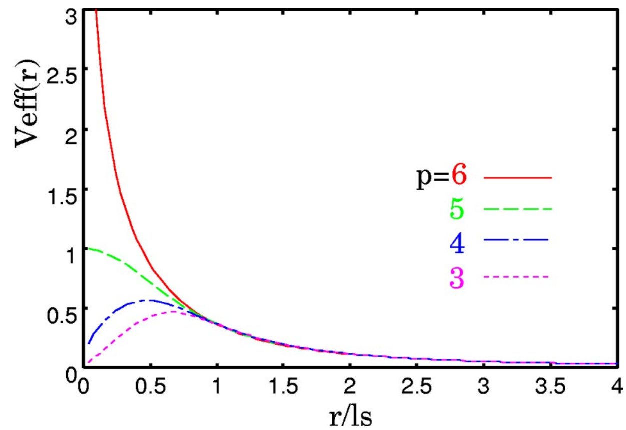

FIG. 1. Effective potential V_{eff} for the radial motion of the probe brane, varying its spatial dimension p. Other parameters are set as $k=l=1$ and $C=0$.

$$V_{\mathrm{eff}}(r)\to\begin{cases}\dfrac{l}{\sqrt{k}}r^{-1/2} & \text{for } r\to 0,\\ 1+C-q+k(q-1)r^{-1} & \\ \quad+\left[\dfrac{l^2}{2(1+C)}-k(q-1)\right]r^{-2} & \text{for } r\to\infty.\end{cases} \tag{22}$$

For $p=5$,

$$V_{\mathrm{eff}}(r)\to\begin{cases}\dfrac{l}{\sqrt{k}}-\left(\dfrac{l^2}{2k\sqrt{k}}+\dfrac{q}{k}\right)r^2 & \text{for } r\to 0,\\ 1+C-q+\left[\dfrac{l^2}{2(1+C)}+k(q-1)\right]r^{-2} & \\ & \text{for } r\to\infty.\end{cases} \tag{23}$$

For $p\leqslant 4$,

$$V_{\mathrm{eff}}(r)\to\begin{cases}\dfrac{l}{\sqrt{k}}r^{(5-p)/2} & \text{for } r\to 0,\\ 1+C-q+\dfrac{l^2}{2(1+C)}r^{-2} & \text{for } r\to\infty.\end{cases} \tag{24}$$

As is pointed out in [16], there exist stable bound orbits in the case of the anti-6-brane.

The behavior of the effective potential is shown in Figs. 1, 2, and 3. Figures 1 and 2 show the effective potential of the p-brane and anti-p-brane for various p, respectively. From this, we can see that there can be a stable bound state in the case of the anti-6-brane, as is expected. Note that the position of the potential minimum, r_{min}, depends on the angular momentum l, and r_{min} can be much larger than l_s if l is sufficiently large. Figure 3 shows the effective potential of the 6-brane for various C. As can be seen, the qualitative features do not depend on C.

Using Eqs. (13), (14), and (20), $\dot{r}$ can be expressed in terms of r,E,l:

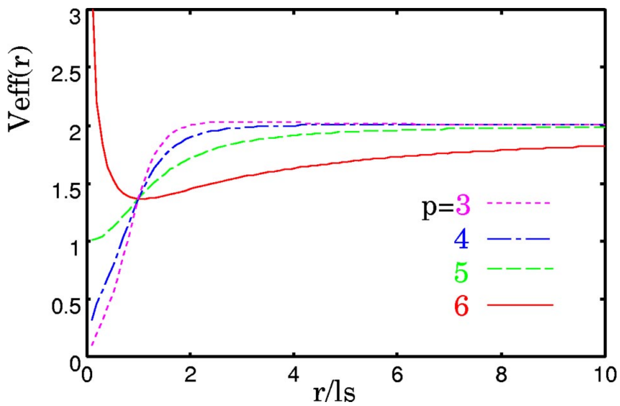

FIG. 2. Effective potential $V_{\rm eff}$ for the radial motion of the probe antibrane, varying its spatial dimension p. Other parameters are set as $k=l=1$ and $C=0$.

$$\dot{r}^2=h^{-1}\left[1-\frac{r^2(1+Ch)^2+hl^2}{r^2(Eh+q)^2}\right]. \tag{25}$$

We can follow the motion of the brane by integrating this equation. Since, as can be seen from Eq. (10), the scale factor on the brane is a function of r, its evolution can also be calculated from this equation as we discuss in the next subsection. Note that this equation corresponds to the Friedmann equation and that this reduces to the Friedmann equation in [19] in the limit of $C\to 0$.

The brane trajectory can be calculated as follows: define $u\equiv 1/r$ and thus,

$$u'\equiv\frac{du}{d\theta}=-r^{-2}\frac{dr}{d\theta}=-r^{-2}\frac{\dot{r}}{\dot{\theta}}=-\frac{p_r}{l}. \tag{26}$$

Eliminating p_r from Eq. (20) using this equation, we obtain,

$$E=h^{-1}[\sqrt{(1+Ch)^2+hl^2(u^2+u'^2)}-q], \tag{27}$$

from which the orbit is obtained as

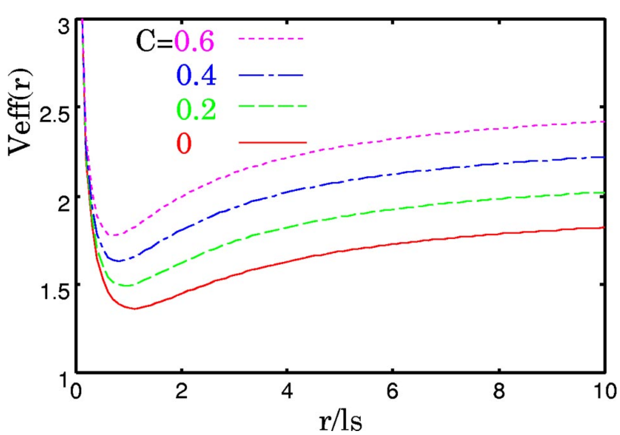

FIG. 3. Effective potential $V_{\rm eff}$ for the radial motion of the probe anti-6-brane, varying the energy scale C of the gauge field on it. Other parameters are set as $k=l=1$.

$$\theta-\theta_0=\int_{1/r_0}^{1/r}\frac{du}{\sqrt{A+Bu^{7-p}-u^2}}, \tag{28}$$

where

$$A=l^{-2}(E^2+2Eq-C^2-2C), \tag{29}$$

$$B=E^2-C^2. \tag{30}$$

Thus the orbit of the probe brane is equivalent to that of a classical nonrelativistic particle in the central potential proportional to r^{p-7}, even when there exists a gauge field on the brane. In particular, for $p=6$, the bound orbit is closed.

IV. COSMOLOGY ON PROBE BRANE

A. Evolution of scale factor

From the induced metric on the brane Eq. (10), the scale factor a is given by

$$a=h^{-1/4}. \tag{31}$$

On the other hand, the cosmological time τ on the brane is expressed as

$$\begin{aligned}\tau&\equiv\int h^{-1/4}\sqrt{1-h(\dot{r}^2+r^2\dot{\theta}^2)}dt\\&=\int h^{-1/4}\frac{1+Ch}{Eh+q}\dot{r}^{-1}dr\\&=\int h^{1/4}\frac{1+Ch}{\sqrt{(Eh+q)^2-(1+Ch)^2-hl^2/r^2}}dr.\end{aligned} \tag{32}$$

Here we used Eqs. (19) and (20) in the second equation and Eq. (25) in the last equation. Thus, from Eqs. (31) and (32), the scale factor a can be obtained as a function of τ.

Here we define two characteristic radii: the gravitational radius $r_{\rm g}$ and gauge-field radius r_c. The former corresponds to the Schwarzschild radius,

$$r_{\rm g}\equiv k^{1/(7-p)}. \tag{33}$$

It should be noted that

$$h(r)\approx\begin{cases}k/r^{7-p} & \text{for } r\ll r_{\rm g},\\ 1 & \text{for } r\gg r_{\rm g}.\end{cases} \tag{34}$$

The latter represents the radius, below which the approximation of the Lagrangian (12) breaks down [$l_{\rm s}^4F_{\mu\nu}F^{\mu\nu}=Ch(r_{\rm c})=1$]:

$$\begin{aligned}r_{\rm c}&\equiv\left(\frac{Ck}{1-C}\right)^{1/(7-p)}\\&\approx(Ck)^{1/(7-p)}.\end{aligned} \tag{35}$$

Hereafter we consider the case $r_g \gg l_s$ because otherwise the scale factor does not change very much in the region where the background solution is reliable ($r \gg l_s$).

Then let us consider the situation where the probe brane goes away from the neighborhood of the source branes (but $r=r_0 \gg l_s, r_c$, of course) to infinity. When $r \ll r_g$, the relation between the scale factor and the cosmological time is simple. In this case Eq. (32) becomes, noting that $h \gg 1$ and $Ch < 1$,

$$\tau \approx \int_{r_0}^{r} h^{1/4} \frac{1+Ch}{Eh} dr$$

$$\approx E^{-1} k^{-3/4} \int_{r_0}^{r} r^{3(7-p)/4} (1 + Ckr^{p-7}) dr. \tag{36}$$

Note that this is independent of q in this limit. First we consider the case $C=0$. Then,

$$\tau = \frac{4}{25-3p} E^{-1} k^{-3/4} (r^{(25-3p)/4} - r_0^{(25-3p)/4}). \tag{37}$$

At late time ($r \gg r_0$), we obtain

$$\tau \propto r^{(25-3p)/4}, \tag{38}$$

from which the evolution of the scale factor is obtained as

$$a(\tau) = h^{-1/4} \approx (kr^{p-7})^{-1/4}$$

$$\propto \tau^{(7-p)/(25-3p)}. \tag{39}$$

Here $(7-p)/(25-3p) = 1/7, 1/5, 3/13, 1/4, 5/19, 3/11$ for $p = 6, 5, \ldots, 1$. Although the expansion becomes faster with smaller p, the acceleration phase cannot be realized.

If $C \neq 0$, a correction term is added,

$$a(\tau) \propto [\tau^{(7-p)/(25-3p)} - ACa_1(\tau)], \tag{40}$$

where A is a constant which depends on E, k, p, and a_1 is, to leading terms,

$$a_1(\tau) = \begin{cases} \tau^{-3(7-p)/(25-3p)} & \text{for } p \geq 4, \\ \tau^{-3/4} \log \tau & \text{for } p = 3, \\ \tau^{-1} (r_0^{-(3-p)/4} - B\tau^{-16/(3-p)(25-3p)}) & \text{for } p \leq 2, \end{cases} \tag{41}$$

where B is also a constant which depends on E, k, p. It should be noted that the effect of the gauge field decreases as the brane expands since its energy density decreases as $h \propto a^{-4}$.

When r becomes much larger than r_g, the scale factor stops to evolve and becomes almost unity. The behavior of the scale factor in the case of no gauge field is shown in Fig. 4. As is expected, the scale factor evolves as a power law and then decelerates quickly to become unity. Figure 5 shows the effect of the gauge field on the brane. As can be seen, the effect is very small even if C is as large as possible and the late-time behavior is independent of the existence of the gauge field.

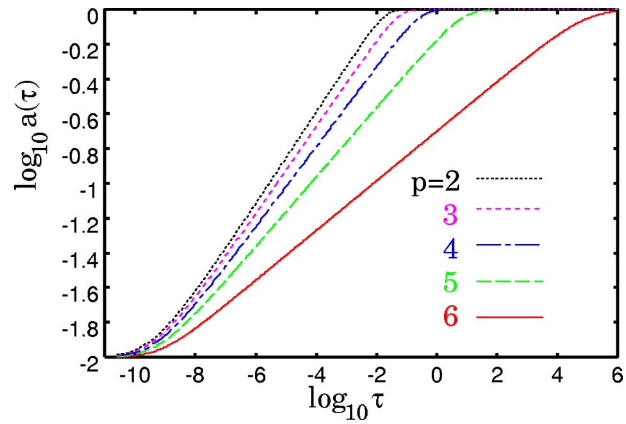

FIG. 4. Evolution of the scale factor $a(\tau)$ without the gauge field on the brane for various p. Other parameters are set as $k = 10^8$, $E = 10^3$, $l = 10$, $q = -1$, $r_0 = 1$.

We can also see the evolution of the scale factor by the effective Friedmann equation which can be derived from Eq. (25):

$$a^{-2}\left(\frac{da}{d\tau}\right)^2 = \frac{(7-p)^2}{16} k^{-2/(7-p)} a^{-2(11-p)/(7-p)}$$
$$\times (1-a^4)^{2(8-p)/(7-p)} (1+Ca^{-4})^{-2}$$
$$\times [(E^2 - C^2) a^{-4} + 2(Eq - C)$$
$$- l^2 k^{-2/(7-p)} a^{-8/(7-p)} (1-a^4)^{2/(7-p)}], \tag{42}$$

which agrees with [20] in the limit of $C \to 0$.

B. High energy limit

Here we consider the probe brane to be a Nambu-Goto brane with a gauge field and the same RR charge as a D-brane, for which the Lagrangian (12) is exact. In this case,

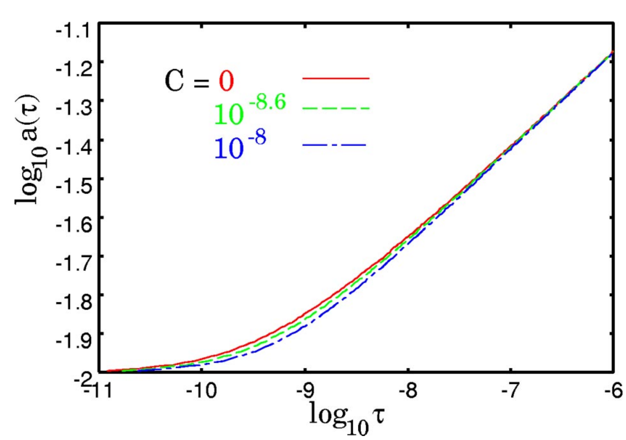

FIG. 5. Evolution of the scale factor $a(\tau)$ with the gauge field on the brane for various C. Other parameters are set as $p = 4$, $k = 10^8$, $E = 10^3$, $l = 10$, $q = -1$, $r_0 = 1$.

we can take the high-energy limit ($Ch \gg 1$). Although, for a D-brane, this limit is in contradiction to the approximation which we used to derive Eq. (12), we could still obtain the tendency to the high-energy effect, as is often done in higher-derivative theory. In this regime, Eq. (32) for $r \ll r_g$ becomes

$$\tau \approx \int_{r_0}^{r} h^{1/4} \frac{C}{\sqrt{E^2-C^2}} dr$$
$$\approx \frac{Ck^{1/4}}{\sqrt{E^2-C^2}} \int_{r_0}^{r} r^{(p-7)/4} dr. \tag{43}$$

For $p \geq 4$,

$$\tau = \frac{4}{p-3} \frac{Ck^{1/4}}{\sqrt{E^2-C^2}} (r^{(p-3)/4} - r_0^{(p-3)/4})$$
$$\xrightarrow{r \gg r_0} \propto r^{(p-3)/4}; \tag{44}$$

then

$$a(\tau) \propto \tau^{(7-p)/(p-3)}, \tag{45}$$

where $(7-p)/(p-3) = 1/3, 1, 3$ for $p = 6, 5, 4$. Thus accelerating expansion is realized for $p = 4$. Next, for $p = 3$,

$$\tau = \frac{Ck^{1/4}}{\sqrt{E^2-C^2}} \log \frac{r}{r_0}; \tag{46}$$

then,

$$a(\tau) = k^{-1/4} r_0 \exp\left(\frac{\sqrt{E^2-C^2}}{Ck^{1/4}} \tau \right). \tag{47}$$

Thus the scale factor increases exponentially. Finally, for $p \leq 2$,

$$\tau = \frac{4}{3-p} \frac{Ck^{1/4}}{\sqrt{E^2-C^2}} (r_0^{-(3-p)/4} - r^{-(3-p)/4}); \tag{48}$$

then,

$$a(\tau) = k^{-1/4} \left(r_0^{-(3-p)/4} - \frac{3-p}{4} \frac{\sqrt{E^2-C^2}}{Ck^{1/4}} \tau \right)^{-(7-p)/(3-p)}. \tag{49}$$

It can be easily shown that the expansion is accelerating in this case. These analyses are confirmed in Fig. 6.

As stated in the previous subsection, the energy density of the gauge field decreases as the brane expands. With the parametrization in Fig. 6, the gauge field is dominant for the whole evolution since C is sufficiently large so that Ch at infinity is still large [$Ch(r=\infty) = C = 1$]. If C is smaller than unity, the late phase will behave like that of the case discussed in the previous subsection, even if accelerating expansion occurs in the early phase. In Fig. 7, we show the cases with intermediate C. We can see the transition from the accelerating phase to the decelerating phase. Of course, the transition occurs earlier with smaller C.

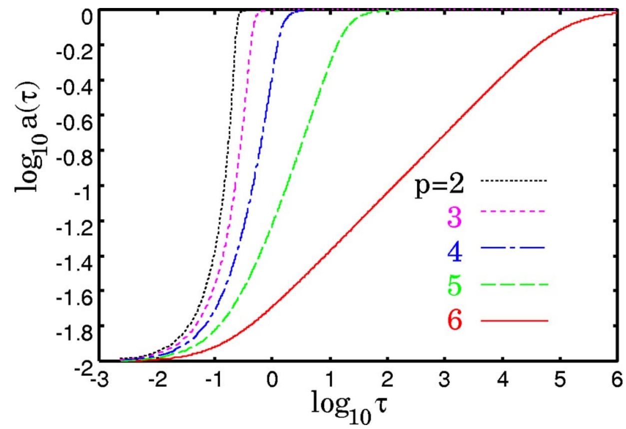

FIG. 6. Evolution of the scale factor $a(\tau)$ of the brane dominated by the gauge field for various p. Other parameters are set as $k = 10^8$, $E = 10^3$, $l = 10$, $q = -1$, $r_0 = 1$, $C = 1$.

C. Einstein frame

Finally, we give the evolution of the scale factor in the Einstein frame. The procedure is almost the same as in the string frame. The induced metric in the Einstein frame is

$$ds^2 = -h^{-(7-p)/8}(1-hv^2)dt^2 + h^{-(7-p)/8} \delta_{ij} dx^i dx^j. \tag{50}$$

Then the cosmological time is

$$\tau = \int_{r_0}^{r} h^{-(7-p)/16} \sqrt{1-hv^2} dr. \tag{51}$$

With $C = 0$, the scale factor evolves as, for $r \gg r_g$,

$$a(\tau) = h^{-(7-p)/16} \propto \tau^{(7-p)^2/(11-p)^2}, \tag{52}$$

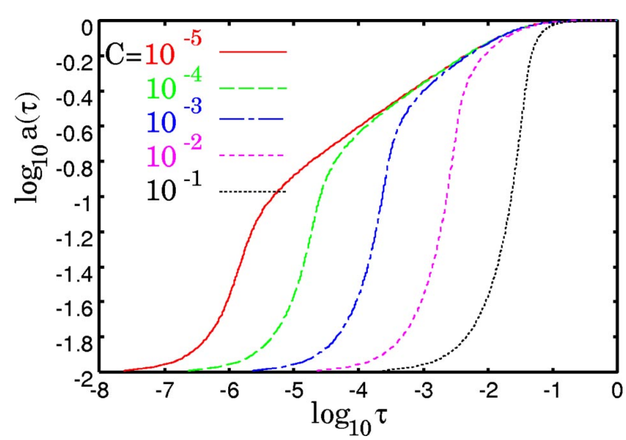

FIG. 7. Evolution of the scale factor $a(\tau)$ of the brane dominated by the gauge field for various C. Other parameters are set as $p = 3$, $k = 10^8$, $E = 10^3$, $l = 10$, $q = -1$, $r_0 = 1$.

where the index is $(7-p)^2/(11-p)^2 = 1/25,1/9,9/49,1/4,25/81,9/25$ for $p=6,5,\ldots,1$. In the high-energy limit $(Ch \gg 1)$, for $p \neq 3$,

$$a(\tau) \propto t^{(7-p)^2/(3-p)^2}, \tag{53}$$

where $(7-p)^2/(3-p)^2 = 1/9,1,9,25,9$ for $p=6,5,4,2,1$. For $p=3$,

$$a(\tau) \propto \exp\left(\frac{\sqrt{E^2-C^2}}{Ck^{1/4}}\tau\right). \tag{54}$$

Thus, the condition that accelerating expansion occurs is the same as in the string frame. It should be noted that the induced metrics in the string frame and the Einstein frame coincide with each other for $p=3$ because the dilaton (3) is constant in this case.

V. DISCUSSION

In the previous section, we dealt with a simple situation that a probe brane goes away from the neighborhood of the source branes to infinity. If the probe brane approaches the source branes, the scale factor decreases as the inverse of that in the previous section. Then the other situations, for example, scattering and bound state of branes, are easy to imagine. In the former case, the brane contracts first, then bounces and finally expands. In the latter case, the brane continues to expand and contract periodically.

In this paper, we followed the dynamics of a probe brane, that is, we neglected the back reaction. This is justified if the probe brane is light compared to the source branes. This means $N \gg 1$, which we assumed in the analyses in Sec. IV. If $N \sim 1$, we have to treat both branes equally and the self-gravity of the branes must be taken into account [27,28].

Our analysis assumes stability of the probe brane. There are possible instabilities due to brane bending and radiation from the brane [16]. Reference [16] gave a preliminary analysis of such instabilities. They found, for $p=6$, that the brane is stable classically against bending and that the radiation is dominated by the one into the bulk dilation field, which can be made sufficiently small by appropriate choice of the string coupling constant. One of the key assumptions they made is to treat the brane motion as nonrelativistic. In other words, their results have been obtained in the large-separation limit: $k/r^{7-p} \ll 1$ (or $r \gg r_g$ in our notation). We obtain some of the expressions for the scale factor for $r \ll r_g$ where the relativistic treatment is necessary in the strict sense but we expect that the relativistic corrections will not change the stability discussed in [16]. Recently, it was shown in [29] that time variations in the background moduli fields generally preclude the existence of stable elliptical orbits.

Finally, although our study is based on the approximated Lagrangian (12), it would be quite interesting and important to study the exact Lagrangian. This will be our future work.

VI. SUMMARY

In this paper, we investigated the evolution of the scale factor of a probe Dp-brane which moves in the background of source Dp-branes. When the probe brane moves away from the source branes, it expands as a power law, whose index depends on the dimension of the brane. If the energy density of the gauge field on the brane is subdominant, the expansion is decelerating irrespective of the dimension of the brane. On the other hand, when the probe brane is a Nambu-Goto brane, the energy density of the gauge field can be dominant, in which case accelerating expansion occurs for $p \leqslant 4$. The accelerating expansion stops when the brane has expanded sufficiently so that the energy density of the gauge field becomes subdominant. Although this is not the case with a probe D-brane, we could still obtain the tendency to a high-energy effect of the Born-Infeld action.

The system which is investigated in this paper is too simple to be our universe. However, further investigation will give understanding of the relation between superstring theory and our universe.

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