Covariant light-front approach for $B \to K^* \gamma$, $K_1 \gamma$, $K_2^* \gamma$ decays

Hai-Yang Cheng and Chun-Khiang Chua

Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China

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Exclusive radiative B decays $B \to K^* \gamma$, $K_1(1270)\gamma$, $K_1(1400)\gamma$, and $K_2^*(1430)\gamma$ are studied in the framework of a covariant light-front quark model. The tensor form factor $T_1(q^2)$ at $q^2=0$, which is relevant to the decay $B \to K^* \gamma$, is found to be 0.24, substantially smaller than what was expected from the conventional light-front model or light-cone sum rules. Taking into account the sizable next-to-leading order (NLO) corrections, the calculated branching ratio of $B \to K^* \gamma$ agrees with experiment, while most of the existing models predict too large $B \to K^* \gamma$ compared to the data. The relative strength of $B \to K_1(1270)\gamma$ and $B \to K_1(1400)\gamma$ rates is very sensitive to the sign of the $K_1(1270)$-$K_1(1400)$ mixing angle. Contrary to the other models in which $K_1(1270)\gamma$ and $K_1(1400)\gamma$ rates are predicted to be comparable, it is found that one of them is strongly suppressed owing to a large cancellation between two different form factor terms. The calculated branching ratio of $B \to K_2^* \gamma$ is in a good agreement with experiment and this may imply the smallness of NLO corrections to this radiative decay mode.

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I. INTRODUCTION

Recently we studied the decay constants and form factors of the ground-state s-wave and low-lying p-wave mesons within a covariant light-front approach [1]. This formalism that preserves the Lorentz covariance in the light-front framework has been developed and applied successfully to describe various properties of pseudoscalar and vector mesons [2]. We extended the covariant analysis of the light-front model in Ref. [2] to even-parity, p-wave mesons. With some explicit examples, we have pointed out in Ref. [1] that relativistic effects could manifest in heavy-to-light transitions at maximum recoil where the final-state meson could be highly relativistic and hence there is no reason to expect that the nonrelativistic quark model is still applicable. For example, the $B \to a_1$ form factor $V_0^{Ba_1}(0)$ is found to be 0.13 in the relativistic light-front model [1], while it is as big as 1.01 in the Isgur-Scora-Grinstein-Wise model [3], a nonrelativistic version of the quark model.

In the present work we wish to apply the covariant light-front approach to the exclusive radiative B decays $B \to K^* \gamma$, $K_1 \gamma$, and $K_2^* \gamma$ involving both s- and p-wave mesons in the final states. They receive dominant contributions from the short-distance electromagnetic penguin process $b \to s\gamma$.[1] The radiative decay $B \to K^* \gamma$ was first measured by CLEO [5] a decade ago and more recently by both B factories BaBar and Belle. The measured branching ratios are

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = \begin{cases} (4.55 \pm 0.70 \pm 0.34) \times 10^{-5} & \text{CLEO [6]}, \\ (4.23 \pm 0.40 \pm 0.22) \times 10^{-5} & \text{BaBar [7]}, \\ (4.01 \pm 0.21 \pm 0.17) \times 10^{-5} & \text{Belle [8]}, \end{cases}$$

$$\mathcal{B}(B^+ \to K^{*+}\gamma) = \begin{cases} (3.76 \pm 0.86 \pm 0.28) \times 10^{-5} & \text{CLEO [6]}, \\ (3.83 \pm 0.62 \pm 0.22) \times 10^{-5} & \text{BaBar [7]}, \\ (4.25 \pm 0.31 \pm 0.24) \times 10^{-5} & \text{Belle [8]}. \end{cases} \tag{1.1}$$

Note that the Belle results are still preliminary. The average branching ratios for the two modes are [9]

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.11 \pm 0.23) \times 10^{-5},$$
$$\mathcal{B}(B^+ \to K^{*+}\gamma) = (4.09 \pm 0.32) \times 10^{-5}. \tag{1.2}$$

The decays $B^+ \to K_1(1270)^+\gamma$ and $B^+ \to K_1(1400)^+\gamma$ have been searched by Belle [10] in the $K^+\rho^0\gamma$ and $K^{*0}\pi^+\gamma$ final states, respectively. Although a sizable signal was observed by Belle, only upper limits were provided due to a lack of ability to distinguish these resonances. As for $B \to K_2^*(1430)\gamma$, CLEO [6] reported the first evidence with the combined result

$$\mathcal{B}(B \to K_2^*\gamma) = (1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5}. \tag{1.3}$$

The most recent Belle measurement [10] yielded

$$\mathcal{B}(B^0 \to K_2^{*0}\gamma) = (1.3 \pm 0.5 \pm 0.1) \times 10^{-5}, \tag{1.4}$$

while BaBar [11] obtained the preliminary results

$$\mathcal{B}(B^0 \to K_2^{*0}\gamma) = (1.22 \pm 0.25 \pm 0.11) \times 10^{-5},$$
$$\mathcal{B}(B^+ \to K_2^{*+}\gamma) = (1.44 \pm 0.40 \pm 0.13) \times 10^{-5}. \tag{1.5}$$

Theoretically, the nonfactorizable corrections to the decay $B \to K^* \gamma$ have been studied in the QCD factorization approach [12] to the next-to-leading order (NLO) in QCD and to the leading order in the heavy quark limit [13–15]. Using the light-cone sum rule (LCSR) result of 0.38 ± 0.06 [16] for

[1]The electromagnetic penguin mechanism $b \to s\gamma$ can also manifest in other two-body radiative decays of bottom hadrons such as $B_s \to \phi\gamma$, $\Lambda_b \to \Sigma^0\gamma$, $\Lambda\gamma$, $\Xi_b \to \Xi\gamma$, $\Omega_b \to \Omega\gamma$. These decays have been studied in Ref. [4].

the form factor $T_1(0)$ to be defined below, it is found in Refs. [13–15] that the NLO corrections yield an enhancement of the $B\to K^*\gamma$ decay rate that can be as large as 80%. The enhancement is so large that the predicted branching ratio disagrees with the observed one (1.2). We shall show in the present work that the covariant light-front approach will lead to a form factor $T_1(0)$ much smaller than what expected from LCSR and the conventional light-front model and yield a significantly improved agreement with experiment.

For $B\to K_1\gamma$ decays, we will first use the covariant light-front model to evaluate the tensor form factors in $B\to K_{1A}$ and $B\to K_{1B}$ transitions, where K_{1A} and K_{1B} are the $^3P_1$ and $^1P_1$ states of K_1, respectively, and then relate them to the physical K_1 states $K_1(1270)$ and $K_1(1400)$. Since the $K_1(1270)$-$K_1(1400)$ mixing angle is large, we shall see that one of the radiative decays $B\to K_1(1270)\gamma$ or $B\to K_1(1400)\gamma$ is strongly suppressed, contrary to the other model predictions in which the aforementioned two decay modes are comparable in their rates.

The paper is organized as follows. The formulism for the tensor form factors evaluated in the covariant light-front model is presented in Sec. II. The numerical results for form factors and decay rates together with discussions are shown in Sec. III. Conclusions are given in Sec. IV followed by an appendix on the heavy quark limit behavior of one of the tensor form factors.

II. FORMALISM

The matrix element for the $B\to K^*\gamma$ transition is given by

$$iM=\langle\overline{K}^*(P'',\varepsilon'')\gamma(q,\varepsilon)|-iH_{\rm eff}|\overline{B}(P')\rangle, \qquad (2.1)$$

where

$$H_{\rm eff}=-\frac{G_{\rm F}}{\sqrt{2}}V_{ts}^*V_{tb}c_{11}Q_{11},$$

$$Q_{11}=\frac{e}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)bF^{\mu\nu}, \qquad (2.2)$$

with $P'^{(\prime\prime)}$ being the incoming (outgoing) momentum, $\varepsilon^{(\prime\prime)}$ the polarization vector of γ (K^*), V_{ij} the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and c_{11} the Wilson coefficient. As will be seen below, the inclusion of nonfactorizable corrections to $B\to K^*\gamma$ will amount to replacing c_{11} by the effective parameter a_{11} to be discussed below in Sec. III. In this work we will calculate the $B\to K^*$ and $B\to K_1,K_2^*$ transition tensor form factors in the covariant light-front quark model and obtain the corresponding radiative decay rates.

Tensor form factors for $B\to K^*$, K_1,K_2^* transitions are defined by

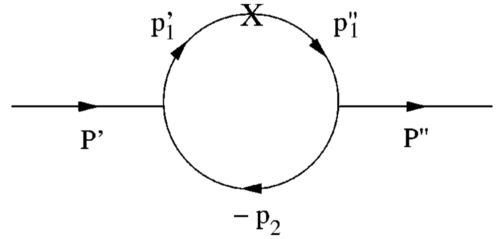

FIG. 1. Feynman diagrams for meson transition amplitudes, where $P'^{(\prime\prime)}$ is the incoming (outgoing) meson momentum, $p_1'^{(\prime\prime)}$ is the quark momentum, p_2 is the antiquark momentum, and X denotes the corresponding $\bar{q}''\sigma_{\mu\nu}(1+\gamma_5)q'$ transition vertex.

$$\begin{aligned}\langle\overline{K}^*(P'',\varepsilon'')|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\overline{B}(P')\rangle &= i\epsilon_{\mu\nu\lambda\rho}\varepsilon''^{\nu*}P^\lambda q^\rho T_1(q^2)+(\varepsilon''^*_\mu P\cdot q-P_\mu\varepsilon''^*\cdot q)T_2(q^2)\\ &\quad+\varepsilon''^*\cdot q\left(q_\mu-P_\mu\frac{q^2}{P\cdot q}\right)T_3(q^2),\\ \langle\overline{K}_{1A,1B}(P'',\varepsilon'')|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\overline{B}(P')\rangle &= i\epsilon_{\mu\nu\lambda\rho}\varepsilon''^{\nu*}P^\lambda q^\rho Y_{A1,B1}(q^2)\\ &\quad+(\varepsilon''^*_\mu P\cdot q-P_\mu\varepsilon''^*\cdot q)Y_{A2,B2}(q^2)\\ &\quad+\varepsilon''^*\cdot q\left(q_\mu-P_\mu\frac{q^2}{P\cdot q}\right)Y_{A3,B3}(q^2),\\ \langle\overline{K}_2^*(P'',\varepsilon'')|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\overline{B}(P')\rangle &= -i\epsilon_{\mu\nu\lambda\rho}\varepsilon''^{\nu\sigma*}P^\sigma P^\lambda q^\rho\frac{U_1(q^2)}{m_B}\\ &\quad-(\varepsilon''^*_{\mu\sigma}P\cdot q-P_\mu\varepsilon''^*_{\sigma\rho}q^\rho)P^\sigma\frac{U_2(q^2)}{m_B}\\ &\quad-\varepsilon''^*_{\sigma\rho}P^\sigma q^\rho\left(q_\mu-P_\mu\frac{q^2}{P\cdot q}\right)\frac{U_3(q^2)}{m_B}, \qquad (2.3)\end{aligned}$$

where $P=P'+P''$, $q=P'-P''$, and the convention $\epsilon^{0123}=1$ is adopted. The physical strange axial-vector $K_1(1270)$ and $K_1(1400)$ are the mixture of K_{1A} and K_{1B} [we follow the Particle Data Group (PDG) [17] to denote the $^3P_1$ and $^1P_1$ states of K_1 by K_{1A} and K_{1B}, respectively] owing to the mass difference of the strange and nonstrange light quarks

$$K_1(1270)=K_{1A}\sin\theta+K_{1B}\cos\theta,$$

$$K_1(1400)=K_{1A}\cos\theta-K_{1B}\sin\theta. \qquad (2.4)$$

The mixing angle θ will be discussed in the next section. For the masses of K_{1A} and K_{1B}, we follow Ref. [18] to determine them from the mass relations $m^2_{K_{1A}}=m^2_{K_1(1270)}+m^2_{K_1(1400)}-m^2_{K_{1B}}$ and $2m^2_{K_{1B}}=m^2_{b_1(1232)}+m^2_{h_1(1380)}$.

To begin with, we consider the transition amplitude given by the one-loop diagram as shown in Fig. 1. We follow the approach of Ref. [2] and use the same notation. The incoming (outgoing) meson has the momentum $P'^{(\prime\prime)}=p_1'^{(\prime\prime)}$

TABLE I. Feynman rules for the vertices ($i\Gamma'_M$) of the incoming mesons-quark-antiquark, where p'_1 and p_2 are the quark and antiquark momenta, respectively. Under the contour integrals to be discussed below, H'_M and W'_M are reduced to h'_M and w'_M, respectively, whose expressions are given by Eq. (2.13). Note that for outgoing mesons, we shall use $i(\gamma_0\Gamma'^{\dagger}_M\gamma_0)$ for the corresponding vertices.

$M(^{2S+1}L_J)$	$i\Gamma'_M$
pseudoscalar ($^1S_0$)	$H'_P\gamma_5$
vector ($^3S_1$)	$iH'_V\left[\gamma_\mu-\frac{1}{W'_V}(p'_1-p_2)_\mu\right]$
axial ($^3P_1$)	$-iH'_{^3A}\left[\gamma_\mu+\frac{1}{W'_{^3A}}(p'_1-p_2)_\mu\right]\gamma_5$
axial ($^1P_1$)	$-iH'_{^1A}\left[\frac{1}{W'_{^1A}}(p'_1-p_2)_\mu\right]\gamma_5$
tensor ($^3P_2$)	$i\frac{1}{2}H'_T\left[\gamma_\mu-\frac{1}{W'_V}(p'_1-p_2)_\mu\right](p'_1-p_2)_\nu$

$+p_2$, where $p_1'^{(\prime\prime)}$ and p_2 are the momenta of the off-shell quark and antiquark, respectively, with masses $m_1'^{(\prime\prime)}$ and m_2. These momenta can be expressed in terms of the internal variables $(x_i,p'_\perp)$,

$$p'^{+}_{1,2}=x_{1,2}P'^{+},\quad p'_{1,2\perp}=x_{1,2}P'_\perp\pm p'_\perp, \tag{2.5}$$

with $x_1+x_2=1$. Note that we use $P'=(P'^-,P'^+,P'_\perp)$, where $P'^{\pm}=P'^0\pm P'^3$, so that $P'^2=P'^+P'^--P'^2_\perp$. In the covariant light-front approach, total four momentum is conserved at each vertex where quarks and antiquarks are off shell. These differ from the conventional light-front approach (see, for example, Refs. [19,20]) where the plus and transverse components of momentum are conserved, and quarks as well as antiquarks are on-shell. It is useful to define some internal quantities

$$M'^2_0=(e'_1+e_2)^2=\frac{p'^2_\perp+m'^2_1}{x_1}+\frac{p'^2_\perp+m^2_2}{x_2},$$
$$\widetilde{M}'_0=\sqrt{M'^2_0-(m'_1-m_2)^2},$$
$$e^{(\prime)}_i=\sqrt{m^{(\prime)2}_i+p'^2_\perp+p'^2_z},\quad p'_z=\frac{x_2M'_0}{2}-\frac{m^2_2+p'^2_\perp}{2x_2M'_0}. \tag{2.6}$$

Here M'^2_0 can be interpreted as the kinetic invariant mass squared of the incoming $q\bar{q}$ system, and e_i the energy of the quark i.

It has been shown in Ref. [21] that one can pass to the light-front approach by integrating out the p^- component of the internal momentum in covariant Feynman momentum loop integrals. We need Feynman rules for the meson-quark-antiquark vertices to calculate the amplitudes depicted in Fig. 1. The Feynman rules for vertices ($i\Gamma'_M$) of ground-state s-wave mesons and low-lying p-wave mesons are summarized in Table I. Note that we use ^3A and ^1A to denote $^3P_1$ and $^1P_1$ states, respectively. It is known that the integration of the minus component of the internal momentum in Fig. 1 will force the antiquark to be on its mass shell [2]. The specific form of the (phenomenological) covariant vertex functions for on-shell quarks can be determined by comparing to the conventional vertex functions [1].

We first consider the tensor form factors for $B\to K^*$ transition. We have

$$\mathcal{B}_{\mu\nu}\varepsilon''^{*\nu}\equiv\langle\overline{K}^*(P'',\varepsilon'')|\overline{s}\sigma_{\mu\lambda}q^\lambda(1+\gamma_5)b|\overline{B}(P')\rangle$$
$$=-i^3\frac{N_c}{(2\pi)^4}\int d^4p'_1\frac{H'_P(iH''_V)}{N'_1N''_1N_2}S_{R\mu\nu}\varepsilon''^{*\nu}, \tag{2.7}$$

where

$$S_{R\mu\nu}=\mathrm{Tr}\left[\left(\gamma_\nu-\frac{1}{W''_V}(p''_1-p_2)_\nu\right)(\not{p}''_1+m''_1)\sigma_{\mu\lambda}q^\lambda(1+\gamma_5)\right.$$
$$\left.\times(\not{p}'_1+m'_1)\gamma_5(-\not{p}_2+m_2)\right], \tag{2.8}$$

$N''_1=p''^2_1-m''^2_1+i\epsilon$, and $N_2=p^2_2-m^2_2+i\epsilon$. By using the identity $2\sigma_{\mu\lambda}\gamma_5=i\epsilon_{\mu\lambda\rho\sigma}\sigma^{\rho\sigma}$, the above trace $S_{R\mu\nu}$ can be further decomposed into

$$S_{R\mu\nu}=q^\lambda S_{\nu\mu\lambda}+\frac{i}{2}q^\lambda\epsilon_{\mu\lambda\rho\sigma}S^{\rho\sigma}_\nu, \tag{2.9}$$

where

$$S_{\nu\mu\lambda}=\mathrm{Tr}\left[\left(\gamma_\nu-\frac{1}{W''_V}(p''_1-p_2)_\nu\right)(\not{p}''_1+m''_1)\sigma_{\mu\lambda}q^\lambda\right.$$
$$\left.\times(\not{p}'_1+m'_1)\gamma_5(-\not{p}_2+m_2)\right]$$
$$=-\epsilon_{\mu\nu\lambda\alpha}2[2(m'_1m_2+m''_1m_2-m'_1m''_1)$$
$$\times p'^{\alpha}_1+m'_1m''_1P^\alpha+(m'_1m''_1-2m'_1m_2)q^\alpha]$$
$$-\frac{2}{W''_V}(4p'_{1\nu}-3q_\nu-P_\nu)$$
$$\times\epsilon_{\mu\lambda\alpha\beta}[(m'_1+m''_1)p'^{\alpha}_1P^\beta$$
$$+(m''_1-m'_1+2m_2)p'^{\alpha}_1q^\beta+m'_1P^\alpha q^\beta]. \tag{2.10}$$

As in Refs. [1,2], we work in the $q^+=0$ frame. For the integral in Eq. (2.7) we perform the p_1^- integration [2], which picks up the residue at $p_2=\hat{p}_2$ and leads to

$$\begin{aligned}
&N_1'^{(\prime\prime)}\to\hat{N}_1'^{(\prime\prime)}=x_1(M'^{(\prime\prime)2}-M_0'^{(\prime\prime)2}),\\
&H_M'^{(\prime\prime)}\to h_M'^{(\prime\prime)},\\
&W''_M\to w''_M,\\
&\int\frac{d^4p_1'}{N_1'N_1''N_2}H_P'H_V''S\to -i\pi\int\frac{dx_2d^2p_\perp'}{x_2\hat{N}_1'\hat{N}_1''}h_P'h_V''\hat{S},
\end{aligned}\tag{2.11}$$

where

$$M_0''^2=\frac{p_\perp''^2+m_1''^2}{x_1}+\frac{p_\perp''^2+m_2^2}{x_2},\tag{2.12}$$

with $p_\perp''=p_\perp'-x_2q_\perp$. In this work the explicit forms of h_M' and w_M' are given by [1]

$$\begin{aligned}
&h_P'=h_V'=(M'^2-M_0'^2)\sqrt{\frac{x_1x_2}{N_c}}\frac{1}{\sqrt{2}\tilde{M}_0'}\varphi',\\
&h_{3A}'=(M'^2-M_0'^2)\sqrt{\frac{x_1x_2}{N_c}}\frac{1}{\sqrt{2}\tilde{M}_0'}\frac{\tilde{M}_0'^2}{2\sqrt{2}M_0'}\varphi_p',\\
&h_{1A}'=h_T'=(M'^2-M_0'^2)\sqrt{\frac{x_1x_2}{N_c}}\frac{1}{\sqrt{2}\tilde{M}_0'}\varphi_p',\\
&w_V'=M_0'+m_1'+m_2,\quad w_{3A}'=\frac{\tilde{M}_0'^2}{m_1'-m_2},\quad w_{1A}'=2,
\end{aligned}\tag{2.13}$$

where φ' and φ_p' are the light-front momentum distribution amplitudes for *s*- and *p*-wave mesons, respectively. There are several popular phenomenological light-front wave functions that have been employed to describe various hadronic structures in the literature. In the present work, we shall use the Gaussian-type wave function [22]

$$\begin{aligned}
&\varphi'=\varphi'(x_2,p_\perp')=4\left(\frac{\pi}{\beta'^2}\right)^{3/4}\sqrt{\frac{dp_z'}{dx_2}}\ \exp\left(-\frac{p_z'^2+p_\perp'^2}{2\beta'^2}\right),\\
&\varphi_p'=\varphi_p'(x_2,p_\perp')=\sqrt{\frac{2}{\beta'^2}}\ \varphi',\quad \frac{dp_z'}{dx_2}=\frac{e_1'e_2}{x_1x_2M_0'}.
\end{aligned}\tag{2.14}$$

The parameter β' is expected to be of order $\Lambda_{\rm QCD}$.

In general, $\hat{p}_1'$ can be expressed in terms of three external vectors P', q, and $\tilde{\omega}$ [$\tilde{\omega}$ being a lightlike vector with the expression $\tilde{\omega}^\mu=(\tilde{\omega}^-,\tilde{\omega}^+,\tilde{\omega}_\perp)=(2,0,0_\perp)$]. In practice, for $\hat{p}_1'$ under integration we use the following rules [2]:

$$\begin{aligned}
\hat{p}_{1\mu}'\doteq{}& P_\mu A_1^{(1)}+q_\mu A_2^{(1)},\\
\hat{p}_{1\mu}'\hat{p}_{1\nu}'\doteq{}& g_{\mu\nu}A_1^{(2)}+P_\mu P_\nu A_2^{(2)}+(P_\mu q_\nu+q_\mu P_\nu)A_3^{(2)}\\
&+q_\mu q_\nu A_4^{(2)},\\
\hat{p}_{1\mu}'\hat{p}_{1\nu}'\hat{p}_{1\alpha}'\doteq{}&(g_{\mu\nu}P_\alpha+g_{\mu\alpha}P_\nu+g_{\nu\alpha}P_\mu)A_1^{(3)}\\
&+(g_{\mu\nu}q_\alpha+g_{\mu\alpha}q_\nu+g_{\nu\alpha}q_\mu)A_2^{(3)}+P_\mu P_\nu P_\alpha A_3^{(3)}\\
&+(P_\mu P_\nu q_\alpha+P_\mu q_\nu P_\alpha+q_\mu P_\nu P_\alpha)A_4^{(3)}\\
&+(q_\mu q_\nu P_\alpha+q_\mu P_\nu q_\alpha+P_\mu q_\nu q_\alpha)A_5^{(3)}\\
&+q_\mu q_\nu q_\alpha A_6^{(3)},
\end{aligned}\tag{2.15}$$

where the symbol $\doteq$ reminds us that the above equations are true only after integration. In the above equation, $A_j^{(i)}$ are functions of $x_{1,2}$, $p_\perp'^2$, $p_\perp'\cdot q_\perp$, and q^2, and their explicit expressions are given by [2]

$$\begin{aligned}
&A_1^{(1)}=\frac{x_1}{2},\quad A_2^{(1)}=A_1^{(1)}-\frac{p_\perp'\cdot q_\perp}{q^2},\\
&A_1^{(2)}=-p_\perp'^2-\frac{(p_\perp'\cdot q_\perp)^2}{q^2},\quad A_2^{(2)}=(A_1^{(1)})^2,\\
&A_3^{(2)}=A_1^{(1)}A_2^{(1)},\quad A_4^{(2)}=(A_2^{(1)})^2-\frac{1}{q^2}A_1^{(2)},\\
&A_1^{(3)}=A_1^{(1)}A_1^{(2)},\quad A_2^{(3)}=A_2^{(1)}A_1^{(2)},\\
&A_3^{(3)}=A_1^{(1)}A_2^{(2)},\quad A_4^{(3)}=A_2^{(1)}A_2^{(2)},\quad A_5^{(3)}=A_1^{(1)}A_4^{(2)},\\
&A_6^{(3)}=A_2^{(1)}A_4^{(2)}-\frac{2}{q^2}A_2^{(1)}A_1^{(2)}.
\end{aligned}\tag{2.16}$$

We do not show the spurious contributions in Eq. (2.15) since they are numerically vanishing [1,2,23]. For the integration in Eq. (2.7) we need only the first two rules in Eq. (2.15), while the third one will be used in the calculation of the $B\to K_2^*$ transition form factors. In general, there are additional rules involving N_2 in Ref. [2] and these may be identified as zero mode contributions to form factors (for different approaches of zero mode contributions, see Ref. [24]). As shown in Eq. (2.10), there is no N_2 term in the trace and hence no zero mode contribution to the $B\to K^*$ form factors. As we shall see, the above statement also holds for $B\to K_1$ and $B\to K_2^*$ form factors.

By using Eqs. (2.7)–(2.15), one arrives at

$$T_1(q^2)=\frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{2h'_P h''_V}{x_2\hat{N}'_1\hat{N}''_1}\left\{m'_1m''_1+x_1(m'_1m_2+m''_1m_2-m'_1m''_1)-\frac{2}{w''_V}[(m'_1+m''_1)A_1^{(2)}]\right\},$$

$$T_2(q^2)=T_1(q^2)+\frac{q^2}{P\cdot q}\frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{2h'_P h''_V}{x_2\hat{N}'_1\hat{N}''_1}$$
$$\times\left\{m'_1m''_1-2m'_1m_2+2A_2^{(1)}(m'_1m_2+m''_1m_2-m'_1m''_1)-\frac{2}{w''_V}[(m''_1-m'_1+2m_2)A_1^{(2)}]\right\},$$

$$T_3(q^2)=\frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{2h'_P h''_V}{x_2\hat{N}'_1\hat{N}''_1}\left\{2m'_1m_2-m'_1m''_1-2A_2^{(1)}(m'_1m_2+m''_1m_2-m'_1m''_1)\right.$$
$$+\frac{2}{w''_V}\{(m''_1-m'_1+2m_2)[A_1^{(2)}+P\cdot q(A_2^{(2)}+A_3^{(2)}-A_1^{(1)})+P\cdot q(m'_1+m''_1)(A_2^{(1)}-A_3^{(2)}-A_4^{(2)})$$
$$\left.+P\cdot q m'_1(A_1^{(1)}+A_2^{(1)}-1)]\}\right\}. \tag{2.17}$$

In order to compare with the conventional light-front model calculation for $T_1(0)$, which is relevant for $B\to K^*\gamma$ decay, we write

$$T_1(0)=\frac{1}{32\pi^2}\int dx d^2p'_\perp \frac{\varphi''(x,p'_\perp)\varphi'(x,p'_\perp)}{\sqrt{\mathcal{A}'^2+p'^2_\perp}\sqrt{\mathcal{A}''^2+p'^2_\perp}}\left\{x^2m_bm_s+x(1-x)(m_bm_q+m_sm_q)+\frac{p'^2_\perp}{\omega''_V}x(m_b+m_s)\right\}, \tag{2.18}$$

where $\mathcal{A}'=m_bx+m_q(1-x)$ and $\mathcal{A}''=m_sx+m_q(1-x)$, $x=x_2$, and m_q is the mass of the spectator quark in the B meson. This is to be compared with the result

$$T_1(0)=\frac{1}{32\pi^2}\int dx d^2p'_\perp \frac{\varphi''(x,p'_\perp)\varphi'(x,p'_\perp)}{\sqrt{\mathcal{A}'^2+p'^2_\perp}\sqrt{\mathcal{A}''^2+p'^2_\perp}}$$
$$\times\left\{x^2m_bm_s+x(1-x)(m_bm_q+m_sm_q)+(1-x)[(1-x)m_q^2+p'^2_\perp]+\frac{p'^2_\perp}{\omega''_V}x(m_b+m_s)\right\} \tag{2.19}$$

obtained in Ref. [25]. It is clear that the terms proportional to $(1-x)m_q^2+p'^2_\perp$ do not exist in our expression for $T_1(0)$. This will affect the numerical result significantly for the $B\to K^*\gamma$ rate as we shall discuss in Sec. III. It is shown in Appendix that our result (2.18) for $T_1(0)$ has the correct heavy quark limit behavior and hence it is more trustworthy than Eq. (2.19).

The calculation for $B\to K_{1A,1B}$ transition form factors can be done in a similar manner. In analogue to Eq. (2.7), we have

$$\mathcal{B}^{^3A}_{\mu\nu}\varepsilon''^{*\nu}=-i^3\frac{N_c}{(2\pi)^4}\int d^4p'_1\frac{H'_P(-iH''_{^3A})}{N'_1N''_1N_2}S^{^3A}_{R\mu\nu}\varepsilon''^{*\nu},$$

$$\mathcal{B}^{^1A}_{\mu\nu}\varepsilon''^{*\nu}=-i^3\frac{N_c}{(2\pi)^4}\int d^4p'_1\frac{H'_P(-iH''_{^1A})}{N'_1N''_1N_2}S^{^1A}_{R\mu\nu}\varepsilon''^{*\nu}, \tag{2.20}$$

where

$$S^{^3A}_{R\mu\nu}=\mathrm{Tr}\left[\left(\gamma_\nu-\frac{1}{W''_{^3A}}(p''_1-p_2)_\nu\right)\gamma_5(\not p''_1+m''_1)\sigma_{\mu\lambda}q^\lambda(1+\gamma_5)(\not p'_1+m'_1)\gamma_5(-\not p_2+m_2)\right],$$

$$S^{^1A}_{R\mu\nu}=\mathrm{Tr}\left[\left(-\frac{1}{W''_{^1A}}(p''_1-p_2)_\nu\right)\gamma_5(\not p''_1+m''_1)\sigma_{\mu\lambda}q^\lambda(1+\gamma_5)(\not p'_1+m'_1)\gamma_5(-\not p_2+m_2)\right]. \tag{2.21}$$

It can be easily shown that $S^{^3A,^1A}_{R\mu\nu}=-S_{R\mu\nu}$ with m''_1 and W''_V replaced by $-m''_1$ and $W''_{^3A,^1A}$, respectively, while only the $1/W''_{^1A}$ term is kept for the $S^{^1A}_R$ case. Consequently, we have, for $i=1,2,3$,

$$Y_{Ai,Bi}(q^2)=T_i(q^2)\ \text{with}\ (m''_1\to-m''_1,h''_V\to h''_{^3A,^1A},\quad w''_V\to w''_{^3A,^1A}), \tag{2.22}$$

where only the $1/W''$ terms in Y_{Bi} form factors are kept. It should be cautious that the replacement of $m''_1\to-m''_1$ should not be applied to m''_1 in w'' and h''. The above simple relation between $B\to K_1$ and $B\to K^*$ transition tensor form factors is similar to that for vector and axial form factors in $P\to A$ and $P\to V$ transitions [1].

Finally we turn to the $B\to K_2^*$ transition given by

$$\mathcal{B}^T_{\mu\nu\lambda}\varepsilon''^{*\nu\lambda}\equiv\langle K_2^*(P'',\varepsilon'')|\bar s\sigma_{\mu\nu}(1+\gamma_5)q^\nu b|B(P')\rangle =-i^3\frac{N_c}{(2\pi)^4}\int d^4p'_1\frac{H'_P(iH''_T)}{N'_1N''_1N_2}S^{PT}_{\mu\nu\lambda}\varepsilon''^{*\nu\lambda}, \tag{2.23}$$

where

$$S^T_{R\mu\nu\lambda}=S_{R\mu\nu}(-q+p'_1)_\lambda. \tag{2.24}$$

The contribution from the $S_{R\mu\nu}(-q)_\lambda$ part is trivial, since q_λ can be taken out from the integration, which is already done in the $B\to K^*$ case. Contributions from the $\hat S_{R\mu\nu}\hat p'_{1\lambda}$ part can be worked out by using Eq. (2.15). Putting all these together leads to

$$\begin{aligned}U_1(q^2)&=\frac{N_c}{16\pi^3}\int dx_2d^2p'_\perp\frac{2M'h'_Ph''_V}{x_2\hat N'_1\hat N''_1}\Bigg\{m'_1m''_1(1-A_1^{(1)}-A_2^{(1)})\\&\quad+2(m'_1m_2+m''_1m_2-m'_1m''_1)(A_1^{(1)}-A_2^{(2)}-A_3^{(2)})\\&\quad-\frac{4}{w''_V}[(m'_1+m''_1)(A_1^{(2)}-A_1^{(3)}-A_2^{(3)})]\Bigg\},\end{aligned}$$

$$\begin{aligned}U_2(q^2)&=U_1(q^2)+\frac{q^2}{P\cdot q}\frac{N_c}{16\pi^3}\int dx_2d^2p'_\perp\frac{2M'h'_Ph''_V}{x_2\hat N'_1\hat N''_1}\\&\quad\times\Bigg\{(m'_1m''_1-2m'_1m_2)(1-A_1^{(1)}-A_2^{(1)})\\&\quad+2(m'_1m_2+m''_1m_2-m'_1m''_1)(A_2^{(1)}-A_3^{(2)}-A_4^{(2)})\\&\quad-\frac{4}{w''_V}[(m''_1-m'_1+2m_2)(A_1^{(2)}-A_1^{(3)}-A_2^{(3)})]\Bigg\},\end{aligned}$$

$$\begin{aligned}U_3(q^2)=&\frac{N_c}{16\pi^3}\int dx_2d^2p'_\perp\frac{2M'h'_Ph''_V}{x_2\hat N'_1\hat N''_1}\Bigg\{(m'_1m''_1-2m'_1m_2)(-1+A_1^{(1)}+A_2^{(1)})\\&+2(m'_1m_2+m''_1m_2-m'_1m''_1)(-A_2^{(1)}+A_3^{(2)}+A_4^{(2)})-\frac{2}{w''_V}[2(m''_1-m'_1+2m_2)(-A_1^{(2)}+A_1^{(3)}+A_2^{(3)})\\&+P\cdot q(m''_1-m'_1+2m_2)(A_1^{(1)}-2A_2^{(2)}-2A_3^{(2)}+A_3^{(3)}+2A_4^{(3)}+A_5^{(3)})+P\cdot q(m'_1+m''_1)(-A_2^{(1)}+2A_3^{(2)}\\&+2A_4^{(2)}-A_4^{(3)}-2A_5^{(3)}-A_6^{(3)})+P\cdot qm'_1(1-2A_1^{(1)}-2A_2^{(1)}+A_2^{(2)}+2A_3^{(2)}+A_4^{(2)})]\Bigg\}.\end{aligned} \tag{2.25}$$

We are now ready to calculate the radiative decay rates. Before proceeding, two remarks are in order: (i) At $q^2=0$ the form factors obey the simple relations $T_2(0)=T_1(0)$, $Y_{A2,B2}(0)=Y_{A1,B1}(0)$, and $U_2(0)=U_1(0)$. (ii) Form factors $T_3(0),Y_{3A,3B}(0),U_3(0)$ do not contribute to the corresponding radiative decay rates. It is straightforward to obtain

TABLE II. The input parameters m_q and β (in units of GeV) in the Gaussian-type wave function (2.14).

m_u	m_s	m_b	β_B	β_{K^*}	β_{K_1,K_2^*}
0.23	0.45	4.4	0.5233	0.2846	0.2979

$$
\begin{aligned}
\mathcal{B}(B\to K^*\gamma)&=\tau_B\frac{G_F^2\alpha m_B^3m_b^2}{32\pi^4}\left(1-\frac{m_{K^*}^2}{m_B^2}\right)^3|V_{tb}V_{ts}^*a_{11}T_1(0)|^2,\\
\mathcal{B}(B\to K_{1A,1B}\gamma)&=\tau_B\frac{G_F^2\alpha m_B^3m_b^2}{32\pi^4}\left(1-\frac{m_{K_{1A,1B}}^2}{m_B^2}\right)^3\\
&\quad\times|V_{tb}V_{ts}^*a_{11}Y_{A1,B1}(0)|^2,\\
\mathcal{B}(B\to K_2^*\gamma)&=\tau_B\frac{G_F^2\alpha m_B^5m_b^2}{256\pi^4m_{K_2^*}^2}\left(1-\frac{m_{K_2^*}^2}{m_B^2}\right)^5|V_{tb}V_{ts}^*a_{11}U_1(0)|^2,
\end{aligned}
\tag{2.26}
$$

where τ_B is the B lifetime. It has been realized recently that non-factorizable strong interaction corrections (i.e., those corrections not related to form factors, such as hard vertex and hard spectator contributions) to $B\to K^*\gamma$ are calculable in the heavy quark limit and amount to replacing the Wilson coefficient c_{11} by the effective parameter a_{11}. Such corrections have been calculated in the QCD factorization framework and in the large energy effective theory up to NLO in α_s and to the leading power in $\Lambda_{\rm QCD}/m_B$ and found to be quite sizable [13–15]. We will return back to this point later.

In the next section, we will give numerical results for form factors, $T_i(q^2),Y_{Ai,Bi}(q^2),U_i(q^2)$, as well as $B\to K^*\gamma$, $K_1\gamma$, $K_2^*\gamma$ decay rates.

III. NUMERICAL RESULTS AND DISCUSSION

To perform numerical calculations we need to first specific some input parameters in the covariant light-front model. The input parameters m_q and β in the Gaussian-type wave function (2.14) are shown in Table II. The constituent quark masses are close to those used in the literature [1,2,20,26,27]. The input parameters β's are fixed by the decay constants whose analytic expressions in the covariant light-front model are given in Ref. [1]. We use $f_B=180$ MeV and $f_{K^*}=230$ MeV to fix β_B and β_{K^*}, respectively. For p-wave strange mesons, we take for simplicity $\beta_{K_1}=\beta_{K_{1A}}=\beta_{K_{1B}}=\beta_{K_2^*}$ [28] and use $f_{K_1(1270)}=175$ MeV extracted from the measured $\tau\to K_1(1270)\nu_\tau$ decays [29] to fix β_{K_1} to be 0.2979 GeV.

As in Refs. [1,2], because of the condition $q^+=0$ we have imposed during the course of calculation, form factors are known only for spacelike momentum transfer $q^2=-q_\perp^2\leqslant 0$, whereas only the timelike form factors are relevant for the physical decay processes. It has been proposed in Ref. [26] to recast the form factors as explicit functions of q^2 in the spacelike region and then analytically continue them to the timelike region. It has been shown recently that, within a specific model, form factors obtained directly from the timelike region (with $q^+>0$) are identical to the ones obtained by the analytic continuation from the spacelike region [24].

In principle, form factors at $q^2>0$ can be evaluated directly in the frame where the momentum transfer is purely longitudinal, i.e., $q_\perp=0$, so that $q^2=q^+q^-$ covers the entire range of momentum transfer [20]. The price one has to pay is that, besides the conventional valence-quark contribution, one must also consider the nonvalence configuration (or the so-called Z graph) arising from quark-pair creation from the vacuum. However, a reliable way of estimating the Z-graph contribution is still lacking unless one works in a specific model, for example, the one advocated in Ref. [24]. Fortunately, this additional non-valence contribution vanishes in the frame where the momentum transfer is purely transverse, i.e., $q^+=0$.

To proceed we find that except for the form factors Y_{B3} and U_2, the momentum dependence of the form factors $T_i,Y_{Ai,Bi},U_i$ in the spacelike region can be well parametrized and reproduced in the three-parameter form

$$F(q^2)=\frac{F(0)}{1-a(q^2/m_B^2)+b(q^2/m_B^2)^2}. \tag{3.1}$$

The parameters a, b, and $F(0)$ are first determined in the spacelike region. We then employ this parametrization to determine the physical form factors at $q^2\geqslant 0$. In practice, the parameters a, b, and $F(0)$ are obtained by performing a 3-parameter fit to the form factors in the range $-20\ \mathrm{GeV}^2\leqslant q^2\leqslant 0$. The obtained a and b parameters are in most cases are not far from unity as expected. However, the parameter b for Y_{B3} and U_2 is rather sensitive to the chosen range for q^2 and can be as large as 6.6 and 8.8, respectively. To overcome this difficulty, we will fit $Y_{B3}(q^2)$ and $U_2(q^2)$ to the form

$$F(q^2)=\frac{F(0)}{(1-q^2/m_B^2)[1-a(q^2/m_B^2)+b(q^2/m_B^2)^2]} \tag{3.2}$$

and achieve a substantial improvement. Note that for the case of $U_2(q^2)$, it is fitted to a smaller range of $-12\ \mathrm{GeV}^2\leqslant q^2\leqslant 0$.

The $B\to K^*$, K_1, K_2^* transition tensor form factors and their q^2 dependence are displayed in Table III and depicted in Fig. 2. The result of $T_1(0)$ is then compared with other model calculations in Table IV. It should be stressed that our $T_1(0)$ is smaller than that obtained from the quark model (QM) [30], the conventional light-front quark model

TABLE III. Tensor form factors of $B\to K^*$, K_1, K_2^* transitions obtained in the covariant light-front model are fitted to the 3-parameter form (3.1) except for the form factors Y_{B3} and U_2 denoted by an asterisk for which the fit formula (3.2) is used. All the form factors are dimensionless.

F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
T_1	0.24	1.00	1.73	0.90	Y_{A1}	0.11	0.15	0.68	0.35
T_2	0.24	0.59	0.92	0.07	Y_{A2}	0.11	0.06	-0.91	0.79
T_3	0.17	0.79	1.72	0.84	Y_{A3}	0.19	0.34	1.02	0.35
Y_{B1}	0.13	0.33	1.94	1.53	U_1	0.19	0.45	2.22	2.13
Y_{B_2}	0.13	0.21	0.83	0.25	U_2	0.19*	0.32*	1.77*	4.32*
Y_{B3}	-0.07*	-0.24*	1.93*	2.33*	U_3	0.16	0.37	2.19	1.80

(LFQM) [25,31],[2] light-cone sum rules (LCSRs) [35–38], and the perturbative QCD approach (PQCD) [39] but is close to two of the lattice calculations [32,34].

The effective parameter $a_{11}(K^*\gamma)$ has been calculated in the framework of QCD factorization to be $|a_{11}(K^*\gamma)|^2 = 0.165^{+0.018}_{-0.017}$ [13] at $\mu = \hat{m}_b$ and $a_{11}(K^*\gamma) = -0.4072 - 0.0256i$ [14] at $\mu = m_b$ (see also Table 3 of Ref. [15]). These effective parameters are larger than the Wilson coefficient c_{11} of order -0.32 at $\mu = m_b$. For $B\to K_1\gamma$ and $K_2^*\gamma$ decays, we shall employ $a_{11} = c_{11}$ as NLO QCD corrections there have not been calculated. In Eq. (2.26), we take $m_b(\hat{m}_b) = 4.4$ GeV for $B\to K^*\gamma$ and $m_b(m_b) = 4.2$ GeV for $B\to K_1\gamma$ and $K_2^*\gamma$ decays.

In Table V, we summarize the calculated branching ratios for the radiative decays $B\to K^*\gamma$, $K_1(1270)\gamma$, $K_1(1400)\gamma$, $K_2^*(1430)\gamma$ in the covariant light-front model. For comparison we also quote experimental results and some other theoretical calculations. For our results and results in LFQM [31], lattice [34], and LCSR [38], we use $|a_{11}(K^*\gamma)|^2 = 0.165 \pm 0.018$ [13]. The theoretical errors in $\mathcal{B}(B\to K^*\gamma)$ arise from $|a_{11}(K^*\gamma)|^2$ and $T_1(0)$. Note that we have assigned a 10% estimated uncertainty for our $T_1(0)$ and that from LFQM [31]. For $B\to K^*\gamma$ rates from heavy quark effective theory (HQET) [42], we have scaled up their results by a factor of $|a_{11}(K^*\gamma)/c_{11}|^2 = 1.78$. Calculations in LCSR [38] and HQET [42] are often expressed in terms of $R \equiv \mathcal{B}(B\to K^{**}\gamma)/\mathcal{B}(b\to s\gamma)$ with K^{**} denoting K_1 or K_2^*. Therefore, the branching ratio of $B\to K^{**}\gamma$ is obtained by multiplying R with $\mathcal{B}(b\to s\gamma) = 3.34\times 10^{-4}$ [9]. For B

[2]This is due to the presence of additional terms proportional to $(1-x)m_q^2 + p'^2_\perp$ in the expression of the form factor $T_1(0)$ in the conventional light-front model [see in Eq. (2.19)]. However, it is shown in the Appendix that this tensor form factor does not have the correct heavy quark limit behavior. In the heavy quark limit, heavy quark spin symmetry allows one to relate the tensor form factors $T_i(q^2)$ to the vector and axial-vector $B\to V$ transition form factors (see Appendix). As shown in Ref. [1], the latter in the covariant light-front model are numerically smaller than the other model results. Therefore, the fact that $T_1(0)$ is smaller in the covariant LF model (see Table III) is consistent with previous form factor calculations in Ref. [1].

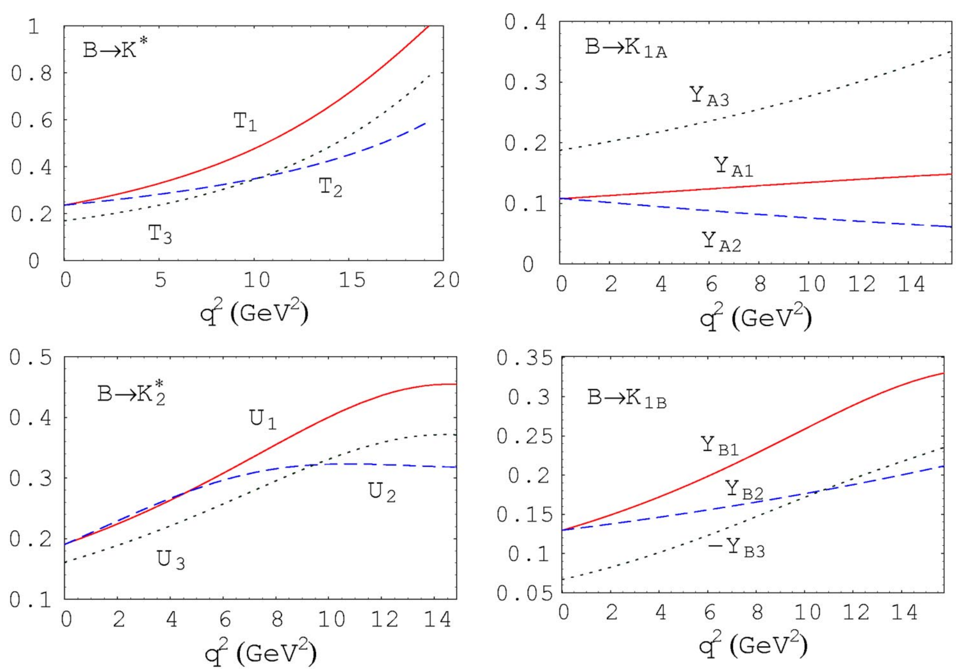

FIG. 2. Tensor form factors $T_i(q^2)$, $Y_{Ai,Bi}(q^2)$, and $U_i(q^2)$ for $B\to K^*$, $B\to K_1$, and $B\to K_2^*$ transitions, respectively.

TABLE IV. Tensor form factor T_1 at $q^2=0$ in this work and in various other models.

Ref.	$T_1(0)$	Ref.	$T_1(0)$
This work	0.24	LCSR [35]	0.32 ± 0.05
QM [30]	0.37 ± 0.09, 0.39	LCSR [36]	0.31 ± 0.04
LFQM [25,31]	0.32 [a]	LCSR [37]	0.38 ± 0.06
Lattice [32]	$0.20\pm0.02\pm0.06$	LCSR [38]	0.32 ± 0.06
Lattice [33]	$0.32^{+0.04}_{-0.02}$	PQCD [39]	0.315 [b] (0.294) [c]
Lattice [34]	$0.25\pm0.05\pm0.02$		

[a]For $\beta_{K^*}=0.32$ GeV.
[b]For $\beta_B=0.40$ GeV.
[c]For $\beta_B=0.42$ GeV.

$\to K_2^*\gamma$, the error in our predicted rate shown in Table V comes from a 10% estimated uncertainty in $U_1(0)$.

As stressed in Refs. [13–15], the NLO correction yields an enhancement of the $B\to K^*\gamma$ rate that can be as large as 80%. Consequently, the prediction in most of the existing models becomes too large as the measured branching ratio is already saturated even before the NLO correction is taken into account. Our prediction of $\mathcal{B}(B\to K^*\gamma)=(3.27\pm0.74)\times10^{-5}$ due to short-distance $b\to s\gamma$ contributions agrees with experiment (see Table V). It is generally believed that long-distance contributions to $B\to K^*\gamma$ is small and not more than 5% (see, e.g., Refs. [43,44], and references therein).

To compute $B\to K_1\gamma$ rates we need to know the $K_1(1270)$-$K_1(1400)$ mixing angle as defined in Eq. (2.4). From the experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki found two possible solutions with a twofold ambiguity, $|\theta|\approx33°$ and $57°$ [18]. A similar constraint $35°\lesssim|\theta|\lesssim55°$ is obtained in Ref. [45] based solely on two parameters: the mass difference of the a_1 and b_1 mesons and the ratio of the constituent quark masses. An analysis of $\tau\to K_1(1270)\nu_\tau$ and $K_1(1400)\nu_\tau$ decays also yields the mixing angle to be $\approx37°$ or $58°$ with a twofold ambiguity [29]. It has been shown in Ref. [29] that the study of hadronic decays $D\to K_1(1270)\pi$, $K_1(1400)\pi$ decays favors the solution $\theta\approx-58°$. However, this is subject to many uncertainties such as the unknown $D\to K_{1A,1B}$ transition form factors and the decay constants of $K_1(1270)$ and $K_1(1400)$.

The physical $B\to K_1(1270)$ and $B\to K_1(1400)$ tensor form factors have the expressions

$$Y_i^{B\to K_1(1270)}(q^2)=Y_{Ai}(q^2)\sin\theta+Y_{Bi}(q^2)\cos\theta,$$

$$Y_i^{B\to K_1(1400)}(q^2)=Y_{Ai}(q^2)\cos\theta-Y_{Bi}(q^2)\sin\theta. \tag{3.3}$$

Since the form factors $Y_{A1}(0)$ and $Y_{B1}(0)$ are similar (see Table III) and since the $K_1(1270)$-$K_1(1400)$ mixing angle is large, it is obvious from Eqs. (3.3) and (2.26) that one of the $B\to K_1\gamma$ decays is strongly suppressed owing to a large cancellation between the $Y_{A1}(0)$ and $Y_{B1}(0)$ terms. In Table V, branching ratios of $B\to K_1\gamma$ are calculated using two different sets of the $K_1(1270)$-$K_1(1400)$ mixing angles

TABLE V. Branching ratios for the radiative decays $B\to K^*\gamma$, $K_1(1270)\gamma$, $K_1(1400)\gamma$, $K_2^*(1430)\gamma$ (in units of 10^{-5}) in the covariant light-front model and in other models. Experimental data are summarized in Sec. I and only the averages for $B\to K^*\gamma$ and $B\to K_2^*\gamma$ are quoted in the table. Experimental limits on $B\to K_1\gamma$ are taken from Ref. [10].

	$B\to K^*\gamma$	$B\to K_1(1270)\gamma$	$B\to K_1(1400)\gamma$	$B\to K_2^*(1430)\gamma$
Expt	4.17 ± 0.19	<9.9	<5.0	1.33 ± 0.20
This work	3.27 ± 0.74[a]	0.02 ± 0.02[b]	0.80 ± 0.12 [b]	1.48 ± 0.30
		(0.04 ± 0.03)[b]	(0.77 ± 0.11) [b]	
		0.77 ± 0.11 [c]	0.08 ± 0.04 [c]	
		(0.84 ± 0.12) [c]	(0.003 ± 0.006) [c]	
Lattice [34]	3.54 ± 1.57[a]			
RQM [40]	4.5 ± 1.5	0.45 ± 0.15	0.78 ± 0.18	1.7 ± 0.6
LFQM [31]	5.81 ± 1.32[a]			
LCSR [38]	5.81 ± 2.27[a]	0.67 ± 0.27[d]	0.30 ± 0.13[d]	1.67 ± 0.67[d]
AP [15]	6.8 ± 2.6			
BB [14,41]	$7.4^{+2.6}_{-2.4}$ [e]			
BFS [13]	$7.9^{+3.5}_{-3.0}$			
HQET [42]	9.99 ± 3.81[f]	1.44 ± 0.53[d]	0.70 ± 0.30[d]	2.07 ± 0.97[d]

[a]Use of $|a_{11}(K^*\gamma)|^2=0.165\pm0.018$ [13] and Eq. (2.26) has been made.
[b]For the $K_1(1270)$-$K_1(1400)$ mixing angle $\theta=-58°(-37°)$.
[c]For the $K_1(1270)$-$K_1(1400)$ mixing angle $\theta=+58°(+37°)$.
[d]Use has been made of $\mathcal{B}(b\to s\gamma)=3.34\times10^{-4}$ [9].
[e]The central value and errors are taken from the complete NLO result for the neutral mode [41].
[f]The original results are scaled up by a factor of $|a_{11}(K^*\gamma)/c_{11}|^2=1.78$.

$\theta=\pm 58°,\pm 37°$.[3] Errors in the rates displayed in Table V stem from 10% estimated uncertainties in $Y_{A1,B1}(0)$. Therefore, the ratio of $B\to K_1(1270)\gamma$ and $K_1(1400)\gamma$ rates is very sensitive to the mixing angle. For example, for $\theta=\pm 58°$ we have

$$\frac{\mathcal{B}(B\to K_1(1270)\gamma)}{\mathcal{B}(B\to K_1(1400)\gamma)}=\begin{cases}10.1\pm 6.2 & \text{for} \quad \theta=+58°,\\ 0.02\pm 0.02 & \text{for} \quad \theta=-58°.\end{cases} \tag{3.4}$$

Evidently, experimental measurement of the above ratio of branching fractions can be used to fix the sign of the mixing angle, and it should be much more clean than the method based on hadronic D decays [29].

It is worth emphasizing that all other models predict comparable $K_1(1270)\gamma$ and $K_1(1400)\gamma$ rates (see Table V). In Refs. [38,42] tensor form factors Y_i are evaluated directly for the physical $B\to K_1(1270)$ and $B\to K_1(1400)$ transitions, while $B\to K_1^{1/2}$ and $B\to K_1^{3/2}$ transition form factors ($K_1^{1/2}$ and $K_1^{3/2}$ being the $P_1^{1/2}$ and $P_1^{3/2}$ states of K_1, respectively) are evaluated first in Ref. [40] and then related to the physical transitions. Hence, measurements of $B\to K_1\gamma$ decays can be utilized to distinguish the covariant light-front model from others.

For $B\to K_2^*\gamma$ decays, the calculated branching ratio of $(1.48\pm 0.30)\times 10^{-5}$ is in a good agreement with the world average of $(1.33\pm 0.20)\times 10^{-5}$. Since the above prediction is for $a_{11}=c_{11}$, this seems to imply that NLO corrections to $B\to K_2^*\gamma$ is not as important and dramatic as in the case of $B\to K^*\gamma$.

IV. CONCLUSION

Exclusive radiative B decays $B\to K^*\gamma$, $K_1(1270)\gamma$, $K_1(1400)\gamma$, and $K_2^*(1430)\gamma$ are studied in the framework of a covariant light-front quark model. Our main conclusions are as follows.

(1) The tensor form factor $T_1(q^2)$ at $q^2=0$, which is relevant to the decay $B\to K^*\gamma$, is found to be 0.24. This is much smaller than what expected from the conventional light-front model or light-cone sum rules but is in a good agreement with a recent lattice result [34]. In the heavy quark limit, the tensor form factors can be related to the vector and axial-vector form factors. Contrary to the conventional light-front model, it is found that the expression of $T_1(q^2)$ in the covariant light-front model has the correct heavy quark limit behavior.

(2) Taking into account the next-to-leading order hard vertex and hard spectator corrections, the predicted branching ratio $\mathcal{B}(B\to K^*\gamma)=(3.27\pm 0.74)\times 10^{-5}$ agrees with experiment, whereas most of the existing models predict too large decay rates of $B\to K^*\gamma$ compared to the data.

(3) The decay rates of $B\to K_1(1270)\gamma$ and $B\to K_1(1400)\gamma$ are very sensitive to the $K_1(1270)$-$K_1(1400)$ mixing angle and hence a measurement of their relative strength will provide an excellent way for determining the sign of the strange axial-vector meson mixing angle. Contrary to the other models in which $K_1(1270)\gamma$ and $K_1(1400)\gamma$ are predicted to be comparable, we found that, depending on the sign of the mixing angle, one of them is strongly suppressed owing to a large cancellation between two different form factor terms. Hence experimental measurements of the ratio of branching fractions will enable us to discriminate between different models.

(4) The predicted branching ratio of $B\to K_2^*\gamma$ is in a good agreement with experiment and this may imply that NLO corrections to $B\to K_2^*\gamma$ are not as important and dramatic as in the case of $B\to K^*\gamma$.

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APPENDIX: HEAVY QUARK LIMIT OF THE FORM FACTOR $T_1(0)$

In the heavy quark limit the tensor form factors $T_i(q^2)$ for $B\to K^*$ transition can be related to vector and axial-vector $B\to K^*$ form factors defined by

$$\langle K^*(P'',\varepsilon'')|V_\mu|B(P')\rangle=-\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta g(q^2),$$

$$\begin{aligned}\langle K^*(P'',\varepsilon'')|A_\mu|B(P')\rangle &\\ =-i\{\varepsilon''^*_\mu f(q^2)+\varepsilon^{*''}\cdot P[P_\mu a_+(q^2) &\\ +q_\mu a_-(q^2)]\}. &\end{aligned} \tag{A1}$$

In the static limit of the b quark, the static b-quark spinor satisfies the equation of motion $\gamma_0 b=b$. Heavy quark spin symmetry implies the relations [46]

$$\langle\overline{K}^*|\overline{s}\gamma_i b|\overline{B}\rangle=\langle\overline{K}^*|\overline{s}i\sigma_{0i}b|\overline{B}\rangle,$$

$$\langle\overline{K}^*|\overline{s}\gamma_i\gamma_5 b|\overline{B}\rangle=-\langle\overline{K}^*|\overline{s}i\sigma_{0i}\gamma_5 b|\overline{B}\rangle. \tag{A2}$$

This gives the form factor relation (see, e.g., Ref. [47] for other form-factor relations)

$$T_1(q^2)=-\frac{1}{2}(m_B-\omega m_{K^*})g(q^2)-\frac{1}{4m_B}f(q^2), \tag{A3}$$

where

$$\omega=\frac{m_B^2+m_{K^*}^2-q^2}{2m_B m_{K^*}}. \tag{A4}$$

Then in the heavy quark limit

[3] Note that by using input parameters in Table II and with four different values of θ, the decay constant $|f_{K_1(1270)}|$ is still within the experimental range (175 ± 19) MeV.

$$T_1(q^2) = -\frac{1}{4} m_b g(q^2) - \frac{1}{4m_b} f(q^2) \tag{A5}$$

for $|q^2| \ll m_B^2$.

From Eq. (2.18) we find that in the heavy quark limit

$$T_1(0) \to \frac{1}{32\pi^2} \int dx d^2 p'_\perp \frac{x m_b m_q \varphi'' \varphi'}{\sqrt{\mathcal{A}'^2 + p'^2_\perp}\sqrt{\mathcal{A}''^2 + p'^2_\perp}}, \tag{A6}$$

where use of $x \to 0$ has been made. It follows from Eq. (B4) of Ref. [1] that

$$g(0) \to -\frac{1}{16\pi^2} \int dx d^2 p'_\perp \frac{\varphi'' \varphi'}{\sqrt{\mathcal{A}'^2 + p'^2_\perp}\sqrt{\mathcal{A}''^2 + p'^2_\perp}} \times \left[x^2 m_b + x m_q + \frac{p'^2_\perp + m_q^2 - x^2 m_b^2}{m_b} \right] \tag{A7}$$

and

$$f(0) \to \frac{1}{32\pi^2} \int dx d^2 p'_\perp \frac{\varphi'' \varphi'}{\sqrt{\mathcal{A}'^2 + p'^2_\perp}\sqrt{\mathcal{A}''^2 + p'^2_\perp}} \times \left[2 x m_b^2 (x m_b - m_q) + 2 \frac{p'^2_\perp + m_q^2 - x^2 m_b^2}{m_b} \right]. \tag{A8}$$

Hence,

$$-\frac{1}{4} m_b g(0) - \frac{1}{4m_b} f(0) \to \frac{1}{32\pi^2} \int dx d^2 p'_\perp \frac{x m_b m_q \varphi'' \varphi'}{\sqrt{\mathcal{A}'^2 + p'^2_\perp}\sqrt{\mathcal{A}''^2 + p'^2_\perp}}. \tag{A9}$$

By comparing Eq. (A9) with Eq. (A6) we see that T_1 has the correct heavy quark limit behavior. It should be stressed that the zero mode contribution to the form factor $f(q^2)$ vanishes in the heavy quark limit. In the conventional light-front model [25,31], the heavy quark limit of $T_1(0)$ contains an additional term $m_q^2 + p'^2_\perp$ in the numerator of Eq. (A6) [see Eq. (2.19)] and hence it does not respect Eq. (A5) in the heavy quark limit.

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