

Bounds on four-fermion contact interactions induced by string resonances

P. Burikham* and T. Han†

Department of Physics, 1150 University Ave., University of Wisconsin, Madison, Wisconsin 53706, USA

F. Hussain‡

The Abdus Salam International Center for Theoretical, Physics, Trieste, Italy

D. W. McKay§

Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045, USA

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Based on tree-level open-string scattering amplitudes in the low string-scale scenario, we derive the massless fermion scattering amplitudes. The amplitudes are required to reproduce those of the standard model at the tree level in the low energy limit. We then obtain four-fermion contact interactions by expanding in inverse powers of the string scale and explore the constraints on the string scale from low energy data. The Chan-Paton factors and the string scale are treated as free parameters. We find that data from the neutral and charged current processes at DESY HERA, Drell-Yan process at the Fermilab Tevatron, and from CERN LEP-II put lower bounds on the string scale M_S , for typical values of the Chan-Paton factors, in the range $M_S \geq 0.9 - 1.3$ TeV, comparable to Tevatron bounds on Z' and W' masses.

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I. INTRODUCTION

String theory, spoken or unspoken, is generally assumed to be the underpinning of the low scale gravity ideas [1,2] explored theoretically and experimentally in recent years. A number of examples of ambitious “top-down” models of string realizations of low scale gravity ideas have been advanced, aiming at consistently achieving the connection to standard model (SM) physics from higher mass scales in certain D-brane scenarios [3]. As yet a fully realistic model such as the SM has not been constructed. On the other hand, one could take a more phenomenological approach, from the “bottom up.” One of the recent endeavors is to obtain the SM tree-level amplitudes at low energies [4–6] based on open-string amplitudes [7,8]. This approach assures the correct low energy phenomenology as given by the SM, yet captures one of the essential features of string theory, namely the string resonances, in a relatively model-independent way. The basic assumptions in this approach are that the fundamental string scale M_S is at the order of 1 TeV, and that the dominant contributions to the low energy processes are due to the exchange of string resonances. Earlier work on phenomenological studies dealt with QED from the M_Z scale to the first few string resonances [4], or neutrino inclusive processes far above the string scale to explore the effects from cosmic neutrinos [5,6]. Phenomenologically, this string-amplitude approach complements the low-scale gravity calculations based on expansions in Kaluza-Klein modes [9], which are argued to be higher orders in string-coupling expansion [3,4].

The purpose of this article is to expand this effort to model both *neutral and charged current* interactions at energies below the string scale.

Data from ep collider HERA experiments at DESY, with lepton-parton center of mass (c.m.) energies receiving a good fraction of the full 320 GeV, the highest energy available in laboratory experiments for deep inelastic scattering, provide one interesting testing ground for low-scale string model ideas. Similarly the CERN e^+e^- collider LEP-II, with c.m. energies up to 200 GeV provides another reasonably sensitive probe of the low energy limit of our string-resonance amplitudes. The full c.m. energy is available to excite string effects in this case. At the Fermilab Tevatron, though the parton-parton collisions get typically only a modest fraction of the 1.8–2 TeV available in the $p\bar{p}$ c.m. energy, there is still sensitivity to 0.5–1 TeV scale physics.

The good agreement between all of the data from the facilities just mentioned and the SM allows bounds to be set on the mass scale of all kinds of new physics effects. For example, leptoquark states are one such effect, and perturbative string resonances can carry the same quantum numbers as the leptoquarks in some channels [5,6]. At the parton level, much of the kinematical range is low enough to justify keeping the lowest order terms in an expansion in inverse string scale. This allows a direct comparison of amplitudes with the existing limits on new physics contact interactions [10–13]. These observations that a comprehensive bound can be applied to a wide class of string-resonance models motivate the work we present here. We hope that exploration of the constraints imposed on the model parameters by the agreement between data and the SM will ultimately shed light on the way string theory signals could emerge as laboratory energies rise above the currently available regime.

This paper is organized as follows. In Sec. II, we summarize the construction of neutral and charged current interactions for SM light fermions based on open-string scattering

*Electronic address: piyabut@pheno.physics.wisc.edu

†Electronic address: than@pheno.physics.wisc.edu

‡Electronic address: hussainf@ictp.trieste.it

§Electronic address: mckay@kuark.phsx.ukans.edu

amplitudes. We then in Sec. III take the low-energy expansion by expanding the string amplitudes in powers of the inverse string scale evaluated at typical kinematic points to obtain the effective four-fermion contactlike interactions. We check that the approximation is good in the kinematic ranges we use. Comparing with the current limits on these interactions, we derive bounds on the string scale M_S . We conclude in Sec. IV.

II. OPEN STRING TREE GRAPH AMPLITUDES

In weakly coupled string theory with a low string scale, one generically expects the string amplitude corrections to the standard model processes to dominate over the graviton corrections, which enter at one loop and are parametrically suppressed by an extra factor of g^2 , a gauge coupling squared [3,4,8]. At energies well below M_S , the stringy corrections can be systematically taken into account by the low-energy expansion of the string amplitudes in terms of s/M_S^2 .

We assume that the tree-level string amplitudes represent the scattering of massless SM particles, as the zero string modes. The first attempt at exploring the low-scale string amplitudes was made to construct a string toy model of QED of electrons and photons [4]. The SM is embedded in a type IIB string theory whose 10-dimensional space has six dimensions compactified on a torus with common period $2\pi R$. There are N coincident D3-branes, on which open strings may end, that lie in the 4 extended dimensions. The extra symmetry of the massless string modes are eliminated by (unspecified) orbifold projection. The paper applies the results to Bhaba scattering and then adds several prescriptions to include some simple processes $e^+e^- \rightarrow Z^0 \rightarrow e^+e^-$ and $q\bar{q} \rightarrow g^* \rightarrow 2$ jets, where g^* represents a string resonance excitation of the gluon. However, this toy model does not attempt to be fully realistic in terms of the SM particle spectrum and their interactions.

Our construction of the tree graph amplitude follows the same pattern as that outlined in Refs. [5,6]. The result is a model containing the SM on the 3-branes and no unacceptable (i.e., unobserved) low energy degrees of freedom. This is accomplished by allowing the group theoretical Chan-Paton factors as free parameters. The masses of gauge bosons W and Z must be introduced by hand, since the string amplitude describes massless particle scattering and we are not consistently modeling the breaking of gauge invariance. Though all the standard model gauge couplings are assumed to unify to a single value at the string scale in this simple construction, we use the physical values of the SM electroweak couplings since we restrict ourselves here to energies below the string scale.

We begin with the general form for a four-fermion amplitude for open strings in such a braneworld framework. The parton level Mandelstam variables are denoted by s , t , and u . The physical scattering process will be identified as $f_1 + f_2 \rightarrow f_3 + f_4$. The s , t , and u channels are labeled (1,2), (1,4), and (1,3), respectively. The ordered amplitude with the convention that all momenta are directed inward reads [8,14,15]:

$$A_{string}(s,t,u) = ig^2 [t\bar{\psi}_1\gamma^\mu\psi_2\bar{\psi}_3\gamma_\mu\psi_4 - s\bar{\psi}_1\gamma^\mu\psi_4\bar{\psi}_3\gamma_\mu\psi_2] \\ \times \left[\frac{S(s,t)}{st} [T(1234) + T(4321)] \right. \\ \left. + (1 \leftrightarrow 4, s \leftrightarrow u) + (1 \leftrightarrow 2, t \leftrightarrow u) \right],$$

where the function $S(x,y)$ is similar to a Veneziano amplitude [16], and is defined by

$$S(x,y) = \frac{\Gamma(1-\alpha'x)\Gamma(1-\alpha'y)}{\Gamma(1-\alpha'x-\alpha'y)}, \quad (1)$$

where the Regge slope parameter $\alpha' = M_S^{-2}$. In the limit $M_S \gg \sqrt{s}$, $S \rightarrow 1$ and the low energy gauge theory expression for the amplitude is regained, as we show below. The factors $T(1234) + T(4321)$ and their $1 \leftrightarrow 4$ and $1 \leftrightarrow 2$ counterparts are proportional to the Chan-Paton factors [17] and involve traces over the group representation matrices, λ , of the fermions at the four vertices. For example, $T(1234) \propto \text{Tr}(\lambda^1\lambda^2\lambda^3\lambda^4)$ with normalization $\text{Tr}(\lambda^a\lambda^b) = \delta^{ab}$ in the adjoint representation of $U(n)$. Typically, with our normalization, the Chan-Paton factors are in the range of -4 to 4 for a general $U(n)$ group. The above general expression serves as the basis for calculating all of the specific helicity and internal quantum number possibilities in the case that the states 3 and 4 have outgoing momenta.

A. Charged current processes

The charged current (CC) string model amplitude in the weak coupling regime receives no contribution from the graviton at one loop. In this sense it is perhaps conceptually cleaner than the neutral current (NC) case [5,6], where the graviton exchange is contained in the one loop amplitude [4]. At energies above the string scale, the extra power of s/M_S^2 in the graviton contribution compensates for the Yang-Mills gauge coupling suppression of the loop amplitude compared to the tree graphs; there the strong gravity dynamics and the string resonance dynamics become comparable. Though we are focusing on the low energy region, where graviton exchange is suppressed, the CC amplitude construction is simpler than that of the NC because there are fewer processes and only one gauge coupling to consider. For this reason we discuss the CC case first in some detail, and then turn to the NC case.

For definitiveness, taking all helicities for the in and out states left-handed (denoted by L), we find the string tree amplitude:

$$A_{string}^{CC}(LL) = ig^2 \left[S(s,t) \frac{s}{t} T_{1234} + S(u,t) \left(-\frac{s}{t} - \frac{s}{u} \right) T_{1324} \right. \\ \left. + S(s,u) \frac{s}{u} T_{1243} \right],$$

where we have further simplified notation by introducing $T_{1234} = T(1234) + T(4321)$ and so forth. The corresponding standard model electroweak (EW) tree amplitude is

$$A_{EW}^{CC} = ig^2 \frac{s}{t - M_W^2}. \quad (2)$$

Here and henceforth, g is identified with the $SU(2)_L$ gauge coupling. We require that the charged current in the t channel contain the W boson as its zero mode and that there is no exotic (leptoquark) zero mode in the u channel. In order to remove the unwanted zero-mode pole, we must require

$$T_{1243} = T_{1324} \equiv T. \quad (3)$$

The low energy gauge theory limit should reproduce the W pole in the t channel in the tree approximation to the string amplitude. Using Eq. (3) and matching the coefficient of the $1/t$ pole to the SM result of Eq. (2), we identify

$$T_{1234} = 1 + T. \quad (4)$$

The tree-level result for the amplitude for $LL \rightarrow LL$, after removing the exotic zero-mode pole in the u channel and identifying the zero-mode pole in the t channel as the W boson, is

$$A_{string}^{CC}(LL) = ig^2 T \frac{s}{ut} f(s, t, u) + ig^2 \frac{s}{t - M_W^2} S(s, t), \quad (5)$$

where

$$f(s, t, u) \equiv uS(s, t) + sS(u, t) + tS(s, u). \quad (6)$$

In the limit $M_S \gg \sqrt{s}$, we have

$$S(s, t) \approx 1 - \frac{\pi^2}{6} \frac{st}{M_S^4}, \quad f(s, t, u) \approx -\frac{\pi^2}{2} \frac{stu}{M_S^4}. \quad (7)$$

The SM tree amplitude is reproduced in the limit $s/M_S^2 \rightarrow 0$. For later convenience we define $V(s, t, u)$ via Eq. (5) by

$$A_{string}^{CC}(LL) = ig^2 \frac{s}{t - M_W^2} V(s, t, u). \quad (8)$$

The above results are also applicable to right-handed antifermion scattering $\bar{R}\bar{R} \rightarrow \bar{R}\bar{R}$. The other helicity combinations including antileptons and antiquarks can be worked out by appropriate crossing. For instance, for the scattering of a left-handed lepton and a right-handed antiquark $L\bar{R} \rightarrow L\bar{R}$, or right-handed antilepton on left-handed quark $\bar{R}L \rightarrow \bar{R}L$, the $s \leftrightarrow u$ and $2 \leftrightarrow 3$ crossed amplitude applies. The amplitudes for this process read

$$\begin{aligned} A_{string}^{CC}(L\bar{R}) &= A_{string}^{CC}(\bar{R}L) = ig^2 \left[S(s, t) \left(-\frac{u}{t} - \frac{u}{s} \right) T_{1234} \right. \\ &\quad \left. + S(t, u) \frac{u}{t} T_{1324} + S(s, u) \frac{u}{s} T_{1243} \right] \\ &= A_{string}^{CC}(LL)(s \leftrightarrow u) = ig^2 \frac{u}{t - M_W^2} \bar{V}(s, t, u), \end{aligned} \quad (9)$$

where the generic label T is not distinguished from that in the LL case above, to avoid clutter in the notation. These expressions are the analogs of those written down for the NC neutrino case in Refs. [5,6], which we expand for the full range of NC cases in the next subsection.

B. Neutral current processes

The open string perturbative amplitude construction for $2 \rightarrow 2$ NC scattering follows exactly the same pattern as described above in the charged current case. The neutral current case involves 4-fermion amplitudes as well as 2-lepton plus 2-gluon external line amplitudes [6]. We find that in the low energy realm the gluon amplitudes contribute negligibly to the constraints. Therefore, we confine ourselves to the 4-fermion construction, again identifying zero-mode poles in the t channel with γ and Z exchange. As before, we require that the Chan-Paton factors are constrained to cancel the exotic zero modes in the other channels. To introduce the SM factors, we adopt the device that fermion labels in the Chan-Paton factors are the guide to constructing the low energy limit. This is because the λ 's of the external legs depend on the $SU(2) \otimes U(1)$ embedding in a larger (unifying) group, and the Chan-Paton traces over λ 's are linked to the quantum numbers of the s , t , and u channels. The connection between the string amplitude zero mode poles and the SM poles, in keeping with this philosophy, is described next.

We consider separately the low energy matching for $2 \rightarrow 2$ amplitudes for (1) all left-handed (L) or all right-handed (R); and (2) $LR \rightarrow LR$ and $RL \rightarrow RL$.

1. $\ell_\alpha q_\alpha \rightarrow \ell_\alpha q_\alpha$; $\alpha = L, R$

The string and SM electroweak tree amplitudes for the like-helicity combinations are

$$\begin{aligned} A_{string}^{NC}(\alpha\alpha) &= ig^2 \left(S(s, t) \frac{s}{t} T_{1234} + S(t, u) \frac{s^2}{ut} T_{1324} \right. \\ &\quad \left. + S(u, s) \frac{s}{u} T_{1243} \right), \end{aligned} \quad (10)$$

$$A_{EW}^{NC}(\alpha\alpha) = 2ie^2 \frac{s}{t} \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\alpha^q}{s_W^2 c_W^2} \right), \quad (11)$$

where $Q_{q,\ell}$ are the electric charge of quark and lepton; $s_W = \sin \theta_W, c_W = \cos \theta_W$. Matching with $e = g \sin \theta_W$ gives

$$T_{1243} = T_{1324} \equiv T, \quad (12)$$

$$T_{1234} = T + 2s_W^2 \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\alpha^q}{s_W^2 c_W^2} \right), \quad (13)$$

which guarantees that there is no zero-mode exotic u -channel pole and that the SM tree amplitude is recovered in the limit $S(s, t) = S(t, u) = S(s, u) \rightarrow 1$ where $s \ll M_S^2$. Because the string models have nothing to say about the electroweak symmetry breaking, we put the effect in by hand in our treatment. We choose to introduce it through the condition on the

Chan-Paton factors, which leads to the t dependence in Eq. (13). In the absence of breaking, all of the gauge bosons would be massless $M_Z, M_W=0$, and Eq. (13) would relate the Chan-Paton factors only through the gauge charges.

Our modified string amplitude now reads

$$A_{string}^{NC}(\alpha\alpha) = ig^2 T \frac{s}{ut} f(s,t,u) + 2ig^2 s_W^2 S(s,t) \frac{s}{t} \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\alpha^q}{s_W^2 c_W^2} \right), \quad (14)$$

where $f(s,t,u)$ was defined in Eq. (7). Our convention for the SM neutral-current couplings is

$$g_L^f = T_{3f} - Q_f \sin^2 \theta_W, \quad g_R^f = -Q_f \sin^2 \theta_W. \quad (15)$$

We have adopted the shorthand that all parameters proportional to Chan-Paton factors are designated by the single symbol T . In fact, in our study of the low energy constraints on the models in the following section, we will make the simplifying assumption that the factors are all equal.

2. $\ell_{\alpha\beta} \rightarrow \ell_{\alpha\beta}$; $\alpha, \beta = L, R$; $\alpha \neq \beta$

The string and SM electroweak tree amplitudes are

$$A_{string}^{NC}(\alpha\beta) = ig^2 \left(S(s,t) \frac{u^2}{st} T_{1234} + S(t,u) \frac{u}{t} T_{1324} + S(u,s) \frac{u}{s} T_{1243} \right), \quad (16)$$

$$A_{EW}^{NC}(\alpha\beta) = 2ie^2 \frac{u}{t} \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\alpha^q}{s_W^2 c_W^2} \right). \quad (17)$$

Again, matching with $e = g \sin \theta_W$ gives

$$T_{1243} = T_{1234} \equiv T, \quad (18)$$

$$T_{1324} = T + 2s_W^2 \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\alpha^q}{s_W^2 c_W^2} \right). \quad (19)$$

The final string amplitude reads

$$A_{string}^{NC}(\alpha\beta) = ig^2 T \frac{u}{st} f(s,t,u) + 2ig^2 s_W^2 S(t,u) \frac{u}{t} \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\beta^q}{s_W^2 c_W^2} \right). \quad (20)$$

This is $s \leftrightarrow u$ crossing from Eq. (14).

To obtain other amplitudes involving antifermions, it is a matter of simple crossing. For example, for the Drell-Yan (DY) process $q\bar{q} \rightarrow \ell\bar{\ell}$, we simply have $s \leftrightarrow t$ crossing of the above formulas in Eqs. (14) and (20).

The amplitudes we have constructed are particularly convenient for comparing to the contact interaction amplitudes analyzed and constrained by data in the literature [11,12]. We turn next to this comparison, deriving constraints on M_S in the process.

III. LINKING STRING AMPLITUDES TO CONTACT INTERACTIONS

In this section, we convert constraints on contact interactions to constraints on the string scale M_S for given T values. In order to compare to data at low energies, we express string deviation from SM electroweak amplitude by $\Delta_{\alpha\beta}$ ($\alpha, \beta = L, R$), namely,

$$A_{string}(\alpha\beta) = A_{EW}(\alpha\beta) + \Delta_{\alpha\beta}. \quad (21)$$

Using Eq. (7), we find for like-helicity fermion scattering ($\alpha\alpha = LL$ and RR)

$$\Delta_{\alpha\alpha} \simeq -\frac{\pi^2}{6} \frac{st}{M_S^4} A_{EW}(\alpha\alpha) - iTg^2 \frac{\pi^2}{2} \frac{s^2}{M_S^4}, \quad (22)$$

where T is the generic parametrized Chan-Paton factor corresponding to the particular process. For unlike-helicity combinations in the neutral current case, $\Delta_{\alpha\beta} = \Delta_{\alpha\alpha}(s \leftrightarrow u)$,

The reduced amplitudes for contact interactions from physics beyond the SM are conventionally parametrized as [10–13]

$$\Delta M_{\alpha\beta}^{\ell q} = \eta_{\alpha\beta}^{\ell q} = \epsilon \frac{4\pi}{\Lambda_{\ell q}^2}. \quad (23)$$

The cutoff $\Lambda_{\ell q}$ is the mass scale at which new physics sets in. It presumably corresponds to the mass of the heavy strongly interacting particles that mediate the new interaction and it is referred as the ‘‘compositeness scale.’’ The sign factor $\epsilon = \pm 1$ allows for constructive or destructive interference between the contact interaction and the SM amplitudes. Typically, in the fit to a given class of interactions, it is designated Λ_{\pm} to distinguish between fit values obtained with $\epsilon = \pm 1$.

The relations between the string contribution and the reduced amplitude parametrization can be found to be, for like-helicity fermion scattering,

$$\Delta M_{\alpha\alpha} = \frac{\Delta_{\alpha\alpha}}{i2s} \simeq -\frac{\pi^2}{12} g^2 \frac{s}{M_S^4} (F + 3T). \quad (24)$$

For unlike-helicity fermion scattering, $\Delta M_{\alpha\beta} = \Delta M_{\alpha\alpha}(s \leftrightarrow u)$. For a Drell-Yan process, which involves antifermions, we have $s \leftrightarrow t$ from Eq. (24). The factor F includes the information for chiral couplings and it is

$$F = \begin{cases} \frac{t}{t - M_W^2} & \text{for charged current,} \\ 2s_W^2 \left(Q_q Q_\ell + \frac{t}{t - M_Z^2} \frac{g_\alpha^\ell g_\beta^q}{s_W^2 c_W^2} \right) & \text{for neutral current } \ell q. \end{cases} \quad (25)$$

It is interesting to note that the leading stringy corrections to the SM amplitudes as in Eq. (22) enter at dimension 8, while the standard parametrization for four-fermion contact interactions as in Eq. (23) is of dimension 6. Due to this additional energy-dependent suppression factor s/M_S^2 , the constraints obtained from low energy data on M_S will thus be weaker than that on $\Lambda_{\ell q}$.

In certain more complicated brane-world models, for example intersecting D-branes [18], there are corrections at dimension 6 from Kaluza-Klein excitations, winding modes as well as string oscillators. They lead to a stronger limit on the lower bound of the string scale, about 2–3 TeV [18].

A. Validity of the approximate amplitudes

With the above setup, we are in position to extract bounds on the string scale from the values of parameters of contact interactions. A global fit of contact interactions to all of the data discussed above plus the low energy data from neutral current and charged current process, including atomic parity violation, is also reported in Ref. [11]. The low energy data dominate these global constraints. As noted earlier, the s/M_S^2 dependence of our string amplitudes severely suppresses stringy effects at very low energies and the low energy data are insensitive to the string scale. We will thus mainly make use of the data at highest energies available like in HERA, Tevatron, and LEP-II.

Our expansion of the factors $S(x, y)$, where $x, y = s, t$, or u , should be valid if bounds on M_S are found to be well above the kinematical region covered by the data. How close can the scale be to the kinematical range of the data before the approximate expansion becomes unreliable? We address this question by computing the CC cross section $e^- p \rightarrow \nu + X$ with the full amplitudes and with the approximated amplitudes. The differential deep-inelastic-scattering (DIS) cross section, in terms of the functions V in $A_{string}^{CC}(LL)$ in Eq. (8) and \bar{V} in $A_{string}^{CC}(LR)$ in Eq. (9), reads

$$\frac{d^2\sigma}{dx dQ^2} = \frac{d\sigma^{SM}}{dQ^2} \{ [u(x, Q^2) + c(x, Q^2)] V^2 + (1-y)^2 [\bar{d}(x, Q^2) + \bar{s}(x, Q^2)] \bar{V}^2 \}, \quad (26)$$

where $d\sigma^{SM}/dQ^2$ is the SM W -exchange Born term differential cross section. In the course of this study, we can probe as well the simple constraint on the model that follows from the measured total cross section [19–21], namely,

$$\sigma(Q^2 > 200 \text{ GeV}^2) = 66.7_{-2.9}^{+3.2} \text{ pb},$$

at $E_{c.m.} = 318 \text{ GeV}$, the ep c.m. energy. The ZEUS Collaboration quotes the value

$$\sigma(Q^2 > 200 \text{ GeV}^2) = 69.0_{-1.3}^{+1.6} \text{ pb}$$

as the SM expectation using its next-to-leading-order (NLO) QCD fit. For example, with $T=1$ one finds the experimental 95% confidence level (C.L.) limit

$$M_S \geq 0.45 \text{ TeV}, \quad (27)$$

whether one uses the full or the approximate amplitude. In general the approximate cross-sections agree with the complete calculation to 3 figures until $M_S \approx E_{c.m.}$, where one finds differences of the order of a percent. For example, with $T=1$ and $M_S = 320 \text{ GeV}$, the full and approximate cross sections are 85.2 pb and 82.8 pb, while with $T=-1$ the cross sections are 62.2 pb and 62.5 pb. The approximation is evidently quite good so long as $M_S > E_{c.m.}$, since the lowest Regge resonance slips into the physical region when $M_S \leq E_{c.m.}$ and should, in principle, be represented by a resonant form with finite width. However, the vanishing of the structure functions as $x \rightarrow 1$ minimizes the impact of the nearby resonance on the DIS cross section as $M_S \rightarrow E_{c.m.}$ from above.

B. Evaluation of lower limits on M_S

Focusing on the chiral amplitudes A_{LL} , which enter in both the NC and CC processes, we combine Eqs. (23) and (24) to express the constraint on M_S at a given T value and Λ bound value as

$$M_S > \left[-\frac{\pi^2}{12} \frac{g^2 s}{\eta} (F + 3T) \right]^{1/4} \text{ for DIS at HERA.} \quad (28)$$

For the DY process at the Tevatron and e^+e^- annihilations at LEP-II, we have $s \leftrightarrow t$ in Eq. (28).

In Table I we show the lower bounds on M_S that follow from the corresponding best fit values of η from the HERA NC data, the Drell-Yan data from Tevatron, and the hadronic cross section from LEP-II quoted in Ref. [11]. These values follow from our NC analysis above. In the table we also use the NC data with the $SU(2)$ relation between the CC and NC amplitudes, namely,

$$\Delta M_{LL}(CC) = \Delta M_{LL}^{ed} - \Delta M_{LL}^{eu},$$

to give corresponding limits on the CC amplitudes. These are not independent constraints, of course, but simply show the impact of the data in the CC sector. We also include the direct CC bound on M_S obtained in the preceding subsection

TABLE I. Lower bounds on the string scale M_S from contact interaction parameters, at a 95% C.L. The Chan-Paton factor T has been taken as ± 1 as indicated.

	HERA NC		Drell-Yan		LEP	
	η (TeV $^{-2}$)	M_S (TeV)/ T	η	M_S/T	η	M_S/T
η_{LL}^{eu}	$-1.18^{+0.53}_{-0.56}$	0.34/+1	$-0.19^{+0.24}_{-0.21}$	0.85/+1	$-0.22^{+0.086}_{-0.084}$	0.32/0 0.50/-1
η_{LL}^{ed}	$1.53^{+1.59}_{-1.35}$	0.29/-1	$0.88^{+0.58}_{-0.73}$	0.34/0 0.57/+1 0.73/-1	$0.26^{+0.095}_{-0.098}$	0.29/0 0.48/+1
η_{CC}	$2.71^{+1.67}_{-1.46}$	0.26/-1	$1.07^{+0.62}_{-0.76}$	0.41/0 0.58/+1 0.73/-1	$0.48^{+0.13}_{-0.13}$	0.33/0 0.45/+1
HERA σ_{CC} :		0.45/+1	(CDF) 0.80	0.53/0 0.75/+1		

from HERA data and the DY bound obtained by Collider Detector at Fermilab (CDF) at the Tevatron on the CC $qqe\nu$ compositeness scale [22], with the corresponding M_S bound. When translating the existing constraints on $\eta_{\alpha\beta}$ to M_S , we need to take into account the different energy dependence as noted earlier. In computing the values of the bounds in Table I, we use the rule of thumb that the average parton energy fraction is $\langle x \rangle \approx 1/3$, so the direct channel HERA parton c.m. energy squared is $s \approx E_{c.m.}^2/3 \approx (0.18 \text{ TeV})^2$. At the Tevatron, where the total c.m. energy was 1.8 TeV, our nominal parton c.m. energy squared is $s \approx E_{c.m.}^2/9 = (0.6 \text{ TeV})^2$. For the momentum transfer squared we take $Q^2 = s/2$. In the following subsections we explain the entries in the table.

C. HERA NC

Limits on the deviation from SM predictions for processes at HERA lead to corresponding bounds on string parameter. From Table IV of Ref. [11], the limits on $\Delta M_{LL} = \eta_{LL}^{eq}$, provided separately for $eeuu$ and $eedd$, are given. At 2σ level (or 95% C.L.), we have the lower bound $\eta_{LL}^{eu} = -2.3/\text{TeV}^2$. We apply the weak isospin constraint that the $eeuu$ and $eedd$ amplitudes have opposite sign, which implies the upper bound $\eta_{LL}^{ed} = 4.7/\text{TeV}^2$. In order to obtain a lower bound on string scale M_S , we need the correct sign of ΔM from our string expression corresponding to each limit on value of η . Consequently, in the $eeuu$ case, the gauge factor $(F+3T) \geq 0$ is required. In the $eedd$ case, the requirement is $(F+3T) \leq 0$. With typical values $s = (0.18 \text{ TeV})^2$, $t/(t-M_Z^2) \approx 1/2$ and $T = +1$ (-1), we find the bounds 0.34 (0.29) TeV as shown in the table. We should comment here that, the typical bounds on masses of leptoquark resonances at HERA are in the range 0.25–0.29 TeV [21], roughly compatible with bounds from our contact interaction analysis. Slightly higher values of $|T|$ produce higher bounds on M_S . For example, with $T = -2$, the value is 0.35 TeV for $eedd$. Clearly larger absolute values of T correspond to larger bounds on M_S , limited only by the requirement that the effective coupling constants remain perturbative, consistent with our string amplitude construction. From Eq. (28), we see that

$M_S \propto (F+3T)^{1/4}$, or roughly proportional to $T^{1/4}$. This is also the case for DY processes at the Tevatron and e^+e^- annihilation at LEP-II.

D. Drell-Yan processes at the Tevatron

We follow the same pattern as described above, now using $s \leftrightarrow t$ of Eq. (24), for limits from DY processes at the Tevatron. For typical values we find the strongest bounds on string scale are 0.85 and 0.73 TeV for modest values $T = +1$ and -1 for $eeuu$ and $eedd$ respectively. An independent search for deviations from the SM in the DY channel $qq\nu l$ at CDF [22], cited in Refs. [12], yields a 95% C.L. upper bound of $0.8/\text{TeV}^2$ on the value of η_{CC} . The corresponding limits on M_S are independent of those derived from the $eedd$ case. Searches for W' and Z' resonances at the Tevatron yield bounds similar to the larger of the bounds just quoted, namely in the range 0.75–0.85 TeV [22]. As in the case of leptoquark resonance searches at HERA, the bounds on the W' and Z' masses at the Tevatron are roughly consistent with the contact interaction bounds we just described. The larger DY bounds rise to 0.86 TeV and 1.04 TeV when the T values are doubled to ± 2 , indicating that increasing the magnitude of T has a marked effect on M_S . In Fig. 1 we show the plot of the lower bound on M_S versus the Chan-Paton parameter T in the range $1 \leq |T| \leq 4$ for the $eeuu$ and $eedd$ cases, which give representative largest lower bounds on M_S for a given T value. In any case, it is fair to say that the resonant bounds and the contact interaction bounds are complementary ways to probe for string physics at the TeV scale.

E. LEP-II

The LEP-II results are from the lowest nominal energy, but have the advantage that all of the c.m. energy can go directly into producing new physics. For LEP-II, we use $s = (0.2 \text{ TeV})^2$ with $t \approx -s/2$. We only consider the cross section for hadron production as stated in Refs. [10,11]. Limits are as listed in Table I. The limits tend to be stronger than in the DIS at HERA case, since the values of η 's and their uncertainties are significantly smaller and the c.m. energy is

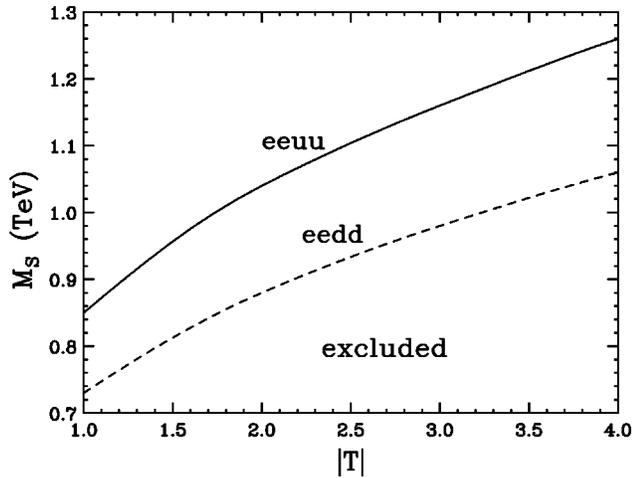


FIG. 1. Relationship between Chan-Paton parameters and lower bounds of the string scale M_S from the DY process at the Tevatron. T is positive and negative for $eeuu$ and $eedd$, respectively. The region under the curves is excluded at 95% C.L.

slightly larger than the characteristic value used in our HERA analysis. A consistent but somewhat weaker limit is given in Ref. [4] with $M_S \geq 0.41$ TeV.

IV. SUMMARY AND CONCLUSIONS

Combining the low energy limit of string amplitudes for NC and CC processes, we find that bounds on the string scale can be obtained that complement and extend previous analyses. In particular, we extend previous models to cover all neutral current phenomena and, for the first time, offer a model of charged current amplitudes in a string resonance framework. The essence of the approach we adopt is that of Ref. [5]. The low energy limit of each string amplitude reproduces the corresponding SM amplitude. This leaves only a limited number of Chan-Paton factors unspecified, and these are treated as free parameters whose values are related by requiring consistency with the perturbative construction of the string amplitudes. In the absence of new physics signals, they are constrained by the agreement between the SM and the data for a given string scale. More generally, the parameter space consists of the string scale M_S and a limited number of free dimensionless parameters denoted generically by T . We refer to this as a “bottom up” approach to probing the string aspect of braneworld.

We have focused in this paper on the match between the low energy limit of the open-string four-fermion amplitudes at typical kinematical region and the constraints on contact interaction parameters determined by data from HERA, Tevatron, and LEP-II. The bounds on the string mass scale

are comparable in every case to those found in specific models or from leptoquark and W' and Z' searches at HERA and Tevatron. This is no surprise, since the accelerator energy and the precision of the measurements dictate the accessible scale in searches for new physics. It is also no surprise that the highest energy data provide the highest values of the lower bound on new physics. The Drell-Yan processes at the Tevatron lead to our strongest constraints, namely

$$M_S \geq \begin{cases} 0.9 \text{ TeV} & \text{for } |T|=1, \\ 1.3 \text{ TeV} & \text{for } |T|=4, \end{cases} \quad (29)$$

as shown in Fig. 1 for the $eeuu$ case.

The relationship between string scale M_S and quantum gravity scale M is model dependent [3,4]. However, $M_S < M$ quite generally, so the bound on M_S applies to M as well. In one simple case of D-brane scenario, the string scale and the quantum gravity scale in the weakly coupled string sector are related by [4]

$$\frac{M}{M_S} = \frac{k}{g^{1/2}}, \quad (30)$$

where the model-dependent factor k is of order 1. Taking the value $k=1$ and the $SU(2)$ gauge coupling at the weak scale for illustration, we obtain from Eqs. (29) and (30) a conservative bound on the gravity scale,

$$M \geq \begin{cases} 1.1 \text{ TeV} & \text{for } |T|=1, \\ 1.6 \text{ TeV} & \text{for } |T|=4, \end{cases} \quad (31)$$

from the Drell-Yan analysis of the Tevatron data. This estimate of the range of values of the scale of gravity in large extra dimensions is competitive with the current accelerator search values and the value from the specific model of Ref. [4]. But again we advise caution because of the model dependence of our estimate.

We conclude that a TeV string scale can measurably modify weak current amplitudes even well below the string scale. The corresponding limits on this scale and the scale of gravity are quite interesting and worth further exploration. Including these considerations in the interpretation of future data will add an extra dimension, or more, to the search for new physics at the TeV scale.

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