Nonleptonic Λ_b decays to $D_s(2317)$, $D_s(2460)$, and other final states in factorization

Alakabha Datta*
Department of Physics, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 1A7

Harry J. Lipkin†
Department of Particle Physics, Weizmann Institute, Rehovot 76100, Israel,
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv, Israel,
and High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4815, USA

Patrick J. O'Donnell‡
Department of Physics, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 1A7
(Received 16 December 2003; published 13 May 2004)

We consider nonleptonic Cabibbo-allowed Λ_b decays in the factorization approximation. We calculate nonleptonic decays of the type $\Lambda_b \to \Lambda_c P$ and $\Lambda_b \to \Lambda_c V$ relative to $\overline{B_d^0} \to D^+ P$ and $\overline{B_d^0} \to D^+ V$ where we include among the pseudoscalar states and the vector states the newly discovered D_s resonances, $D_s(2317)$ and $D_s(2460)$. In the ratio of Λ_b decays to $D_s(2317)$ and $D_s(2460)$ relative to the $\overline{B_d^0}$ decays to these states, the poorly known decay constants of $D_s(2317)$ and $D_s(2460)$ cancel, leading to predictions that can shed light on the nature of these new states. In general, we predict the Λ_b decays to be larger than the corresponding $\overline{B_d^0}$ decays and in particular we find the branching ratio for $\Lambda_b \to \Lambda_c D_s(2460)$ can be between four to five times the branching ratio for $\overline{B_d^0} \to D^+ D_s(2460)$. This enhancement of Λ_b branching ratios follows primarily from the fact that more partial waves contribute in Λ_b decays than in $\overline{B_d^0}$ decays. Our predictions are largely independent of model calculations of hadronic inputs like form factors and decay constants.

DOI: 10.1103/PhysRevD.69.094002 PACS number(s): 13.30.Eg

I. INTRODUCTION

Nonleptonic decays are widely used to obtain information about the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the standard model (SM), as well as to obtain insights about the nonperturbative aspects of QCD. Nonleptonic decays in the B and Λ_b systems are interesting since the heavy mass of the b quark relative to the scale of soft nonperturbative physics allows for simplifications and makes tractable the difficult problem of calculating nonleptonic decays.

The nonleptonic decays of the Λ_b baryon have received relatively less attention than those of the B meson. In the Λ_b baryon, the spin of the baryon is carried by the b quark with the light diquark in a spin- and isospin-singlet state. This fact plays an important role in Λ_b decays [1] and leads to simplification of the nonperturbative dynamics involved in these decays. Because of this spin correlation between the b and the Λ_b, polarized Λ_b decays can provide important information about the weak interaction of the b quark [2]. Nonleptonic Λ_b decays can therefore be used to test the SM and to obtain insights into nonperturbative QCD.

In this work we consider Cabibbo-allowed Λ_b decays. Cabibbo-allowed Λ_b and B decays are usually calculated using factorization. We will also concentrate only on the factorizable part and discuss briefly nonfactorizable effects later in this section.

The factorizable amplitude is expressed in terms of form factors and decay constants. However the form factors and, barring a few cases, the decay constants are unknown hadronic inputs. Therefore, predictions for nonleptonic Λ_b decays depend on model calculation of form factors and decay constants and can have a wide range even within the factorization assumption [3]. Our purpose in this paper is to obtain predictions for Λ_b decays, within factorization, using the heavy m_b limit and using experimental inputs.

The method we use is the following: instead of directly calculating the Λ_b decays we consider instead the ratio of Cabibbo-allowed Λ_b decays relative to the corresponding Cabibbo-allowed B decays. The branching ratios for the Λ_b decays can then be obtained by simply using the experimental numbers for the Cabibbo-allowed B decays. One obvious advantage of considering such ratios is that the dependence on decay constants drop out in the ratio. Furthermore, in the heavy m_b limit, these ratios can be expressed as ratios of squared form factors. In the heavy m_b limit all form factors can be related to one single form factor and a dimensional constant representing the effective mass of the light degrees of freedom in the Λ_b baryon. These ratios of form factors are obtained using a mild assumption about the q^2 behavior of the form factors and the measurement of the branching ratio $\mathrm{BR}[\Lambda_b \to \Lambda_c \pi^-]/\mathrm{BR}[\overline{B_d^0} \to D^+ \pi^-]$. Our predictions turn out to be minimally dependent on hadronic inputs such as form factors and decay constants.

Another advantage of calculating ratios of branching ratios is that some of the nonfactorizable amplitudes cancel in the ratio. To see how this happens consider the decays $\Lambda_b \to \Lambda_c P$ and $\overline{B_d^0} \to D^+ P$. Now the underlying quark transition

*Email address: datta@physics.utoronto.ca
†Email address: harry.lipkin@weizmann.ac.il
‡Email address: odonnell@physics.utoronto.ca

is $b \to cP$. The corrections to factorization can arise from gluon emission between the b or the c quark and the quark constituents of P. However, these corrections are the same for Λ_b and $\overline{B}_d^0$ decays and so cancel in the ratio of their amplitudes. Gluon emissions involving the spectator quark in $\overline{B}_d^0$ and the spectator diquark in Λ_b may also be similar given the fact that the diquark in Λ_b belongs in the $\overline{3}$ under color $SU(3)_c$ as does the spectator antiquark in $\overline{B}_d^0$ and so both the spectators may have similar color interactions. Furthermore, within factorization, the small perturbative corrections to the form factors will also cancel.

Nonleptonic decays involving the newly discovered $D_s(2317)$ [4] and $D_s(2460)$ [5] states are of particular interest. It was shown in Ref. [6] that nonleptonic $\overline{B}_d^0$ decays involving these states can provide clues to the true nature of these states which is still not known [7–10]. It will be therefore interesting to see if these new D_s resonances show up in the Λ_b decays and how the rates for these decays compare to the $\overline{B}_d^0$ decays involving the new D_s states. As shown in Ref. [6] nonleptonic $\overline{B}_d^0$ decays to $D_s(2317)$ and $D_s(2460)$ involve the poorly known decay constants of these new states. However, in the ratio of Λ_b decays to $D_s(2317)$ and $D_s(2460)$ relative to the $\overline{B}_d^0$ decays to these states the decay constants of $D_s(2317)$ and $D_s(2460)$ cancel, leading to robust predictions that can shed additional light on the nature of these new states.

A. Masses and form factors

This assertion that the diquark in Λ_b belongs in the $\overline{3}$ under color $SU(3)_c$ as does the spectator antiquark in $\overline{B}_d^0$ leads to similar color interactions has been dramatically confirmed by relations between hadron masses based on a simple QCD-based argument that goes beyond simple models for the spectator diquarks and the spectator antiquark [11,12].

The hadrons under consideration all consist of a quark, denoted by q_i, of any flavor i and "light quark state" having either the quantum numbers of a $\overline{3}$ color diquark denoted by ud or a $\overline{3}$ color antiquark denoted by $\overline{u}$. While we use the notations ud and $\overline{u}$ for these light quark states, they can apply to any more complicated light quark configuration containing the same quantum numbers.

Consider the following four states of a quark of flavor i bound to a ud or $\overline{u}$ configuration. These are the pseudoscalar and vector mesons

$$|P_i\rangle = |q_i\overline{u}\rangle_{S=0}, \quad |V_i\rangle = |q_i\overline{u}\rangle_{S=1} \tag{1}$$

and the isoscalar and isovector baryons with spins 1/2 and 3/2, respectively,

$$|B_i^0\rangle = |q_i(ud)_{I=0}\rangle_{S=1/2}, \quad |B_i^1\rangle = |q_i(ud)_{I=1}\rangle_{S=3/2}. \tag{2}$$

Interesting mass relations between these hadrons were obtained [11] from the following QCD-motivated assumptions:

(1) The effective mass of any constituent in a hadron depends on the hadron wave function only via the color-electric field seen by the constituent. The color electric fields are very simply related in these hadrons.

(2) The color-electric field seen by the light quark systems ud and $\overline{u}$ are independent of the flavor of the quark q_i.

(3) The color-electric field seen by the quark q_i is independent of whether the color $\overline{3}$ light quark system is a ud diquark or a $\overline{u}$ antiquark.

(4) The color-magnetic interaction between the quark q_i and the spin-zero diquark vanishes in the baryon state $|B_i^0\rangle$.

(5) The color-magnetic contribution to the meson mass cancels out in the linear combination of masses [11]:

$$\tilde{M}_i = \frac{3M(V_i) + M(P_i)}{4}. \tag{3}$$

(6) The hyperfine splitting between the meson states $|P_i\rangle$ and $|V_i\rangle$ is inversely proportional to the effective mass m_i^{eff} of the quark of flavor i and similarly for the hyperfine splitting between the baryon states $|B_i^0\rangle$ and $|B_i^1\rangle$. However, this cancels out in the combination of Eq. (3).

These immediately give for any two quark flavors i and j

$$\tilde{M}_i - \tilde{M}_j = M(B_i^0) - M(B_j^0) \equiv m_i^{eff} - m_j^{eff} \tag{4}$$

and

$$\frac{M(V_i) - M(P_i)}{M(V_j) - M(P_j)} = \frac{M(B_i^1) - M(B_i^0)}{M(B_j^1) - M(B_j^0)} \equiv \frac{m_j^{eff}}{m_i^{eff}}. \tag{5}$$

Equations (4) and (5) give all the mass relations between mesons and baryons previously obtained [11,13–16] from the Sakharov-Zeldovich model [17] improved by De Rujula, Georgi, and Glashow [18]. In particular, we note that the change in baryon masses when the b quark in a Λ_b is changed into a c quark to make a Λ_c,

$$\langle m_b^{eff} - m_c^{eff}\rangle_{bar} = M(\Lambda_b) - M(\Lambda_c) = 3339 \text{ MeV} \tag{6}$$

is exactly equal to the change in meson masses when the b quark in a B meson is changed into a c quark to make a D when the appropriate average of pseudoscalar and vector mesons is taken to cancel out the hyperfine interaction:

$$\langle m_b^{eff} - m_c^{eff}\rangle_{mes} = \frac{3(M_{B*} - M_{D*}) + M_B - M_D}{4} = 3342 \text{ MeV}. \tag{7}$$

The fact that the change in the hadron mass produced by the quark transition $b \to c$ is the same when the quark is bound to a ud diquark and to a $\overline{u}$ antiquark suggests that the diquark and antiquark are spectators in the transition and will also effect the transition $b \to c$ in the same way when it is produced by the emission of a W in a weak decay.

We now note that rearranging Eq. (6) gives the dimensional constant $\bar{\Lambda}$ defined in Ref. [19] to represent the effective mass of the light degrees of freedom in the Λ_b and Λ_c baryon:

$$\bar{\Lambda}=m_{\Lambda_b}-m_b=m_{\Lambda_c}-m_c. \tag{8}$$

The value $\bar{\Lambda}=575$ MeV was estimated in Ref. [11] using quark masses that fit both meson and baryon masses.

B. Nonfactorization

Although we will use a factorization assumption there is a question of the correctness of such an assumption and what corrections would enter from nonfactorization. Nonfactorizable effects are known to be important for hyperon and charmed-baryon nonleptonic decays [20–22]. An unambiguous signal for the presence of nonfactorizable effects in Λ_b decays would be the observation of the decay $\Lambda_b\to\Sigma_c P$ or $\Lambda_b\to\Sigma_c V$. This is because, for the factorizable contribution, the light diquark in the Λ_b baryon remains inert during the weak decay. Thus, since the light diquark is an isosinglet, and since strong interactions conserve isospin to a very good approximation, the above Λ_b decays are forbidden within the factorization assumption [1].

One way to estimate the size of nonfactorizable corrections is to use the pole model. In this model, one assumes that the nonfactorizable decay amplitude receives contributions primarily from one-particle intermediate states, and that these contributions then show up as simple poles in the decay amplitude. Estimates of such pole diagrams in Λ_b decays have been found to be small and so are neglected in our analysis [3]. Note that these pole diagrams arise only through weak interactions involving the spectator quark and so small estimates of the pole diagram confirms the assumption of small spectator interaction in $\Lambda_b(\bar{B}^0_d)$ decays [23].

In Sec. II we discuss $\Lambda_b\to\Lambda_c P$ decays and in Sec. III we discuss $\Lambda_b\to\Lambda_c V$ decays. We will focus on those processes for which factorization is expected to be a good approximation, namely color-allowed decays. Finally in Sec. IV we present our summary.

II. $\Lambda_b\to\Lambda_c P$ DECAYS

We begin our analysis by studying the nonleptonic decay $\Lambda_b\to\Lambda_c P$. The general form for this amplitude can be written as

$$\mathcal{M}_P=A(\Lambda_b\to\Lambda_c P)=i\bar{u}_{\Lambda_c}(a+b\gamma_5)u_{\Lambda_b}. \tag{9}$$

In the rest frame of the parent baryon, the decay amplitude reduces to

$$A(\Lambda_b\to\Lambda_c P)=i\chi^\dagger_{\Lambda_c}(S+P\vec{\sigma}\cdot\hat{p})\chi_{\Lambda_b}, \tag{10}$$

where $\hat{p}$ is the unit vector along the direction of the daughter baryon momentum, and the S and P wave amplitudes are given by

$$S=\sqrt{2m_{\Lambda_b}(E_{\Lambda_c}+m_{\Lambda_c})}a$$

and

$$P=-\sqrt{2m_{\Lambda_b}(E_{\Lambda_c}-m_{\Lambda_c})}b,$$

where E_{Λ_c} and m_{Λ_c} are, respectively, the energy and mass of the final-state baryon Λ_c. The decay rate is then given by

$$\Gamma=\frac{|\vec{p}|}{8\pi m_{\Lambda_b}{}^2}(|S|^2+|P|^2), \tag{11}$$

where $|\vec{p}|$ is the magnitude of the momentum of the decay products in the rest frame of the Λ_b.

We will use factorization in order to estimate various nonleptonic amplitudes. The starting point is the SM effective Hamiltonian for hadronic B decays [24]:

$$H^q_{eff}=\frac{G_F}{\sqrt{2}}\left[V_{ub}V^*_{uq}(c_1O^q_1+c_2O^q_2)-\sum_{i=3}^{10}V_{tb}V^*_{tq}c^t_iO^q_i\right]+\text{H.c.}, \tag{12}$$

where

$$O^q_1=\bar{q}_\alpha\gamma_\mu Lc_\beta\bar{c}_\beta\gamma^\mu Lb_\alpha,\quad O^q_2=\bar{q}\gamma_\mu Lc\bar{c}\gamma^\mu Lb,$$

$$O^q_{3(5)}=\bar{q}\gamma_\mu Lb\sum_{q'}\bar{q}'\gamma^\mu L(R)q',$$

$$O^q_{4(6)}=\bar{q}_\alpha\gamma_\mu Lb_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu L(R)q'_\alpha,$$

$$O^q_{7(9)}=\frac{3}{2}\bar{q}\gamma_\mu Lb\sum_{q'}e_{q'}\bar{q}'\gamma^\mu R(L)q',$$

$$O^q_{8(10)}=\frac{3}{2}\bar{q}_\alpha\gamma_\mu Lb_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu R(L)q'_\alpha. \tag{13}$$

In the above, q can be either a d or an s quark, depending on whether the decay is a $\Delta S=0$ or a $\Delta S=-1$ process, $q'=d$, u, s or c, with $e_{q'}$ the corresponding electric charge, and $R(L)=1\pm\gamma_5$. The values of the Wilson coefficients c_i can be found in Ref. [25].

We now apply the effective Hamiltonian to specific exclusive Λ_b and B decays. We begin with $\Lambda_b\to\Lambda_c\pi^-$ and $\bar{B}^0\to D^+\pi^-$ which is a $b\to c\bar{u}d$ transition. Factorization allows us to write

$$A(\Lambda_b\to\Lambda_c\pi^-)=if_\pi q^\mu\langle\Lambda_c|\bar{c}\gamma_\mu(1-\gamma_5)b|\Lambda_b\rangle X_\pi,$$

$$A(\bar{B}^0\to D^+\pi^-)=if_\pi q^\mu\langle D^+|\bar{c}\gamma_\mu(1-\gamma_5)b|\bar{B}^0\rangle X_\pi, \tag{14}$$

$$X_\pi=\frac{G_F}{\sqrt{2}}V_{cb}V^*_{ud}a_2.$$

The pseudoscalar decay constant f_π is defined as

$$i f_\pi q^\mu = \langle \pi | \overline{d} \gamma^\mu (1-\gamma_5) u | 0 \rangle \tag{15}$$

and $a_2 = c_2 + c_1/N_c$.

Now, the vector and axial-vector matrix elements between the Λ_b and Λ_c baryons can be written in the general form

$$\langle \Lambda_c | \overline{c} \gamma^\mu b | \Lambda_b \rangle = \overline{u}_{\Lambda_c} \left[f_1 \gamma^\mu + i \frac{f_2}{m_{\Lambda_b}} \sigma^{\mu\nu} q_\nu + \frac{f_3}{m_{\Lambda_b}} q^\mu \right] u_{\Lambda_b},$$

$$\langle \Lambda_c | \overline{c} \gamma^\mu \gamma_5 b | \Lambda_b \rangle = \overline{u}_{\Lambda_c} \left[g_1 \gamma^\mu + i \frac{g_2}{m_{\Lambda_b}} \sigma^{\mu\nu} q_\nu + \frac{g_3}{m_{\Lambda_b}} q^\mu \right] \gamma_5 u_{\Lambda_b}, \tag{16}$$

where the f_i and g_i are Lorentz-invariant form factors. Heavy-quark symmetry imposes constraints on these form factors. In our approach we will only consider the b as heavy and consider corrections up to order $1/m_c$. In the $m_b \to \infty$ limit (but with $1/m_c$ corrections), one obtains the relations [19]

$$f_1 = g_1 = \left[1 + \frac{\overline{\Lambda}}{2m_{\Lambda_c}} \left(1 - \frac{\overline{\Lambda}}{m_{\Lambda_c}} \right) \frac{\omega}{(\omega+1)} \right] \xi_B(\omega) + \frac{\eta(\omega)}{2m_c},$$

$$f_2 = g_2 = f_3 = g_3 = -\frac{\overline{\Lambda}}{2m_{\Lambda_c}(\omega+1)} \left(1 - \frac{\overline{\Lambda}}{m_{\Lambda_c}} \right) \xi_B(\omega), \tag{17}$$

where $\xi_B(\omega)$ is the Isgur-Wise function for the $\Lambda_b \to \Lambda_c$ transition, $\overline{\Lambda}$ is defined in Eq. (8), $\eta(\omega)$ represents the correction from the kinetic energy of heavy quark in the baryon, and

$$\omega = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2}{2 m_{\Lambda_b} m_{\Lambda_c}}.$$

We point out that it is not necessary to estimate the quantity $\eta(\omega)$ for our calculation as we only use the relation $f_1 = g_1$. Estimates of $\eta(\omega)$ are found to be negligible [26] and so we will set $\eta(\omega) = 0$.

The dimensional constant $\overline{\Lambda}$ representing the effective mass of the light degrees of freedom in the Λ_b and Λ_c baryon is estimated from Ref. [11] with $\overline{\Lambda} = 575$ MeV. Now from Eq. (17) we see that in the $m_c \to \infty$ limit only the form factors f_1 and g_1 are non zero and the form factors f_2, g_2, f_3, and g_3 are suppressed by $O(1/m_c)$. We will use this fact later on in our calculations. Using Eqs. (14) and (17), the amplitudes a and b of Eq. (9) can be written as

$$a_\pi = f_\pi X_\pi \left[(m_{\Lambda_b} - m_{\Lambda_c}) f_1(q^2 = m_\pi^2) + f_3 \frac{m_\pi^2}{m_{\Lambda_b}} \right],$$

$$b_\pi = f_\pi X_\pi \left[(m_{\Lambda_b} + m_{\Lambda_c}) g_1(q^2 = m_\pi^2) - g_3 \frac{m_\pi^2}{m_{\Lambda_b}} \right]. \tag{18}$$

In Eq. (18) we can drop the suppressed contributions from the $f_3(g_3)$ form factors and using the heavy quark effective theory (HQET) relation $f_1 = g_1$ the quantities a_π and b_π can be expressed in terms of only one form factor. The S and P wave amplitudes are then written as

$$S = f_\pi X_\pi [(m_{\Lambda_b}^2 - m_{\Lambda_c}^2)] \sqrt{1 - \frac{m_\pi^2}{(m_{\Lambda_b} + m_{\Lambda_c})^2}} f_1(q^2 = m_\pi^2),$$

$$P = f_\pi X_\pi [(m_{\Lambda_b}^2 - m_{\Lambda_c}^2)] \sqrt{1 - \frac{m_\pi^2}{(m_{\Lambda_b} - m_{\Lambda_c})^2}} f_1(q^2 = m_\pi^2). \tag{19}$$

The vector and axial-vector matrix elements between the $\overline{B^0}$ and D^+ mesons can be written in terms of form factors [27]

$$\langle D^+(p_D) | J_\mu | \overline{B_d^0} \rangle = \left[(p_B + p_D)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2), \tag{20}$$

where $q = p_B - p_D$. From Eq. (14) one then obtains

$$A(\overline{B_d^0} \to D^+ \pi^-) = f_\pi X_\pi (m_B^2 - m_D^2) F_0(q^2 = m_\pi^2). \tag{21}$$

Note that the form of this amplitude is similar to the one in Eq. (19) with the important difference that for the Λ_b decays there are two partial waves allowed by angular momentum conservation.

We are interested here in the ratio

$$R_\pi = \frac{\mathrm{BR}[\Lambda_b \to \Lambda_c \pi^-]}{\mathrm{BR}[\overline{B_d^0} \to D^+ \pi^-]}. \tag{22}$$

We can define similar ratios R_K, R_{D_s}, R_D, and $R_{D_s(2317)}$. In passing we note that it is useful to also consider ratios of nonleptonic to semileptonic (SL) decays,

$$\mathrm{SL}_{\Lambda_b}=\frac{\Gamma[\Lambda_b\to\Lambda_c M]}{d\Gamma[\Lambda_b\to\Lambda_c l\nu]/d\omega},$$

$$\mathrm{SL}_{\overline{B}_d^0}=\frac{\Gamma[\overline{B}_d^0\to DM]}{d\Gamma[\overline{B}_d^0\to Dl\nu]/d\omega},$$

$$\mathrm{SL}_{\Lambda_b\overline{B}_d^0}=\frac{d\Gamma[\Lambda_b\to\Lambda_c l\nu]/d\omega}{d\Gamma[\overline{B}_d^0\to Dl\nu]/d\omega}, \tag{23}$$

where M is a P or a V meson. The semileptonic $\Lambda_b\to\Lambda_c l\nu$ decay distribution [26] as well as the nonleptonic $\Lambda_b\to\Lambda_c M$ transition in factorization can be expressed in terms of the $\Lambda_b\to\Lambda_c$ form factors in Eq. (16). Now using Eq. (17) and the estimate of $\overline{\Lambda}$ the quantity SL_{Λ_b} is independent of form factors and can therefore be used to check for the validity of factorization in $\Lambda_b\to\Lambda_c M$ transitions. One can use the ratio $\mathrm{SL}_{\overline{B}_d^0}$ to check for factorization in $\overline{B}_d^0$ decays. However the structure of the $1/m_{c,b}$ corrections are not so simple here [28]. Finally the ratio $\mathrm{SL}_{\Lambda_b\overline{B}_d^0}$ can be used to express the ratio of $\Lambda_b\to\Lambda_c$ form factor and $\overline{B}_d^0\to D$ form factor as a function of ω.

For the decays $\Lambda_b\to\Lambda_c(\pi^-,K^-)$ there are no penguin contributions. However, for the decays $\Lambda_b\to\Lambda_c(D_s^-,D^-,D_s(2317))$ there are penguin contributions and the penguin operators affect the Λ_b and B decays differently [2]. For the decay $\Lambda_b\to\Lambda_c D_s^-$ we obtain

$$\begin{aligned}A(\Lambda_b\to\Lambda_c D_s^-)&=if_{D_s}q^\mu\langle\Lambda_c|\overline{c}\gamma_\mu(1-\gamma_5)b|\Lambda_b\rangle X_{D_s}\\&\quad+if_{D_s}q^\mu\langle\Lambda_c|\overline{c}\gamma_\mu(1+\gamma_5)b|\Lambda_b\rangle Y_{D_s},\end{aligned} \tag{24}$$

where

$$X_{D_s}=\frac{G_F}{\sqrt{2}}\left[V_{cb}V_{cs}^*a_2-\sum_{q=u,c,t}V_{qb}V_{qs}^*(a_4^q+a_{10}^q)\right],$$

$$Y_{D_s}=-\frac{G_F}{\sqrt{2}}\left[\sum_{q=u,c,t}V_{qb}V_{qs}^*(a_6^q+a_8^q)\right]\chi_{D_s}, \tag{25}$$

with

$$\chi_{D_s}=\frac{2m_{D_s}^2}{(m_s+m_c)(m_b-m_c)}, \tag{26}$$

and for even i, $a_i=c_i+c_{i-1}/N_c$.

In the above equations we have used

$$if_{D_s}q^\mu=\langle D_s|\overline{s}\gamma^\mu(1-\gamma_5)c|0\rangle, \tag{27}$$

where $q^\mu\equiv p^\mu_{\Lambda_b}-p^\mu_{\Lambda_c}=p^\mu_{D_s}$ is the four-momentum transfer. One can then show that

$$\langle D_s^-|\overline{s}(1\pm\gamma_5)c|0\rangle=\mp\frac{f_{D_s}m_{D_s}^2}{m_s+m_c},$$

$$\langle\Lambda_c|\overline{c}(1\pm\gamma_5)b|\Lambda_b\rangle=\frac{q^\mu}{(m_b-m_c)}\langle\Lambda_c|\overline{c}\gamma_\mu(1\mp\gamma_5)b|\Lambda_b\rangle. \tag{28}$$

This then leads to

$$a_{D_s}=f_{D_s}(X_{D_s}+Y_{D_s})\left[(m_{\Lambda_b}-m_{\Lambda_c})f_1+f_3\frac{m_{D_s}^2}{m_{\Lambda_b}}\right],$$

$$b_{D_s}=f_{D_s}(X_{D_s}-Y_{D_s})\left[(m_{\Lambda_b}+m_{\Lambda_c})g_1-g_3\frac{m_{D_s}^2}{m_{\Lambda_b}}\right], \tag{29}$$

and

$$\begin{aligned}S&=f_{D_s}(X_{D_s}+Y_{D_s})[(m_{\Lambda_b}^2-m_{\Lambda_c}^2)]\\&\quad\times\sqrt{1-\frac{m_{D_s}^2}{(m_{\Lambda_b}+m_{\Lambda_c})^2}}f_1(q^2=m_{D_s}^2),\\P&=f_{D_s}(X_{D_s}-Y_{D_s})[(m_{\Lambda_b}^2-m_{\Lambda_c}^2)]\\&\quad\times\sqrt{1-\frac{m_{D_s}^2}{(m_{\Lambda_b}-m_{\Lambda_c})^2}}f_1(q^2=m_{D_s}^2).\end{aligned} \tag{30}$$

The corresponding B decay, $\overline{B}_d^0\to D^+D_s^-$, is

$$A(\overline{B}_d^0\to D^+D_s^-)=f_{D_s}(X_{D_s}+Y_{D_s})(m_B^2-m_D^2)F_0(q^2=m_{D_s}^2). \tag{31}$$

Similar expressions can be written for the pair of decays $\Lambda_b\to\Lambda_c D^-$ and $\overline{B}_d^0\to D^+D^-$ with obvious changes. Note that from Eqs. (25) and (26) the quantity Y_{D_s} or χ_{D_s} is formally suppressed by $1/m_b$ though with a large coefficient. Taking the effective quark masses $m_b=5.050$ GeV, $m_c=1.710$ GeV, and $m_s=0.602$ GeV [11] we find $\chi_{D_s}\sim 1$, which shows the effect of the large coefficient. However, to simplify our discussion we will neglect Y_{D_s}. Given the fact that the penguins are smaller than the tree amplitude, the error from the neglect of Y_{D_s} is of the same order as the subleading $1/m_b$ effects that we have neglected. We should point out that for CP-violating studies the quantities X_{D_s} and Y_{D_s} play an important role [2]. However, here we are interested in decay rates only and not CP-violating observables.

Using the values of the particle masses as well as the lifetimes of the Λ_b and B_d^0 [29] we obtain

$$R_\pi=1.73\frac{f_1^2(q^2=m_\pi^2)}{F_0^2(q^2=m_\pi^2)}. \tag{32}$$

Now using Eq. (32) and experimental information on R_π allows us to extract the form factor ratio

$$r(q^2=m_\pi^2)=f_1(q^2=m_\pi^2)/F_0(q^2=m_\pi^2).$$

There has been a preliminary measurement of $\Lambda_b \to \Lambda_c \pi^-$ by Collider Detector Facility (CDF) [30] with the branching ratio $[6.0\pm1.0(\mathrm{stat})\pm0.8(\mathrm{syst})\pm2.1(\mathrm{BR})]\times10^{-3}$. Using the Particle Data Group (PDG) value for $\overline{B_d^0}\to D^+\pi^-$, which is $(2.76\pm0.25)\times10^{-3}$ [29], and taking the central value of the measurements, we obtain $R_\pi\approx2.17$. Using Eq. (32), this then leads to

$$\frac{f_1(q^2=m_\pi^2)}{F_0(q^2=m_\pi^2)}=1.12. \tag{33}$$

In the heavy m_c and m_b limit we can relate the form factors f_1 and F_0 to the Isgur-Wise functions for the $\Lambda_b\to\Lambda_c$ and $B\to D$ transition, $\xi_B(\omega_B)$ and $\xi_M(\omega_M)$,

$$f_1(m_\pi^2)\approx f_1(0)=\xi_B(\omega_B^{max}),$$

$$F_0(m_\pi^2)\approx F_0(0)=F_1(0)=\frac{m_B+m_D}{2\sqrt{m_B m_D}}\xi_M(\omega_M^{max}), \tag{34}$$

which gives

$$\xi_B(\omega_B^{max})=1.4\xi_M(\omega_M^{max}). \tag{35}$$

In the heavy m_c and m_b limit $\omega_B=\omega_M$. However for actual masses $\omega_B^{max}=1.458$ and $\omega_M^{max}=1.588$, which indicates that $m_c\to\infty$ is not a very good limit. Keeping in mind that $\xi_{B,M}(\omega=1)=1$, Eq. (35) indicates that the baryon Isgur-Wise function falls off slower than the mesonic counterpart.

To make predictions for the ratio R_P for the other decays we would need the ratio of form factors $r(q^2=m_P^2)=f_1(q^2=m_P^2)/F_0(q^2=m_P^2)$. This requires a dynamical input that will be our only assumption for the calculation of the decays besides factorization.

We assume a general parametrization of the form factors for the region of q^2 that we are interested in,

$$f_1(q^2)=f_1(0)\,\eta_B\left(\frac{q^2}{M_{B*}^2}\right),$$

$$F_0(q^2)=F_0(0)\,\eta_M\left(\frac{q^2}{M_{M*}^2}\right)$$

where M_{B*} and M_{M*} are some heavy masses that scale as m_b. In other words the difference $M_{B*}-M_{M*}$ vanishes as $m_b\to\infty$. Furthermore $\eta_{B,M}(0)=1$ by definition. Assuming q^2 to be smaller than M_{B*}^2 and M_{M*}^2 we can write

TABLE I. $R_P=\mathrm{BR}[\Lambda_b\to\Lambda_c P^-]/\mathrm{BR}[\overline{B_d^0}\to D^+P^-]$ with experimental input.

R_P	Theory	Experiment
R_π	2.17	2.17 [a]
R_K	2.14	–
R_D	1.79	–
R_{D_s}	1.75	–
$R_{D_s(2317)}$	1.58	–

[a]Reference [30].

$$\eta_B\left(\frac{q^2}{M_{B*}^2}\right)=1+\alpha_B\frac{q^2}{M_{B*}^2}+\cdots,$$

$$\eta_M\left(\frac{q^2}{M_{B*}^2}\right)=1+\alpha_M\frac{q^2}{M_{B*}^2}+\cdots, \tag{36}$$

and so

$$\frac{\eta_B\left(\frac{q^2}{M_{B*}^2}\right)}{\eta_M\left(\frac{q^2}{M_{B*}^2}\right)}=1+\alpha_B\frac{q^2}{M_{B*}^2}-\alpha_M\frac{q^2}{M_{B*}^2}+\cdots. \tag{37}$$

Note that the often used pole model for form factors is just one example of the general parametrization in Eq. (36), where M_{B*} and M_{M*} can be identified with the excited baryon and meson states.

Now the largest q^2 we will be interested in is $q^2\sim4\ \mathrm{GeV}^2$ and so taking $M_{M*,B*}$ around 5–6 GeV we expect the second term in Eq. (36) to be around 10–15%. Furthermore, we expect α_B and α_M to be of the same sign as the form factors increase with q^2. This implies further cancellation in the second term in Eq. (37) and so to a good approximation

$$\frac{\eta_B\left(\frac{q^2}{M_{B*}^2}\right)}{\eta_M\left(\frac{q^2}{M_{M*}^2}\right)}=1. \tag{38}$$

So in the heavy m_b limit we can write for the form factor ratio r

$$r(q^2)\approx r(q^2=0)\approx r(q^2=m_\pi^2). \tag{39}$$

Hence the measurement of $r(q^2=m_\pi^2)$ allows us to make predictions for other decays that are presented in Table I.

III. $\Lambda_b \to \Lambda_c V$ DECAYS

We now turn to the decays $\Lambda_b \to \Lambda_c V$ where $V = \rho, K^* a_1, D^*, D_s^*, D_s(2460)$. The general decay amplitude can be written as [20,2]

$$\mathcal{M}_V = A(\Lambda_b \to \Lambda_c V) = \overline{u}_{\Lambda_a} \varepsilon_\mu^* \left[\frac{p_{\Lambda_b}^\mu + p_{\Lambda_c}^\mu}{m_{\Lambda_b}} (a + b\gamma_5) + \gamma^\mu (x + y\gamma_5) \right] u_{\Lambda_b}, \tag{40}$$

where ε_μ^* is the polarization of the vector meson. In the rest frame of the Λ_b, we can write $p_V = (E_V, 0, 0, |\vec{p}|)$ and $p_{\Lambda_c} = (E_{\Lambda_c}, 0, 0, -|\vec{p}|)$, and Eq. (40) can be reduced to [20]

$$\mathcal{M}_V = \chi_f^\dagger [S\vec{\sigma} + P_1 \hat{p} + iP_2 \hat{p} \times \vec{\sigma} + D(\vec{\sigma}\cdot\hat{p})\hat{p}] \cdot \vec{\epsilon} \chi_i, \tag{41}$$

where $\hat{p}$ is a unit vector in the direction of the vector meson momentum. The amplitudes for the three helicity states of the vector meson can be written as

$$\mathcal{M}(+1) = \frac{P_2 - S}{\sqrt{2}} \chi_f^\dagger [\vec{\sigma}\cdot(\vec{\epsilon_1} + i\vec{\epsilon_2})]\chi_i,$$

$$\mathcal{M}(-1) = \frac{P_2 + S}{\sqrt{2}} \chi_f^\dagger [\vec{\sigma}\cdot(\vec{\epsilon_1} - i\vec{\epsilon_2})]\chi_i, \tag{42}$$

$$\mathcal{M}(0) = \frac{E_V}{m_V} \chi_f^\dagger [(S + D)\vec{\sigma}\cdot\hat{p} + P_1]\chi_i.$$

In terms of the quantities defined in Eq. (40) we then have

$$S = -\sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})} y,$$

$$P_1 = \sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})} \frac{p}{E_V} \left[\frac{m_{\Lambda_b} + m_{\Lambda_c}}{E_{\Lambda_c} + m_{\Lambda_c}} x + 2a \right],$$

$$P_2 = -\sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})} \frac{px}{E_{\Lambda_c} + m_{\Lambda_c}}, \tag{43}$$

$$D = \sqrt{2m_{\Lambda_b}(E_{\Lambda_c} + m_{\Lambda_c})} \frac{p^2}{E_V(E_{\Lambda_c} + m_{\Lambda_c})} [2b - y].$$

We note from Eq. (42) that for light V, $E_V \sim m_{\Lambda_b}$ and so as $m_b \to \infty$ the amplitude with longitudinally polarized V dominates. Hence in this limit only two combinations of partial waves contribute. We also note that the longitudinal amplitude $\mathcal{M}(0)$ is of the same form as Eq. (10) for $\Lambda_b \to \Lambda_c P$. Hence in the $m_b \to \infty$ and a light V limit we can write the decay rate for $\Lambda_b \to \Lambda_c V$, following Eq. (11), as

$$\Gamma_{V0} = \frac{|\vec{p}|}{8\pi m_{\Lambda_b}^2} \left[(|S + D|^2 + |P_1|^2) \frac{E_V^2}{m_V^2} \right]. \tag{44}$$

The complete expression for the decay rate with finite m_b and V not necessarily light is given by

$$\Gamma_V = \frac{|\vec{p}|}{8\pi m_{\Lambda_b}^2} \left[(|S + D|^2 + |P_1|^2) \frac{E_V^2}{m_V^2} + (|S|^2 + |P_2|^2) \right], \tag{45}$$

where $|\vec{p}|$ is the magnitude of the momentum of the decay products in the rest frame of the Λ_b.

We use factorization to calculate the coefficients a, b, x, and y in Eq. (40) for various decays. Consider first the decay $\Lambda_b \to \Lambda_c \rho$. We define the decay constant g_ρ as

$$m_\rho g_\rho \varepsilon_\mu^* = \langle \rho | \overline{d} \gamma_\mu u | 0 \rangle, \tag{46}$$

and so we obtain

$$A(\Lambda_b \to \Lambda_c \rho) = m_\rho g_\rho \{ \varepsilon_\mu^* \langle \Lambda_c | \overline{c} \gamma^\mu (1 - \gamma_5) b | \Lambda_b \rangle X_\rho \}, \tag{47}$$

$$X_\rho = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2,$$

with $a_2 = c_2 + c_1/N_c$, and a, b, x, and y in Eq. (40) given by

$$a_\rho = m_\rho g_\rho f_2 X_\rho,$$

$$b_\rho = -m_\rho g_\rho g_2 X_\rho,$$

$$x_\rho = m_\rho g_\rho \left[f_1 - \frac{m_{\Lambda_c} + m_{\Lambda_b}}{m_{\Lambda_b}} f_2 \right] X_\rho,$$

$$y_\rho = -m_\rho g_\rho \left[g_1 + \frac{m_{\Lambda_b} - m_{\Lambda_c}}{m_{\Lambda_b}} g_2 \right] X_\rho. \tag{48}$$

For the general decay $\Lambda_b \to \Lambda_c V$ the quantities a, b, x, and y have the same form as Eq. (48) and we can then write

$$S + D = 2m_V g_V m_{\Lambda_b} \left[f_1 + f_2 \frac{m_V^2}{(m_{\Lambda_b} - m_{\Lambda_c}) m_{\Lambda_b}} \right] \frac{K_1}{K_2},$$

$$P_1 = 2m_V g_V m_{\Lambda_b} \left[f_1 - f_2 \frac{m_V^2}{(m_{\Lambda_b} + m_{\Lambda_c}) m_{\Lambda_b}} \right] \times \frac{m_{\Lambda_b}^2 - m_{\Lambda_c}^2}{m_{\Lambda_b}^2 + m_{\Lambda_c}^2} \frac{K_4}{K_3 K_1},$$

$$P_2 = -m_V g_V m_{\Lambda_b}[f_1 - f_2]\frac{m_{\Lambda_b} - m_{\Lambda_c}}{m_{\Lambda_b}}\frac{K_4}{K_1},$$

$$S = m_V g_V m_{\Lambda_b}[f_1 + f_2]\frac{m_{\Lambda_b} + m_{\Lambda_c}}{m_{\Lambda_b}} K_1, \tag{49}$$

where

$$K_1 = \sqrt{1 - \frac{m_V^2}{(m_{\Lambda_b} + m_{\Lambda_c})^2}},$$

$$K_2 = 1 + \frac{m_V^2}{(m_{\Lambda_b}^2 - m_{\Lambda_c}^2)},$$

$$K_3 = 1 - \frac{m_V^2}{(m_{\Lambda_b}^2 + m_{\Lambda_c}^2)},$$

$$K_4 = \sqrt{1 - \frac{2m_V^2(m_{\Lambda_b}^2 + m_{\Lambda_c}^2)}{(m_{\Lambda_b}^2 - m_{\Lambda_c}^2)^2}\left[1 - \frac{m_V^2}{2(m_{\Lambda_b}^2 + m_{\Lambda_c}^2)}\right]}. \tag{50}$$

In the light V and $m_b \to \infty$ case $K_{1,2,3,4} \to 1$ and the dependence on the form factor f_2 drops out. Also, only the first two combination of partial waves, $S+D$, and P_1 contribute. In the heavy m_b limit and identifying the light $V=\rho$, as an example, we can write, using the relations in Eq. (17) and dropping terms suppressed by m_ρ^2/E_ρ^2,

$$|M|^2(\Lambda_b \to \Lambda_c \rho^-) = (G_F\sqrt{2})^2 |V_{cb}V_{ud}^*|^2 a_2^2 m_\rho^2 f_\rho^2 f_1(m_\rho^2)^2 m_{\Lambda_b}^2 \times \frac{E_\rho^2}{m_\rho^2}\left[1 + \frac{(m_{\Lambda_b}^2 - m_{\Lambda_c}^2)^2}{(m_{\Lambda_b}^2 + m_{\Lambda_c}^2)^2}\right].$$

The corresponding expression for $\overline{B}^0 \to D^+\rho^-$ is within the factorization assumption [31],

$$|M|^2(\overline{B}^0 \to D^+\rho^-) = (G_F\sqrt{2})^2 |V_{cb}V_{ud}^*|^2 a_2^2 m_\rho^2 f_\rho^2 F_1(m_\rho^2)^2 m_B^2 \frac{p^2}{m_\rho^2}. \tag{51}$$

From Eq. (51) we see, that unlike the pseudoscalar case, the form factor $F_1(q^2)$ appears. However, $F_0(q^2=0) = F_1(q^2=0)$ and for the values of q^2 we are interested in we will make the assumption $F_1(q^2) \approx F_0(q^2)$. We therefore obtain for the ratio of form factors

$$r(q^2) = \frac{f_1^2(q^2)}{F_0(q^2)} \approx \frac{f_1^2(q^2)}{F_1(q^2)} \approx r(q^2=0). \tag{52}$$

TABLE II. $R_V = \mathrm{BR}[\Lambda_b \to \Lambda_c V^-]/\mathrm{BR}[\overline{B_d^0} \to D^+V^-]$ for $f_2=0$.

R_V	Theory (Γ_V)	Theory (Γ_{V0})
R_ρ	1.75	1.68
R_{K*}	1.82	1.72
R_{a_1}	2.08	1.89
R_{D*}	3.21	2.58
$R_{D_s^*}$	3.47	2.74
$R_{D_s(2460)}$	4.76	3.50

We can now use the experimental input for $r(q^2=m_\pi^2)$ from Eq. (32) to make predictions for the various $\Lambda_b \to \Lambda_c V$ decays.

It is clear from Eq. (49) that as m_V gets larger the effect of the form factor f_2 becomes important and we have to introduce additional model dependency by requiring the value of f_2. However f_2 is suppressed by $1/m_c$ and so we will present our predictions in two cases. In the first case we shall take the $m_c \to \infty$ limit and so $f_2=0$. However, we will use the measured values of the various particle masses thereby including finite m_c effects. Hence, the only assumption that we make here is that $m_c \to \infty$ is applicable only as far as the form factor f_2 is concerned. For the second case we estimate f_2/f_1 using Eq. (17) with $m_c=1.710$, and $\overline{\Lambda}=0.575$ GeV [11] and $\xi_B(\omega) \approx 1$.

We present our results in Table II with $f_2=0$ while in Table III we present results with $f_2 \neq 0$. The second column in Tables II and III uses the full decay rate in Eq. (45) while column three uses the decay rate with only the longitudinal polarization as given in Eq. (44). From Table II we make the following observations. When the vector meson V is light, then there is little difference between the entries in column two and column three, indicating the dominance of the longitudinally polarized contribution. With higher m_V the contributions from the transverse polarization components become important. The second observation is that, for light V, $R_V \leqslant 2$, as only two partial waves corresponding to the longitudinal vector polarization contribute. However, with increasing m_V the various quantities $K_{1,2,3,4}$ become important and in particular the partial wave P_1 increases. The net effect is that, even with only the longitudinal vector polarization, the Λ_b decay rate is more than the corresponding B rate by more than a factor of two for charm final states. Finally we see that the branching ratio for $\Lambda_b \to \Lambda_c D_s(2460)$ is between

TABLE III. $R_V = \mathrm{BR}[\Lambda_b \to \Lambda_c V^-]/\mathrm{BR}[\overline{B_d^0} \to D^+V^-]$ for $f_2 \neq 0$.

R_V	Theory (Γ_V)	Theory (Γ_{V0})
R_ρ	1.75	1.68
R_{K*}	1.81	1.72
R_{a_1}	2.07	1.88
R_{D*}	3.17	2.56
$R_{D_s^*}$	3.43	2.71
$R_{D_s(2460)}$	4.68	3.46

four to five times that of the corresponding B mode. This is simply from the fact that more partial waves contribute in the Λ_b decays and the fact that the $\Lambda_b \to \Lambda_c$ form factor is larger than the corresponding $\overline{B_d^0} \to D^+$ form factor as suggested by experiment [30]. From Table III we see that the effects of non zero f_2 from finite m_c effects are rather small.

IV. SUMMARY

In this paper we have considered nonleptonic Cabibbo-allowed Λ_b decays in the factorization approximation. We have discussed possible nonfactorizable effects and how experiments can be used to test look for them. We calculated decays of the type $\Lambda_b \to \Lambda_c P$ and $\Lambda_b \to \Lambda_c V$ relative to $\overline{B_d^0} \to D^+ P$ and $\overline{B_d^0} \to D^+ V$ where we included among the pseudoscalar states (P) and the vector states (V) the newly discovered D_s resonances, $D_s(2317)$ and $D_s(2460)$. Using a preliminary measurement of the branching ratio for $\Lambda_b \to \Lambda_c \pi^-$ and a mild assumptions about the q^2 behavior of form factors we made predictions for several Λ_b decays relative to the corresponding $\overline{B_d^0}$ decays. In general we found the Λ_b decays to be larger than the corresponding $\overline{B_d^0}$ decays and in particular we found $\Lambda_b \to \Lambda_c D_s(2460)$ can be between four to five times $\overline{B_d^0} \to D^+ D_s(2460)$. This enhancement of Λ_b can be understood from the fact that more partial waves contribute in Λ_b decays than in $\overline{B_d^0}$ decays and the fact that the $\Lambda_b \to \Lambda_c$ form factor is larger than the corresponding $\overline{B_d^0} \to D^+$ form factor.

ACKNOWLEDGMENTS

We thank Shin-Shan Yu for useful discussions and bringing the CDF measurements to our notice. This work was financially supported by NSERC of Canada.

[1] A. Datta, H.J. Lipkin, and P.J. O'Donnell, Phys. Lett. B **450**, 250 (1999).

[2] W. Bensalem, A. Datta, and D. London, Phys. Lett. B **538**, 309 (2002); Phys. Rev. D **66**, 094004 (2002).

[3] See, e.g., H.Y. Cheng, Phys. Rev. D **56**, 2799 (1997); M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, and A.G. Rusetsky, *ibid.* **57**, 5632 (1998); Fayyazuddin and Riazuddin, *ibid.* **58**, 014016 (1998); J.P. Lee, C. Liu, and H.S. Song, *ibid.* **58**, 014013 (1998); X.H. Guo, Mod. Phys. Lett. A **13**, 2265 (1998); R. Mohanta, A.K. Giri, M.P. Khanna, M. Ishida, S. Ishida, and M. Oda, Prog. Theor. Phys. **101**, 959 (1999); M.R. Khodja, Riazuddin, and Fayyazuddin, Phys. Rev. D **60**, 053005 (1999).

[4] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **90**, 242001 (2003).

[5] CLEO Collaboration, D. Besson *et al.*, Phys. Rev. D **68**, 032002 (2003).

[6] A. Datta and P.J. O'Donnell, Phys. Lett. B **572**, 164 (2003); C.H. Chen and H.N. Li, Phys. Rev. D **69**, 054002 (2004).

[7] T. Barnes, F.E. Close, and H.J. Lipkin, Phys. Rev. D **68**, 054006 (2003); H.J. Lipkin, Phys. Lett. B **580**, 50 (2004).

[8] R.N. Cahn and J.D. Jackson, Phys. Rev. D **68**, 037502 (2003); E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003); A.P. Szczepaniak, Phys. Lett. B **567**, 23 (2003); K. Terasaki, Phys. Rev. D **68**, 011501(R) (2003).

[9] W.A. Bardeen, E.J. Eichten, and C.T. Hill, Phys. Rev. D **68**, 054024 (2003).

[10] T.E. Browder, S. Pakvasa, and A.A. Petrov, Phys. Lett. B **578**, 365 (2004).

[11] M. Karliner and H.J. Lipkin, Phys. Lett. B **575**, 249 (2003).

[12] M. Karliner and H.J. Lipkin, hep-ph/0307243.

[13] I. Cohen and H.J. Lipkin, Phys. Lett. **93B**, 56 (1980).

[14] Harry J. Lipkin, Phys. Lett. B **233**, 446 (1989); Nucl. Phys. **A507**, 205c (1990).

[15] Harry J. Lipkin, Nucl. Phys. **A478**, 307c (1988).

[16] H.J. Lipkin, in *Proceedings of the 6th Conference on the Intersections of Particle and Nuclear Physics*, Big Sky, Montana, 1997, edited by T. W. Donnelly, AIP Conf. Proc. No. 412 (AIP, Woodbury, 1997), p. 504.

[17] Ya.B. Zeldovich and A.D. Sakharov, Yad. Fiz **4**, 395 (1996) [Sov. J. Nucl. Phys. **4**, 283 (1967)].

[18] A. De Rujula, H. Georgi, and S.L. Glashow, Phys. Rev. D **12**, 147 (1975).

[19] H. Georgi, B. Grinstein, and M.B. Wise, Phys. Lett. B **252**, 456 (1990).

[20] S. Pakvasa, S.P. Rosen, and S.F. Tuan, Phys. Rev. D **42**, 3746 (1990).

[21] R. Pérez-Marcial, R. Huerta, A. Garćia, and M. Avila-Aoki, Phys. Rev. D **40**, 2955 (1990); **44**, 2203(E) (1991).

[22] H.Y. Cheng and B. Tseng, Phys. Rev. D **46**, 1042 (1992); **48**, 4188 (1993); Q.P. Xu and A.N. Kamal, *ibid.* **46**, 270 (1992); J.G. Körner and M. Kramer, Z. Phys. C **55**, 659 (1992); A. Datta, hep-ph/9504428.

[23] A. Datta, H.J. Lipkin, and P.J. O'Donnell, Phys. Lett. B **529**, 93 (2002).

[24] A.J. Buras, hep-ph/9806471.

[25] R. Fleischer, Z. Phys. C **58**, 483 (1993); **62**, 81 (1994); G. Kramer, W.F. Palmer, and H. Simma, *ibid.* **66**, 429 (1995); Nucl. Phys. **B428**, 77 (1994); N.G. Deshpande and X.-G. He, Phys. Lett. B **336**, 471 (1994).

[26] J.G. Körner and B. Melic, Phys. Rev. D **62**, 074008 (2000).

[27] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[28] M.E. Luke, Phys. Lett. B **252**, 447 (1990).

[29] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[30] Kevin Pitts, talk presented at Lepton-Photon Conference, Fermilab, 2003. Note that the number quoted in the talk is incorrect by a factor of 10. We thank Shin-Shan Yu for pointing it out to us.

[31] A. Abd El-Hady, A. Datta, and J.P. Vary, Phys. Rev. D **58**, 014007 (1998).