Neutralino pair production and three-body decays at e^+e^- linear colliders as probes of *CP* violation in the neutralino system

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In the *CP*-invariant supersymmetric theories, the steep *S*-wave (slow *P*-wave) rise of the cross section for any nondiagonal neutralino pair production in e^+e^- annihilation, $e^+e^-\to\tilde{\chi}^0_i\tilde{\chi}^0_j$ $(i\neq j)$, near threshold is accompanied by the slow *P*-wave (steep *S*-wave) decrease of the fermion invariant mass distribution of the three-body neutralino decay, $\tilde{\chi}^0_i\to\tilde{\chi}^0_j f\overline{f}$ $(f=l$ or $q)$, near the end point. These selection rules, unique to the neutralino system due to its Majorana nature, guarantee that the observation of simultaneous sharp *S*-wave excitations of the production cross section near threshold and the lepton or quark invariant mass distribution near the end point is qualitative, unambiguous evidence for *CP* violation in the neutralino system.

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I. INTRODUCTION

Most supersymmetric extensions of the standard model (SM) based on some soft supersymmetry (SUSY) breaking mechanism contain several *CP* phases, whose large values tend to render lepton and quark electric dipole moments (EDMs) too large to satisfy stringent experimental constraints [1]. Such *CP* crises are generic in supersymmetric theories, but may be resolved by pushing the masses of some sparticles, especially the first and second generation sfermions, above a few TeV, by arranging for internal cancellations or by simply setting phases to be extremely small [2]. On the other hand, new sources of *CP* violation beyond the SM are required to explain the non-zero baryon asymmetry in the Universe in the standard big bang framework [3]. Therefore, it is crucial to look for new signatures of *CP* violation in such SUSY scenarios with some large phases, as long as they are consistent with the stringent EDM and other low-energy constraints. In this light, detailed analyses of the neutralino sector at future e^+e^- linear collider experiments [4] can prove particularly fruitful [5–9], because in most supersymmetric theories neutralinos belong to the class of light supersymmetric particles [10] and the neutralino system contains two non-trivial *CP* violating phases.

There are many different ways for probing *CP* violation in the neutralino system. The imaginary parts of the complex parameters in the neutralino mass matrix could most directly and unambiguously be determined by measuring suitable *CP* violating observables by exploiting initial beam polarization and angular correlations between neutralino production and decay at future high-energy colliders [6–9]. But their experimental measurements will be quite difficult. The presence of *CP* violating phases can also be identified through by their impact on *CP*-even quantities such as neutralino masses, branching ratios and so on. However, since these quantities are already non-zero in the *CP* conserving case, the detection of the presence of non-trivial *CP* phases will require a careful quantitative analysis of a number of physical observables, especially for small *CP*-odd phases giving rise to very small deviations from the *CP*-conserving values [1]. On the other hand, the rise of excitation curves near threshold for non-diagonal neutralino pair production in e^+e^- collisions is altered qualitatively in *CP*-noninvariant theories [5,6] by allowing a steep *S*-wave increase of all pairs simultaneously. Thus, as demonstrated in Ref. [6], precise measurements of the threshold behavior of the non-diagonal neutralino pair production processes may give clear indications of non-zero *CP* violating phases in the neutralino sector if at least three different neutralino states are accessible kinematically.

In the present paper we provide a new powerful method for probing *CP* violation in the neutralino system, which is based on a combined analysis of the threshold excitations of neutralino pair production in e^+e^- annihilation and the fermion invariant mass distribution near the end point of three-body neutralino fermionic decays:

$$e^+e^-\to\tilde{\chi}^0_i\tilde{\chi}^0_j \quad (i\neq j) \ \text{ and } \ \tilde{\chi}^0_i\to\tilde{\chi}^0_j f\overline{f} \ (f=l,q).$$

(The three-body decay process includes clean $\mu^+\mu^-$ and e^+e^- decay channels with little background, which allow a clear reconstruction of the kinematical configuration with good precision.) This method relies on selection rules, unique to the neutralino system due to its Majorana nature in *CP*-invariant theories, and it can work effectively if the branching ratios of the three-body neutralino fermionic decays are not suppressed. (Once two-body decays of the neutralino $\tilde{\chi}^0_i$ into *Z*, Higgs bosons or sfermions are open, the new method is ineffective.)

Before demonstrating the new method for probing *CP* violation in the neutralino system in detail, we describe briefly the mixing for neutral gauginos and Higgsinos in *CP*-noninvariant theories with non-vanishing phases in Sec. II. In Sec. III we introduce the selection rules for the production of neutralino pairs and the neutralino to neutralino transition via a (virtual) vector boson or sfermion exchange. Then, we prove that in any *CP*-invariant SUSY theory, if the production cross section for any non-diagonal neutralino pair in e^+e^- annihilation increases steeply in *S* waves (slowly in *P* waves) near threshold, the lepton or quark invariant mass distribution of the decay $\tilde{\chi}^0_i\to\tilde{\chi}^0_j f\overline{f}$ $(f=l$ or $q)$ should decrease slowly in *P* waves (steeply in *S* waves) near the end point. Thus, observation of simultaneous sharp *S*-wave excitations of both the production of any non-diagonal neutralino pair $\tilde{\chi}_i^0\tilde{\chi}_j^0$ near threshold and the fermion invariant mass distribution of the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}$ near the end point will be qualitative, unambiguous evidence for *CP* violation in the neutralino system. A quantitative demonstration of the method based on a specific set of relevant SUSY parameters is given in the last part of Sec. III. Finally, conclusions are drawn in Sec. IV.

II. NEUTRALINO MIXING

In the minimal supersymmetric extension of the standard model (MSSM), the mass matrix of the spin-1/2 partners of the neutral gauge bosons, $\tilde{B}$ and $\tilde{W}^3$, and of the neutral Higgs bosons, $\tilde{H}_1^0$ and $\tilde{H}_2^0$, takes the form

$$\mathcal{M}=\begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix} \tag{1}$$

in the $\{\tilde{B},\tilde{W}^3,\tilde{H}_1^0,\tilde{H}_2^0\}$ basis. Here M_1 and M_2 are the fundamental SUSY breaking U(1) and SU(2) gaugino mass parameters, and μ is the Higgsino mass parameter. As a result of electroweak symmetry breaking by the vacuum expectation values of the two neutral Higgs fields v_1 and v_2 ($s_\beta = \sin\beta$, $c_\beta = \cos\beta$ where $\tan\beta = v_2/v_1$), non-diagonal terms proportional to the *Z*-boson mass m_Z appear and the gauginos and Higgsinos mix to form the four neutralino mass eigenstates $\tilde{\chi}_i^0$ $(i=1\text{–}4)$. In general the mass parameters M_1, M_2 and μ in the neutralino mass matrix (1) can be complex. By re-parametrization of the fields, M_2 can be taken real and positive, while the U(1) mass parameter M_1 is assigned the phase Φ_1 and the Higgsino mass parameter μ the phase Φ_μ.

The neutralino mass eigenvalues $m_i \equiv m_{\tilde{\chi}_i^0}$ $(i=1,2,3,4)$ can be chosen positive by a suitable definition of the mixing matrix *N*, rotating the gauge eigenstate basis $\{\tilde{B},\tilde{W}^3,\tilde{H}_1^0,\tilde{H}_2^0\}$ to the mass eigenstate basis of the Majorana fields $\tilde{\chi}_i^0$ $(i=1\text{–}4)$. In general the matrix *N* involves six angles and 10 phases, and can be written as [6,11]

$$N=\mathrm{diag}\{e^{i\alpha_1},\ e^{i\alpha_2},\ e^{i\alpha_3},\ e^{i\alpha_4}\}\mathsf{R}_{34}\mathsf{R}_{24}\mathsf{R}_{14}\mathsf{R}_{23}\mathsf{R}_{13}\mathsf{R}_{12}, \tag{2}$$

where R_{jk} are rotations in the complex $[jk]$ plane characterized by a mixing angle θ_{jk} and a (Dirac) phase β_{jk}. One of (Majorana) phases α_i is nonphysical and, for example, α_1 may be chosen to vanish. None of the remaining nine phases can be removed by rotating the fields since neutralinos are Majorana fermions. The neutralino sector is *CP* conserving if both μ and M_1 are real, which is equivalent to $\beta_{ij}=0$ (mod π) and $\alpha_i=0$ (mod $\pi/2$). Majorana phases $\alpha_i=\pm\pi/2$ do not signal *CP* violation but merely indicate different intrinsic *CP* parities of the neutralino states in *CP*-invariant theories [12].

III. NEUTRALINO PAIR PRODUCTION AND THREE-BODY DECAYS

Both the production processes, $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ $(i,j=1\text{–}4)$, and the three-body neutralino decays, $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}$, are generated by the five mechanisms: *s*-channel *Z* exchange and *t*- and *u*-channel $\tilde{f}_{L,R}$ exchanges with $\tilde{f}=\tilde{e}$ for the production processes. After appropriate Fierz transformations of the sfermion exchange amplitudes and with the fermion masses neglected, the transition matrix element of the production process $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and that of the three-body fermionic neutralino decays $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}$ can be written as

$$T(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0) = \sum_{\alpha,\beta=L,R} Q_{\alpha\beta}[\overline{v}(e^+)\gamma_\mu P_\alpha u(e^-)] \times[\overline{u}(\tilde{\chi}_i^0)\gamma^\mu P_\beta v(\tilde{\chi}_j^0)], \tag{3}$$

$$D(\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}) = \sum_{\alpha,\beta=L,R} Q'_{\alpha\beta}[\overline{u}(f)\gamma^\mu P_\alpha v(\bar{f})] \times[\overline{u}(\tilde{\chi}_j^0)\gamma_\mu P_\beta u(\tilde{\chi}_i^0)], \tag{4}$$

that is to say, as a sum of the products of a $\tilde{\chi}^0$ vector or axial vector current and a fermion vector or axial vector current, respectively. We refer to Refs. [6] and [8] for the expressions of the generalized bilinear charges $Q_{\alpha\beta}$ and $Q'_{\alpha\beta}$, just mentioning that the bilinear charges become independent of the kinematical variables when two neutralinos are at rest. Therefore, in this static limit, both the production and the decays can be considered to proceed via a static vector boson exchange.

Some general properties of the bilinear charges $Q_{\alpha\beta}$ and $Q'_{\alpha\beta}$ in Eqs. (3) and (4) can be derived in *CP*-invariant theories by applying *CP* invariance and the Majorana condition for neutralinos to the transition matrix elements. In *CP*-invariant theories, the production of a neutralino pair through a vector or axial vector current with positive intrinsic *CP* parity satisfies the *CP* relation [9,13]

$$1=\eta^i\eta^j(-1)^L, \tag{5}$$

in the non-relativistic limit of two neutralinos, where $\eta^i=\pm i$ is the intrinsic *CP* parity of $\tilde{\chi}_i^0$ and *L* is the orbital angular momentum of the produced neutralino pair. The selection rule (5) reflects the fact that if two neutralinos $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ have the same or opposite *CP* parity, the current for the neutralino pair production must be pure axial-vector or pure vector form, respectively; cf. [13]. Because the axial-vector current and the vector current involve the combination of *u* and *v* spinors for the two Majorana particles, the axial vector corresponds to the *P* wave $(L=1)$ and the vector to the *S* wave $(L=0)$.

On the other hand, the neutralino decay, $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 + V$, where V stands for the final two-fermion system forming a vector current as in Eq. (4), satisfies the *CP* relation

$$\eta^i = \eta^j(-1)^L \quad \text{or equivalently } 1 = -\eta^i \eta^j (-1)^L, \tag{6}$$

in the non-relativistic limit of two neutralinos, where L is the orbital angular momentum of the final state of $\tilde{\chi}_j^0$ and V. We emphasize first that the neutralino to neutralino transition current is pure axial-vector or pure vector form for the two neutralinos of the same or opposite *CP* parity, respectively, as in the production case. However, because two u-spinors are associated with the currents in the neutralino to neutralino transition, the axial-vector corresponds to S-wave excitation while the vector corresponds to P-wave excitation, giving rise to the relative minus sign between Eqs. (5) and (6).

One immediate consequence of the selection rules (5) and (6) is that, in *CP*-invariant theories, if the production of a pair of neutralinos with the same (opposite) *CP* parity through a vector or axial vector current is excited slowly in P waves (steeply in S waves) [12], then the neutralino to neutralino transition via such a vector or axial vector current is excited sharply in S waves (slowly in P waves). More explicitly, the power of the selection rules (5) and (6) can clearly be seen by inspecting the expressions for the S-wave excitations of the total cross section $\sigma\{ij\}$ $(i \neq j)$ near threshold and of the fermion invariant mass distribution of the three-body neutralino decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}$ (with the fermion masses neglected) near the end point:

$$\sigma\{ij\} \approx \frac{4\pi\alpha^2 m_i m_j}{(m_i+m_j)^4}\beta\{|\Im(G_R)|^2 + |\Im(G_L)|^2\} + \mathcal{O}(\beta^3), \tag{7}$$

$$\frac{d\Gamma\{ij\}}{dz_{ff}} \approx \frac{2\alpha^2}{\pi}\left(\frac{m_j}{m_i}\right)^{3/2}(m_i - m_j)\beta'\{|\Re(G_R')|^2 + |\Re(G_L')|^2\} + \mathcal{O}(\beta'^3), \tag{8}$$

where $\beta = \sqrt{1-(m_i+m_j)^2/s}$ and $\beta' = \sqrt{1-z_{ff}^2}$ with the dimensionless variable $z_{ff} = m_{ff}/m_{ff}^{\max}$, the ratio of the fermion invariant mass m_{ff} to its maximal value $m_{ff}^{\max} = m_i - m_j$. Here, the coupling dependent parts, each of which is connected with the chirality of the neutralino current, are given by

$$G_R^{(\prime)} = -\frac{Q_f}{2c_W^2}D^{(\prime)}(N_{i3}N_{j3}^* - N_{i4}N_{j4}^*) - \frac{Q_f^2}{c_W^2}F_R^{(\prime)}N_{i1}N_{j1}^*,$$

$$G_L^{(\prime)} = \frac{(I_3^f - Q_f s_W^2)}{2c_W^2 s_W^2}D^{(\prime)}(N_{i3}N_{j3}^* - N_{i4}N_{j4}^*) + \frac{1}{s_W^2 c_W^2}F_L^{(\prime)}N_{i2}' N_{j2}'^*, \tag{9}$$

with $N_{i2}' = (I_3^f - Q_f)s_W N_{i1} - I_3^f c_W N_{i2}$, and the kinematic functions are given by

$$D = (m_i+m_j)^2/[(m_i+m_j)^2 - m_Z^2],$$

$$F_{L,R} = (m_i+m_j)^2/(m_{\tilde{e}_{L,R}}^2 + m_i m_j),$$

$$D' = (m_i-m_j)^2/[(m_i-m_j)^2 - m_Z^2],$$

$$F_{L,R}' = (m_i-m_j)^2/(m_{\tilde{f}_{L,R}}^2 - m_i m_j). \tag{10}$$

In *CP*-invariant theories, all the (complex) rotation matrices R_{jk} in Eq. (2) become real and orthogonal. Therefore, if the neutralinos $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ have the same *CP* parity, then the Majorana phase difference, $\alpha_i - \alpha_j$, is 0 or π, and so $N_{ik}N_{jl}^*$ is real. On the contrary, if the neutralino pair have the opposite *CP* parity, the phase difference $\alpha_i - \alpha_j$ is $\pm\pi/2$ and so $N_{ik}N_{jl}^*$ is purely imaginary. Consequently, in *CP*-invariant theories the cross section of a non-diagonal neutralino pair rises steeply in S waves only when the produced neutralinos have opposite parity, as dictated by the first *CP* relation (5) and as clearly indicated by Eq. (7). One important implication of the selection rule is that, even if the $\{ij\}$ and $\{ik\}$ pairs are excited steeply in S waves, the pair $\{jk\}$ must be excited slowly in P waves characterized by the slow rise $\sim\beta^3$ of the cross section [5,6]. In contrast to the production case, the characteristic sharp S-wave decrease of the fermion invariant mass distribution near the end point is possible only if the neutralinos have the same *CP* parity, as dictated by the second *CP* relation (6) and as clearly indicated by Eq. (8).

However, in the *CP*-noninvariant theories the orbital angular momentum is no longer restricted by the selection rules (5) and (6). *The production of all non-diagonal pairs can simultaneously be excited steeply in S waves near threshold, and the corresponding neutralino to neutralino transition can be excited steeply in S waves even if the production cross section of the same non-diagonal neutralino pair is excited steeply in S waves.* Consequently, *CP* violation in the neutralino system can clearly be signaled in two ways by (i) the sharp S-wave excitations of the production of three non-diagonal $\{ij\}$, $\{ik\}$ and $\{jk\}$ pairs near threshold [6] or by (ii) the simultaneous S-wave excitations of the production of any non-diagonal $\{ij\}$ neutralino pair in e^+e^- annihilation, $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$, near threshold and of the fermion invariant mass distribution of the neutralino three-body decays, $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 f\bar{f}$, near the end point.

It is noteworthy that only the light neutralinos $\tilde{\chi}_{1,2}^0$ among the four neutralino states, which are expected to be lighter than sfermions and gluinos in many scenarios, may be kinematically accessible in the initial phase of e^+e^- linear colliders. In this situation, the method based on the threshold behaviors of the production of three different non-diagonal neutralino pairs for probing *CP* violation is not available. On the contrary, the combined analysis of the threshold excitation of the production process, $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$, and the fermion invariant mass distribution of the decay, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\bar{f}$, near the end point can still serve as one of the most powerful probes of *CP* violation in the neutralino system even in the initial phase of e^+e^- linear colliders.

In order to illustrate the method for probing *CP* violation numerically, we take a parameter set for the fundamental SUSY parameters:[1]

$$\tan\beta = 10, \quad |M_1| = 100 \ \text{GeV}, \quad M_2 = 150 \ \text{GeV},$$

$$|\mu| = 400 \ \text{GeV}, \quad \Phi_\mu = 0, \tag{11}$$

and we choose two different values, $\{0,\pi\}$ for the phase Φ_1, in the *CP*-invariant case, and one value, $\pi/2$, in the *CP* non-invariant case. (The parameter point with such a large phase $\Phi_1 = \pi/2$ might already have been excluded by the stringent EDM constraints. Nevertheless, this point is taken just for illustrative purposes in the present work; the indirect EDM limits depend also on many parameters of the theory outside the neutralino sector.) We take the slepton masses, $m_{\tilde{l}_L} = 250$ GeV and $m_{\tilde{l}_R} = 200$ GeV and consider the three-body leptonic decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 l^+ l^-$, especially with $l = e, \mu$, for the purpose of illustration. We note that the neutralinos $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ have the same (opposite) *CP* parity for $\Phi_1 = 0$ ($\Phi_1 = \pi$). As expected from the selection rules (5) and (6) in the *CP*-invariant case, Fig. 1 clearly shows that if the production of the neutralino pair $\tilde{\chi}^0_1\tilde{\chi}^0_2$ in e^+e^- annihilation increases slowly in *P* waves (steeply in *S* waves) near threshold, then the lepton invariant mass distribution of the decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 l^+ l^-$ decreases steeply in *S* waves (slowly in *P* waves) near the end point for the neutralino pair of the same (opposite) *CP* parity with $\Phi_1 = 0$ ($\Phi_1 = \pi$). On the contrary, in the *CP*-noninvariant case ($\Phi_1 = \pi/2$) the production and decay are excited steeply both in *S* waves near threshold.

Before concluding, we note that it will be crucial to know the relevant sfermion masses beforehand with good precision in order to use the method for probing *CP* violation properly; the overall structure of the energy dependence of the neutralino pair production and of the three-body decay distributions depends strongly on the sfermion masses [14], unless they are very heavy.

[1]Analyses of electric dipole moments strongly suggest that *CP* violation in the Higgsino sector will be very small in the MSSM if this sector is non-invariant at all [1,2].

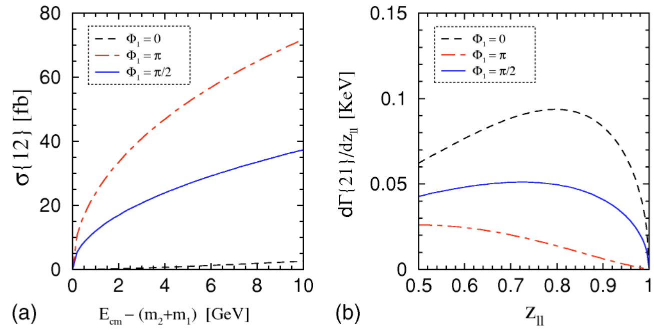

FIG. 1. (a) The threshold behavior of the neutralino production cross sections $\sigma\{12\}$ near the threshold and (b) the lepton invariant mass distribution of the decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 l^+ l^-$ near the end point, illustrated for the parameter set $\tan\beta = 10$, $|M_1| = 100$ GeV, $M_2 = 150$ GeV, $|\mu| = 400$ GeV and $\Phi_\mu = 0$ as well as the slepton masses, $m_{\tilde{l}_L} = 250$ GeV and $m_{\tilde{l}_R} = 200$ GeV.

IV. CONCLUSIONS

We have shown that only in *CP*-noninvariant theories the production of any non-diagonal neutralino pair $\tilde{\chi}^0_i\tilde{\chi}^0_j$ $(i \neq j)$ in e^+e^- annihilation near threshold and the fermion invariant mass distribution of the three-body neutralino fermionic decay $\tilde{\chi}^0_i \to \tilde{\chi}^0_j f\bar{f}$ near the end point can simultaneously be excited steeply in *S* waves.

In light of the possibility that only the two light neutralinos $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ among the four neutralinos can be accessed kinematically in the initial phase of e^+e^- linear colliders, the combined analysis of the production of the neutralino pair $\tilde{\chi}^0_1\tilde{\chi}^0_2$ in e^+e^- annihilation near threshold and the neutralino decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 f\bar{f}$ near the end point of its fermion invariant mass could provide a first qualitative indication of *CP* violation in the neutralino system.

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