

Universality, vortices, and confinement: Modified SO(3) lattice gauge theory at nonzero temperature

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We investigate the adjoint SU(2) lattice gauge theory in 3 + 1 dimensions with the Wilson plaquette action modified by a \mathbb{Z}_2 monopole suppression term. For the zero-twist sector we report on indications for the existence of a finite temperature transition decoupled from the unphysical bulk transitions.

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I. INTRODUCTION

Pure (3 + 1)-dimensional SU(N) lattice gauge theories in the fundamental representation show a finite-temperature deconfinement phase transition [1,2] together with the spontaneous breaking of a global \mathbb{Z}_N center symmetry [3,4] governing the critical indices, which, e.g., for SU(2) correspond to the universality class of the three-dimensional (3D) Ising model [5]. Lattice universality arguments are commonly used to claim that the same should hold for any possible lattice action discretization, in particular with different gauge group representations. On the other hand, if confinement of quarks can be ascribed to peculiar (topological) excitations of the continuum Yang-Mills fields, it is not clear how center symmetry breaking can lead to an effective theory of quantum chromodynamics (QCD) [6]. At finite-temperature vortices classified through $\pi_1(\text{SU}(N)/\mathbb{Z}_N) \sim \mathbb{Z}_N$ along a compactified dimension can of course provide a suitable degree of freedom for the pure Yang-Mills case [7], but their fate in the presence of fermions remains challenging. Moreover, in supersymmetric (SUSY) Yang-Mills theory [8,9] and in the Georgi-Glashow model [10] confinement is driven by magnetic monopoles. Recent work regarding Yang-Mills theories based on exceptional groups [11] conjectures that vortices might not be at all necessary to have a confining theory. What role do vortices play then in SU(N)? An investigation of the SU(2) lattice gauge theory in the *center blind* adjoint representation, i.e., $\text{SO}(3) \sim \text{SU}(2)/\mathbb{Z}_2$, might offer some interesting insight. The presence of finite temperature effects for such a model has been debated for a long time. Two main problems have been faced. First, Polyakov's center symmetry breaking mechanism is available only for the half-integer representations. For integer ones the \mathbb{Z}_2 local invariance makes the fundamental Polyakov loop of no use. The adjoint Polyakov loop, on the other hand, *cannot* be, strictly speaking, an order parameter for a transition, assuming it existed, since gluons will screen adjoint quarks at some distance [~ 1.25 fm for SU(2)] [12,13]. However, it can still be taken as a signature to distinguish, at finite temperature, a

confined although screened phase from a deconfined one. Anyhow, the problem of establishing a well defined criterion for confinement in this case remains an open and interesting one.

Second, and most importantly, lattice artifacts lead to first order bulk phase transitions at strong coupling, preventing the continuum limit to be reached within the ordinary confined phase [14,15]. Similar phase diagrams are shared by SU(N) theories with $N \geq 3$ [16]. For SU(2) \mathbb{Z}_2 monopoles were observed to drive the bulk transition [17,18]. Another interesting development came as the vortex free energy, measured on the lattice already in the fundamental theory [19], was recently computed also in the adjoint theory [20], thus suggesting how center vortices and twist sectors are entering a center-blind theory. The latter work, however, falls short of any attempt to investigate the theory at finite temperature, stumbling on ergodicity problems of the algorithm at the bulk phase transition, where tunneling among different twist sectors becomes strongly suppressed. Thus, a thorough finite temperature investigation of the SO(3) theory taking into account the topological excitations is still missing. Several attempts searching for a decoupling of the finite temperature transition from the bulk transitions were originally undertaken by Datta and Gavai (see Refs. [21–23] and further citations therein). These authors used the fundamental-adjoint mixed action modified by \mathbb{Z}_2 monopole and vortex suppression terms within the Villain-type formulation. By studying the specific heat a finite temperature transition consistent with the Ising universality class was found. In the pure center-blind adjoint case including only monopole suppression it could, however, only be established for the smallest time extension $N_t = 2$. Moreover, the relevance of different twist sectors had not yet appeared in the literature.

Here we want to go a step further employing Wilson's adjoint action formulation modified by a similar \mathbb{Z}_2 monopole suppression. In this case the action itself is manifestly center blind. The corresponding phase structure looks in many respects similar to the Villain case, but it differs—as we shall show—substantially in the fundamental-adjoint coupling plane. The areas along the two axes are completely

separated by a bulk phase transition and thus the proof of universality—if the latter is really fulfilled—becomes more complicated. In our investigation the twist variables will prove an important ingredient to understand the phase structure of the model.

The outline of the paper is as follows. In Sec. II we introduce the model and discuss its phase structure at vanishing temperature with standard bulk observables. In Sec. III twist variables will be used in order to show that the first order bulk transition seems to weaken to second order in a certain $\beta_A - \lambda$ range. In Sec. IV we report on the existence of the finite temperature transition and check scaling for the critical temperature. Section V contains our conclusions. Reports of this work at early stages have been published in Refs. [24,25].

II. ADJOINT ACTION MODEL WITH Z_2 MONOPOLE SUPPRESSION

We study the SU(2) lattice gauge theory with a mixed fundamental-adjoint representation Wilson action modified by a chemical potential term suppressing Z_2 monopoles

$$S = \sum_P \left[\frac{4}{3} \beta_A \left(1 - \frac{\text{Tr}_F^2 U_P}{4} \right) + \beta_F \left(1 - \frac{\text{Tr}_F U_P}{2} \right) \right] + \lambda \sum_c (1 - \sigma_c), \quad (1)$$

$$\frac{1}{g^2} = \frac{\beta_F}{4} + 2 \frac{\beta_A}{3},$$

where $\sigma_c = \prod_{P \in \partial c} \text{sgn}(\text{Tr}_F U_P)$ taken as a product around elementary 3-cubes c defines the Z_2 charge. For these monopoles a density can be introduced as

$$M = 1 - \left\langle \frac{1}{N_c} \sum_c \sigma_c \right\rangle \quad (2)$$

normalized such that it tends to unity in the strong coupling region [N_c denotes the number of 3-cubes on the four-dimensional (4D) lattice]. We will be particularly interested in the pure adjoint theory, i.e., $\beta_F = 0$. In the latter case one can analyze the model with the link variables represented by both SO(3) or SU(2) matrices, exploiting the property $\text{Tr}_A = \text{Tr}_F^2 - 1$ for the Wilson term or picking a random SU(2) representative of the SO(3) link to construct the Z_2 monopole contribution. As expected, nothing changes in the phase diagram, the integration over the fundamental links simply doubling the integration domain in the partition function. A standard Metropolis algorithm has been used to update the links in both cases and we use either one according to the best performance in the case at hand. Although σ_c is constructed in terms of fundamental representation plaquettes, it is a *natural* SO(3) quantity. In fact, for every given SO(3) link variable, the corresponding SU(2) representative can always be determined up to a sign. But the latter is cancelled in the

product of plaquettes around any 3-cube, since each link occurs twice for each edge. In other words,

$$U_\mu(x) \rightarrow -U_\mu(x) \Rightarrow \sigma_c \rightarrow \sigma_c, \quad \forall \mu, x, c. \quad (3)$$

This ensures that for $\beta_F = 0$ the action we study is center blind in the entire $\beta_A - \lambda$ plane.

Previous finite temperature investigations were mainly done in the Villain discretization for the SO(3) term in the action, introducing an independent Z_2 -valued plaquette variable σ_P [22,23,26]. Two terms with chemical potentials were added in order to suppress completely the effect of the lattice artifacts, Z_2 monopoles and vortices, whose densities in this case are given by

$$\tilde{M} = 1 - \left\langle \frac{1}{N_c} \sum_c \tilde{\sigma}_c \right\rangle, \quad \tilde{\sigma}_c = \prod_{P \in \partial c} \sigma_P, \quad (4)$$

$$\tilde{E} = 1 - \left\langle \frac{1}{N_l} \sum_l \tilde{\sigma}_l \right\rangle, \quad \tilde{\sigma}_l = \prod_{P \in \hat{\partial} l} \sigma_P, \quad (5)$$

where c and l label the N_c 3-cubes and the N_l products of plaquettes have a link in common, respectively. We can see that the monopole suppression term in Eq. (2) looks formally identical with the one used in connection with the Villain-type action, but its realization is different and leads to a different phase structure. The Villain discretization (on \mathbb{R}^4) can be proven to be equivalent to the standard *fundamental* Wilson action for $\lambda_V \rightarrow \infty$ [22,23,26] [see Eq. (6)]. On the other hand, one can show that the limit $\lambda \rightarrow \infty$ of the Wilson adjoint formulation can be mapped, in the trivial twist sector, to some positive plaquette model [33]. Such a mapping, however, as we shall see below, is not equivalent to a continuous connection with the standard fundamental action theory. A simple inspection of the $\beta_{A(V)} - \beta_F$ phase diagram in these two limits shows that the differences are conspicuous. Therefore, λ_V and λ must not simply be identified [20,24,25]. One should always bear in mind that in the Villain case the SO(3) invariance under $U_\mu(x) \rightarrow -U_\mu(x)$ is not realized in the action itself but only once the auxiliary σ_P variables are integrated out. In Fig. 1 the $\beta_F - \beta_A$ phase diagram of our model (2) at $T=0$ is shown for rather strong monopole suppression $\lambda = 1.0$. Phase I is connected with the ordinary confinement phase of the standard SU(2) model within the fundamental representation, whereas phase II extending to $\beta_A \rightarrow \infty$ occurs completely decoupled from phase I for $\beta_A \geq 0$. Indeed, simulating the model with the lattice size up to 12^4 the average plaquette variable exhibits a strong discontinuity across and metastable states on top of the transition line. At $\beta_A = 0.0$ the latter was located at $\beta_F \approx 0.96$. By studying the fundamental representation Polyakov loop and its susceptibility for lattice size 4×12^3 the finite-temperature phase transition was seen at $\beta_F \approx 1.35$, i.e., within phase I as one would expect. The phase structure clearly differs from that of the Villain case, where the transition line for increasing β_F extends along the latter axis up to $\beta_F \rightarrow \infty$ (see Ref. [23]). Therefore, universality of physics observed for $\beta_F = 0$ in comparison with $\beta_A = 0$ remains an open question.

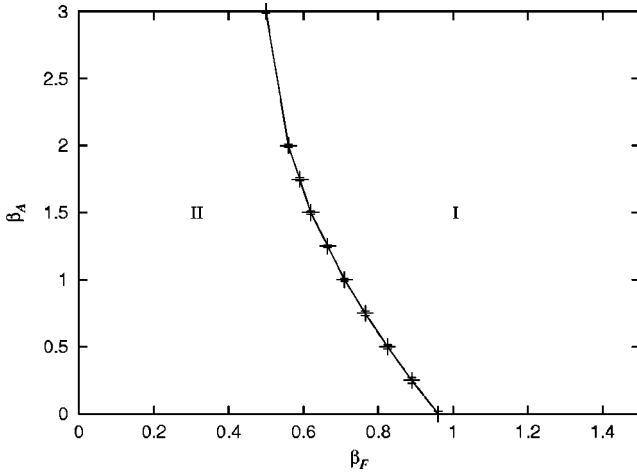


FIG. 1. The bulk phase transition of the mixed fundamental-adjoint action model (1) with monopole suppression for $\lambda=1.0$.

For $\beta_F=0$ —as one turns on the monopole suppression—the bulk transition moves down from $\beta_A \approx 2.5$ to lower β_A values intersecting the λ axis at $\lambda \approx 0.92$ as one can see from Fig. 2. Phases I and II are denoted as in Fig. 1. The bulk transition is characterized by the condensation of \mathbb{Z}_2 monopoles within phase I ($M > 0$) and by their suppression in phase II ($M = 0$). It has been located by monitoring the monopole density M (Fig. 3), the plaquette, and the twist variables (for the definition of the latter see below) as a function of β_A for varying λ . Figure 3 for a lattice size of 4×12^3 (i.e., for nonzero temperature) shows the transition to be discontinuous for small λ turning into a continuous one (probably second order) at larger λ and lower β_A , respectively (see the next paragraph). The end point of the first-order branch (at $\beta_A \approx 1.2$ for $N_t = 4$) seems to be identical with the lower λ endpoint of the finite temperature transition to be reported in Sec. IV.

For $\beta_A > 0$, $\lambda \geq 1.0$ no bulk transition is observed anymore. Our β_A - λ phase diagram looks very similar to that of the Villain case in the β_A - λ_V plane [22,26]. But note that in the Wilson case phase II seems to be disconnected from phase I.

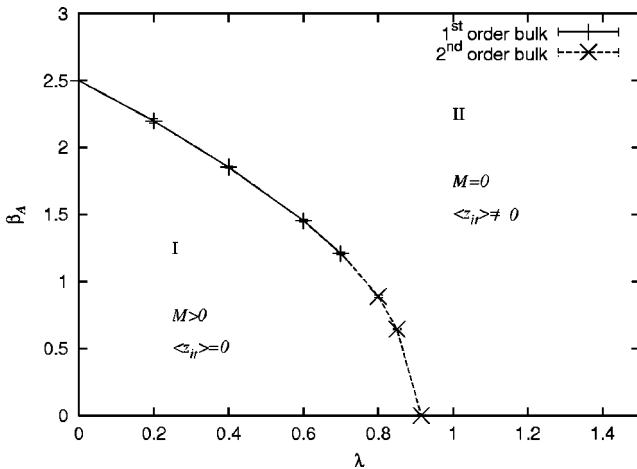


FIG. 2. The bulk phase transition in the β_A - λ plane as seen for lattice size 4×12^3 .

In any case the properties of the different phases should be established by means of observables well defined in the continuum theory, e.g., the glueball spectrum or the screening mass. Such a project is currently under consideration.

III. TWIST SECTORS AND TUNNELING

Vortex free energies related to 't Hooft spatial loops were recently measured on the lattice within the fundamental and the adjoint representation of $SU(2)$ [19,20]. In the $SO(3)$ case the Villain discretization was used and it was stressed that on the torus T^4 the following equivalence holds:

$$\sum_{\text{t.s.}} Z_{SU(2)} \equiv A \sum_{\sigma_P = \pm 1} \int (DU) e^{\beta_V \sum_P \sigma_P \text{Tr}_F U_P} \prod_c \delta(\tilde{\sigma}_c - 1), \quad (6)$$

where the left-hand side is summed over all the twist sectors. To obtain the equivalence of this modified $SO(3)$ Villain model with the $SU(2)$ Wilson action case with standard periodic boundary conditions the global constraint

$$N_{\mu\nu} \equiv \prod_{P \in \text{plane } \mu\nu} \sigma_P = +1 \quad (7)$$

has to be satisfied. It is straightforward to see that an observable able to distinguish between trivial and nontrivial twist sectors is given by

$$z_{\mu\nu} \equiv \frac{1}{L_\rho L_\sigma} \sum_{\rho\sigma} \prod_{P \in \text{plane } \mu\nu} \text{sgn Tr}_F U_P \quad (\epsilon_{\rho\sigma\mu\nu} = 1). \quad (8)$$

These *twist variables* are again truly $SO(3)$ observables since, due to the boundary conditions, the signs of the links in the fundamental representation drop out in the product, the plane extending over the whole length of the space and time directions.

A connection between the existence of nontrivial twist sectors, the presence of center vortices, and, in the deconfined phase, the occurrence of a state characterized by a value $-1/3$ for the adjoint Polyakov loop L_A (first observed in [27,21]) was proposed by de Forcrand and Jahn [20]. The latter authors found the twist sectors to become metastable under local updates close to the bulk transition, i.e., the suppression of \mathbb{Z}_2 monopoles causes the tunneling among different vortex vacua to become exponentially suppressed with increasing volume. The authors were thus prevented to go further and to investigate the possible extension of such connection into the finite-temperature case.

In the Wilson case, turning on the $SO(3)$ invariant \mathbb{Z}_2 monopole suppression term, we observed, for small volumes ($V=4^4$) and on top of the strong first order branch of the bulk phase transition ($\lambda \leq 0.7$), tunneling between different distinct twist sectors [24,25] as well as a dynamical relation between the adjoint Polyakov loop and the twist observable,

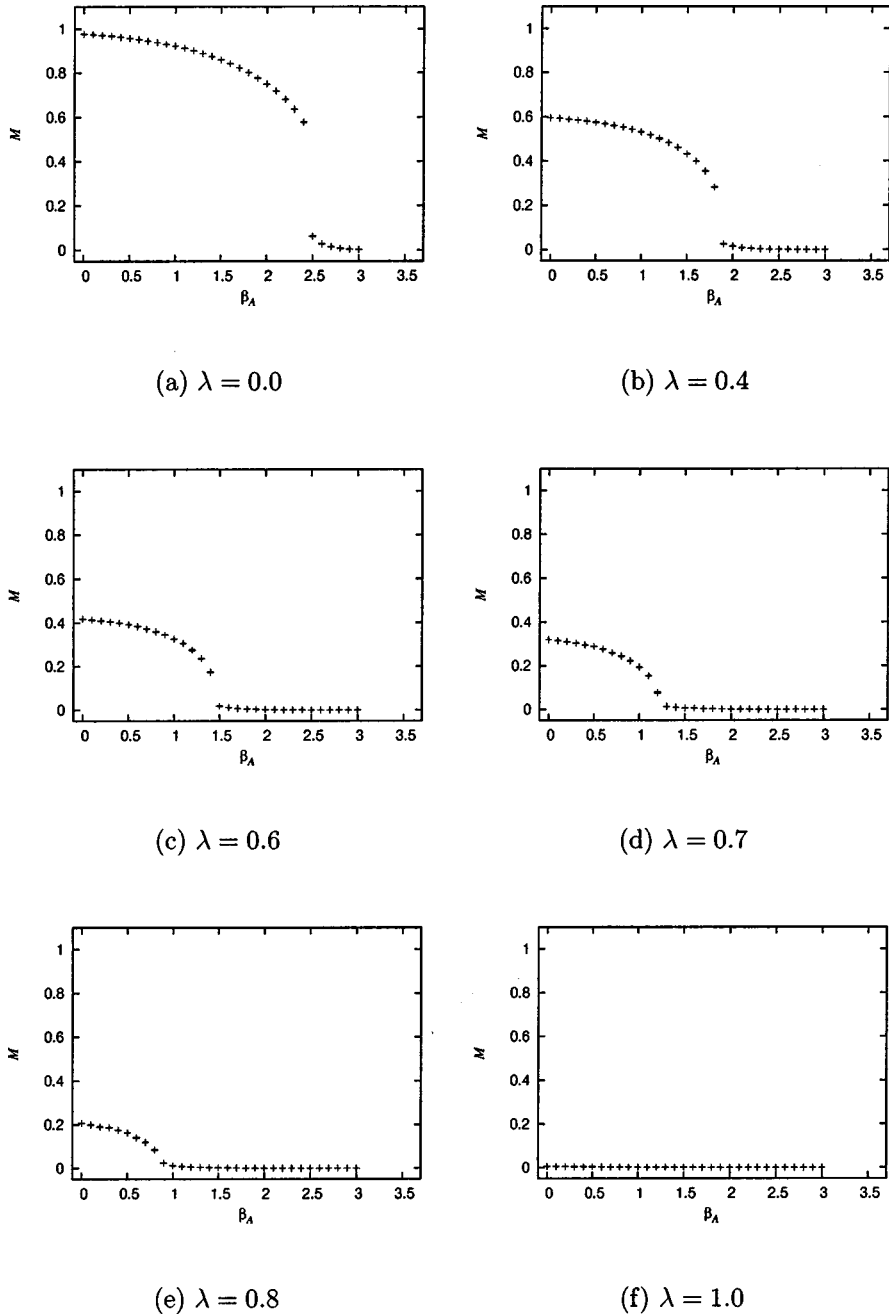


FIG. 3. Monopole density as a function of β_A for different values of λ ($V=4 \times 12^3$) with a statistics of $O(10^5)$ configurations.

whose nontrivial value signals the presence of the state $L_A^- = -\frac{1}{3}$ also in this case. For increasing lattice volume on top of the bulk phase transition at low λ values the tunneling between the twist sectors becomes more and more suppressed. Therefore, in agreement with Ref. [20], for small values of the chemical potential the phase transition seems really first order as we have already argued earlier by monitoring other observables such as the average plaquette or the \mathbb{Z}_2 monopole density.

However, as soon as the chemical potential is increased the situation changes. The observables are no more discontinuous, thus suggesting a very weak first order or second order phase transition. Indeed, for $\lambda > 0.75$, on top of the

transition, tunneling between the twist sectors is still evident but no suppression is seen for increasing volume as Fig. 4 shows for the lattice size 12^4 . The twists oscillate between -1 and $+1$ but the adjoint Polyakov loop fluctuates close to zero and not between $+1$ and $-1/3$, in contrast to what was found in Ref. [20] for $\lambda_V=0$ and in our case at small λ . The behavior of the twist variables reminds of the fundamental Polyakov loop in the $SU(2)$ theory in the fundamental representation, or of the magnetization in an Ising-like system. This justifies the definition of an order parameter $\langle \tilde{z} \rangle$ such as

$$\langle \tilde{z} \rangle \equiv \left\langle \frac{1}{3} (|z_{xt}| + |z_{yt}| + |z_{zt}|) \right\rangle. \quad (9)$$

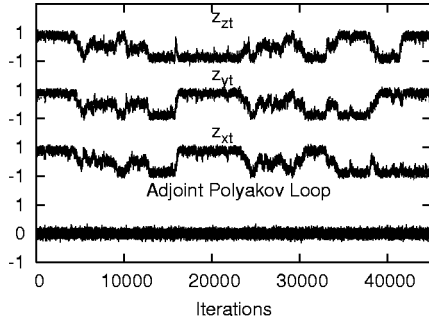


FIG. 4. Monte Carlo history of the adjoint Polyakov loop and of the three electric twist variables at $\lambda = 0.858, \beta_A = 0.65$ for $V = 12^4$.

Figure 5 shows the behavior of the susceptibility of this new observable \tilde{z}

$$\chi_{\text{twist}} = N_s^3 (\langle \tilde{z}^2 \rangle - \langle \tilde{z} \rangle^2) \quad (10)$$

for three different volumes 8^4 to 12^4 . A finite-size scaling test for the maximum values of the susceptibility with the fit ansatz [28]

$$f(N_s) = a [N_s \log(N_s)]^{1/4 w} \quad (11)$$

shows that the peaks increase with the lattice size with the exponent $w \approx 2.4 \pm .2$ which is close to the value of the 4D Ising universality class being consistent with the theoretical

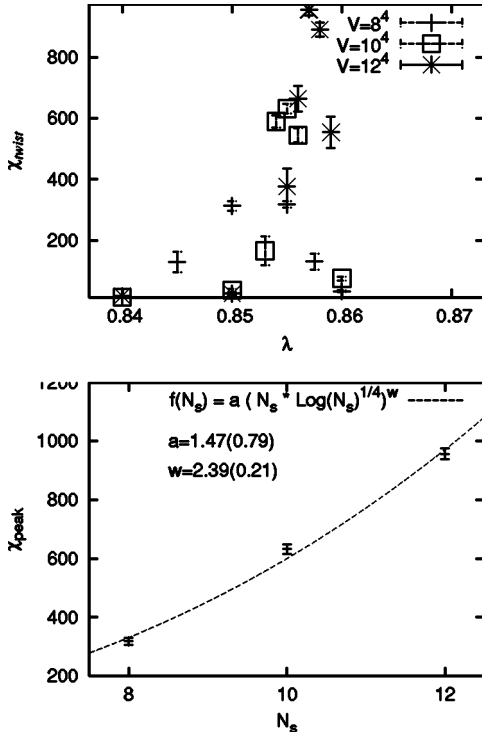


FIG. 5. (upper) Susceptibility χ_{twist} as a function of λ for volumes $8^4, 10^4$ and 12^4 for $\beta_A = 0.65$. (lower) Finite-size scaling of the peak values of χ_{twist} as a function of the linear lattice size N_s . The dashed line shows the finite-size scaling function fitted to the data.

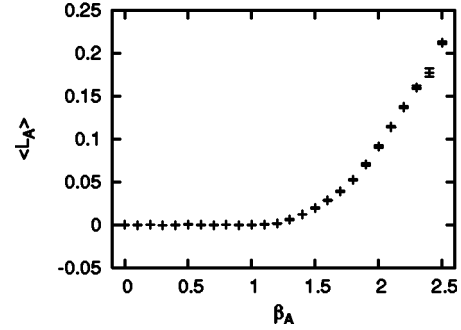
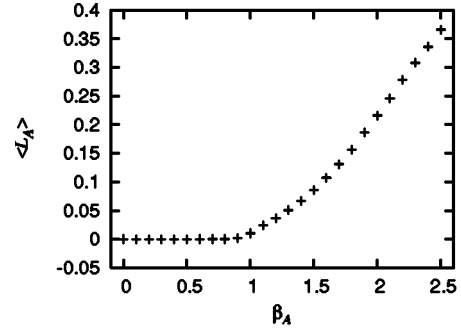


FIG. 6. Adjoint Polyakov loop $\langle L_A \rangle$ as a function of β_A in the trivial twist sector for lattice sizes 4×16^3 (upper) and 6×16^3 (lower), both for $\lambda = 1.0$.

predictions for the pure \mathbb{Z}_2 monopole system at $\beta_A = 0$. More statistics and larger volumes are needed to confirm this result. Moreover, the transition does not show any scaling with N_τ , thus behaving still consistently as a $T=0$ bulk transition.

For larger chemical potential ($\lambda \geq 1.0$) and positive β_A , i.e., away from the bulk transition (in phase II), tunneling is no more observed and the twist sectors become rigid. Keeping the local update algorithm and turning on the \mathbb{Z}_2 monopole potential with $\lambda \geq 1$ we can thus study the theory within the trivial twist sector, i.e., for the case of a zero number (modulo 2) of (extended) vortices.

IV. INDICATION FOR A PHASE TRANSITION AT $T \neq 0$

At finite temperature along a compactified dimension the periodic boundary condition on the gauge fields allow for gauge transformations that can be classified through $\pi_1(SO(3)) \sim \mathbb{Z}_2$. In the fundamental representation this would indeed translate into a gauge transformation that differs by a nontrivial element of the center at the boundaries. Such transformations can induce a selection rule within the Hilbert space. The vacuum state and the higher excited states can exist in a superposition of the two ($d+1$ for a gauge theory discretized on a d -dimensional torus) different topological states. The creation of a vortex in the vacuum, as measured by the 't Hooft loop, simply means taking the expectation value between two vacuum states belonging to different topological sectors. A symmetry breaking argument can thus be applied to the vacuum state in the Hilbert space, although center symmetry is always unbroken. The observable \tilde{z} is behaving such that it averages to zero if, at finite

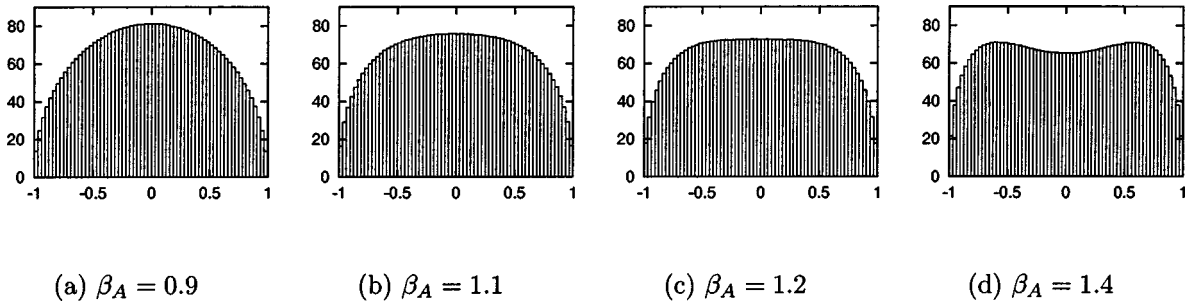


FIG. 7. Spatial distribution of the fundamental Polyakov loop $\langle L_F \rangle$ in the trivial twist sector at $\lambda = 1.0$, $V = 4 \times 16^3$ and at various β_A values.

temperature, the transitions occur among states that exist in a superposition of the different topologically distinct eigenstates, while it takes a nonzero value otherwise.

We will identify the trivial topological sector, in which we shall be particularly interested, with $z_{xt} = z_{yt} = z_{zt} = 1$. This corresponds to vacuum expectation value between the (unique even on a torus) topologically trivial state, i.e., no creation/annihilation of extended vortices (modulo 2).

For what we have shown in the previous chapter, we have a local way to implement this constraint in the action. Indeed, since ergodicity in the $\beta_A - \lambda$ plane is recovered along the bulk phase transition when it weakens, one simply needs to study the system in the broken phase fixing the trivial twist sector. More than a constraint it actually simply seems a dynamical feature of the (3+1)-dimensional adjoint theory. Setting $\lambda \geq 1$ and moving parallelly to the β_A axis, we have studied the phase structure for zero ($N_\tau = N_s$) and non-zero temperature ($N_\tau \ll N_s$). The linear spatial lattice size was taken up to $N_s \leq 16$. We have determined the distribution of the fundamental Polyakov loop, governing the expectation value of the adjoint Polyakov loop, as an indicator for a physical temperature effect. Appropriate initial conditions were used to specify the trivial twist sector monitored throughout the simulations. Far enough from the bulk transition the local updates are keeping it fixed. For sake of completeness we have studied the other twist sectors as well given the negative value of the Polyakov loop associated with these sectors. For $\lambda \geq 1$ and on the asymmetric lattice ($N_\tau = 4$) a clear finite- T phenomenon appears. The adjoint Polyakov loop still averages to zero for low enough β_A , while at higher values it eventually starts growing, remaining always positive (see Fig. 6), consistently with the fixed trivial twist sector and with its interpretation as the exponen-

tial of the free energy of an (infinitely massive) adjoint quark.

For increasing extent of the lattice in the imaginary time direction the onset of the growth is clearly seen to be shifted to larger β_A . Alternatively one can also consider the spatial distribution of the fundamental Polyakov loop. For low β_A it shows a broad peak around $L_F(\vec{x}) = 0$, consistent with the Haar measure distribution $\sim \sqrt{1 - L_F^2}$, but with increasing β_A two symmetric peaks show up, consistent with $\langle L_A \rangle > 0$ (cf. Fig. 7). The same picture occurs for $N_t = 6$ but with a shifted coupling as one can see from Fig. 8. In order to highlight this shift somewhat more quantitatively we fitted these distributions, produced with a statistics of $O(10^4$ to $10^5)$ configurations and rescaled by the Haar measure, with some high order polynomials (up to 12th order). We verified that the odd derivatives are all zero within the errors and that the second derivative departs from zero to positive values (see Fig. 9). The corresponding departure points β_A^0 should be taken as lower bounds for the critical values β_A^{crit} . We show them in Table I.

Although we did not determine the critical β_A values we see that the departure points already indicate a reasonable scaling behavior in N_t . Anyway larger volumes, larger N_τ 's, and also other observables will be needed to confirm the universality with the fundamental case.

Moreover, although a throughout numerical study is still required, our data indicate that the point at which the finite-temperature lines meet the bulk transition line coincides with the point where the bulk transition weakens or changes its order from first to second. This is consistent with the hypothesis that for large enough volume the bulk phase transition would decouple from the deconfinement effect and be always

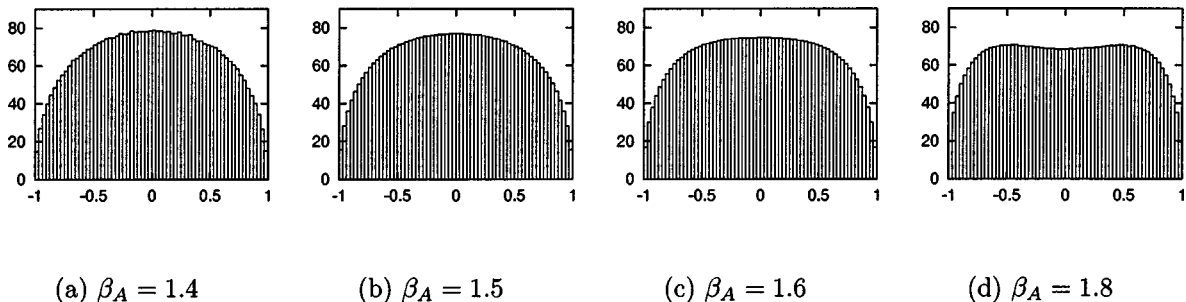


FIG. 8. Same as in Fig. 7 for $V = 6 \times 16^3$.

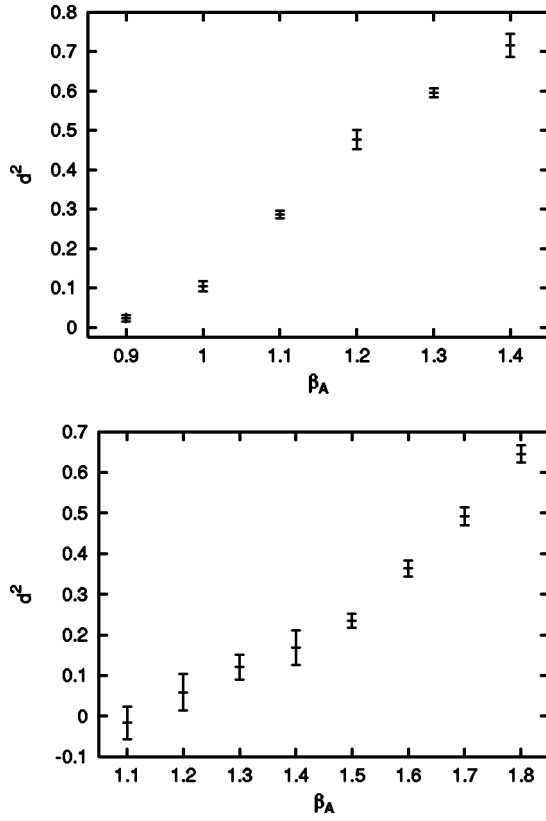


FIG. 9. Second derivative of the fitting polynomial for the Haar measure rescaled fundamental Polyakov loop distributions at various β_A in the trivial twist sector ($V=16^3, \lambda=1.0$); the upper figure for $N_\tau=4$, the lower one for $N_\tau=6$.

weak first or second order even at $\lambda=0$.

Unfortunately, a direct numerical verification seems unviable since from the estimates given in Ref. [20] it follows that it should occur at very large volumes [$V \approx O(700^4)$ for the Villain case].

V. CONCLUSIONS

We have carried out a thorough study of pure SU(2) lattice gauge theory in the Wilson adjoint representation, decoupling the strong coupling bulk effects from the continuum limit by introducing a chemical potential term suppressing Z_2 monopoles. As stressed in Sec. II, our formulation is not, at least not trivially, equivalent to the Villain one used in Refs. [20,22,26] in the whole parameter range we have explored. We have, moreover, included the study of the twist sectors in our analysis, which had not yet appeared in the literature when the previous studies in Ref. [22] were carried out. In this respect, we have extended the work of Ref. [20] to the

TABLE I. The β_A^0 -values as explained in the text estimated from distributions of the fundamental Polyakov loop (see Fig. 9).

N_τ	β_A^0	Statistics
4	0.92 ± 0.08	$O(10^5)$
6	1.25 ± 0.15	$O(10^4)$

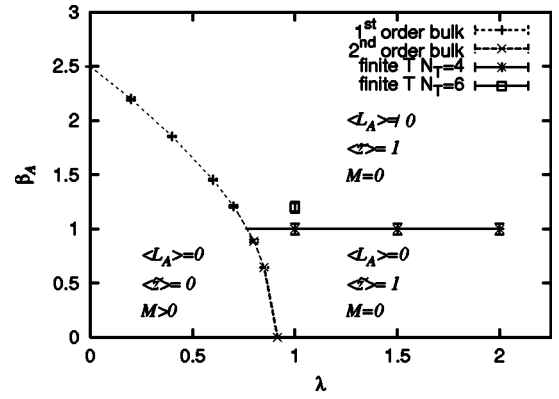


FIG. 10. Phase diagram in the $\beta_A - \lambda$ plane for $N_\tau=4$ (continuous line) and $N_\tau=6$ (single point).

finite temperature case. At this point it would be interesting to study also the behavior of the Villain discretization with the Z_2 monopole suppression term used in Eq. (1). By monitoring mainly the twist variables and the adjoint Polyakov loop we studied first the character of the bulk transition. The latter turned out to become weaker with increasing chemical potential λ , turning possibly into a second order transition. At strong monopole suppression $\lambda > 1$ no bulk transition was found anymore. It is this area in the phase diagram where we started to search for a finite temperature transition. We have found convincing indications for such a transition at two values of N_τ from the adjoint Polyakov loop and from a typical change of the distributions of the fundamental Polyakov loop variable.

Due to the strong suppression of tunneling between different twist sectors the investigations were carried out with a local update algorithm within the fixed trivial twist sector. Of course, a final answer should take into account also tunnelings between the sectors to be achieved, e.g., by multicritical updates [29] or even more promising with parallel tempering [30]. The results of the present investigation are collected in Fig. 10 where we have also included two further points for the finite-temperature transition seen for $N_\tau=4$ at larger values $\lambda=1.5, 2.0$ in an analogous way as demonstrated for $\lambda=1.0$.

In the meantime we are carrying out an additional investigation with the Pisa disorder parameter (see, e.g., Ref. [31]) adapted for the SO(3) case, which hopefully will enable us to determine the universality class of the transition. Preliminary data at least support the existence of a finite-temperature transition decoupled from the bulk transition line and at β_A values compatible with our results quoted here. We shall report on this in a forthcoming letter.

A quantitative study of the observed finite-temperature physical transition could also be viable relying on pure thermodynamic quantities, as already done in Ref. [22], through the Lee-Yang zeros technique or by studying the free energy of a Z_2 vortex [19,32]. More careful investigations of this kind will hopefully tell us something about the real nature of the finite-temperature transition we have reported here. But the mere occurrence of such a transition is a prerequisite to a still missing proof of universality between SU(2) and SO(3) lattice gauge theories.

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