

Z* resonances: Phenomenology and models

Byron K. Jennings*

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

Kim Maltman†

Department of Mathematics and Statistics, York University, 4700 Keele Street, Toronto, Ontario, Canada M3J 1P3
and CSSM, University of Adelaide, Adelaide, 5005, Australia

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We explore the phenomenology of, and models for, the Z^* resonances, the lowest of which is now well established, and called the Θ . We provide an overview of three models which have been proposed to explain its existence and/or its small width, and point out other relevant predictions and potential problems, for each. The relation to what is known about KN scattering, including possible resonance signals in other channels, is also discussed.

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I. INTRODUCTION

Strangeness +1 baryon resonances (Z^* 's) have been treated with considerable disdain in the past (see, for example, the comments by the Particle Data Group [1] for 1992, the last year they were discussed). Even at that time there were candidates for Z^* resonances [2,3]. It is interesting to notice that the paper [4] following the latter of these references increased the spin-orbit force by a factor of 3 in a cloudy bag model calculation in a desperate attempt to reduce the need for the Z^* resonances. The results of the 1985 analysis were largely confirmed in a later analysis [5] by the same group and are roughly consistent with Ref. [2]. More recently it was shown that the poles found in the 1992 analysis correspond to peaks in the time delay and speed plots [6].

Theoretically, multiquark states were considered long ago in the bag model, and the masses of Z^* configurations calculated in some detail for the negative parity sector [7]. However, these states typically suffer from the presence of a fall-apart mode and are usually associated with poles in the p matrix [8] rather than with real resonances (poles in the t matrix). Even that association has been questioned [9].

In the Skyrme and chiral soliton models of the nucleon, states with exotic quantum numbers occur naturally through the presence of solutions corresponding to higher flavor representations. In the $SU(2)_F$ case, an early embarrassment for these models, was the prediction of $I=J=5/2, 7/2, \dots$, states. These states arise via projection from the same intrinsic state as the ground state. Since the $I=J=5/2, Y=1$ state, in particular, was not seen, it was assumed to be an artifact of the model. (The model is natural in the $N_c=\infty$ limit, but would in general require $1/N_c$ corrections in the real world.)

The $SU(3)_F$ version of the model also predicts a number of higher states, these occurring in various exotic multiplets, $\mathbf{10}_F$, $\mathbf{27}_F$, $\mathbf{35}_F$, etc. The $SU(2)$ $I=J=5/2$ state lies in an $SU(3)_F$ $\mathbf{35}_F$ and the $I=J=7/2$ in an $\mathbf{81}_F$. The Z^* reso-

nances with isospin 0, 1, and 2 lie in the $\overline{\mathbf{10}}_F$, $\mathbf{27}_F$, and $\mathbf{35}_F$ representations, respectively. (In pentaquark models these are the only representations with strangeness +1.) The existence of such a $\mathbf{10}_F$ state was noted long ago [10,11]. Since such resonances did not correspond to 3-quark states they tended to be ignored. This changed with the work of Ref. [12] and the follow-up work of Refs. [13,14] where a narrow state in the 1500–1600 MeV region was predicted (see also Refs. [15,16]). A narrow strangeness +1 state was then found experimentally in this energy region [17–23] generating a good deal of subsequent theoretical discussion [15,16,24–51]. It is now necessary to understand the nature of this state and the implications of its existence. The existence of a new narrow resonance in a region of the baryon spectrum thought previously to be reasonably well understood [52] raises questions about how good this understanding actually is. In particular, it raises the possibility (or, perhaps, likelihood) that states with nonexotic quantum numbers may be either structurally similar to the recently observed exotic (pentaquark) state, or contain significant admixtures of such exotic configuration(s).

In this paper we consider the phenomenology of the Z^* resonances, including the recently discovered Θ . We explore the implications of a number of models compatible with the existence of a Z^* resonance in the region of the Θ for the as-yet-undetermined quantum numbers of the Θ , and consider other potentially observable Z^* states predicted by those models.

The rest of the paper is organized as follows. In Sec. II we summarize the current experimental situation for the Z^* resonances and related KN scattering results. In Sec. III we discuss the results from various soliton model calculations, with an emphasis on implications for possible states beyond the $\Theta(1540)$. In Sec. IV we discuss models based on an explicit pentaquark structure, involving interquark interactions mediated by either effective Goldstone boson, or effective color magnetic, exchange, and compare the two approaches. In Sec. V we discuss briefly recent QCD sum rule and lattice explorations of the $I=0,1$ Z^* sectors. Finally, in

*Electronic address: jennings@triumf.ca

†Electronic address: kmaltman@yorku.ca

TABLE I. The experimental Z^* resonances. The mass range quote for the lowest resonance corresponds to the range of central values found in the experiments of Refs. [17–23]. The limit on the width is a conservative one, compatible with the limits reported in all of Refs. [29–31]. The parameters of the higher states are from Ref. [5].

Mass (MeV)	Width (MeV)	Quantum numbers
1528–1555	<6	$(I, J^P) = (0?, ?)$
1788	340	D03
1811	236	P13
1831	190	P01
2074	503	D15

Sec. VI we draw conclusions and suggest directions for future work.

II. THE Z^* PHENOMENOLOGY

In this section we summarize the phenomenology of the Z^* resonances. We begin with a brief reminder of the basic results from KN scattering. It is known that both s -wave phase shifts are repulsive at low energies [5]. This implies that the central part of the KN interaction will produce no KN potential model resonances. In the p -wave sector, the $P01$ and $P13$ waves are attractive while the $P03$ and $P11$ waves are repulsive. This suggests a spin-orbit potential with different signs in the two isospin sectors. These qualitative features are correctly described in a number of approaches: the cloudy bag model [4], the meson exchange picture [53,54], and both the quark Born term [55], and resonating group [56] approaches to the nonrelativistic quark model.

The 1982 version of the ‘‘Review of Particle Properties’’ [2] lists five Z^* resonances $Z_0(1780)$ (P01), $Z_0(1865)$ (D03), $Z_1(1900)$ (P13), $Z_1(2150)$, and $Z_1(2500)$. The last two have no spin or parity assignment. An analysis of K^+p scattering in 1985 [3] found evidence for three states: P13 at 1780 MeV, P11 at 1720 MeV, and D15 at 2160 MeV. A more complete analysis [5], going to higher energies and also including $I=0$, found four resonances. Their properties are listed in lines 2–5 of Table I. For the last two experimental determinations, the mass is the pole location, while for the first it is the Breit-Wigner peak location. The two should not be expected to be identical. Reference [5] also gives Argand plots which show strong forward looping for the first three of these states. Reference [6] shows speed and time delay plots which are also consistent with the resonance interpretation.

In addition to the phase shift analysis, there are a number of recent photoproduction experiments [17,19,20,22] all of which see a narrow resonance at $m \sim 1540$ MeV, with a width consistent with experimental resolution. Explicitly the results are $m = 1540 \pm 10$ MeV, $\Gamma < 25$ MeV [17]; $m = 1543 \pm 5$ MeV, $\Gamma < 22$ MeV [19]; $m = 1540 \pm 5$ MeV, $\Gamma < 25$ MeV [20]; and $m = 1555 \pm 10$ MeV, $\Gamma < 26$ MeV [22]. Two of the experiments, Refs. [20,22], report negative results in their searches for a Θ^{++} signal. A narrow signal in the same region has also been seen in K^+Xe scattering [18], as

well as in a recent reanalysis of old $\nu/\bar{\nu}$ bubble chamber data [21]. The former finds $m = 1539 \pm 2$ MeV, $\Gamma < 9$ MeV [18], the latter $m = 1533 \pm 5$ MeV, $\Gamma < 20$ MeV [21]. The HERMES Collaboration has also presented evidence for a narrow Θ^+ in $eD \rightarrow pK_s X$ at a beam energy of 27.6 GeV, with $m = 1528 \pm 3 \pm 2$ MeV, $\Gamma < 20$ MeV [23], and no sign of a Θ^{++} signal. The failure to observe a Θ^{++} in $\gamma p \rightarrow K^- K^+ p$, in both the SAPHIR [20] and CLAS [22] experiments, if correct, rules out the proposed isotensor assignment suggested in Ref. [25]. An $I=0$ assignment is most natural in light of these results. However, an $I=1$ assignment is still possible since, in $\gamma p \rightarrow \bar{K}KN$, three different reduced isospin matrix elements appear if $I_\Theta = 1$, allowing a cancellation to occur in the $K^- K^+ p$ production amplitude. The absence in the HERMES experiment, however, is very unnatural for an $I=1$ Θ in the higher multiplicity production environment and hence, it seems to us, strongly favor $I=0$.

In addition to the direct upper limits on the width already noted, indirect upper limits have been obtained using information from elastic scattering. Reference [29] gives $\Gamma < 6$ MeV, while Ref. [30] finds a limit of a few MeV, and prefers a width of an MeV or smaller. Bounds of $< 1 - 4$ MeV (from a consideration of K^+d scattering data) and $< 0.9 \pm 0.3$ MeV (from a consideration of the signal in the DIANA experiment [18]) have also been obtained in Ref. [31].

In settling on the entries for the mass and width of the $\Theta(1540)$ in Table I, we have taken a conservative approach and quoted the full range of central values for the mass obtained in the various experiments, and the largest of the reported upper bounds. It is likely that the width is significantly smaller than this upper bound.

The example of the Θ shows that important information can be obtained from knowing that no resonance has been seen in a given energy region in the existing KN database. Indeed, we must take into account not only the resonances that have been claimed, but also the absence of any other resonance signals. The KN phase shift analysis [5] saw no resonances below 1788 MeV. A resonance could have been missed if it was too narrow [as in the case of the $\Theta(1540)$] or if it was very wide. A medium-width resonance should have been seen, if it exists. Similarly, Refs. [19,20,22] would have seen a resonance if it were narrow, at least if its mass were below ~ 1800 MeV. References [20,22], in particular, were able to rule out $I=2$ states in this range as a result of their increased sensitivity to the $+2$ charge state.

The nominal threshold for the $K\Delta$ channel is 1725 MeV. A state with a mass significantly below this value can decay only to KN or, with additional phase space suppression, $K\pi N$. We would expect such a state to have been seen, if it exists. Above ~ 1725 MeV, the opening of the $K\Delta$ channel could make an $I=1$ or 2 state broad. The K^*N threshold is at 1830 MeV and gives an additional open channel for higher mass states with $I=0$ or 1.

We conclude that below ~ 1725 MeV there is likely only one $Y=2$ resonance, the $\Theta(1540)$. Above ~ 1725 MeV, a resonance may have been missed only if it is very broad and decays predominately into a channel other than KN .

III. THE CHIRAL SOLITON MODEL

Here we review the results obtained from the Skyrme and Chiral soliton models. We rely on Refs. [12–16,36,38,40]. Our aim is (i) to explore the extent to which these models make predictions different from those of the pentaquark models presented in the next section and (ii) to see what additional experimental information would serve to best test the soliton model approach.

It is worth noting that there has been some recent debate over the validity of the rigid rotor approximation for the quantization of the relevant collective modes in the soliton picture, in particular concerning the relation of this approximation to the large N_c limit of QCD [37,39,41,42]. Potential problems with the rigid rotor approximation for $S = +1$ exotic states were noted long ago [59]. More recently, the question of whether it is safe to neglect the couplings between collective rotational modes and other degrees of freedom at large N_c was raised in connection with the observation that the splitting of exotic from nonexotic baryon states does not go to zero as $N_c \rightarrow \infty$ [37]. That the rigid rotor approximation is not necessarily exact in the exotic sector, even in the large N_c limit, is seen explicitly in the toy model constructed by Poblitsa [41]. Cohen [42] has also provided arguments showing that, in general, in the exotic sector, the exotic collective rotational modes need not decouple from the vibrational modes, even as $N_c \rightarrow \infty$. Significant vibrational-rotation coupling was also seen explicitly, for $N_c = 3$, in the results of Ref. [13]. None of these observations, however, rules out the possibility that treating the collective rotations as the dominant degrees of freedom in the low energy part of the spectrum might be a good phenomenological approximation. Vibrational-rotational mixing is, in fact, likely to be most important for states with nonexotic quantum numbers, where nonexotic radially excited configurations and low-lying exotic configurations with the same quantum numbers may lie close together. This expectation is borne out in the explicit calculations of Ref. [13].

There is a consensus among the various soliton model calculations that a state with P01, $Y=2$ quantum numbers and a relatively narrow width should occur in the 1500–1600 MeV mass range. This state lies in a $\overline{\mathbf{10}}_F$. A relatively narrow result for the width of this state was reported in Ref. [12], though the precise value quoted (15 MeV) has been subsequently questioned [57]. A corrected version of the DPP calculation, given by Jaffe [58], yields instead $\Gamma_\theta \sim 30$ MeV. The narrow width results in part from a cancellation [12,13] between the contributions of operators proportional to parameters G_0 and G_1 which are, respectively, leading and next-to-leading order in the $1/N_c$ expansion. It has been shown, however, that in the large N_c counting, the coefficient of the $1/N_c$ operator matrix element receives an $O(N_c)$ enhancement relative to the leading order operator contribution, so the two canceling contributions are formally of the same order in N_c [43]. A width for the Θ significantly less than the width of the Δ is, therefore, quite natural in the soliton picture [43]. To satisfy the $\Gamma_\Theta < 6$ (~ 1 ?) MeV experimental bound, however, the numerical cancellation has to be rather close. Thus, though a relatively narrow (\sim few

10's of MeV) width is natural, a *very* narrow (~ 1 MeV) width is less so, since it requires a rather close fine-tuning of the magnitudes of the parameters G_0 and G_1 .

In addition to the lowest-lying, P01, $\overline{Y}=2$ state, there should be other nearby members of the $\overline{\mathbf{10}}_F$ multiplet. The first state of interest here is the P11, $Y=1$ member, having nucleon quantum numbers.

In the original work of Ref. [12] (DPP), the $\overline{\mathbf{10}}_F$ N state was identified with the $N(1710)$, which identification was used to fix the average mass of the $\overline{\mathbf{10}}_F$ multiplet. (This is no longer true of the most recent update [40]; we will discuss the updated version below.) The splitting within the $\overline{\mathbf{10}}_F$ was determined by the then-current value (~ 45 MeV [61]) of the nucleon sigma term. The $\overline{\mathbf{10}}_F$ assignment for the $N(1710)$ is subject to several possible objections. In Ref. [13], for instance, once $SU(3)_F$ breaking was taken into account, the N state in this region was found to have sizable components not only of the $\overline{\mathbf{10}}_F$ configuration, but also of the radially excited $\mathbf{8}_F$ state, as well as states in the $\mathbf{27}_F$ and $\mathbf{35}_F$ multiplets. There are also potential problems associated with the predicted decay widths: the soliton model predicts the $\Delta\pi$ decay width to be a factor of 2.2 smaller than the $N\eta$ decay width for a pure $\overline{\mathbf{10}}_F$ state, whereas the $N(1710)$ has a large $\Delta\pi$ but small $N\eta$ branching fraction [62]. (A large relative $N\eta$ branching fraction is also predicted in the positive parity pentaquark scenario of Ref. [28].) In addition, the more complete calculation of Refs. [14,15] shows two states close together in this mass range, one coming from the rotation, and one from the vibration of the soliton. (The claim of Ref. [28] that the soliton model does not have a nearby $\mathbf{8}_F$ with which the $\overline{\mathbf{10}}_F$ can mix ignores the presence of the vibrational states. The Roper would be predominately a vibrational state in the soliton picture.) The vibrational ($\mathbf{8}_F$) and rotational ($\overline{\mathbf{10}}_F$) states mix strongly in the analysis of Refs. [13,15]. This mixing will, no doubt, have an important effect on the predicted branching ratios. The analysis of Ref. [63] does, in fact, suggest that there are two N states in this region. Two states are probably necessary in both the soliton and positive parity pentaquark models (see the next section for a discussion of the pentaquarks).

The recent report by the NA49 Collaboration of an exotic Ξ^{--} state with $m = 1862 \pm 2$ MeV and $\Gamma < 18$ MeV [60], necessitated a refitting of the $SU(3)_F$ -breaking parameters of the original DPP analysis [40]. This refitting turns out to require a different assignment for the $N(1710)$, which is perhaps welcome in light of the comments above. The reason is as follows. The NA49 state can be naturally interpreted as the $I = 3/2$ $S = -2$ $\overline{\mathbf{10}}_F$ partner, $\Xi_{3/2}$, of the Θ . This state was originally predicted to have a mass of 2070 MeV by DPP. Assuming the $\overline{\mathbf{10}}_F$ assignment is correct, and taking the Θ and $\Xi_{3/2}$ masses as input, one obtains an alternate solution for the symmetry-breaking parameters of the model. Taking the result of the ChPT analysis for the light quark mass ratio $2m_s/(m_d + m_u) = 24.4 \pm 1.5$ [64] as input, this solution corresponds to a nucleon sigma term of 77 ± 5 MeV, somewhat higher than, though not incompatible with, the more recent experimental determination of Ref. [65], $\sigma_N = 64 \pm 7$ MeV.

TABLE II. Predictions for the masses of exotic states in the $J^P=3/2^+$, 27_F multiplet in the DPP implementation of $SU(3)_F$ breaking in the soliton model picture. The results of Ref. [36], given in the third column, are based on the original DPP parameter set, the results denoted “updated” on the modified set obtained using the Θ and $\Xi_{3/2}$ masses as input. Masses are given in MeV.

State	(I,S)	Ref. [36]	Updated
Θ_1	(1,1)	1595	1628
Γ_{27}	(2,-1)	1904	1727
Ξ_{27}	($\frac{3}{2}$, -2)	2052	1908
Ω_{27}	(1,-3)	2200	2088

The interpretation of the NA49 signal as the $\overline{10}_F$ partner of the Θ is thus phenomenologically acceptable in the soliton picture. With this interpretation, the $\overline{10}_F$ N state lies between 1650 and 1690 MeV, once one takes into account mixing with the ground state nucleon, and hence is no longer to be identified with the $N(1710)$ [40] (though mixing with the radially excited 8_F state, not considered in Ref. [40], may complicate this picture).

It is worth stressing that, in the approach of Refs. [12,40], once the Θ and $\Xi_{3/2}$ masses are employed as input, *all* parameters in the model are fully determined. Precise predictions then follow for the locations of other exotic baryon states. Of particular interest for testing the soliton picture are those exotic states lying in the next lowest multiplet, having $J^P=3/2^+$, 27_F quantum numbers. These states were considered in detail in Ref. [36], using the original DPP parametrization for the symmetry-breaking terms. It turns out that the modified fit necessitated by the NA49 observation significantly alters the predictions for most of these exotic states. The results for the masses from Ref. [36], together with the modified results obtained using the updated parametrization of Ref. [40], are given in Table II [66,67]. We note that (i) the $I=1$ $J^P=3/2^+$ Z^* resonance, denoted Θ_1 , lies rather close to the Θ ($m_{\Theta_1} - m_{\Theta} < 90$ MeV), (ii) while the position of the Θ_1 is only modestly altered by the updated parametrization, the masses of the remaining 27_F , $3/2^+$ exotics are all significantly lowered, and (iii) with the updated parametrization, the exotic $J^P=3/2^+$ $I=3/2$ cascade state Ξ_{27} is predicted to lie only 46 MeV above the analogous 10_F $\Xi_{3/2}$ state. The lowering of the masses of the 27_F exotics will have a significant impact, through reduced phase space, on the prediction for the widths of these states [36]. The presence of a 27_F $\Xi_{3/2}$ state should be detectable through the existence of a $\Xi^* \pi$ decay branch, which is $SU(3)_F$ forbidden for a 10_F state, but allowed for a 27_F state.

While fitting the Θ and $\Xi_{3/2}$ masses fixes the parameters of the DPP version of the soliton approach, one should bear in mind that $SU(3)_F$ breaking is treated only to first order in m_s in Refs. [12,36,40]. It has been argued elsewhere that higher-order-in- m_s corrections may not be negligible [15,16]. The somewhat high value of σ_N obtained from the updated linear-in- m_s fit may also argue for the presence of higher order corrections. In order to get a feel for the uncertainties associated with such differences in implementation of

TABLE III. Results for the masses (in MeV) of the nearby exotic states lying above the Θ . Results in the column labeled “Linear” correspond to the linear-in- m_s treatment with the updated values of the fit parameters obtained using the Θ and $\Xi_{3/2}$ masses as input. Results in the column labeled “All Orders” are those from Ref. [15], and correspond to the all-orders-in- m_s diagonalization explained in the text.

State	(F, J^P, I, S)	Linear	All orders
$\Xi_{3/2}$	($\overline{10}, \frac{1}{2}^+, \frac{3}{2}, -2$)	1862 (fit)	1780
Θ_1	($27, \frac{3}{2}^+, 1, 1$)	1628	1650
Γ_{27}	($27, \frac{3}{2}^+, 2, -1$)	1727	1690
Ξ_{27}	($27, \frac{3}{2}^+, \frac{3}{2}, -2$)	1908	1850
Ω_{27}	($27, \frac{3}{2}^+, 1, -3$)	2089	2020

$SU(3)_F$ breaking, we compare the results of the updated fit above to those of Ref. [15]. The latter were obtained using the same leading-order-in- N_c $O(m_s)$ $SU(3)_F$ -breaking operator as in DPP, and one of the two next-to-leading-order-in- N_c $O(m_s)$ operators, but diagonalizing to all orders rather than truncating at first order in m_s .

The comparison of the results for the nearby exotic states in the two approaches is given in Table III. We see that the sensitivity to the treatment of $SU(3)_F$ breaking is rather modest, the largest discrepancy being 82 MeV. This occurs for the case of the $\overline{10}_F$ $\Xi_{3/2}$ state, which comes out somewhat low in comparison with experiment in the approach of Ref. [15]. One should, however, bear in mind that the $\Xi_{3/2}$ mass was used as input in fixing the model parameters in Ref. [40] while the value, 1780 MeV [15], obtained in Ref. [15] was a prediction, made in advance of the NA49 observation. The value of Ref. [15] is rather similar to the estimate, ~ 1750 MeV, given in the pentaquark model of Ref. [28]. The agreement between two such apparently different models is quite surprising. One should also bear in mind that the “all-orders” treatment of Ref. [15] includes only the higher-order-in- m_s effects generated by diagonalizing the lead-order-in- m_s operators. Additional effects associated with higher-order-in- m_s effective operators have been neglected. For the sake of both (i) verifying that it is possible to reproduce the observed $\Xi_{3/2}$ mass in the all-orders-diagonalization approach and (ii) determining the size of the shifts in the masses associated solely with the higher order diagonalization corrections, it would be interesting to add the remaining next-to-leading-order-in- N_c $SU(3)_F$ -breaking operator employed in DPP to the analysis of Ref. [15]. Note that any pentaquark model which predicts the Θ will also predict exotic Ξ states obtained from the Θ by interchanging the strange quark and one species of light quark $\bar{s} \Rightarrow \bar{u}$, $u \Rightarrow s$ or similarly for the d . The existence of such states thus does not, by itself, distinguish between the soliton and pentaquark pictures.

Let us return to the Θ_1 , which is the next lowest lying Z^* resonance after the Θ in all of the soliton model analysis. We have seen that there is only very modest sensitivity to the treatment of $SU(3)_F$ breaking in the predicted mass of the

Θ_1 in the two rigid rotor approaches discussed above. A somewhat higher mass, ~ 148 MeV above the Θ , is obtained in the bound state approach to strangeness in the soliton model [38], though one should bear in mind, as pointed out by the authors of Ref. [38], that somewhat larger-than-expected $SU(3)_F$ -breaking modifications of the parameters of the model are necessary to accommodate the Θ in this approach. An interesting observation made in Ref. [38] is that, independent of the details of the implementation of the bound state approach, a particular linear combination of the splittings of the $I=1$ 27_F , $J^P=3/2^+$ and 27_F , $J^P=1/2^+$ Z^* resonances from the Θ is determined solely by the pionic moment of inertia, I_π , which is very well constrained by the Δ - N splitting. One thus has the following sum rule relating the Z^* splittings to the Δ - N splitting

$$\begin{aligned} & \frac{2}{3}[m_{27_F,3/2^+} - m_\Theta] + \frac{1}{3}[m_{27_F,1/2^+} - m_\Theta] \\ &= \frac{2}{3}[m_\Delta - m_N] = 195 \text{ MeV}. \end{aligned} \quad (1)$$

Since the 27_F , $1/2^+$ Z^* state lies significantly above the 27_F $3/2^+$ (Θ_1) state, for any phenomenological acceptable parametrization of the model, an $I=1$ Θ_1 partner of the Θ appears unavoidable below ~ 1700 MeV in this framework. This feature is thus common to both the bound state and rigid rotor versions of the chiral soliton model.

Whether or not a relatively low-lying state such as the θ_1 should be observable in existing (or future) experiments depends on its width. If one employs only the leading-order-in- N_c operator of DPP, one obtains an estimate $\Gamma_{\theta_1} \approx 80$ MeV [36]. It seems to us it would be surprising if a $S=+1$ state with $\Gamma \sim 80$ MeV had not been seen in either the production or scattering experiments discussed in the previous section [68]. Even the higher estimates of ~ 1650 – 1690 MeV for the mass are likely to be problematic, since the first $Y=2$, P13 state seen in the data is the one at 1811 MeV. The spin-isospin excitation energy for the Θ thus appears to be a factor of ~ 2 or more too small in the soliton model approach, assuming the earlier experimental results are correct. As we will see in the next section, the pentaquark picture also predicts a low lying $I=1$ excitation of the Θ , and hence suffers from the same apparent problem.

The next Z^* state, with $(I, J^P) = (1, 1/2^+)$ quantum numbers, is predicted to lie significantly higher than the corresponding $(1, 3/2^+)$ state (for example, at 2030 MeV in the rigid rotor approximation using the original DPP parameter set [36], 1861 MeV using the updated parameter values, and 1830 MeV in the bound state approach [38]). Its width is likely to be rather large [68]. With a large width, it would become important to consider corrections to the rigid collective coordinate rotation approximation, which might significantly affect the prediction for the mass. A broad state could also easily have escaped detection in the scattering experiments.

Whether or not the soliton model can quantitatively accommodate the putative D03 and D15 resonances is not yet clear. Reference [15] suggests, based on an analogy with

nonexotic channels, that the D03 and D15 resonances may be quadrupole excitations of the lower P01 and P13 resonances. It also (i) suggests that the 1831 P01 state may be the radial excitation of the Θ and (ii) argues for the presence of a low-lying S01 resonance, which has not been seen. More detailed calculations of the excited exotic states in the soliton model are needed in order to see whether these expectations are actually borne out, in quantitative terms.

Finally, apropos the proposed isotensor assignment for the Θ [25], we comment that the lowest-lying $I=2$ state comes out very high in the soliton model (~ 1950 MeV in Ref. [15] and 2035 MeV in the bound state approach [38]). Rather similar values (~ 1980 MeV) are obtained in either the Goldstone-boson-exchange or color-magnetic-exchange versions of the pentaquark picture. Thus in all of these approaches the isotensor assignment is strongly disfavored.

In summary, the soliton model accounts fairly well for the observed properties of the Θ , provided the resonance quantum numbers turn out to be $(I, J^P) = (0, 1/2^+)$. Potential problems for the approach are the need for a second nucleon state near 1710 MeV, and the location of the $Y=2$, $(I, J^P) = (1, 3/2^+)$ state. The exotic $\Xi_{3/2}$ is predicted to lie somewhat low in one version of the model [15] but can be accommodated with not-unreasonable parameter values. Improved experimental data in the energy region of the problematic states would be useful, as would experimental searches for the other predicted exotics, and explicit calculations for the location of the lowest exotic negative parity states.

If the quantum numbers of the $\Theta(1540)$ are other than $(I, J^P) = (0, 1/2^+)$ the soliton model is in serious, and probably terminal, trouble: not only will it have predicted a state that has not been found, it will have failed to predict a state that has been.

IV. Z* RESONANCES AS PENTAQUARKS

In order to construct a Z^* state similar to the $\Theta(1540)$ one in the quark model, one requires a configuration with a minimum of four light (u, d) quarks and one \bar{s} quark. There is a long history of interest in, and quark-model-based studies of, channels where such pentaquark configurations might occur [4,7,54–56,69–73]. The discovery of the Θ has sparked renewed interest in this [24,27–29,32–35,46], as well as other [26,44,47–51], approaches. We discuss here two versions of the quark model approach: one in which the spin-dependent qq interactions are generated by effective Goldstone boson exchange between constituent quarks [74] (the GB case), and one in which they are generated by effective color-magnetic exchange (the CM case). The bag model and nonrelativistic constituent quark model represent two different implementations of the CM approach. We refer to the interactions in both the GB and CM cases, collectively, as “hyperfine” interactions.

In the GB case, the effective interaction has the form

$$\begin{aligned} H_{\text{GB}} &= \sum_{i<j=1,\dots,4} H_{\text{GB}}^{ij} \\ &= -C_{\text{GB}} \sum_{i<j=1,\dots,4,F} [\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F][\vec{\sigma}_i \cdot \vec{\sigma}_j] f_F(r_{ij}), \end{aligned} \quad (2)$$

where the sum on i, j runs over the four light quarks, that on F runs over the octet of pseudo-Goldstone bosons (π, K, η), and the form of $f_F(r_{ij})$ employed in the model may be found in Ref. [71]. Note that in Refs. [24,35] an approximate, “schematic” version of H_{GB} was employed, in which the spatial dependence of the interaction was omitted. As we will see below, this approximation can lead to a significant overestimate of the hyperfine attraction available in the positive parity sector, and hence should be treated with some caution. (Reference [33] performs a similar schematic treatment of the CM interaction.) As in Ref. [71] we do not include GB-induced interactions between the light quarks and \bar{s} in the putative pentaquark states in order to avoid incorporating interactions which would correspond to the exchange of Goldstone bosons in the Goldstone boson two-particle subchannel $\ell\bar{s}$ ($\ell = u, d$).

In the CM case, the effective interaction has the form

$$H_{CM} = \sum_{i < j = 1, \dots, 5} H_{CM}^{ij} \\ = -C_{CM} \sum_{i < j = 1, \dots, 5} [\vec{F}_i^c \cdot \vec{F}_j^c][\vec{\sigma}_i \cdot \vec{\sigma}_j] f(r_{ij}) \mu_{ij}, \quad (3)$$

where the sum now runs over all pairs (with 5 labeling the \bar{s} quark), $\vec{F}_i^c = \vec{\lambda}_i/2$ for $i = 1, \dots, 4$, and $\vec{F}_5^c = -\vec{\lambda}_5^*/2$. The factor μ_{ij} is defined to be 1 if ij is a light quark pair. In the $SU(3)_F$ limit $\mu_{i\bar{s}} \equiv \hat{\mu}$ is also equal to 1. Phenomenologically, one requires $\hat{\mu} \approx 0.6$ in order to account for $\Lambda - \Sigma$ splitting in the model. We will consider both zero range and finite range versions of $f(r_{ij})$ in what follows.

Before quoting results for the negative and positive parity hyperfine expectations in the models, it is worthwhile pointing out certain generic features associated with the pentaquark picture. First note that Z^* resonances with $I = 0, 1, 2$ lie in the $4q$ flavor multiplets $[f]_F^{4q} = [22], [31]$ and $[4]$, respectively. Combining these with the antiquark flavor tableau [11] one obtains the $SU(3)_F$ representations

$$\begin{aligned} [22] \otimes [11] &= \overline{\mathbf{10}} \oplus \mathbf{8}, \\ [31] \otimes [11] &= \mathbf{27} \oplus \mathbf{8} \oplus \mathbf{10}, \\ [4] \otimes [11] &= \mathbf{35} \oplus \mathbf{10}. \end{aligned} \quad (4)$$

The Z^* states lie in the first of the flavor multiplets on the right-hand side (RHS) in all cases. These are the only possible Z^* flavor classifications possible in the pentaquark picture. In the absence of flavor-dependent interactions between the antiquark and the quarks (as is the case for both the GB and CM models outlined above), the multiplets on the RHS of Eqs. (4) are degenerate in the $SU(3)_F$ limit. (If one allows configurations containing s quarks, one also has $[211] \otimes [11]$, which contains degenerate $\mathbf{8}_F$ and $\mathbf{1}_F$ pentaquark configurations.) The nonexotic states of the $\overline{\mathbf{10}}_F, \mathbf{27}_F$, and $\mathbf{35}_F$ will thus mix strongly with the corresponding members of the accompanying $\mathbf{8}_F$ and/or $\mathbf{10}_F$ multiplets once $SU(3)_F$ breaking is turned on. As pointed out in Ref. [28], a natural

expectation is that this mixing might turn out to be “ideal,” i.e., to diagonalize the $s\bar{s}$ pair number. As noted above, such mixing between members of exotic and non-exotic flavor multiplets also occurs in the soliton model, though the multiplets corresponding to the degenerate sets of the pentaquark scenario are typically not exactly degenerate in the soliton model case.

Second, note that, in the positive parity sector, where one presumably has one unit of orbital excitation, one must combine the 5-quark total spin S_T with the orbital $L = 1$ to form the total angular momentum J . In the absence of spin-orbit forces, the states of different J formed from the same S_T will be degenerate. Since, empirically, spin-orbit splittings in the baryon spectrum are typically rather small, there is a possibility of relatively nearby spin-orbit partners for any state in the pentaquark picture. A quantitative estimate has been made in Ref. [46], assuming the $\vec{L} \cdot \vec{S}$ forces to be generated by effective gluon exchange plus scalar confinement. Assuming either of the scenarios of Refs. [27,28] for the Θ structure, a splitting of order 10's of MeV between the Θ and its $J^P = 3/2^+$ partner is found, with a conservative upper bound of 150 MeV.

Finally we comment on the expected widths of pentaquark Z^* states. Those states with s -wave NK quantum numbers lying above NK threshold have fall-apart modes and hence will not correspond to resonances. In contrast, for pentaquark states lying above KN threshold, but with p - or d -wave KN quantum numbers, the centrifugal barrier may inhibit the decay to KN . A p -wave KN state at 1540 MeV, in a square well of hadronic size (~ 0.8 fm), for example, has a tunneling width of ~ 280 MeV, while a corresponding d -wave state has a width of only ~ 20 MeV. Taking into account the square of the overlap with KN , the width of a pentaquark state can be significantly smaller than the KN tunneling width. However, especially in the p -wave case, at 1540 MeV, one is relatively near the top of the barrier and so states with significantly higher mass for which fall-apart p -wave modes exist are expected to be very broad. Note, however, that such p -wave fall-apart modes are available in the positive parity sector only for those states where the quark spin S_T is $1/2$. Pentaquark states with $S_T = 3/2$ or $5/2$ would require a tensor interaction in order to decay to KN in a p wave, and hence need not be undetectably broad, at least if they do not lie too far above the relevant p -wave fall-apart threshold (ΔK or NK^* for $S_T = 3/2$ and ΔK^* for $S_T = 5/2$).

We now consider the hyperfine expectations for possible Z^* states in the GB and CM models, in both the negative and positive parity sectors. In the negative parity case, all five quarks can be put in the lowest spatial orbital. For positive parity, an orbital excitation is required, and we consider states for which the orbital symmetry, classified by the S_4 of the four light quarks, is $[31]_L$.

A. Negative parity Z^* pentaquarks

In the $SU(3)_F$ limit, with all five quarks in the lowest spatial orbital, one can factor out the common spatial matrix element and determine the hyperfine expectations by standard group-theoretic methods. These results are easily

TABLE IV. The lowest eigenvalues of $\langle H_{\text{GB}} \rangle$ and $\langle H_{\text{CM}} \rangle$ in those negative parity Z^* channels allowed by the Pauli principle for the four light quarks. The results are in units of either C_{GB} or C_{CM} times the common spatial matrix element.

(I, J)	GB	CM	CM
		$\hat{\mu}=1$	$\hat{\mu}=0.6$
(0,1/2)	-9.33	-4.67	-3.33
(0,3/2)	-9.33	0.33	-0.33
(1,1/2)	-8.00	-1.44	-0.78
(1,3/2)	-5.33	-3.00	-1.27
(1,5/2)	0.00	3.33	2.80
(2,1/2)	2.67	7.33	6.27
(2,3/2)	2.67	3.33	3.87

checked by direct computation. $SU(3)_F$ breaking may also be implemented by both group-theoretic methods and direct computation, providing a check on the reliability of the calculations. In quoting results, we will suppress the constants $C_{\text{GB,CM}}$ and spatial matrix elements throughout.

The results for the GB case are given in column 2 of Table IV. For reference, note that the expectation in the N is -14 , which corresponds to a hyperfine energy of ~ -420 MeV with standard values for the parameters of the model. The results for the CM case are given in columns 3 and 4, which correspond to the $SU(3)_F$ limit ($\hat{\mu}=1$) and $\hat{\mu}=0.6$, respectively. For reference, the corresponding expectations in the N and K are -2 and -4 for $\hat{\mu}=1$ and -2 and -2.4 for $\hat{\mu}=0.6$. For $I=1$, where configurations with the spin of the four light quarks, $S_\ell=0,1$ or 2 are all Pauli allowed, and hence two possible states with $J=1/2$ or $3/2$ exist, we show only the lower of the two eigenvalues.

From the table, we see that, in all channels, the hyperfine expectation is either repulsive or significantly less attractive than in KN . This is true for both the GB and CM cases. Taking the nonhyperfine contributions into account, the models both predict the negative parity states to lie considerably above KN threshold, and also significantly above 1540 MeV. One should of course bear in mind that the model treatments of the one-body energies are subject to significant uncertainties. In particular, the response of the vacuum to the presence of an additional quark-antiquark pair is modeled only rather crudely in the bag model, and not at all in the GB and CM versions of the constituent quark model. However, even if one is willing to argue that one-body energies are significantly overestimated, there is no possible negative parity channel to which it is possible to consistently assign the Θ , for reasons which we now explain.

For the CM case, the most attractive hyperfine expectation occurs for $(I, J)=(0, 1/2)$. Such a configuration has a potential s -wave KN fall-apart mode and hence must be either bound or nonresonant. The Θ , which is not bound, can therefore not be assigned to the $(0, 1/2)$ channel. This argument can be avoided only for states with $J=3/2$ or $5/2$, which require a d wave to decay to KN . At 1540 MeV, such a state would be very narrow, especially once the overlap with KN was taken into account. The most attractive by far

of these higher-spin channels is that with $(I, J)=(1, 3/2)$. It is, however, impossible to assign the Θ to this channel since, if one did, the more attractive $(0, 1/2)$ channel would lie below KN threshold. This is ruled out experimentally. Thus, no possible negative parity assignment for the Θ remains in the CM case.

In the GB case, the most attractive hyperfine interaction occurs for $(I, J)=(0, 1/2)$ and $(0, 3/2)$, which are degenerate. A $(0, 3/2)$ state, which requires a d -wave KN decay, would be narrow if located at 1540 MeV. The accompanying $(0, 1/2)$ configuration, with its fall-apart s -wave KN mode would be non-resonant and not a classification problem. However, as we will see in the next subsection, the optimal GB hyperfine attraction in the positive parity sector is sufficiently strong that the lowest Z^* state has positive parity. Since no Z^* resonance is observed below the Θ , it follows that a negative parity assignment is ruled out also in the GB case.

We conclude that, should the Θ turn out to have negative parity, it will be necessary to abandon both the CM and GB models (as well as the current implementations of the soliton model) for any future applications in the multi-quark sector.

B. Positive parity Z^* pentaquarks

In the positive parity sector, a large number of independent Pauli-allowed states exist having $[31]_L$ light quark orbital symmetry. Since, for a color singlet $4\ell\bar{s}$ state, the light quark color is necessarily $[211]_C$, the joint spin-isospin-orbital symmetry of the four light quarks must be $[31]_{ISL}$. Once one takes into account the coupling of the spin of the \bar{s} to S_ℓ to form S_T , one finds 4 such independent $[31]_{ISL}$ states in the $(I, S_T)=(0, 1/2)$ channel, 3 in the $(0, 3/2)$ channel, 6 in the $(1, 1/2)$ channel, 5 in the $(1, 3/2)$ channel, 2 each in the $(2, 1/2)$ and $(2, 3/2)$ channels, and 1 each in the $(0, 5/2)$, $(1, 5/2)$ and $(2, 5/2)$ channels. The $\ell\bar{s}$ interactions in the CM model couple states with the same S_T , but different S_ℓ , while such couplings are absent in the version of the GB model employed here. While one can simply construct the full set of states in each channel, compute the hyperfine expectations, and diagonalize the resulting matrix, a physical understanding of the origin of the lowest possible eigenvalues, and a good approximation to the structure of the corresponding lowest-lying eigenstates, can be obtained from simple physical arguments based on a consideration of attractive correlations accessible in the $4\ell\bar{s}$ sector.

In the GB model, the only attractive s -wave $\ell\ell$ correlations are those with $I=S=0$, $C=\bar{3}$ and $I=S=1$, $C=\bar{3}$. In these configurations, the hyperfine pair expectations, suppressing C_{GB} and the spatial matrix elements, are -8 and $-4/3$, respectively. There are no attractive $\ell\ell\ell$ configurations, apart from the nucleon. Organizing the four light quarks into two $I=S=0$, $C=\bar{3}$ pairs, thus takes optimal advantage of the strong hyperfine attraction in that channel. As pointed out in Refs. [28,29], such a two-cluster configuration, coupled to net color 3_C [the ‘‘Jaffe-Wilczek (JW) correlation’’], is forbidden unless the two clusters are in a relative p wave. Neglecting interactions between the clusters, as well as further cross-cluster light-quark antisymmetrization

effects (both suppressed by the relative p wave between the clusters), the GB hyperfine expectation for such a configuration is -16 , which is now more attractive than in the N . This configuration is possible only in the $(I, S_T) = (0, 1/2)$ channel. The $(I, S_\ell) = (0, 0)$ configuration constructed from two $I = S = 1, C = \bar{3}$ pairs, which is also present in the $(I, S_T) = (0, 1/2)$ channel, has a hyperfine attraction of $-8/3$ in the same approximation, and can mix with the JW correlated state to reduce the hyperfine expectation even further. We thus expect the hyperfine expectation in the GB model to (i) be minimized in the $(I, S_T) = (0, 1/2)$ channel, (ii) be less than -16 , and (iii) correspond to a state dominated by the JW correlation. We will see that these expectations are borne out by the results of the full calculations given below. In terms of the light quark $S_4 \downarrow S_2 \times S_2$ substate labels in the spin, isospin, color and orbital sectors, the JW state, neglecting cross-cluster antisymmetrization, is

$$|JW\rangle = |[211]_{CAA}|[22]_{IAA}|[22]_{SAA}|[31]_{LSS}\rangle. \quad (5)$$

With our phase conventions, the ISC overlap of this state with the $(I, S_T) = (0, 1/2)$ $N_{123}K_{45}$ configuration is $1/2\sqrt{6}$. The p -wave KN decay width will thus be naturally small [$\sim 280(1/24)f^2$ MeV $\approx 10f^2$ MeV, where f is a spatial overlap factor] for a state at 1540 MeV dominated by the JW correlation. The next most attractive correlation is that involving one $I = S = 0, C = \bar{3}$ and one $I = S = 1, C = \bar{3}$ light quark pair. This configuration has $(I, S_\ell) = (1, 1)$, and produces degenerate $(I, S_T) = (1, 1/2)$ and $(1, 3/2)$ configurations when combined with the \bar{s} quark. The hyperfine expectation is $-28/3$, before additional mixing is included.

The situation, though somewhat more complicated, is similar in the CM case. Here the only attractive $\ell\ell$ correlations are those with $I = S = 0, C = \bar{3}$ and $I = 0, S = 1, C = 6$. The hyperfine expectations, again suppressing C_{CM} and spatial matrix elements, are -2 and $-1/3$, respectively. A strongly attractive light quark configuration is again formed by constructing the JW correlation, whose CM hyperfine expectation is -4 . The JW correlation has $(I, S_\ell) = (0, 0)$, and hence is present only in the $(I, S_T) = (0, 1/2)$ channel. The $(I, S_\ell) = (0, 1)$ correlation produced by combining one $I = S = 0, C = \bar{3}$ and one $I = 0, S = 1, C = 6$ pair has a less attractive light quark hyperfine expectation $-7/3$ and can also contribute to the $(I, S_T) = (0, 1/2)$ channel. As first noted by Karliner and Lipkin [27] (KL), however, with CM interactions, coupling the \bar{s} spin to the $S = 1$ pair in such a way as to make the total spin of the three-quark correlation $1/2$ leads to a reversal of the ordering of the hyperfine energies of the two correlated light quark states, once the $\ell\bar{s}$ interactions are taken into account. Indeed, in the $SU(3)_F$ limit, the hyperfine expectation of the JW correlated state, including the \bar{s} , remains unchanged at -4 , while that of the KL correlated state is lowered to $-17/3$. For $\hat{\mu} = 0.6$ the expectations are -4 for the JW state and $-13/3$ for the KL state. The $\ell\bar{s}$ interactions not only make the KL correlation lower in energy than the JW correlation, but also couple the two correlated configurations. This effect is especially important for

the $SU(3)_F$ -broken case, $\hat{\mu} = 0.6$, where the JW and KL configurations are close to degenerate. The CM hyperfine matrix in the JW, KL subspace of the $(I, S_T) = (0, 1/2)$ channel is, suppressing C_{CM} and the spatial matrix elements

$$\begin{pmatrix} -4 & -2\sqrt{3}\hat{\mu} \\ -2\sqrt{3}\hat{\mu} & -\frac{7}{3} - \frac{10}{3}\hat{\mu} \end{pmatrix}. \quad (6)$$

The lowest eigenvalue is -8.4 for $\hat{\mu} = 1$ and -6.2 for $\hat{\mu} = 0.6$, in both cases significantly lower than expectation for either the JW or KL configuration. The optimal combination is a roughly equal admixture of the JW and KL correlations. The ISC overlap with $N_{123}N_{45}$ for such a configuration is $\sim -1/5$ for our phase conventions, again providing a natural explanation for the narrow width of a state dominated by such a configuration and lying at 1540 MeV. The above results of course neglect cross-cluster interactions, as well as additional antisymmetrization-induced effects between the clusters. They also neglect the presence of other attractive configurations (for example, $\ell\ell\ell\bar{s}$ with $I = 1/2$, color $\bar{3}$ and spin 1 produced by coupling the \bar{s} to a $\ell\ell\ell$ configuration with $I = 1/2$, color $\mathbf{8}$ and $S_\ell = 1$). These are less attractive than the JW and KL configurations, but can also mix with the above combination of JW and KL states. As we will see below, the results of the full calculation (which includes all cross-cluster interactions and is fully antisymmetrized in the coordinates of the four light quarks) are in good agreement with the estimates just given for the optimal hyperfine expectation. This observation suggests that the optimized combination of KL and JW correlations dominates the lowest-lying state in the $(I, S_T) = (0, 1/2)$ channel.

We now present the results obtained by constructing the full set of completely antisymmetrized states allowed in each channel for the light quark $[31]_L$ configuration and computing and diagonalizing the resulting hyperfine matrix. In each case we quote only the lowest eigenvalue in the channel in question. Details of the construction and calculations will be presented elsewhere [75].

We first comment briefly on the structure of the spatial matrix elements in the $[31]_L$ sector. Using the $S_2^{12} \times S_2^{34}$ labeling, the $[31]_L$ S_4 irrep has a basis $\{|SS\rangle, |SA\rangle, |AS\rangle\}$. Writing the hyperfine matrix element between two fully antisymmetrized states $|[n]\rangle$ and $|[m]\rangle$ as

$$\begin{aligned} \langle [n] | H_{GB,CM} | [m] \rangle &= 6 \langle [n] | H_{GB,CM}^{12} | [m] \rangle \\ &+ 4 \langle [n] | H_{GB,CM}^{45} | [m] \rangle, \end{aligned} \quad (7)$$

one finds that the matrix elements involve the following, in general nonzero, spatial matrix elements $\langle m | f(r_{12}) | m \rangle$ and $\langle m | f(r_{45}) | m \rangle$, with $m = SS, SA, AS$, and $\langle SS | f(r_{45}) | SA \rangle$. In the ‘‘schematic’’ approximation all the diagonal matrix elements are set equal to 1 while the off-diagonal 45 matrix element is set equal to zero. In general, however, the $\langle AS | f(r_{12}) | AS \rangle$ matrix element will be suppressed relative to the $\langle SS | f(r_{12}) | SS \rangle$ and $\langle SA | f(r_{12}) | SA \rangle$ matrix elements (it, in fact, must vanish for zero range interactions). In the

TABLE V. The lowest eigenvalues of $\langle H_{GB} \rangle$ for positive parity Z^* channels. The results are in units of $C_{GB} \langle SS | f(r_{12}) | SS \rangle$. The heading “schematic” refers to the schematic treatment (neglect) of the spatial dependence in H_{GB} , the heading “realistic” to the use of the explicit spatial dependence described in Ref. [71].

(I, J)	Schematic	Realistic
(0,1/2)	-28.0	-21.9
(0,3/2)	-9.3	-10.5
(0,5/2)	4.0	1.7
(1,1/2)	-21.3	-17.1
(1,3/2)	-21.3	-17.1
(1,5/2)	0.0	-0.9
(2,1/2)	2.7	0.8
(2,3/2)	-8.0	-6.2
(2,5/2)	-8.0	-6.2

GB model, the explicit form of the spatial dependence used in the model results in a suppression

$$\frac{\langle AS | f(r_{12}) | AS \rangle}{\langle SS | f(r_{12}) | SS \rangle} \simeq 0.3 \tag{8}$$

if one employs a Gaussian wave function with p -wave excitations in the light quark coordinates. This result is much closer to the zero range than to the schematic limit. The relations among the other matrix elements are also not, in general, well-approximated by the schematic approximation.

In generating results for the GB case we have employed the actual spatial dependence employed by the proponents of the model, but also quote the results in the “schematic” limit for comparison. The results are given in Table V.

For the CM case we quote results for both a “zero range” and “finite range” treatment of the spatial dependence. For the zero range case we employ $f(\vec{r}_{ij}) = \delta^3(\vec{r}_{ij})$, while for the finite range case we employ a Gaussian with width $\sim 1/3$ fm. The results are then quoted with an overall factor of $C_{CM} \langle SS | f(r_{12}) | SS \rangle$ factored out. The results are given in Table VI. In both the GB and CM cases, one can use the

TABLE VI. The lowest eigenvalues of $\langle H_{CM} \rangle$ for positive parity Z^* channels. ZR and FR denote the zero range and finite range versions of the CM spatial dependence, respectively. The results are in units of $C_{CM} \langle SS | f(r_{12}) | SS \rangle$.

(I, J)	$\hat{\mu} = 1$	$\hat{\mu} = 1$	$\hat{\mu} = 0.6$	$\hat{\mu} = 0.6$
	(ZR)	(FR)	(ZR)	(FR)
(0,1/2)	-6.86	-8.26	-5.14	-6.23
(0,3/2)	-3.82	-3.11	-2.27	-2.01
(0,5/2)	2.58	3.03	2.01	2.51
(1,1/2)	-5.60	-7.84	-3.92	-5.81
(1,3/2)	-2.19	-2.66	-1.46	-1.78
(1,5/2)	2.58	3.09	2.22	2.61
(2,1/2)	1.08	-0.49	1.88	0.19
(2,3/2)	0.09	-2.02	1.17	-0.47
(2,5/2)	3.33	3.37	3.07	2.91

schematic limit to test the reliability of the calculation since, in that limit, the expectations can again be determined using group-theoretic methods.

C. Comments on the pentaquark results

Although the crudeness of the treatment of vacuum response in the models prevents the one-body energies, and hence also the absolute location of any particular state, from being reliably predicted, the relative orderings, as well as the splittings, in the models are well determined, and hence amenable to comparison with experimental data. In the GB model, the excitation energy to promote one of the light quarks to a p wave is ~ 250 MeV [71]. In the CM case, for correlations of the type expected to dominate the most favored pentaquark channel, the excitation energy is expected to be ~ 210 MeV [27]. We find that, in both the GB and CM cases, the increase in the hyperfine energy in going from the negative to the positive parity sector is more than enough to compensate for the orbital excitation energy. The lowest lying pentaquark state in both models is thus predicted to have positive parity. In both cases this state has $I=0$, $S_T=1/2$, and is to be identified with the Θ . Thus, depending on the sign of any possible spin-orbit force, the quantum numbers of the Θ are predicted to be $I=0$, with J^P either $1/2^+$ or $3/2^+$. As we will explain below, other phenomenological input strongly favors the former assignment. The $1/2^+$ state also lies lowest for the quantitative estimate of the $\vec{L} \cdot \vec{S}$ splitting given in Ref. [46]. Note that the “schematic” approximation is, in general, rather unreliable. In particular, in the $(I, S_T) = (0, 1/2)$ channel, to which the Θ must be assigned, it leads to a 77% overestimate of the size of the hyperfine attraction, relative to KN , in the GB case.

The next lowest positive parity states in the GB model correspond to the degenerate pair $(I, S_T) = (1, 1/2)$ and $(1, 3/2)$, predicted to lie at ~ 1685 MeV, before spin orbit interactions are taken into account. Spin orbit splitting will make either the P11 or P13 state lowest in the first case, and either the P11 or P15 state lowest in the second. No resonance has been reported in this region in any of these channels, though a KN P13 resonance is claimed at 1811 MeV. The first spin-isospin excitation of the Θ thus appears in the same vicinity as in the soliton model, and hence also corresponds to an excitation energy which is, on current evidence, too small by a factor of ~ 2 . The other attractive hyperfine state is that with $(I, S_T) = (0, 3/2)$, predicted to lie at ~ 1855 MeV. Since a P01 resonance is seen in this region (at 1831 MeV), the GB model naturally accommodates such a state, provided the spin-orbit couplings favor the low spin state in the $I=0$ sector. This identification would then simultaneously require identifying the Θ with the $J^P = 1/2^+$ configuration. The lowest of the negative parity configurations not having an s-wave fall-apart mode is predicted to have quantum numbers $I=0$, $J^P = 3/2^-$ and a hyperfine expectation ~ 345 MeV less attractive than the Θ . Taking into account the orbital excitation energy, one expects this state to lie at ~ 1640 MeV. The D03 resonance claimed experimentally is located at 1788 MeV, so the GB prediction gives a prediction for the negative parity excitation energy which is a factor of

>2 too small, though the quantum numbers of the lowest-lying negative parity state are in agreement with the experimental claim.

In the CM model, the next lowest positive parity state after the Θ has $(I, S_T) = (1, 1/2)$. Depending on the range of the effective interaction, it lies between ~ 30 and 95 MeV above the Θ . Although it has a p -wave fall-apart mode, which is suppressed only by barrier penetration, the underlying KN tunneling width (at least for a square well of hadronic size) does not grow rapidly enough with energy to make such a state broad unless it lies in the upper part of this range. For what would appear to be the more realistic (finite range) version of the model, therefore, one would expect to have seen this state experimentally. The first spin-parity excitation of the Θ (which should correspond to either the P11 or P13 wave) is thus again predicted to lie significantly too low in the spectrum, at least for the finite range version of the CM model. The next positive parity state has $(I, S_T) = (0, 3/2)$, and is predicted to lie between 1755 and 1855 MeV, depending on the range of the interaction. If the $J=1/2$ state is favored by the spin-orbit couplings, then, as in the GB case, this allows the identification of this state with the claimed P01 resonance at 1831 MeV, and forces the choice $J^P = 1/2^+$ for the spin parity of the Θ on us. Finally, with the CM interaction, the $(I, S_T) = (1, 3/2)$ state is predicted to lie in the range 1815 to 1870 MeV. The lowest state, after spin-orbit coupling, will have either P11 or P15 quantum numbers. No such state is seen, though it is predicted to lie significantly above the p -wave ΔK threshold, and so may be rather broad. The lowest negative parity configuration without an s -wave fall-apart mode, as for the GB case, is predicted to have quantum numbers $(I, J^P) = (0, 3/2^-)$. Taking into account the difference of the hyperfine splitting relative to the Θ and the estimated orbital excitation energy [27], it is expected to lie ~ 240 MeV above the Θ , i.e., near 1780 MeV. It is thus natural to identify it with the claimed D03 resonance at 1788 MeV.

V. QCD SUM RULE AND LATTICE STUDIES

In this section we comment briefly on the results obtained in recent QCD sum rule [47–49] and lattice [50,51,76] studies. References [49–51] all report evidence for a negative parity assignment for the Θ . The two analysis frameworks both have a rigorous relation to QCD so, if all approximations were under control in these studies, the question of the parity of the Θ would be settled, and all of the models discussed above would be ruled out. We show that it not yet possible to reach such a conclusion.

A. Lattice studies

Reference [50] is a quenched study with Wilson gauge and fermion actions $L \sim 2$ fm and lattice spacing a varying between 0.171 and 0.093 fm. Finite size effects were investigated and a linear extrapolation in a performed. The range of light quark masses m_q studied corresponds to m_π in the range ~ 400 – 650 MeV. A linear extrapolation to physical m_q was employed. The $J^P = 1/2^\pm$, $I=0,1$ channels were all

investigated. In most cases a single interpolating field was used, but on the largest lattice, and at the largest quark mass, variable linear combinations of two interpolating fields were employed. Studying the resulting 2×2 correlation matrix allowed a convincing separation of the scattering state from the nonscattering state, at this m_q [77]. In the full analysis, the lowest Z^* resonance was found to occur in the $I=0$ $J^P = 1/2^-$ channel. One should bear in mind that the result quoted in Eq. (3.2) of Ref. [50],

$$m_{I=0, J^P=1/2^-} = 1539 \pm 50 \text{ MeV}, \quad (9)$$

corresponds to the smallest value of a , and *not* to the result of the continuum extrapolation shown in Fig. 4 of Ref. [50]. The latter is not explicitly quoted but, reading from the figure, would correspond to roughly

$$m_{I=0, J^P=1/2^-} = 1465 \pm 115 \text{ MeV}. \quad (10)$$

The continuum-extrapolated $I=0$, $1/2^+$ state, again from Fig. 4, has a mass

$$m_{I=0, J^P=1/2^+} \approx 1.9 m_{I=0, J^P=1/2^-} [50]. \quad (11)$$

Reference [51] is also a quenched study with unimproved Wilson gauge and fermion action. A single lattice spacing, $a=0.068$ fm, and lattice size, $L \approx 2.2$ fm were employed. The values of m_q used correspond to m_π in the range 600 – 1000 MeV and, again, a linear chiral extrapolation was assumed. A single interpolating field with $I=0$, coupling to both the $J^P = 1/2^\pm$ channels, was employed and the projection onto individual parities performed. It is claimed that two plateaus are seen in the effective mass plot, one corresponding to the KN scattering state, and one to the relevant Z^* resonance, though this claim has been questioned [76]. Again, the $1/2^-$ mass is found to be the lower of the two, with

$$m_{I=0, J^P=1/2^-} = 1760 \pm 90 \text{ MeV} \quad (12)$$

and

$$m_{I=0, J^P=1/2^+} = (1.5 \pm 0.1) m_{I=0, J^P=1/2^-}. \quad (13)$$

Note that the chirally extrapolated N and K masses come out somewhat high in the simulation (1050 ± 30 and 520 ± 10 MeV, respectively), so if one estimates the Z^* resonance masses using the chirally extrapolated values of the ratios to the threshold mass $m_N + m_K$, as in Ref. [50], the $I=0$, $1/2^-$ mass would be reduced to 1610 MeV, reducing the disagreement with the estimate of Ref. [50].

The third lattice study [76] is again quenched, with only a single lattice spacing, but employs, instead of Wilson fermions, overlap fermions. Much lighter quark masses are reached than in the other simulations, with a minimum pion mass of $m_\pi \approx 180$ MeV. Both the $I=0$, $J^P = 1/2^\pm$ channels were considered. The KN scattering states, and also the ghost state in the positive parity channel, were all clearly identified, but no signal for either a positive *or* negative parity Θ was seen.

We comment here that it is almost certainly crucial to push the simulations down to the low m_q values reported in Ref. [76]. Given the expected intimate relation between pentaquark configurations and states of the excited baryon spectrum, a natural place to look for guidance on this issue is results of recent lattice studies of the S11 and Roper resonances [78]. If one takes only those parts of the results of Ref. [78] corresponding to the range of m_q employed in either Ref. [50] or Ref. [51], one finds that (i) the negative parity N^* lies significantly lower than the positive parity N^* for all such m_q and (ii) if one makes a linear extrapolation of the ratios m_{S11}/m_N and m_{Roper}/m_N to physical m_q , the resulting “prediction” for the physical ratio m_{Roper}/m_{S11} is ≈ 1.32 for extrapolation based on the range employed in Ref. [51] and ≈ 1.25 for extrapolation based on the range employed in Ref. [50], in both cases in serious disagreement with experiment [79]. The source of the problem turns out to be the use of the linear chiral extrapolation: the results of the actual simulation are far from linear below $m_\pi \approx 400$ MeV, displaying a crossover of the positive and negative parity levels around $m_\pi \approx 240$ MeV and producing a central value for the mass ratio $m_{\text{Roper}}/m_{S11} \approx 0.94$ at physical m_q , in excellent agreement with experiment. Whether or not a similar low- m_q behavior is to be expected for the Z^* signals found using pentaquark interpolating fields is not known at present. The N^* results, however, clearly signal the potential dangers in assuming the validity of a linear extrapolation from the large quark masses used in the simulations of Refs. [50,51]. This indicates to us that reaching a definitive conclusion on the parity of the Θ is not yet possible on the basis of current simulations.

B. QCD sum rule studies

All three QCD sum rule studies in the literature employ the Borel transformed dispersive sum rule formulation, with a single-pole-plus-continuum ansatz for the spectral function and factorization estimates for condensates of dimension $D = 6$ and higher. Reference [47] considers the $J = 1/2$ $I = 0, 1$ and 2 channels. No parity projection is performed, so both negative and positive parity Z^* states can, in principle, contribute to the spectral function of the correlators employed. The continuum threshold parameter, s_0 , is taken to lie between 3.6 and 4.4 GeV^2 , while the Borel mass, M , is varied over the range 1.5–2.5 GeV. Since s_0/M^2 is less than or ≈ 1 over much of this range, significant continuum contributions will be present. Masses of 1560, 1590, and 1530 MeV are quoted for the $I = 0, 1$ and 2 Z^* states, respectively, with errors of order 150 MeV in all cases.

Reference [48] considers the $I = 0$, $J = 1/2$ channel, again without parity projection. Two interpolating fields, combined with a variable relative coefficient, are employed. The Borel mass is varied over the range $2 \text{ GeV}^2 < M^2 < 3 \text{ GeV}^2$, while $s_0 = 4.0 \pm 0.4 \text{ GeV}^2$, so continuum contributions will be less significant than in Ref. [47]. A result $m = 1550 \pm 100$ MeV is quoted for the Θ mass.

Reference [49] (SDO) also considers only the $I = 0$, $J = 1/2$ channel, but performs a parity projection to separate the $P = +$ and $P = -$ cases. A single interpolating field is

employed, with $1 \text{ GeV} < M < 2 \text{ GeV}$, and $3.24 \text{ GeV}^2 < s_0 < 4.0 \text{ GeV}^2$. It is argued that no Z^* signal is seen in the positive parity channel, while a mass of ~ 1500 MeV is quoted in the negative parity channel.

We now comment on these analyses. The first point of relevance is that, taking the expressions and OPE parameter values given in each reference, and varying s_0 to look for a stability window in M for the physical output (Z^* mass), one finds that no such stability window exists so long as one imposes the constraint of spectral positivity. The absence of such a stability window typically signals the existence of problems with the approximations used on either the OPE or spectral integral side of the sum rules (or both). Such problems can arise from an overly-simple form of the spectral ansatz and/or poor convergence with D of the integrated OPE series. One similarly finds no stability of the putative Z^* mass with respect to the relative coefficient of the two interpolating fields used in Ref. [48], suggesting that the lowest state has not been successfully separated from the other spectral contributions. In the cases where no parity projection has been performed, the lack of stability might result from the presence of reasonably isolated low-lying states in both parity channels. In such a situation, the form of the spectral ansatz means that the spectral contribution of the higher of the two low-lying $P = \pm$ states must be approximated as part of the continuum contribution. Such an approximation can be rather inaccurate, especially if (as for the pentaquark correlators) the continuum version of the spectral function is a strongly increasing function of s . In such a situation, however, the results of the analysis could still be interpreted, qualitatively, as indicating the need for low-lying spectral strength, and hence of a low-lying pentaquark configuration.

A more serious potential problem is the convergence with D of the integrated OPE series. In general, as the number of elementary fields in the composite interpolating operator grows, OPE contributions of a given dimension correspond to higher and higher numbers of loops, and hence receive stronger and stronger numerical loop suppression factors in their coefficients. At a given Borel mass M , this means that higher D contributions for pentaquark correlators will typically be much more important relative, for example, to the well-determined $D = 0$ (perturbative) and $D = 4$ contributions than is the case for ordinary meson and baryon correlators. This is a significant potential problem since higher D condensates are typically not known phenomenologically and end up being estimated by the vacuum saturation/factorization approximation (VSA). This approximation is known to be rather crude (being in error by a factor of $\sim 1.5 - 4$ for the $D = 6$ contributions to various combinations of vector and axial vector correlators for which reliable determinations from data exist [80,81]), and hence can be the source of significant theoretical systematic errors if higher D contributions are dominant. One can, of course, in principle, simply go to larger M in order to further suppress higher D contributions relative to well-known low-dimension ones, but doing so without simultaneously increasing s_0 leads to a spectral integral dominated by continuum contributions, and hence to large errors on any extracted resonance parameters.

TABLE VII. Contributions to the OPE side of the sum rule of Eq. (14) for those terms ($D \leq 6$) included in the analysis of Ref. [49]. The upper sign for the chirally odd (odd dimension) contributions corresponds to the positive parity case, the lower sign to the negative parity case.

D	s_0 (GeV)		
	2.56	3.24	4.0
0	0.00016	0.00052	0.00138
1	± 0.00009	± 0.00027	± 0.00065
3	± 0.00149	± 0.00339	± 0.00673
4	0.00040	0.00083	0.00149
5	∓ 0.00315	∓ 0.00576	∓ 0.00943
6	0.00029	0.00047	0.00070

Increasing s_0 , however, is typically not an option since one requires a realistic ansatz for the spectral function in the region below s_0 and, if s_0 is large, that region of the spectrum can no longer be sensibly approximated by a single narrow resonance contribution.

In discussing the question of the convergence with D of the OPE series, we concentrate on the SDO analysis since the projection onto the separate parity channels makes it more likely that the single-pole-plus-continuum ansatz can be safely employed. After transferring ‘‘continuum’’ spectral contributions to OPE side, the SDO sum rules are of the form

$$|\lambda_{\pm}|^2 e^{-m_{\pm}^2/M^2} = \frac{1}{M^{12}} \left[\sum_{D=2n} c_D \frac{\langle O_D \rangle}{M^D} \pm \sum_{D=2n+1} c_D \frac{\langle O_D \rangle}{M^{D+1}} \right], \quad (14)$$

where m_{\pm} is the mass of the $J^P = 1/2^{\pm}$ Z^* state, λ_{\pm} is the strength of its coupling to the interpolating field and the c_D depend on s_0/M^2 . A similar relation holds for the derivative with respect to $1/M^2$. The ratio of these two expressions, from which $|\lambda_{\pm}|^2$ cancels, is used to determine m_{\pm} . In SDO the sums on the RHS include terms up to and including $D = 6$. We will now argue that, at the scales employed in the analysis, large contributions of $D > 6$ are *necessarily* present, so that the conclusions based on including terms only up to $D = 6$ are not reliable.

To see this, recall that spectral positivity requires that the coefficient of the exponential on the left-hand side (LHS) of Eq. (14) is positive. If the spectral ansatz is sensible (a necessary condition for the extracted resonance parameters to have physical meaning), a negative value for the truncated sum on the OPE side of the equation *necessarily* implies that numerically non-negligible positive OPE contributions have been neglected in the truncation employed. In Table VII we list the contributions to the OPE side of Eq. (14) for $M = 1.5$ GeV (the midpoint of the SDO range) and all s_0 employed by SDO. We see that (i) the $D = 5$ contribution is dominant and (ii) the truncated OPE sum is negative for $s_0 = 2.56$ and 3.24 GeV². The former observation shows that the series of the dominant chirally odd (odd dimension) contributions shows no sign of convergence for any of the s_0

considered while the latter demonstrates unambiguously that $D > 6$ OPE contributions cannot be neglected in the positive parity channel. Since those same contributions enter the negative parity sum rule, with either the same or opposite sign, the truncated series for the negative parity case is also shown to be unreliable. Thus, unfortunately, at those scales where one can hope to make a sensible ansatz for the $s < s_0$ part of the spectral function, the convergence of the OPE is too slow with D to allow a determination of the separate negative and positive parity Z^* masses using the Borel sum rule technique.

One might consider trying to redo the sum rule analysis in the finite energy sum rule (FESR) framework where, through the choice of the weight function, one has control over the dimensions of the terms in the OPE which contribute (up to corrections suppressed by additional powers of α_s). It is known that the ‘‘pinch-weighted’’ version of such FESR’s are very well satisfied at the scales in question, in channels where they have been tested [81,82]. However, with the higher D chirally odd contributions being numerically dominant it is almost certainly the case that the integrated contributions associated with the radiative corrections in the Wilson coefficients for those higher D terms will not be numerically negligible, even if one has chosen the weight in such a way that the leading contribution integrates to zero. It thus appears to us unlikely that the results of such a FESR analysis would be free of sizable theoretical systematic uncertainties. In view of the above comments, we conclude that it is not possible to fix the parity of the Θ through the arguments which currently exist in the literature based on QCD sum rules.

VI. SUMMARY AND DISCUSSION

Although there exist quantitative differences among the three models discussed above, we have seen that their predictions for the spectrum of Z^* resonances are, somewhat surprisingly, in qualitative agreement. Specifically (i) the lowest lying Z^* state is predicted to have positive, not negative, parity; (ii) the most natural quantum number assignment for this state is $I = 0$, $J^P = 1/2^+$; (iii) the first positive parity excitation of the Θ is predicted to lie significantly lower than indicated by current experimental data; and (iv) the lowest-lying $I = 2$ Z^* excitation is predicted to occur rather high in the spectrum, near 2 GeV.

Although, the arguments presented so far do not rule out a $J^P = 3/2^+$ assignment for the Θ in either the GB or CM versions of the pentaquark picture, such an assignment would create problems in accounting for the claimed P01 state at 1831 MeV. An even more compelling argument in favor of the $J^P = 1/2^+$ assignment is given below. Quantum numbers other than $(I, J^P) = (0, 1/2^+)$ for the Θ would thus represent a terminal problem for all three models.

We reiterate that existing lattice and QCD sum rule analyses do not yet provide a reliable framework for establishing the parity of the Θ . In the case of the lattice simulations, a crucial improvement will be future work at light m_q . This is necessary in order to avoid relying on a linear extrapolation of results obtained using m_q values for which the analogous

extrapolation is known to be unreliable in the N^* sector. In the case of the sum rule analyses, the issue is the convergence in D of the integrated OPE contributions. We have explained why we are pessimistic about the possibility of improving the current situation and obtaining a reliable sum rule analysis in the Θ channel.

Even if the quantum numbers of the Θ do turn out to be correctly predicted, a significant potential problem, common to all three models, is the small predicted excitation energy for the first spin-isospin excitation. That an $I=1$ Z^* excitation is expected no more than 150 MeV above the θ in all versions of the chiral soliton approach, as well as in both versions of the pentaquark approach, represents a surprising commonality between the models. However, it appears to us unlikely that such a low-lying state would have escaped detection in both the KN scattering experiments and recent photoproduction experiments, especially since it is not expected to be particularly broad in either the soliton or pentaquark pictures. It would, of course, be highly desirable to perform a dedicated search for Z^* states lying above the Θ , not only to verify that such a low-lying state has not, in fact, been missed, but also to expand our empirical knowledge of the Z^* spectrum. It would also be useful to have predictions in the soliton picture for the locations of any negative parity Z^* states.

A way in which the soliton and pentaquark predictions may differ lies in the potential existence of spin-orbit partners of the pentaquark states. However, in those cases where the pentaquark decay is inhibited only by the p -wave centrifugal barrier, the spin-orbit splitting may push the partner states up sufficiently far that either the width becomes very large or the higher state no longer resonates. Whether such additional states should actually show up in the spectrum or not is thus likely dependent on specific details, rather than the basic qualitative features, of the underlying models. An understanding of the source of the spin-orbit splitting is probably required to make any progress on this question.

The soliton and pentaquark approaches have many similarities outside the Z^* sector as well. Both predict an exotic $I=3/2$ Ξ state lying in the same $\overline{\mathbf{10}}_F$ as the Θ . Both approaches also require more than one N state in the vicinity of the PDG $N(1710)$. In the case of the soliton model, the lowest vibrational excitation is expected to dominate the Roper. Mixing between another vibrational excitation and the $\overline{\mathbf{10}}_F$ rotational excitation is likely required in order to account for the dominantly nonstrange decay modes of the $N(1710)$. In the pentaquark picture, as pointed out by Jaffe and Wilczek [28], one expects (i) degenerate $\mathbf{8}_F$ and $\overline{\mathbf{10}}_F$ N states in the $SU(3)_F$ limit and (ii) (probably) ideal mixing of these states after $SU(3)_F$ breaking is turned on. The combination with no hidden strangeness should then lie below the

Θ , and is a natural candidate to dominate the Roper (which comes out consistently high in $3q$ quark model treatments), while the orthogonal combination should have hidden strangeness and lie a similar distance *above* the Θ . This second state should, however, as in the soliton model case, decay dominantly to states containing strange particles. Another N state with these properties is thus required in the vicinity of the $N(1710)$. The $N(1710)$ might then be dominated by a three-light-quark radial excitation configuration. In either picture one sees that the positive parity excited baryon spectrum becomes considerably more complicated than previously thought, once one takes the existence of the Θ into account.

We would like to stress that the existence of flavor multiplets degenerate in the $SU(3)_F$ limit is a very general feature, common to *any* version of the pentaquark picture having effective quark-antiquark interactions independent of flavor. The existence of the Θ , combined with the observation that the nonstrange, ideally mixed combination of the $\mathbf{8}_F$ and $\overline{\mathbf{10}}_F$ must lie below the Θ makes it inevitable that the pentaquark configuration will play a major role in the structure of the Roper. In fact, since there are no states significantly below 1540 MeV except for the Roper and ground state nucleon, both of which have $J^P=1/2^+$, this argument, in combination with the $I=0$ classification, seems to us to leave no option other than a $J^P=1/2^+$ assignment for the Θ in the pentaquark picture. In addition, with typical nonstrange-strange splittings, one would expect the nonstrange state to lie somewhat below the actual Roper (a feature seen also in the soliton model [15]). To move it up to the observed location would then require some mixing with a lower state, i.e., the ground state N . This then implies that some level of pentaquark admixture *must* be present in the $N(939)$. A mixing of the $\overline{\mathbf{10}}_F$ into the nucleon is also seen in the soliton model (see, e.g., the results quoted in Ref. [36]). Similar arguments will hold for the Λ , Σ and Ξ channels (and even the Δ channel if we take the degenerate $\mathbf{27}_F \oplus \mathbf{8}_F \oplus \overline{\mathbf{10}}_F$ excited states into account). Again we expect the first positive parity excited states to have large pentaquark components, and the ground states to have at least some level of pentaquark admixture. The existence of the Θ seems to us to leave no way out of these conclusions. This implies that the quark model treatment of the positive parity baryons must be revisited; certainly for the excited states and probably also for the ground states.

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