Fermion masses and mixing in intersecting brane scenarios

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We study the structure of Yukawa couplings in intersecting D6-branes wrapping a factorizable 6-torus compact space T^6. Models with a MSSM-like spectrum are analyzed and found to fail in predicting the quark mass spectrum because of the way in which the family structures for the left-handed, right-handed quarks, and, eventually, the Higgs fields are ''factorized'' among the different tori. In order to circumvent this, we present a model with three supersymmetric Higgs doublets which satisfies the anomaly cancellation condition in a more natural way than the previous models, where quarks were not treated universally regarding their brane assignments or some particular branes were singled out, being invariant under orientifold projection. In our model, the family structures of all standard model particles arise in one of the tori and can naturally lead to universal strength Yukawa couplings which accommodate the quark mass hierarchy and mixing angles.

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I. INTRODUCTION

Uncovering the nature and origin of the fermion families and the observed pattern of fermion mass hierarchies and mixings is one of the most fundamental issues in high-energy physics. In the framework of the standard model (SM), the vacuum expectation value (VEV) of the Higgs field responsible for electroweak symmetry breaking generates the fermion masses through Yukawa couplings. However, the SM does not address the origin of these couplings, and the observed values for the fermion masses are considered as initial ''input'' parameters [1]. In addition, the electroweak Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix arising from the matrices that diagonalize the up- and down-quark mass matrices is determined experimentally to have, again, a hierarchical structure [1] where the third generation mixing is mostly with the second generation rather than the first. Something similar to the hierarchies and mixings happens for the neutrinos and a huge amount of effort was expended in order to understand this ''flavor problem'' of the structure of the fermion masses and mixing. Phenomenological studies considered ''textures'' [2] in the form of mass matrices leading to approximately correct relations, and attempts to understand the presence of such ''textures'' then followed in different flavor models [3] or within grand unified theories (GUTs) (see [4] and references therein).

Despite the insight which can be gained from these phenomenological studies of the fermion mass matrices, arguably the true resolution to the flavor problem lies in the domain of the underlying fundamental theory of which the SM would be the low-energy effective theory. Since at present superstrings or M theory is the only candidate for a truly fundamental quantum theory of all interactions, studies of the flavor structure of the Yukawa couplings within four-dimensional superstring models are well motivated. In particular, the couplings of the effective Lagrangian in superstring theory are in principle calculable and not input parameters, which allows us to address the flavor problem quantitatively without introducing *ad hoc* assumptions. Indeed, the structure of fermion masses has been studied in a number of semirealistic heterotic string models such as Abelian Z_n orbifolds [5,6], which have a beautiful geometric mechanism to generate a mass hierarchy [7,8], and the resulting renormalizable Yukawa couplings can be explicitly computed [9,10] as functions of the geometrical moduli. An important result of such studies was to demonstrate that the trilinear superpotential couplings at the string scale are generally either zero or $\mathcal{O}(1)$, such that they can provide a natural explanation for the top quark Yukawa coupling [11], while other mechanisms utilizing higher-dimensional nonrenormalizable operators do generate the lighter Yukawa couplings [11,12].

With the advent of Dirichlet D-branes in type-II and type-I string theories, the phenomenological possibilities of string theory have widened in several respects. Type-I and type-IIB orientifold models [13–16], where the gauge groups of the effective low-energy Lagrangian arise from sets of coincident D-branes and where the matter fields arise from open strings which must start and end upon D-branes, were proposed and investigations into their general phenomenological features have been possible. In [16,17] a classification of the matter fields has been extracted based on general grounds and formulas for the soft terms and renormalizable Yukawa couplings were derived. This has enabled a number of studies for the patterns of soft breaking parameters [16,18,19] and Yukawa textures [20,21]. Nevertheless, the study of the structure of the renormalizable Yukawa matrices and its viability within these scenarios of D-branes at singularities has proved to be unable to explain the experimental data, since they would generally lead to a variant of the ''democratic'' texture of Yukawa, and one has to break this ''democracy'' by perturbative higher-order effects or nonrenormalizable operators, the nature of which is still unclear [21]. However, recent studies of the flavor problem within ''intersecting D-brane'' models [22–26] seemed more promising [27,28]. In these models, chiral fields to be identified with SM fermions reside at different brane intersections and there is a natural origin for the replication of quark-lepton generations. In fact, most models are toroidal or orbifold (orientifold) compactifications of type-II string theory with Dp-brane wrapping intersecting cycles on the compact space, and typically the branes would intersect a multiple number of times, giving rise to the family structure. Moreover, Yukawa couplings between three chiral fields arise from open string instantons stretching a world sheet with triangle shape in whose vertices lie the chiral fields. Each world sheet contributes semiclassically to the Yukawa coupling weighted by e^{-A}, where A is the world sheet area. This exponential weighting makes very natural the appearance of hierarchies in Yukawa couplings of different fermions with a pattern controlled by the size of the triangles.

Yet the simple model presented in [27] which is based on D6-brane wrapping cycles on an orientifold of $T^2\times T^2\times T^2$ and has the chiral spectrum of the minimal supersymmetric standard model (MSSM) does not really give acceptable fermion masses. It leads to a mass spectrum of two massless and one massive eigenvalue for the Yukawa matrices. This reproduces the leading effect of one generation being much heavier than the other two and, thus, should be considered only as a starting point for a deeper phenomenological description where small variations in the setup might give rise to smaller but nonvanishing masses for the rest of quarks. In fact, we trace this problem of mass degeneracy to the factorizable form $T^2\times T^2\times T^2$ of the compactified space and to the fact that the family structures of the quark doublets, the quark singlets, and the Higgs bosons arise, each in one of the tori different from the others. This leads to a Yukawa matrix of special ''factorizable'' form $(a_i b_j)$ which has always two vanishing eigenvalues.

We argue in this paper that one can get more interesting Yukawa structures assuming three generations of supersymmetric Higgs fields $(H_i^u,H_i^d)_{i=1,2,3}$. This allows the generation of family structures for the quark doublets, singlets, and Higgs fields to take place in only one of the tori. In fact, several models with three families, including Higgs fields, have been constructed [30,31] and were favored by unification of the gauge couplings in heterotic string. Importantly for our analysis, having three families of Higgs fields would allow one easily to satisfy the Ramond-Ramond (RR) tadpole cancellation condition which requires the number of fundamentals to equal that of antifundamentals for $SU(2)$. This is because the Higgs fields can account for the six $SU(2)$ doublets needed to be added to the three lepton left-handed doublets in order to equal the nine antidoublets of the three families of chiral left-handed quark color triplets $3(3,\bar{2})$. This offers a natural solution to the anomaly cancellation condition without the need to put the left-handed quarks in different brane intersections [24,29] or to assume some specific properties satisfied by some of the branes [27]. Moreover, having three Higgs doublets introduces more Yukawa couplings, which introduces more flexibility in the computation of the mass matrices [31]; hence, one can accommodate the observed quark masses and their mixing angles.

The structure of the paper is as follows. In the next section we briefly review the different models for intersecting branes leading to a MSSM-like spectrum and state what gauge symmetry they have. Following this, we describe our model of three supersymmetric Higgs fields and the way it satisfies the anomaly cancellation condition. A brief discussion on how to determine the string scale in this class of models is presented in Sec. III. Section IV is devoted to a detailed analysis of the quark masses and mixings. Our conclusions are given in Sec. V.

II. INTERSECTING BRANE MODELS

In this section we start with reviewing the construction of MSSM-like models from intersecting D-branes. We also set some notation that we will use throughout the paper. The intersecting D-brane scenario offers an interesting way to get chiral fermions. Consider a bunch of N Dp-branes and another set of M Dp-branes $(p>3)$, both containing Minkowski space and intersecting at some angle in the $(p-3)$ extra dimensions. One then gets massless chiral fermions transforming as $(N,\bar{M})$ under gauge group $U(N)\times U(M)$ which allows us to represent the SM fermions. In addition, if the extra six dimensions are compact, the intersection of a couple of branes is in general multiple and the replication of generations is natural. Recently, a particularly interesting class of models yielding ''just'' the massless fermion spectrum of the SM was constructed [24,27]. These models consider D6-branes in type-IIA string theory compactified on a factorizable 6-torus $T^2\times T^2\times T^2$. One can wrap a D6-brane on a 1-cycle of each T^2 so it expands a three-dimensional cycle on the whole T^6. We denote the wrapping numbers of the D6_a-brane on the ith T^2 by (n_a^i,m_a^i). If one minimizes the volume of these 3-cycles in their homology class, they are described by hyperplanes quotiened by a torus lattice and this implies that the number of times two branes D6_a and D6_b intersect in T^6 is given by the signed intersection number

$$I_{ab}=(n_a^1 m_b^1-m_a^1 n_b^1)(n_a^2 m_b^2-m_a^2 n_b^2)(n_a^3 m_b^3-m_a^3 n_b^3). \quad (1)$$

In addition to this, one performs an ''orientifold'' projection on the torus represented by the product $\Omega\times R$, where Ω is the world sheet parity operator and R is the reflection operator with respect to one of the axes of the tori. The set of fixed points under $\Omega\times R$ forms an orientifold plane—namely, a subspace of spacetime where the orientation of the string can flip. This set has eight components and corresponds to $O6$

planes wrapped on the 3-cycle with wrapping numbers $(n_i, m_i) = (1,0)$. Now each D-brane α has a mirror image under $\Omega \times R$ denoted by $\alpha^\star$. If the brane wraps a cycle $[\Pi_\alpha] = (n^i_\alpha, m^i_\alpha)_{i=1,2,3}$ and $\epsilon^{(i)}_\alpha$ represents the transversal distance of the brane α from the origin in the ith torus in clockwise sense from the direction defined by $[\Pi_\alpha]$, then the mirror image brane $\alpha^\star$ would wrap a cycle $[\Pi_{\alpha^\star}] = (n^i_\alpha, -m^i_\alpha)_{i=1,2,3}$ for rectangular tori, and the corresponding translation shift from the origin in the ith torus is given by $\epsilon^{(i)}_{\alpha^\star} = -\epsilon^{(i)}_\alpha$.

There were, so far, two ways of embedding the standard model gauge group into products of unitary and symplectic gauge groups, and both ways used four stacks a,b,c,d (and their orientifold mirrors) of D6-branes called, respectively, the baryonic, left, right, and leptonic branes. Both methods (models) succeed in getting a MSSM-like spectrum free of anomalies. However, in order to do so, in the first model two left-handed quarks were doublets and one left-handed quark was an antidoublet, while in the second model one of the branes (b) was singled out, being invariant under the orientifold projection. We will summarize in the next subsection the setup of these two models. Then we will present in the following subsection our setup to generate the SM-like spectrum with the aid of three supersymmetric Higgs doublets.

A. Models with a MSSM-like spectrum

In the first model (see [24] for details)—let us call it model A—one gets initially the gauge symmetry

$$\text{model A:} \quad U(3)\times U(2)\times U(1)\times U(1), \tag{2}$$

resulting from the following number of branes in the corresponding stacks:

$$N_a=3, \quad N_b=2, \quad N_c=1, \quad N_d=1.$$

Then one would embed $SU(3)_c$ into $U(3)$ and $SU(2)_L$ into $U(2)$. In order to yield the desired SM spectrum, it is enough to select the wrapping numbers (n^i_α, m^i_α) for the four sets of D6-branes in such a way that the intersection wrapping numbers are given by

$$I_{ab}=1, \quad I_{ab*}=2,$$
$$I_{ac}=-3, \quad I_{ac*}=-3,$$
$$I_{bd}=0, \quad I_{bd*}=-3,$$
$$I_{cd}=-3, \quad I_{cd*}=3, \tag{3}$$

all other intersections vanishing. The massless fermion spectrum residing at the intersections is shown in Table I, where the N_R represents a right-handed neutrino and the hypercharge generator is defined as

$$Q_Y=\frac{1}{6}Q_a-\frac{1}{2}Q_c+\frac{1}{2}Q_d. \tag{4}$$

In this model one adopts the choice of splitting the left-handed quarks into one quark represented by the intersection

TABLE I. Standard model spectrum and $U(1)$ charges in the first model (A).

Intersection	Matter fields		Q_a	Q_b	Q_c	Q_d	Y
(ab)	Q_L	$(3,\overline{2})$	1	-1	0	0	1/6
(ab^*)	q_L	$2(3,2)$	1	1	0	0	1/6
(ac)	U_R	$3(\overline{3},1)$	-1	0	1	0	$-2/3$
(ac^*)	D_R	$3(\overline{3},1)$	-1	0	-1	0	1/3
(bd^*)	L	$3(1,\overline{2})$	0	-1	0	-1	$-1/2$
(cd)	E_R	$3(1,1)$	0	0	-1	1	1
(cd^*)	N_R	$3(1,1)$	0	0	1	1	0

(ab) and the other two reprsented by (ab^*) for consistency requirements. In fact, as already mentioned, the RR tadpole cancellation condition, which is stronger than the gauge anomaly cancellation condition, requires the same number of doublets and antidoublets. This choice, then, allows the left-handed quarks not to be universal under the $U(1)_b$ charge, so that if two left quarks were $U(2)$ doublets and the other one was $U(2)$ antidoublet, then taking the SM leptons as $U(2)$ antidoublets allows us to satisfy the requirement without the need of extra doublets. As to the Higgs field sector, the Higgs fields would come from the intersection between $b(b^*)$ and $c(c^*)$ branes, and there are four possible varieties of them $[(h_i, H_i)_{i=1,2}]$ since we have two varieties of left quarks (Q_L, q_L) and two varieties of right quarks (U_R, D_R).

The second model (see [27,28] for details), to be called model B, presents a slight variation where $N_b=1$ but b^*, the mirror of b, lies on top of it $(b=b^*)$, so it can actually be considered as a stack of two branes which, under Ω projection, yields a $USp(2)=SU(2)$ gauge group. So the initial gauge group is

$$\text{model B:} \quad U(3)\times SU(2)\times U(1)\times U(1). \tag{5}$$

With the intersection numbers

$$I_{ab}=3,$$
$$I_{ac}=-3, \quad I_{ac*}=-3,$$
$$I_{db}=3,$$
$$I_{dc}=-3, \quad I_{dc*}=-3, \tag{6}$$

and all other intersection numbers being zero, one gets the spectrum shown in Table II. The N_R denotes the right-handed neutrino and the hypercharge generator is defined as $Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$. Notice that we do not have here the Q_b anomaly condition since doublets and antidoublets in $SU(2)$ are the same [there is no $U(1)_b$ to differentiate between them]. A particular class of configurations satisfying the conditions (6) is presented in Table III and the intersections (bc) and (bc^*) can be identified with the MSSM Higgs particles H^u, H^d.

TABLE II. Standard model spectrum and $U(1)$ charges in the second model (B).

Intersection	SM matter fields	$SU(3)\times SU(2)$	Q_a	Q_c	Q_d	Y
(ab)	Q_L	3(3,2)	1	0	0	1/6
(ac)	U_R	$3(\bar{3},1)$	−1	1	0	−2/3
(ac^*)	D_R	$3(\bar{3},1)$	−1	−1	0	1/3
(db)	L	3(1,2)	0	0	1	−1/2
(dc)	N_R	3(1,1)	0	1	−1	0
(dc^*)	E_R	3(1,1)	0	−1	−1	1

B. Models with three supersymmetric Higgs doublets

As discussed above, the first model (A) treats the left quarks differently as regards their location at the brane intersections. Moreover, the intersection numbers [Eq. (3)] are not "symmetric" among the branes and their mirrors (e.g., $I_{bd}=0$, $I_{bd*}=3$). As to the second model (B), although it also reproduces an SM-like spectrum (with a right-handed neutrino), it singles out one of the branes by requiring its invariance under orientifold action ($b\equiv b^*$). The origin of these assumptions lies, as already said, in the consistency requirement that the number of fundamentals should be equal to the number of antifundamentals even for $SU(2)$.

We are proposing now another way to satisfy this condition. We consider, as in the first model (A), a stack of two branes (b) giving rise to $U(2)$ gauge symmetry. We will treat the three left-handed quarks universally and consider that we have chiral quarks in $3(3,\bar{2})$ under $SU(3)\times SU(2)$. The full model must contain then nine fields (1,2), three of which correspond to left-handed leptons. As to the remaining six doublets, we do not need extra doublets to be accounted for if we take the natural assumption of three generations of Higgs particles $(H_i^u,H_i^d)_{i=1,2,3}$. In fact, in both models (A) and (B) the u Higgs field and the d Higgs field are assigned opposite $U(1)$ charges so that anomaly would not be affected by including them. However, no reason prohibits them from having the same $U(1)_b$ charge, and thus they can provide the extra six doublets necessary for anomaly cancellation.

Our model will be purely toroidal, with no orientifold projection and so no mirror branes added. We shall consider that the SM particles reside at the intersections among four stacks of branes, $N_a=3$, $N_b=2$, $N_c=1$, $N_d=1$. The intersection numbers would be then

$$|I_{ab}|=3 \text{ representing } Q_L, \quad |I_{bc}|=3 \text{ representing } H^u,$$
$$|I_{ac}|=3 \text{ representing } U_R,$$
$$|I_{bd}|=6=3+3 \text{ representing } H^d,L,$$
$$|I_{ad}|=3 \text{ representing } D_R, \quad |I_{cd}|=3 \text{ representing } E_R. \tag{7}$$

In this third model—let us call it model C—we used the fact that the Higgs field H^d and the lepton L have the same $SU(3)\times SU(2)\times U(1)_Y$ quantum numbers, and so one can consider getting both of them at the intersection of the same branes (b and d). Requiring that the observed hypercharge generator be a linear combination of the four $U(1)$'s one finds the general solution

$$Q_Y=\left(\frac{2}{3}+\beta\alpha\right)Q_a+\left(\frac{1}{2}+\beta\alpha\right)Q_b+\alpha Q_c+\gamma(1+\beta\alpha)Q_d, \tag{8}$$

where $\beta^2=\gamma^2=1$ defined in Table IV and α is arbitrary. Notice that with the choice $\alpha=-\frac{1}{2}$, $\beta=+1$, $\gamma=1$, we get the hypercharge defined in Eq. (4).

We will give in Sec. IV an example of D6-brane wrapping numbers realizing the conditions (7). As can be seen from Table IV, all $U(1)$ gauge groups are anomaly free. One should consider also mixed gauge and gravitational anomalies. However, we expect that these anomalies are canceled by a generalized Green-Schwarz mechanism and that three combinations of the $U(1)$'s would get massive with mass roughly of the order of the string scale, while the hypercharge Y combination would stay massless. The symmetries whose gauge bosons become massive would disappear as gauge symmetries from the low-energy effective field theory, but remain as global symmetries unbroken in perturbation theory. In this respect, $U(1)_a$ and $U(1)_d$ represent, respectively, the global baryonic and leptonic number symmetries. However, assigning Q_d charges to the Higgs field H^d might lead to a breaking of the lepton number symmetry when the Higgs field acquires a VEV. Notice that in this model we do not have a right-handed neutrino as a chiral fermion from intersecting branes. Also, once we assume the Higgs fields H^u,H^d come in a number of generations equal to that of the SM particles, then the gauge anomalies would be canceled

TABLE III. D6-brane wrapping numbers giving rise to the chiral spectrum of the MSSM in the second model (B).

N_i	(n_α^1,m_α^1)	(n_α^2,m_α^2)	(n_α^3,m_α^3)
$N_a=3$	(1,0)	(1,3)	(1,−3)
$N_b=1$	(0,1)	(1,0)	(0,−1)
$N_c=1$	(0,1)	(0,−1)	(1,0)
$N_d=1$	(1,0)	(1,3)	(1,−3)

TABLE IV. Standard model spectrum and $U(1)$ charges in the third model (C).

Intersection	Matter fields	$SU(3)\times SU(2)$	Q_a	Q_b	Q_c	Q_d	Q_Y
(ab)	Q_L	$3(3,\bar{2})$	1	−1	0	0	1/6
(ac)	U_R	$3(\bar{3},1)$	−1	0	β	0	−2/3
(ad)	D_R	$3(\bar{3},1)$	−1	0	0	γ	1/3
(bd)	L	3(1,2)	0	+1	0	$-\gamma$	−1/2
(bd)	H_1^d	3(1,2)	0	+1	0	$-\gamma$	−1/2
(bc)	H_2^u	3(1,2)	0	+1	$-\beta$	0	1/2
(cd)	E_R	3(1,1)	0	0	$-\beta$	γ	1

automatically. Thus, in this model there is no relation between the number of colors and the number of families as was the case in the previous models (A) and (B) [27,24].

III. STRING SCALE IN THE MODEL WITH THREE HIGGS DOUBLETS

The toroidal models are in general nonsupersymmetric and might have tachyons at intersections. However, it is possible to vary the compact radii in order to get rid of the tachyons. Also, one can adjust the radii so that there is one massless scalar at any given intersection, which means that one gets $N=1$ supersymmetry (SUSY) at that specific intersection [24]. For instance, in the MSSM-like model (B), if the ratios of radii in the second and third torus are equal, then the *same* $N=1$ SUSY is preserved at all intersections and the model is locally $N=1$ supersymmetric, having a MSSM spectrum with a minimal Higgs set [27].

For nonsupersymmetric models, stabilization of the hierarchy of the weak scale can be achieved by lowering the string scale down to a few TeV [32,33], while for supersymmetric models there are several arguments in favor of string scales in the "intermediate" range $M_I \approx 10^{10-14}$ GeV [20]. Such arguments provide an explanation to the observed experimental neutrino masses [35] or a means to attack the hierarchy problem of unified theories in supergravity models by getting the gravitino mass around the weak scale $m_{3/2} \approx M_W$ in a natural way without invoking any hierarchically suppressed nonperturbative effect [36]. Also, for intermediate string scale scenarios, charge and color breaking constraints on the acceptable region of parameter space for soft supersymmetry breaking terms become less important [37,38]. In addition, the observed ultrahigh-energy (10^{20} eV) cosmic rays can be explained, for intermediate string scale, as products of long-lived massive string mode decays [39].

In order to compute the string scale at which the running coupling constants intersect in model (C) with three Higgs doublets, one uses the one-loop running equation for the gauge coupling:

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(M_Z)} + \frac{b_i^{NS}}{2\pi}\ln\frac{M_S}{M_Z} + \frac{b_i^S}{2\pi}\ln\frac{Q}{M_S}, \tag{9}$$

where $\alpha_i = g_i^2/4\pi$, with $i=2,3,Y$, and b_i's are the coefficients of the β functions, where M_Z represents the overall nonsupersymmetric scale while $M_S \approx 500$ GeV represents an overall supersymmetric scale [40]. On the other hand, from Eq. (8) we have the following relation at the string scale M_I:

$$\frac{1}{\alpha_Y(M_I)} = \frac{\alpha^2}{\alpha_1^c(M_I)} + \frac{(1+\beta\alpha)^2}{\alpha_1^d(M_I)} + \frac{(1/2+\beta\alpha)^2}{\alpha_2(M_I)} + \frac{(2/3+\alpha\beta)^2}{\alpha_3(M_I)}. \tag{10}$$

For the SM content with one Higgs doublet, the nonsupersymmetric β functions are given by $b_3^{NS}=7$, $b_2^{NS}=19/6$, and $b_Y^{NS} = -C^2 \times 41/6$, where C is the normalization constant of the $U(1)_Y$ hypercharge [$C^2=3/5$ in $SU(5)$ GUT]. As for the supersymmetric β functions, considering three supersymmetric generations of standard particles, two Higgs doublets, and an arbitrary number of extra particles, we have

$$b_3^S = 3 - \frac{1}{2}n_3, \tag{11}$$

$$b_2^S = -1 - \frac{1}{2}n_2, \tag{12}$$

$$b_Y^S = -C^2(11+q), \tag{13}$$

where

$$q = \sum_{i=1}^{n_1} Y_i^2 + 2\sum_{j=1}^{n_2} Y_j^2 + 3\sum_{k=1}^{n_3} Y_k^2, \tag{14}$$

and n_1, n_2, and n_3 are the number of extra $SU(3)\times SU(2)$ singlets, $SU(2)$ doublets, and $SU(3)$ triplets, respectively, with masses close to M_S and hypercharges Y_I. From Eqs. (9) and (10), one finds

$$\ln\frac{M_I}{M_S} = \frac{1}{\left[\left(\frac{2}{3}+\alpha\beta\right)^2 b_3^S + \left(\frac{1}{2}+\alpha\beta\right)^2 b_2^S - b_Y^S\right]} \times \left\{ 2\pi\left(\frac{1}{\alpha_Y(M_Z)} - \frac{\alpha^2}{\alpha_1^c(M_I)} - \frac{(1+\beta\alpha)^2}{\alpha_1^d(M_I)} - \frac{\left(\frac{1}{2}+\alpha\beta\right)^2}{\alpha_2(M_Z)} - \frac{\left(\frac{2}{3}+\alpha\beta\right)^2}{\alpha_3(M_Z)} \right) + \left[b_Y^{NS} - \left(\frac{1}{2}+\alpha\beta\right)^2 b_2^{NS} - \left(\frac{2}{3}+\alpha\beta\right)^2 b_3^{NS} \right] \ln\frac{M_S}{M_Z} \right\}. \tag{15}$$

We shall use the experimental values [41] $M_Z = 91.187$ GeV, $\alpha_3(M_Z) = 0.1184$, $\alpha_2(M_Z) = 0.0338$, $\alpha_Y(M_Z) = 0.01016$ given in the modified minimal subtraction ($\overline{\text{MS}}$) scheme. The fact that we have four extra Higgs doublets with respect to the case of the MSSM means that we should take $n_2=4$ and $n_1=n_3=0$. Now, assuming that $\alpha=1/2$, $\beta=+1$ and $C^2=3/5$, one obtains

$$\ln\frac{M_I}{M_S} = 40.56 - \frac{0.17}{\alpha_1^c(M_I)} - \frac{1.59}{\alpha_1^d(M_I)} - 1.89\ln\frac{M_S}{M_Z}. \tag{16}$$

Thus for $\alpha_1^c(M_I) \sim \alpha_1^d(M_I) \sim 0.1$, one finds $M_I \simeq 10^{12}$ GeV. One could also check that the curves of α_2 and α_3 cross at approximately this intermediate scale which is, as emphasized above, an attractive possibility. However, we should mention here that in this class of toroidal or orientifold models there may exist extra chiral fields that would change the gauge couplings and might lead to a lower string scale. Although this possibility is indeed crucial in nonsupersymmetric models in order to avoid any hierarchy between the string scale and the electroweak scale, it is not essential in our model with supersymmetric content. The model could still be consistent if these additional fields are decoupled from our spectrum or the possible threshold corrections are small [33,34].

IV. ANALYSIS OF FERMION MASSES AND MIXING

In [27], it was shown that a Yukawa coupling between fields at the intersections of factorizable 3-cycles Π_a, Π_b, and Π_c on a factorizable T^6 is given by

$$Y_{ijk}=h_{qu}\sigma_{abc}\prod_{r=1}^{n}\vartheta\begin{bmatrix}\delta^{(r)}\\ \phi^{(r)}\end{bmatrix}(\kappa^{(r)}). \tag{17}$$

Here, each triplet of intersection (i,j,k) is described by the multi-indices

$$i=(i^{(1)},i^{(2)},i^{(3)})\in\Pi_a\cap\Pi_b,\quad i^{(r)}=0,\ldots,|I_{ab}^{(r)}|-1,$$
$$j=(j^{(1)},j^{(2)},j^{(3)})\in\Pi_c\cap\Pi_a,\quad j^{(r)}=0,\ldots,|I_{ca}^{(r)}|-1,$$
$$k=(k^{(1)},k^{(2)},k^{(3)})\in\Pi_b\cap\Pi_c,\quad k^{(r)}=0,\ldots,|I_{bc}^{(r)}|-1, \tag{18}$$

where (r) is an index indicating the rth torus, and $I_{ab}=\Pi_{r=1}^{n}I_{ab}^{(r)}=\Pi_{r=1}^{n}(n_a^{(r)}m_b^{(r)}-n_b^{(r)}m_a^{(r)})$, where $I_{ab}^{(r)}$ denotes the intersection number of cycles a and b on the rth torus and I_{ab} is the total intersection number; $\sigma_{abc}=\mathrm{sgn}(I_{ab}I_{bc}I_{ca})$, h_{qu} stands for the quantum contribution to the instanton amplitude, and ϑ is the complex theta function:

$$\vartheta\begin{bmatrix}\delta\\ \phi\end{bmatrix}(\kappa)=\sum_{l\in\mathbb{Z}}e^{\pi i(\delta+l)^2\kappa}e^{2\pi i(\delta+l)\phi}. \tag{19}$$

We have

$$\delta^{(r)}=\frac{i^{(r)}}{I_{ab}^{(r)}}+\frac{j^{(r)}}{I_{ca}^{(r)}}+\frac{k^{(r)}}{I_{bc}^{(r)}}+\frac{d^{(r)}(I_{ab}^{(r)}\epsilon_c^{(r)}+I_{ca}^{(r)}\epsilon_b^{(r)}+I_{bc}^{(r)}\epsilon_a^{(r)})}{I_{ab}^{(r)}I_{bc}^{(r)}I_{ca}^{(r)}}+\frac{s^{(r)}}{d^{(r)}}, \tag{20}$$

$$\phi^{(r)}=(I_{ab}^{(r)}\theta_c^{(r)}+I_{ca}^{(r)}\theta_b^{(r)}+I_{bc}^{(r)}\theta_a^{(r)})/d^{(r)}, \tag{21}$$

$$\kappa^{(r)}=\frac{J^{(r)}}{\alpha'}\frac{|I_{ab}^{(r)}I_{bc}^{(r)}I_{ca}^{(r)}|}{(d^{(r)})^2}, \tag{22}$$

where $d^{(r)}=gcd(I_{ab}^{(r)},I_{bc}^{(r)},I_{ca}^{(r)})$ and $s^{(r)}\equiv s(i^{(r)},j^{(r)},k^{(r)})\in\mathbb{Z}$ is a linear function of the integers $i^{(r)}$, $j^{(r)}$, and $k^{(r)}$. Here $\epsilon^{(r)}$ represents the "shifts" in the rth torus while the phase $\theta^{(r)}$ accounts for adding Wilson lines around the D-brane wrapping 1-cycles in the rth torus. $J^{(r)}$ represents the Kahler structure of the rth torus and so $\kappa^{(r)}$ would be proportional to its area. Once we have determined the Yukawa couplings, one can compute the quark masses and mixings to see whether the model reproduces the observed hierarchical structure.

A. Models with one supersymmetric Higgs doublet

In the first model (A) [24], the case of a minimal set of Higgs fields similar to the MSSM was shown to give masses to the top, charm, and bottom quarks while the strange, down, and up quarks remained massless. It was argued that, with a double-Higgs-field system, the observed hierarchy of fermion masses would be a consequence of the different values of the Higgs fields and the hierarchical values of Yukawa couplings, coming from geometrical considerations.

As to the second model (B) [27] and with the wrapping numbers shown in Table III, one gets one Higgs doublet H^u (H^d) at the intersection of bc ($bc*$). Yukawa couplings of the form

$$Y_{ij}^{U}Q_L^iH_uU_R^j,\quad Y_{ij*}^{D}Q_L^iH_dD_R^{j*} \tag{23}$$

were computed, and only the third generation of the quarks are massive. It was argued that a smaller perturbation of this setup can give rise to smaller but nonvanishing masses for the rest of the quarks.

However, examining the model in depth would trace the problem of having two zero eigenvalues in the Yukawa matrices to the "factorizable" form that they take when the family replications for the left-handed quarks and the right-handed quarks come from different tori. For the case of Table III we see that the index $i_{ab}^{(r)}=0,\ldots,|I_{ab}^{(r)}|-1$, denoting the left quarks, would span the values 0,1,2 only in the second torus, while the index $i_{ac(c*)}^{(r)}=0,\ldots,|I_{ac(c*)}^{(r)}|-1$, denoting the right quarks, would have its family structure only in the third torus. In such cases and neglecting Wilson line effects, the Yukawa couplings would always be of the form

$$Y_{ij}\sim\vartheta^{(1)}\begin{bmatrix}\delta(0)\\ 0\end{bmatrix}(\kappa^{(1)})\times\vartheta^{(2)}\begin{bmatrix}\delta(i)\\ 0\end{bmatrix}(\kappa^{(2)})\times\vartheta^{(3)}\begin{bmatrix}\delta(j)\\ 0\end{bmatrix}\times(\kappa^{(3)}) \tag{24}$$

and so it is of a "factorizable" form

$$Y_{ij}\sim a_ib_j.$$

Such matrices always have two zero eigenvalues since, for instance, the second and third columns are proportional to the first one.

Could we get more interesting phenomenology if the family structures of both left-handed and right-handed quarks arise in the same torus? The answer is no, if we restrict ourselves to one Higgs doublet. In fact, if we adopt the wrapping numbers shown in Table V, where the branes b, c, $c*$ are on top of each other in the second torus, we find that the conditions (6) are satisfied, and we have one massless nonchiral Higgs doublet arising at the intersection of the brane b and the brane c (or $c*$) in the first and third tori. In other

TABLE V. Alternative example of D6-brane wrapping numbers in the second model (B) leading to a chiral spectrum of the MSSM. The family structure of both the left-handed and right-handed quarks arises in the second torus.

N_i	(n^1_α,m^1_α)	(n^2_α,m^2_α)	(n^3_α,m^3_α)
$N_a=3$	(1,0)	(1,3)	(1,0)
$N_b=1$	(0,1)	(1,0)	(0,−1)
$N_c=1$	(1,1)	(1,0)	(1,1)
$N_d=1$	(1,0)	(1,3)	(1,0)

words, there is a minimal Higgs sector with a μ parameter determined by the distance between the branes b and c along the second torus.

In order to compute the Yukawa structure, we now note that the family structure for both the left-handed and the right-handed quarks originates in the second torus, and so, neglecting Wilson lines effect, we shall get

$$Y_{ij}\sim\vartheta^{(1)}\begin{bmatrix}\delta(0)\\0\end{bmatrix}(\kappa^{(1)})\times\vartheta^{(2)}\begin{bmatrix}\delta^{(2)}(i,j)\\0\end{bmatrix}(\kappa^{(2)})\times\vartheta^{(3)}\begin{bmatrix}\delta(0)\\0\end{bmatrix}(\kappa^{(3)}), \tag{25}$$

where $\delta^{(2)}(i,j)=(i+j)/3+\lambda$, and λ is a constant determined by the shifts ϵ_a, ϵ_b, ϵ_c. However, using the periodicity of theta function,

$$\vartheta^{(r)}\begin{bmatrix}\delta+1\\\phi\end{bmatrix}(\kappa)=\vartheta^{(r)}\begin{bmatrix}\delta\\\phi\end{bmatrix}(\kappa), \tag{26}$$

we get the following form for the Yukawa matrix:

$$Y_{ij}\sim\begin{pmatrix}a&b&c\\b&c&a\\c&a&b\end{pmatrix}. \tag{27}$$

A matrix of this form has a spectrum such that two of the eigenvalues are always opposite in sign, so it leads to two degenerate states and cannot reproduce the hierarchy in the masses of the quarks. Having spotted the origin of the problem, we now move to our third model (C).

B. Model with three supersymmetric Higgs doublets

We saw in the previous subsection that having the family structures of the left-handed quarks and the right-handed quarks to arise from different tori leads to a mass matrix with two vanishing eigenvalues. Also, having one Higgs doublet in the setup would lead to a phenomenologically unacceptable form for the mass matrices. One could also check that getting Higgs fields doublet replication in one torus different from the torus where the family structure arises for the quarks is similar to the one Higgs doublet situation. In this case, their effects are factored out. Thus, we are led naturally to seek a situation where we have more than one Higgs doublet and where the family structures of all left-handed quarks, right-handed quarks, and Higgs fields arise in one of the tori. Recalling that the assumption of three supersymmetric Higgs fields is also a ‘‘normal’’ choice in order to cancel anomalies, we consider the model (C), with three Higgs doublets. Since we are interested only in the quark sector, we adopt the intersection numbers (7) of the model (C), seeking to generate family structures for the left and right quarks, as well as for the Higgs fields, in the second torus, say. No constraint is imposed on where the family structure for the leptons would arise. This means that we are not trying here to interpret the lepton masses and mixing, in particular that the model does not contain right-handed neutrinos or Majorana neutrinos because lepton number is a symmetry, and so the question of neutrino masses should be addressed differently.

TABLE VI. Example of D6-brane wrapping numbers in the third model (C). The family structure of the standard model particles arises in the second torus.

N_i	(n^1_α,m^1_α)	(n^2_α,m^2_α)	(n^3_α,m^3_α)
$N_a=3$	(1,0)	(2,3)	(1,0)
$N_b=2$	(0,1)	(1,3)	(0,1)
$N_c=1$	(0,1)	(1,0)	(0,1)
$N_d=1$	(1,−1)	(3,3)	(1,−1)

The conditions (7) are obtained with the wrapping numbers shown in Table VI where the branes b and c are on top of each other in the first and third tori, and so one gets massless non-chiral H^u Higgs doublets. Moreover, we see that the family structures of all the standard model particles (the left- and right-handed quarks, the left- and right-handed leptons, and the Higgs fields) arise in the second torus. The branes b and d give an intersection number equal to 6, so we can identify the first three intersections as the three H^d Higgs doublets while the last three intersections would be the three left lepton doublets. In this way the Higgs doublets H^d, like the left leptons, are chiral but this does not lead to a problem in constructing the MSSM superpotential since the chiral H^d can still form a μ term with the nonchiral H^u. An approach to deal with chirality issues is to compactify over T^2/Z^2 instead of T^2 [23]. However, one should compute the detailed spectrum to check when this would generically project onto chiral matter. We do not follow this approach here, but instead seek an assignment of wrapping numbers that leads to chiral fermions. We found that such an assignment could be obtained provided we allow multiwrapping cycles for the branes. As an example, in Table VI we use a cycle (3,3) for the brane d in the second torus. Since the wrapping numbers are not coprime, the brane d is multiwrapped 3 times over the cycle (1,1) in this torus. Normally, multiwrapping leads to an enhancement of the gauge symmetry [42] (look also at [23,43] where similar multiwrapped assignments were used for D4-branes and, hence, for the whole compact dimension of the brane). Nonetheless, in our case the multiwrapping occurs only in the second torus. Even if this partial multiwrapping in the second torus enhanced the world volume gauge group from $U(1)_d$ to $U(1)^3_d$ (with generators Q^a_d, $a=1,2,3$), our discussion regarding the hypercharge and the anomaly cancellation [Eq. (8)] would stay valid with $Q_d = \Sigma_{a=1}^{3} Q_d^a$ [43]. Thus, we shall not examine further the effects of multiwrapping, especially that our model should be considered as a step towards building a more realistic one. Actually, the wrapping numbers in Table VI, as is the case in Tables III and V (see [27,44] for a discussion of this point), show that the corresponding brane content by itself does not satisfy the RR tadpole conditions $\Sigma_a N_a \Pi_a = 0$, which would read as follows:

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 0, \quad \sum_a N_a n_a^1 m_a^2 m_a^3 = 0,$$

$$\sum_a N_a m_a^1 n_a^2 n_a^3 = 0, \quad \sum_a N_a m_a^1 n_a^2 m_a^3 = 0,$$

$$\sum_a N_a n_a^1 m_a^2 n_a^3 = 0, \quad \sum_a N_a m_a^1 m_a^2 n_a^3 = 0,$$

$$\sum_a N_a n_a^1 n_a^2 m_a^3 = 0, \quad \sum_a N_a m_a^1 m_a^2 m_a^3 = 0. \tag{28}$$

Yet, since tadpole cancellation conditions are closely connected to cancellation of anomalies and our model does cancel the anomalies related to the gauge groups in Table IV, it is not so surprising that with a slight change in the setup one could satisfy the RR tadpole conditions. In fact and in the spirit of bottom to top approach, the model should be seen as a submodel embedded in a bigger one where extra RR sources are included. These may either involve some hidden and possibly nonfactorizable extra branes with no neat intersections with the SM branes or some Neveu-Schwarz–Neveu-Schwarz (NS–NS) background fluxes, with none of these possibilities adding a ‘‘net’’ chiral matter content [45]. We shall not dwell on the details of such embedding which might lead to extra matter, expectedly, heavy and disconnected from the SM sector. Rather, we shall take our setup and examine what interesting geometrical explanations for the fermion masses it might lead to. The quark Yukawa coupling with the H^d Higgs field would then be proportional to a product of three theta functions (neglecting again the Wilson line phase):

$$\vartheta^{(1)}\begin{bmatrix}\delta(0)\\0\end{bmatrix}(\kappa^{(1)})\times\vartheta^{(2)}\begin{bmatrix}\delta(i,j,k)\\0\end{bmatrix}(\kappa^{(2)})\times\vartheta^{(3)}\begin{bmatrix}\delta(0)\\0\end{bmatrix}(\kappa^{(3)}), \tag{29}$$

with $i,j,k=0,1,2$. The index k runs only over the first three I_{bd} intersections identified with the H^d Higgs doublets. Thus the quark Yukawa couplings for both the H^u and H^d Higgs fields would be proportional to

$$Y_{ijk} \sim \vartheta^{(2)}\begin{bmatrix}\delta(i,j,k)\\0\end{bmatrix}(\kappa^{(2)}) \tag{30}$$

and so we will restrict, henceforth, our discussion to the second torus. For the U-quark Yukawa coupling $Y^u H^u Q_L U_R$ we have $|I_{ab}| = |I_{bc}^{(2)}| = |I_{ac}| = 3$. This is similar to the case of elliptic fibration discussed in [27,46] where the intersection numbers are not coprime and only the triplets of intersection satisfying the selection rule

$$i+j+k \equiv 0 \bmod 3 \tag{31}$$

are connected by an instanton. We then get the Yukawa couplings

$$Y_{ij1} \sim \begin{pmatrix} A & 0 & 0\\ 0 & 0 & B\\ 0 & C & 0\end{pmatrix}, \quad Y_{ij2} \sim \begin{pmatrix} 0 & 0 & C\\ 0 & A & 0\\ B & 0 & 0\end{pmatrix},$$
$$Y_{ij3} \sim \begin{pmatrix} 0 & B & 0\\ C & 0 & 0\\ 0 & 0 & A\end{pmatrix}, \tag{32}$$

with

$$A = \vartheta\begin{bmatrix}\epsilon/3\\0\end{bmatrix}(3J/\alpha'), \quad B = \vartheta\begin{bmatrix}(\epsilon-1)/3\\0\end{bmatrix}(3J/\alpha'),$$
$$C = \vartheta\begin{bmatrix}(\epsilon+1)/3\\0\end{bmatrix}(3J/\alpha'), \tag{33}$$

and where we have $\epsilon = \epsilon_a + \epsilon_b + \epsilon_c$. For the D-quark Yukawa coupling $Y^d H^d Q_L D_R$ one would get the same result with a different ϵ shift $\epsilon' = \epsilon_a + \epsilon_b + \epsilon_d$. However, as we shall see, a numerically good fit is obtained around $\epsilon \simeq \epsilon' \simeq 0$, and to fix the ideas let us take $\epsilon = \epsilon'$. Thus, we get the U-quark mass matrix

$$M_{ij}^u = h_{qu}\begin{pmatrix} Av_1^u & Bv_3^u & Cv_2^u\\ Cv_3^u & Av_2^u & Bv_1^u\\ Bv_2^u & Cv_1^u & Av_3^u\end{pmatrix} \tag{34}$$

and the D-quark mass matrix

$$M_{ij}^d = h_{qu}\begin{pmatrix} Av_1^d & Bv_3^d & Cv_2^d\\ Cv_3^d & Av_2^d & Bv_1^d\\ Bv_2^d & Cv_1^d & Av_3^d\end{pmatrix}, \tag{35}$$

where h_{qu} includes the quantum fluctuation factor and we expect it to be similar for the u and d quarks since leading effects would come from QCD loops [28] and $v_i^{u,d}$ is the VEV for the Higgs $H_i^{u,d}$ with

$$\sum_{i=1}^{3} (v_i^u)^2 + (v_i^d)^2 = (174)^2 \ \ (\mathrm{GeV})^2. \tag{36}$$

The quark masses are obtained by diagonalizing the above mass matrices,

$$U_L M^u U_R^\dagger = d_U,$$
$$D_L M^d D_R^\dagger = d_D, \qquad (37)$$

where U_L, U_R, D_L, D_R are unitary matrices and

$$d_U = diag(m_t, m_c, m_u),$$
$$d_D = diag(m_b, m_s, m_d),$$

while the CKM matrix is given by

$$\mathrm{CKM} = U_L D_L^\dagger. \qquad (38)$$

We have seven free parameters consisting of the six Higgs VEVs with the constraint (36), the area of the torus, and the shift ϵ. We will not include the unknown overall multiplicative factor h_{qu} which is of order $\mathcal{O}(1)$. This set of parameters can be fixed by the quark masses and one mixing angle and the model has to predict the remaining two mixing angles in the CKM matrix. This might be a nontrivial task since one has to span the whole range of all of these free parameters very carefully. Here we will consider some examples and try to show that for a particular choice of these free parameters one may obtain a well studied Yukawa texture, like for instance the universal strength Yukawa (USY) couplings (see Ref. [47], and references therein). Let us start with the case of approximately symmetric matrices M^u, M^d—i.e., $B \approx C$. In this case, the shape of theta function for a fixed area argument shows that it is centered symmetrically around $\epsilon = 0$, and so we will span the ϵ parameter around this value. Also, in order to generate the mass spectrum one could put the mass matrices in the form

$$M_{ij}^{u,d} = h_{qu} A v_3^{u,d} \begin{pmatrix} v_1^{u,d}/v_3^{u,d} & \alpha_1 & \alpha_2 v_2^{u,d}/v_3^{u,d} \\ \alpha_2 & v_2^{u,d}/v_3^{u,d} & \alpha_1 v_1^{u,d}/v_3^{u,d} \\ \alpha_1 v_2^{u,d}/v_3^{u,d} & \alpha_2 v_1^{u,d}/v_3^{u,d} & 1 \end{pmatrix}, \qquad (39)$$

where $\alpha_1 = B/A$ and $\alpha_2 = C/A$. So one could generate the spectrum provided that

$$\{v_1^u, v_2^u, v_3^u\} \propto \{m_u, m_c, m_t\},$$
$$\{v_1^d, v_2^d, v_3^d\} \propto \{m_d, m_s, m_b\}, \qquad (40)$$

and $\alpha_1, \alpha_2 \ll 1$. The conditions (40) with the constraint (36) would determine the range in which we should vary the VEVs. With such considerations one finds that the choice of parameters,

$$v_1^u \simeq 63 \text{ MeV}, \quad v_2^u \simeq 0.95 \text{ GeV}, \quad v_3^u \simeq 174 \text{ GeV},$$
$$v_1^d \simeq 8.5 \text{ MeV}, \quad v_2^d \simeq 136 \text{ MeV}, \quad v_3^u \simeq 4.2 \text{ GeV},$$
$$\epsilon \simeq 0.002, \quad \text{area} \simeq 18.71, \qquad (41)$$

gives the quarks mass spectra

$$d_U = \{m_t = 173.9 \text{ GeV}, \quad m_c = 1.02 \text{ GeV},$$
$$m_u = 4.3 \text{ MeV}\},$$
$$d_D = \{m_b = 4.19 \text{ GeV}, \quad m_s = 136 \text{ MeV},$$
$$m_d = 8.2 \text{ MeV}\}, \qquad (42)$$

which are in the experimentally acceptable range [1], and a CKM matrix with diagonal elements near the unity, and $(V_{CKM})_{12} \simeq 0.216$. However, $(V_{CKM})_{13} \sim (V_{CKM})_{23} \sim 10^{-4}$–$10^{-5}$.

One can also look for other structures different from the "hierarchical" ($\alpha \ll 1$) texture—for example, the case of $\alpha_{1,2} \sim v_1/v_3 \sim v_2/v_3 \sim 1$ leads to a nearly democratic Yukawa texture which is known to accommodate the observed masses and mixing. However, our checks with real-valued VEVs indicate that our configuration leads to the correct masses and one mixing angle only while the other two mixing angles are smaller than their experimental values. This is similar to Z_3-heterotic situation in [31], where another mechanism, Fayet-Iliopoulos breaking, was called for in order to address the question of the complete quark mixing. However, with complex VEVs and a democratic texture, one gets the USY texture

$$Y^{u,d} = \lambda_{u,d} \begin{pmatrix} e^{i\varphi_{13}^{u,d}} & 1 & e^{i\varphi_{23}^{u,d}} \\ 1 & e^{i\varphi_{23}^{u,d}} & e^{i\varphi_{13}^{u,d}} \\ e^{i\varphi_{23}^{u,d}} & e^{i\varphi_{13}^{u,d}} & 1 \end{pmatrix}. \qquad (43)$$

This type of Yukawa matrix, where all Yukawa couplings have the same modulus and the flavor dependence being all contained in the phases, has been recently studied in [47]. It was shown that with very small values of the phases $\sim 10^{-3}$–10^{-2} one could generate the right values of the quark masses and the CKM mixing angles. It is interesting to note that this class of couplings is motivated by horizontal symmetries [47] and also arises in the models with two large extra dimension [48]. Here we find another motivation for the USY couplings.

V. CONCLUSIONS

We have shown how simple configurations of D-branes wrapping a compact space may give a good quantitatively description of quark masses and mixing. In particular, one finds that with a three supersymmetric Higgs doublets model the anomaly cancellation condition could be solved easily without introducing extra matter doublet fields or putting assumptions on the quarks' brane assignment or on the branes themselves. In this class of models, it turns out that the string scale is of order 10^{12} GeV which is an interesting scale for generating neutrino masses and many other phenomenological issues. With real Higgs VEVs, the model can easily account for the quark masses and one of the CKM mixing angles. However, with complex VEVs one can get Yukawa couplings in the form of USY textures which can accommodate the masses and the three CKM mixing angles with very small phases. It would be worthwhile to study the leptonic sector from this perspective.

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