PHYSICAL REVIEW D **69**, 095003 (2004)

Neutral top-pion and lepton flavor violating processes

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(Received 22 December 2003; published 12 May 2004)

In the context of top-color-assisted techicolor (TC2) models, we study the contributions of the neutral top-pion π_t^0 to the lepton flavor violating processes $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_l$. We find that the present experimental bound on $\mu \to e\gamma$ gives severe constraints on the free parameters of TC2 models. Taking into account these constraints, we consider the processes $l_i \to l_j l_k l_l$ generated by top-pion exchange at the tree level and the one loop level, and obtain the branching ratio $\mathrm{Br}(\mu \to 3e) \simeq 2.87 \times 10^{-14}$, $1.1 \times 10^{-15} \leqslant \mathrm{Br}(\tau \to 3e) \simeq \mathrm{Br}(\tau \to 2e\mu) \leqslant 4.4 \times 10^{-15}$, $3.1 \times 10^{-15} \leqslant \mathrm{Br}(\tau \to 2\mu e) \simeq \mathrm{Br}(\tau \to 3\mu) \leqslant 1.5 \times 10^{-14}$ in most of the parameter space.

DOI: 10.1103/PhysRevD.69.095003 PACS number(s): 12.60.Cn, 13.35.Dx, 14.80.Cp, 14.80.Mz

I. INTRODUCTION

The standard model (SM) accommodates fermion and weak gauge boson masses by including a fundamental Higgs scalar *H*. However, the SM cannot explain the dynamics responsible for the generation of mass. Furthermore, the scalar sector suffers from the problems of triviality and unnaturalness. Thus, the SM can only be an effective field theory below some high-energy scale. New physics should exist at energy scales around TeV.

The cause of electroweak symmetry breaking (EWSB) and the origin of fermion masses are important problems of current particle physics. Given the large value of the top quark mass and the sizable splitting between the masses of the top and bottom quarks, it is natural to wonder whether m_t has a different origin from the masses of other quarks and leptons. There may be a common origin for EWSB and top quark mass generation. Much theoretical work has been carried out in connection to the top quark and EWSB. Top-color-assisted technicolor (TC2) models [1], flavor-universal TC2 models [2], top see-saw models [3], and top flavor see-saw models [4] are four of such examples. The common feature of these kinds of models is that top-color interactions are assumed to be chiral critically strong at the scale about 1 TeV, and it is coupled preferentially to the third generation. In TC2 models, EWSB is mainly generated by TC interactions or other strong interactions. The top-color interactions also make small contributions to EWSB and give rise to the main part of the top quark mass. Then, the presence of the physical top pions in the low-energy spectrum is an inevitable feature of these kinds of models. Thus, studying the possible signatures of the top pions at present and future high- or low-energy colliders can help the collider experiments to search for top pions, test top-color scenario, and further to probe EWSB mechanism.

It is well known that the individual lepton numbers L_e, L_μ, and L_τ are automatically conserved and the tree-level lepton flavor violating (LFV) processes are absent in SM, due to unitary of the leptonic analog of Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and the masslessness of the three neutrinos. However, the solar neutrino experiments [5] and the atmospheric neutrino experiments [6] confirmed by reactor and accelerator experiments [7] provide very strong evidence for mixing and oscillation of the flavor neutrinos, which presently provide the only direct observation of physics that cannot be accommodated within the SM and imply that the separated lepton numbers are not conserved. Thus, the SM requires some modification to account for the pattern of neutrino mixing, in which the LFV processes such as $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_l$ are allowed. The observation of these LFV processes would be a clear signature of new physics beyond the SM. The fact and the improvement of their experimental measurements force one to make more elaborate theoretical calculations in the framework of some specific models beyond the SM and see whether the LFV effects can be tested in the future experiments. For instance, these LFV processes have been widely studied in a model independent way in Ref. [8], in the SM with extended right-handed and left-handed neutrino sectors [9], in supersymmetric models [10], in the general two *Higgs* doublet model (2HDM) type III [11], in the *Zee* model [12], and in the top-color models [13].

The aim of this paper is to study the contributions of the neutral top pion π_t^0 predicted by TC2 models to the LFV processes $l_i \to l_j \gamma$ and $l_i \to l_j l_k l_l$ and see whether π_t^0 can give significant effects on these processes. The paper is organized as follows: in Sec. II we give the flavor-diagonal (FD) and flavor-changing (FC) couplings of π_t^0 to the three family leptons and calculate their contributions to the LFV process $l_i \to l_j \gamma$. Using the experimental upper limit of the LFV process $\mu \to e\gamma$, we give the constraints on the flavor mixing factors. The tree-level and one loop-level contributions of π_t^0 to the branching ratios $\mathrm{Br}(l_i \to l_j l_k l_l)$ are calculated in Sec. III. The conclusions are given in Sec. IV.

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II. THE FLAVOR MIXING FACTORS OF π_t^0 AND THE LFV PROCESSES $l_i \to l_j \gamma$

A. The couplings of the neutral top pion π_t^0 to leptons

For TC2 models [1], TC interactions play a main role in breaking the electroweak symmetry. Top-color interactions make small contributions to EWSB, and give rise to the main part of the top quark mass, $(1-\varepsilon)m_t$, with the parameter $\varepsilon \ll 1$. Thus, there is the following relation:

$$\nu_\pi^2 + F_t^2 = \nu_W^2, \tag{1}$$

where ν_π represents the contributions of TC interactions to EWSB, $\nu_W = \nu/\sqrt{2} \simeq 174$ GeV. Here $F_t \simeq 50$ GeV is the physical top-pion decay constant, which can be estimated from the Pagels-Stokar formula. This means that the masses of the gauge bosons W and Z are given by absorbing the linear combination of the top pions and technipions. The orthogonal combination of the top pions and technipions remains unabsorbed and physical [14]. However, the absorbed Goldstone linear combination is mostly the technipions while the physical linear combination is mostly the top pions, which are usually called physical top pions $(\pi_t^\pm, \pi_t^0)$. The existence of the physical top pions in the low-energy spectrum can be seen as characteristic of the top-color scenario, regardless of the dynamics responsible for EWSB and other quark masses.

The FD couplings of the neutral top pion π_t^0 to leptons can be written as

$$\frac{m_l}{\nu}\bar{l}\gamma^5 l \pi_t^0, \tag{2}$$

where $l = \tau$, μ, or e. For TC2 models, the underlying interactions, top-color interactions, are nonuniversal and therefore do not posses GIM mechanism. The nonuniversal gauge interactions result in the new FC coupling vertices when one writes the interactions in the mass eigenbasis. Thus, the top pions can induce the new FC scalar coupling vertices [15]. The FC couplings of π_t^0 to leptons can be written as

$$\frac{m_\tau}{\nu}k_{\tau i}\bar{\tau}\gamma^5 l_i \pi_t^0, \tag{3}$$

where l_i $(i=1,2)$ is the first (second) lepton e (μ), $k_{\tau i}$ is the flavor mixing factor, which is the free parameter.

Using the FD and FC couplings of the neutral top pion π_t^0 to fermions, we have studied the FC process $\mu^+\mu^- \to \overline{t}c$ mediated by π_t^0 exchange [16]. We find that π_t^0 can generate several hundred $\overline{t}c$ events and the signals of π_t^0 might be detected via the process $\mu^+\mu^- \to \overline{t}c$ in the first muon collider. In the next subsection, we will study the contributions of π_t^0 to the LFV processes $l_i \to l_j \gamma$ and see whether the experimental upper limits of these processes can give significant constraints on the flavor mixing factor $k_{\tau i}$.

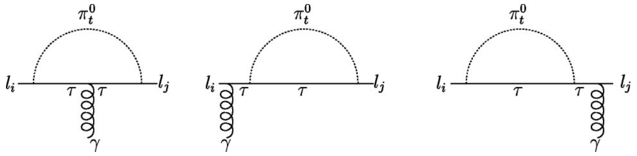

FIG. 1. Feynman diagrams contribute to the LFV processes $l_i \to l_j \gamma$ due to the neutral top-pion π_t^0 exchange in TC2 models.

B. The LFV processes $l_i \to l_j \gamma$

The observation of neutrino oscillations [5,6] implies that individual lepton numbers $L_{e,\mu,\tau}$ are violated, suggesting the appearance of the LFV processes, such as $l_i \to l_j \gamma$ and $l_i \to l_i l_j l_l$. The branching ratios of these processes are extremely small in the SM with right-handed neutrinos. For example, Ref. [17] has showed $\mathrm{Br}(\mu \to e\gamma) < 10^{-47}$. Such small branching ratios are unobservable.

The present experimental upper limits are [18]

$$\mathrm{Br}(\tau \to \mu\gamma) < 1.1 \times 10^{-6},$$
$$\mathrm{Br}(\tau \to e\gamma) < 2.7 \times 10^{-6}, \tag{4}$$
$$\mathrm{Br}(\mu \to e\gamma) < 1.1 \times 10^{-11}.$$

These bounds are expected to be improved by a few orders of magnitude in the future. For example, in an experiment under preparation at PSI [19], it is planned to reach a sensitivity

$$\mathrm{Br}(\mu \to e\gamma) < 1 \times 10^{-14}. \tag{5}$$

Thus, these processes are ideal tools to search for new physics. The observation of any rate for one of these processes would be a signal of new physics.

The relevant Feynman diagrams for the contributions of the neutral top pion π_t^0 to the LFV processes $l_i \to l_j \gamma$ are shown in Fig. 1. The internal fermion lines may be τ, μ, or e. However, the internal fermion propagator provides a term proportional to m_f^2 in the numerator, which is not cancelled by the m_f^2 in the denominator since the heavy π_t^0 mass dominates the denominator. Thus, we only take the internal fermion line as the τ fermion line.

Using Eq. (2), Eq. (3), and other relevant Feynman rules, the decay widths of the LFV processes $l_i \to l_j \gamma$ can be written as

$$\Gamma(\tau \to m\gamma) = \frac{m_\tau^5 k_{\tau m}^2 \alpha_e}{2048\nu^4\pi^4}\left[F_1^2 - \frac{1}{2}m_\tau^2(F_2^2 + F_2F_3) - m_\tau F_1F_2\right], \tag{6}$$

where $m = \mu$ or e, F_i are

$$F_1 = B_0 + m_{\pi_t}^2 C_0 - 2C_{24} + m_\tau^2(C_{11} - C_{12}) - B_0^* - B_1', \tag{7}$$

$$F_2 = 2m_\tau(-C_{21} - C_{22} + 2C_{23}), \tag{8}$$

$$F_3 = 2m_\tau(C_{22} - C_{23}), \tag{9}$$

$$\Gamma(\mu\to e\gamma)=\frac{m_\tau^4 m_\mu k_{\tau\mu}^2 k_{\tau e}^2\alpha_e}{2048\nu^4\pi^4}\left[F'^2_1-m_\mu F_1'F_2' -\frac{1}{2}m_\mu^2(F'^2_2+F_2'F_3')\right], \tag{10}$$

with

$$F_1'=m_\mu^2(C_{11}-C_{12})+m_\tau(m_\mu-m_\tau)C_0-2C_{24}+B_0+m_{\pi_t}^2C_0 -\frac{m_\tau}{m_\mu}B_0^*+\frac{m_\tau-m_\mu}{m_\mu}B_0'-B_1', \tag{11}$$

$$F_2'=2[(m_\tau-m_\mu)(C_{11}-C_{12})-m_\mu(C_{21}+C_{22}-C_{23})], \tag{12}$$

$$F_3'=2[m_\tau C_{12}+m_\mu(C_{22}-C_{23})]. \tag{13}$$

The standard Feynman integrals B_n, C_0, and C_{ij} can be written as

$$C_{ij}=C_{ij}(-p_l,p_\gamma,m_{\pi_t},m_\tau,m_\tau),$$

$$C_0=C_0(-p_l,p_\gamma,m_{\pi_t},m_\tau,m_\tau), \tag{14}$$

$$B_0=B_0(p_\gamma,m_\tau,m_\tau),\quad B_0^*=B_0(-p_m,m_{\pi_t},m_\tau),$$

$$B_1'=B_1'(-p_l,m_{\pi_t},m_\tau), \tag{15}$$

where $m_{\pi_t}$is the mass of the top pion, the variable P_m ($m=\mu$ or e) is the momentum of the find state lepton, P_l is the momentum of the initial state lepton, and $l=\tau$ or μ, which corresponds the lepton τ decay $\tau\to e(\mu)\gamma$ and the lepton μ decay $\mu\to e\gamma$, respectively. In the above equations, we have assumed that the masses of the final state lepton equal to zero.

For TC2 models, the top-color interactions only contact with the third generation. The new particles, such as gauge boson Z' and top pions $\pi_t^{0,\pm}$, treat the fermions in the third generation differently from those in the first and second generations and treat the fermions in the first generation the same as those in the second generation. So, in our calculation, we will assume that the mixing factor $k_{\tau\mu}$ is equal to the mixing factor $k_{\tau e}$. In this case, we have $\Gamma(\tau\to\mu\gamma)\simeq\Gamma(\tau\to e\gamma)$ for $m_\mu\simeq 0, m_e\simeq 0$. The corresponding branching ratios Br$(l_i\to l_j\gamma)$ can be written as

$$\mathrm{Br}(\tau\to\mu\gamma)\simeq\mathrm{Br}(\tau\to e\gamma)=\mathrm{Br}^{exp}(\tau\to e\nu_e\overline{\nu}_\tau)\frac{\Gamma(\tau\to e\gamma)}{\Gamma(\tau\to e\nu_e\overline{\nu}_\tau)}, \tag{16}$$

$$\mathrm{Br}(\mu\to e\gamma)=\frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\nu_e\overline{\nu}_\mu)}, \tag{17}$$

with

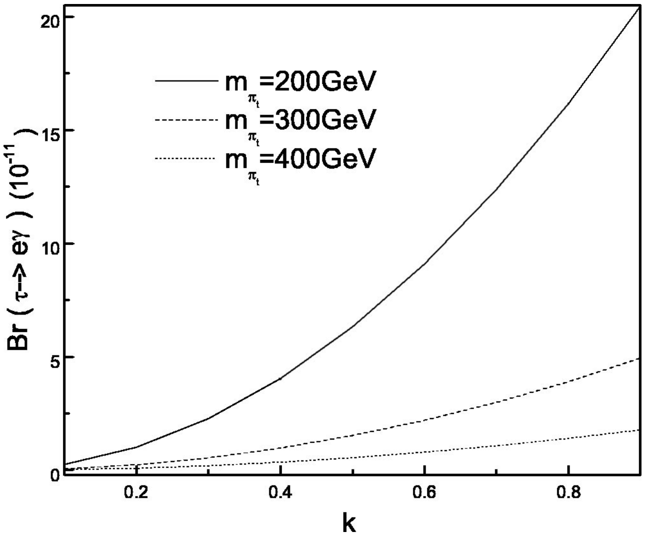

FIG. 2. The branching ratio Br$(\tau\to e\gamma)$ as a function of the flavor mixing factor k for three values of the top-pion mass m_{π_t}.

$$\Gamma(\tau\to e\nu_e\overline{\nu}_\tau)=\frac{m_\tau^5G_F^2}{192\pi^3},\quad \Gamma(\mu\to e\nu_e\overline{\nu}_\tau)=\frac{m_\mu^5G_F^2}{192\pi^3}, \tag{18}$$

where the Fermi coupling constant $G_F=1.16637\times10^{-5}$ GeV^{-2} and the precision measured branching ratio Br$^{exp}(\tau\to e\nu_e\overline{\nu}_\tau)=(17.83\pm0.06)\%$ [18].

To obtain numerical results, we take the SM parameters as $\alpha_e(m_Z)=\frac{1}{128.8}$, $m_\tau=1.777$ GeV, $m_\mu=0.105$ GeV [18]. The limits on the mass m_{π_t} of the top pion may be obtained via studying its effects on various experimental observables [20]. It has been shown that m_{π_t} is allowed to be in the range of a few hundred GeV depending on the models. As numerical estimation, we take the top-pion mass m_{π_t} and the mixing factor $k=k_{\tau\mu}=k_{\tau e}$ as free parameters.

We plot the branching ratios Br$(\tau\to e\gamma)$ and Br$(\mu\to e\gamma)$ as a function of the mixing factor k for three values of the top-pion mass in Figs. 2 and 3, respectively. To compare the value of Br$(\mu\to e\gamma)$ given by π_t^0 exchange with its current experimental limit, we have used the horizontal solid line to denote Br$(\mu\to e\gamma)=1.1\times10^{-11}$ in Fig. 3. One can see from Figs. 2 and 3 that the branching ratios of the LFV processes $l_i\to l_j\gamma$ are Br$(\tau\to\mu\gamma)\simeq$Br$(\tau\to e\gamma)<2\times10^{-10}$ and Br$(\mu\to e\gamma)<1.4\times10^{-7}$ in most of the parameter space of TC2 models. The branching ratio Br$(\tau\to l\gamma)$ is at least four orders of magnitude below the present experimental bound on $\tau\to l\gamma$, which is far from the reach of present or next generation experiments. The branching ratio Br$(\mu\to e\gamma)$ can be above or below the present experimental bound on $\mu\to e\gamma$, which depends on the value of the top-pion mass m_{π_t} and the mixing factor k. In most of the parameter space, the value of Br$(\mu\to e\gamma)$ is in the range of 2.5×10^{-12} to 1.4×10^{-7}.

Using the present experimental bound on the LFV process $\mu\to e\gamma$, we can give the constraints on the mixing factor k

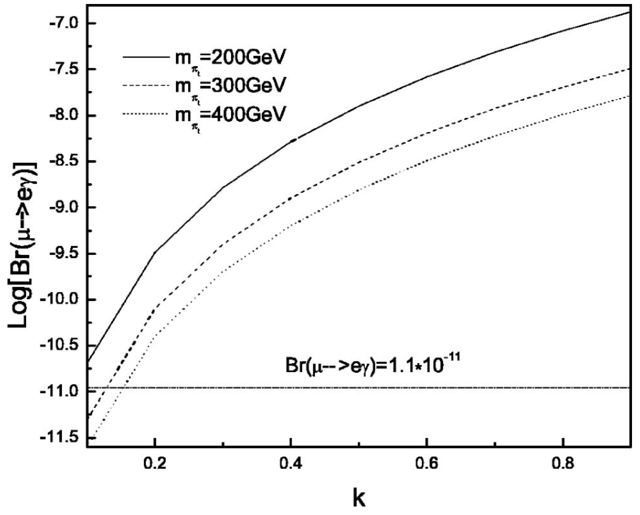

FIG. 3. The branching ratio Br($\mu \to e\gamma$) as a function of the flavor mixing factor k for three values of the top-pion mass m_{π_t}.

for 150 GeV$\leq m_{\pi_t} \leq$400 GeV. The numerical results are showed in Fig. 4. From Fig. 4 we can see that the mixing factor k increases as m_{π_t} increases. If we demand that the top-pion mass is smaller than 400 GeV, then there must be $k \leq 0.21$. Thus, the present experimental upper bound of the LFV process $\mu \to e\gamma$ gives severe constraints on the free parameters m_{π_t} and k of TC2 models. In the next section, we will take the $\mu \to e\gamma$ constraints into account and calculate the branching ratios of the LFV processes $l_i \to l_j l_k l_l$.

III. THE LFV PROCESSES $l_i \to l_j l_k l_l$

In TC2 models, the LFV processes $l_i \to l_j l_k l_l$ can be generated at the tree level and also can be induced via photon penguin diagrams at the one-loop level, as shown in Fig. 5. For the diagrams Figs. 5(b),(c),(d), we have taken $k=l$.

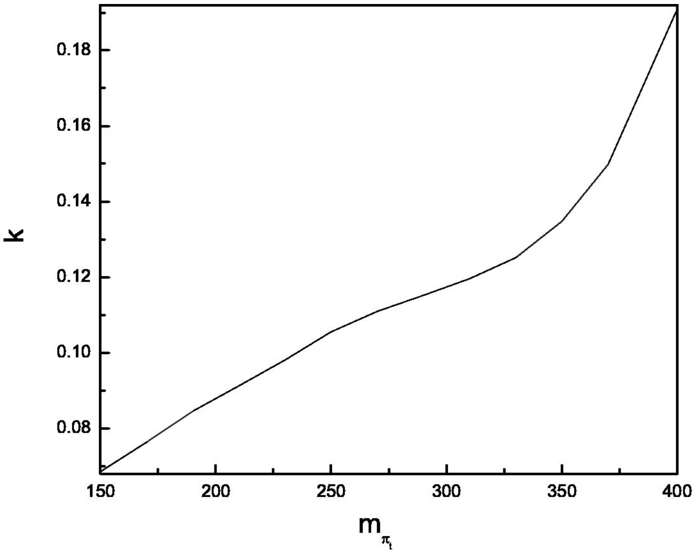

FIG. 4. The flavor mixing factors k as a function of top-pion mass m_{π_t} for Br($\mu \to e\gamma$)$=1.2\times 10^{-11}$.

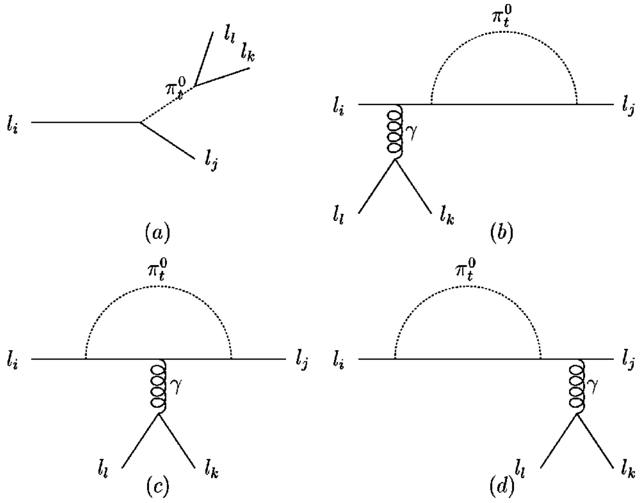

FIG. 5. The tree-level and one-loop Feynman diagrams contribute to the LFV processes $l_i \to l_j l_k l_l$ induced by π_t^0 exchange.

Let us first consider the contributions of the neutral top pion π_t^0 to the LFV processes $l_i \to l_j l_k l_l$ via Fig. 5(a). For the decay $\mu \to 3e$, it is induced by the FC scalar coupling $\pi_t^0 \overline{\mu e}$. However, the top-color interactions only contact with the third generation fermions. The flavor mixing between the first and second generation fermions is very small [15]. In numerical estimation, we will assume $k_{\mu e} \simeq 0$. So, the branching ratio Br($\mu \to 3e$) induced by π_t^0 exchange at the tree-level is zero. The LFV processes $\tau \to 2\mu e$ and $\tau \to 2e\mu$ can only be generated via the FC couplings $\pi_t^0 \tau e$ and $\pi_t^0 \tau\mu$. The decay widths of the processes $\tau \to l_i l_j l_k$ are given by

$$\Gamma(\tau \to 3e) = \frac{m_\tau^7 m_e^2}{1042\pi^3 m_\pi^4 \nu^4} k^2, \tag{19}$$

$$\Gamma(\tau \to 3\mu) = \frac{m_\tau^7 m_\mu^2}{1042\pi^3 \nu^4 m_\pi^4} k^2, \tag{20}$$

$$\Gamma(\tau \to 2\mu e) = \frac{m_\tau^7 m_\mu^2}{3072\pi^3 m_\pi^4 \nu^4} k^2, \tag{21}$$

$$\Gamma(\tau \to 2e\mu) = \frac{m_\tau^7 m_e^2}{3072\pi^3 m_\pi^4 \nu^4} k^2, \tag{22}$$

where m_l ($l=\mu$, e, or τ) represents the lepton mass and $k=k_{\tau\mu}=k_{\tau e}$.

Now we consider the one-loop contributions of the neutral top pion π_t^0 to the LFV processes $l_i \to l_j l_k l_l$ via the photonic penguin diagrams shown in Figs. 5(b),(c),(d). Same as Fig. 1, the internal fermion line of the photonic penguin diagrams only is the τ fermion line. Comparing Fig. 5 with Fig. 1, one can use the branching ratios Br($l_i \to l_j \gamma$) to express the branching ratios $\mathrm{Br}(l_i \rightarrow l_j l_k l_l)$ [10]. The one-loop expressions of the branching ratios $\mathrm{Br}(l_i \rightarrow l_j l_k l_l)$ can be written as:

$$\begin{aligned}\mathrm{Br}^{1\text{-}loop}(\tau\rightarrow 3\mu) &= \mathrm{Br}^{1\text{-}loop}(\tau\rightarrow 3e) = \mathrm{Br}^{1\text{-}loop}(\tau\rightarrow 2\mu e)\\ &= \mathrm{Br}^{1\text{-}loop}(\tau\rightarrow 2e\mu)\\ &\simeq \frac{\alpha_e}{3\pi}\left(\ln\frac{m_\tau^2}{m_\mu^2}-\frac{11}{4}\right)\mathrm{Br}(\tau\rightarrow e\gamma),\end{aligned} \tag{23}$$

$$\mathrm{Br}^{1\text{-}loop}(\mu\rightarrow 3e) \simeq \frac{\alpha_e}{3\pi}\left(\ln\frac{m_\tau^2}{m_\mu^2}-\frac{11}{4}\right)\mathrm{Br}(\mu\rightarrow e\gamma). \tag{24}$$

For the LFV processes $\tau\rightarrow l_i l_j l_k$, we have assumed $m_\mu \simeq 0$, $m_e \simeq 0$ and taken $\mathrm{Br}(\tau\rightarrow e\gamma) \simeq \mathrm{Br}(\tau\rightarrow\mu\gamma)$. The expressions of the $\mathrm{Br}(\tau\rightarrow e\gamma)$ and $\mathrm{Br}(\mu\rightarrow e\gamma)$ have been given in Eqs. (16) and (17), respectively.

Comparing the one-loop contributions of π_t^0 to the $l_i \rightarrow l_j l_k l_l$ with the tree-level contributions, we find that the branching ratios $\mathrm{Br}(\tau\rightarrow 2e\mu)$, $\mathrm{Br}(\tau\rightarrow 3e)$, and $\mathrm{Br}(\mu\rightarrow 3e)$ given by the one-loop diagrams mediated by π_t^0 exchange are larger than those generated by the tree-level diagrams at least by four orders of magnitude. This is because the FD coupling $\pi_t^0 ee$ is proportional to m_e/ν, which can strongly suppress the values of these branching ratios. However, for the processes $\tau\rightarrow 3\mu$ and $\tau\rightarrow 2\mu e$, two kinds of contributions are comparable. Thus, we will ignore the tree-level contributions of π_t^0 exchange to the $\tau\rightarrow 3e$, $\tau\rightarrow 2e\mu$, and $\mu\rightarrow 3e$ in the following numerical estimation.

Taking into account the constraints of the current experimental upper bound $\mathrm{Br}(\mu\rightarrow e\gamma) \leqslant 1.1\times 10^{-11}$ on the free parameters m_{π_t} and k, we find that the branching ratio $\mathrm{Br}(\mu\rightarrow 3e)$ is approximately equal to 2.87×10^{-14}, which might be observable in the planned experiments of the next generation. Combining the tree-level and one-loop contributions, we have $\mathrm{Br}(\tau\rightarrow 3e) \simeq \mathrm{Br}(\tau\rightarrow 2e\mu)$ and $\mathrm{Br}(\tau\rightarrow 3\mu) \simeq \mathrm{Br}(\tau\rightarrow 2\mu e)$ in TC2 models, which are shown in Fig. 6 as functions of the top-pion mass m_{π_t}. In Fig. 6, we have used Br_1 and Br_2 represent $\mathrm{Br}(\tau\rightarrow 3e)$ and $\mathrm{Br}(\tau\rightarrow 3\mu)$, respectively. One can see from Fig. 6 that the branching ratios slowly decrease as m_{π_t} increases. As long as $m_{\pi_t} < 400$ GeV, we have $\mathrm{Br}(\tau\rightarrow 3e) \simeq \mathrm{Br}(\tau\rightarrow 2e\mu) \geqslant 1.1\times 10^{-15}$ and $\mathrm{Br}(\tau\rightarrow 3\mu) \simeq \mathrm{Br}(\tau\rightarrow 2\mu e) \geqslant 3.1\times 10^{-15}$. Even if we take $m_{\pi_t} = 200$ GeV, the branching ratio $\mathrm{Br}(\tau\rightarrow 3\mu)$ can only reach 1.54×10^{-14}, which is far below the experimental bound on $\tau\rightarrow l_i l_j l_k$ (10^{-6} or 10^{-7}) [18,21].

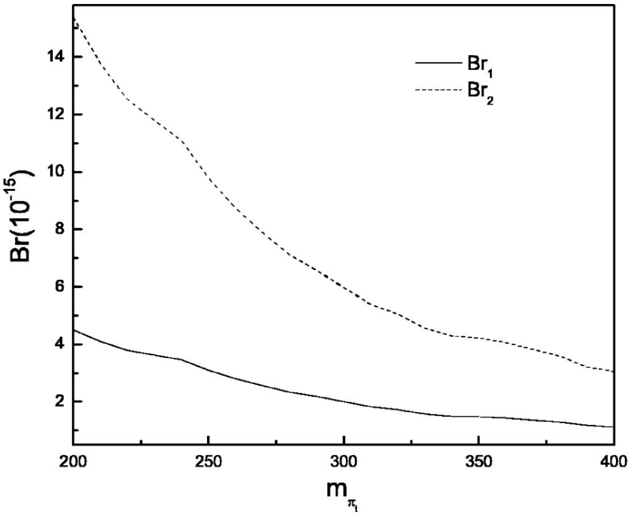

FIG. 6. The branching rations $\mathrm{Br}(l_i \rightarrow l_j l_k l_l)$ as functions of the top-pion mass m_{π_t}. We have assumed $\mathrm{Br}_1 = \mathrm{Br}(\tau\rightarrow 3e)$ and $\mathrm{Br}_2 = \mathrm{Br}(\tau\rightarrow 3\mu)$.

IV. CONCLUSIONS

The presence of physics top pions in the low-energy spectrum is a common feature of top-color models. The physics top pions have large Yukama couplings to the third family fermions and can induce the FC scalar couplings, which might give significant contributions to the FC processes. The effects of the top pion on these processes are governed by its mass m_{π_t} and the relevant flavor mixing factors.

In this paper we study the contributions of the neutral top pion π_t^0 predicted by TC2 models on the LFV processes $l_i \rightarrow l_j\gamma$ and $l_i \rightarrow l_j l_k l_l$. We find that the branching ratio $\mathrm{Br}(\tau\rightarrow e\gamma)$ is approximately equal to the branching ratio $\mathrm{Br}(\tau\rightarrow\mu\gamma)$ and is smaller than 2×10^{-10} in all parameter space of TC2 models. The present experimental bound on $\mu\rightarrow e\gamma$ produces severe constraints on the top-pion mass m_{π_t} and the mixing factor k. Based on these constraints on the free parameters of TC2 models, we further calculate the contributions of π_t^0 to the LFV processes $l_i \rightarrow l_j l_k l_l$ at the tree-level and one-loop level. For the LFV processes $\mu\rightarrow 3e$, $\tau\rightarrow 3e$, and $\tau\rightarrow 2e\mu$, the contributions coming from photonic penguin diagrams are larger than those from the tree-level top-pion exchange at least by four orders of magnitude. While two kinds of contributions are comparable for the processes $\tau\rightarrow 3\mu$ and $\tau\rightarrow 2\mu e$. If we take the top-pion mass $m_{\pi_t} \leqslant 400$ GeV, we have $\mathrm{Br}(\tau\rightarrow 3e) \simeq \mathrm{Br}(\tau\rightarrow 2e\mu) \geqslant 1.1\times 10^{-15}$ and $\mathrm{Br}(\tau\rightarrow 2\mu e) \simeq \mathrm{Br}(\tau\rightarrow 3\mu) \geqslant 3.1\times 10^{-15}$, which cannot be observable in the near future experiments. However, the branching ratio $\mathrm{Br}(\mu\rightarrow 3e)$ is approximately equal to 2.8×10^{-14}, which may be observable in the planned experiments of the next generation.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (90203005).

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