Universal texture of quark and lepton mass matrices with an extended flavor 2↔3 symmetry

Yoshio Koide*
Department of Physics, University of Shizuoka, 52-1 Yada, Shizuoka, 422-8526 Japan
(Received 16 December 2003; published 5 May 2004)

Against the conventional picture that the mass matrix forms in the quark sectors will take somewhat different structures from those in the lepton sectors, on the basis of the idea that all the mass matrices of quarks and leptons have the same texture, a universal texture of quark and lepton mass matrices is proposed by assuming a discrete symmetry Z_3 and an extended flavor $2\leftrightarrow 3$ symmetry. The texture is described by three parameters (including the phase parameter). The neutrino masses and mixings are investigated according to this ansatz.

DOI: 10.1103/PhysRevD.69.093001 PACS number(s): 12.15.Ff, 11.30.Hv, 14.60.Pq

I. INTRODUCTION

From the point of view of quark and lepton unification, the idea that their matrix forms are described by a universal texture is very attractive. In particular, in contrast to the conventional picture that the mass matrix forms in the quark sectors will take somewhat different structures from those in the lepton sectors, it is interesting to investigate whether or not all the mass matrices of quarks and leptons can be described in terms of the same mass matrix form as in the neutrinos. Recently, a quark and lepton mass matrix model based on a discrete symmetry Z_3 and a flavor $2\leftrightarrow 3$ symmetry has been proposed [1]. In the model (we will refer to it as model I hereafter), the quark and lepton mass matrices M_f are given by the texture

$$M_f = P_f \hat{M}_f P_f, \tag{1.1}$$

where $\hat{M}_f$ is a real matrix with the form

$$\hat{M}_f = a_f \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + b_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & x_f \\ 0 & x_f & 1 \end{pmatrix} = \begin{pmatrix} 0 & a_f & a_f \\ a_f & b_f & b_f x_f \\ a_f & b_f x_f & b_f \end{pmatrix}, \tag{1.2}$$

and P_f is a phase matrix defined by

$$P_f = \mathrm{diag}(e^{i\delta_1^f},\ e^{i\delta_2^f},\ e^{i\delta_3^f}). \tag{1.3}$$

[As seen in the expression in Eq. (1.2), the model is essentially based on a two Higgs doublet model.] As we see in the next section, model I can give interesting results in the quark and lepton mass matrix phenomenology. However, in the model, $2\leftrightarrow 3$ symmetry was required only for the mass matrix $\hat{M}_f$, not for the fields ν_{Li}. The phase matrix P_f in Eq. (1.1), which breaks the $2\leftrightarrow 3$ symmetry, has been introduced from a phenomenological point of view.

In the present model, we propose a universal texture of quark and lepton mass matrices,

$$M_f = a_f \begin{pmatrix} 0 & e^{-i\phi_f} & 1 \\ e^{-i\phi_f} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + b_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & e^{-2i\phi_f} & 1 \\ 0 & 1 & 1 \end{pmatrix}, \tag{1.4}$$

which is similar to the matrix (1.1), but does not include the phenomenological phase matrix (1.3). [The phases $e^{-i\phi}$ and $e^{-2i\phi}$ in the matrix (1.4) are introduced by an extended $2\leftrightarrow 3$ symmetry which will be discussed in Sec. III.] In comparison with model I where the texture (1.1) has five parameters a_f, b_f, x_f, $\delta_1^f-\delta_2^f$, and $\delta_2^f-\delta_3^f$, the present model (1.4) has only three parameters a_f, b_f, and ϕ_f, so that the three mass eigenvalues can completely determine the three parameters a_f, b_f, and ϕ_f. As a result, for example, we will obtain the prediction

$$|V_{cb}| \simeq \frac{m_s}{m_b} + \frac{m_c}{m_t}, \tag{1.5}$$

different from model I, where $|V_{cb}|$ was given by $|V_{cb}| = \cos(\delta_3-\delta_2)/2$, where $\delta_i = \delta_i^u - \delta_i^d$, and the value $|V_{cb}|$ was freely adjustable using the parameter $\delta_3-\delta_2$.

In the next section, Sec. II, we will give a brief review of model I, because the present model is closely related to model I. In Sec. III, by introducing an extended flavor $2\leftrightarrow 3$ symmetry, we will propose a new universal texture of the quark and lepton mass matrices and we will investigate quark mass matrix phenomenology. In Sec. IV, we will discuss the neutrino mass matrix M_ν on the basis of the new universal texture. Predictions of $\sin^2 2\theta_{atm}$ and $|(V_\ell)_{13}|^2$ are given only in terms of the charged lepton mass ratios, independently of the parameters in M_ν. Predictions of $R = \Delta m^2_{solar}/\Delta m^2_{atm}$ and $\tan^2\theta_{solar}$ depend on two adjustable parameters in M_ν. We will give predictions for some typical values of the parameters. Finally, Sec. V is devoted to a summary and discussion.

*Email address: koide@u-shizuoka-ken.ac.jp

 ©2004 The American Physical Society

II. TWO HIGGS DOUBLET MODEL WITH A Z_3 SYMMETRY

In the present section, we give a brief review of model I [1].

We assume that under a discrete symmetry Z_3 the quark and lepton fields ψ_L, which belong to 10_L, $\overline{5}_L$, and 1_L of SU(5) $(1_L=\nu_R^c)$, are transformed as

$$\psi_{1L}\to\psi_{1L}, \quad \psi_{2L}\to\omega\psi_{2L}, \quad \psi_{3L}\to\omega\psi_{3L}, \tag{2.1}$$

where $\omega=e^{2i\pi/3}$. [Although we use the terminology of SU(5), at present, we do not consider the SU(5) grand unification.] Then the bilinear terms $\overline{q}_{Li}u_{Rj}$, $\overline{q}_{Li}d_{Rj}$, $\overline{\ell}_{Li}\nu_{Rj}$, $\overline{\ell}_{Li}e_{Rj}$, and $\overline{\nu^c_{Ri}}\nu_{Rj}$ $[\nu_R^c=(\nu_R)^c=C\overline{\nu_R}^T$ and $\overline{\nu_R^c}=\overline{(\nu_R^c)}]$ are transformed as follows:

$$\begin{pmatrix} 1 & \omega^2 & \omega^2 \\ \omega^2 & \omega & \omega \\ \omega^2 & \omega & \omega \end{pmatrix}. \tag{2.2}$$

Therefore, if we assume two SU(2) doublet Higgs scalars H_A and H_B, which are transformed as

$$H_A\to\omega H_A, \quad H_B\to\omega^2 H_B, \tag{2.3}$$

we obtain the mass matrix form

$$M_f=\begin{pmatrix} 0 & a_{12}^f & a_{13}^f \\ a_{12}^f & 0 & 0 \\ a_{13}^f & 0 & 0 \end{pmatrix}\langle H_A^0\rangle+\begin{pmatrix} 0 & 0 & 0 \\ 0 & b_{22}^f & b_{23}^f \\ 0 & b_{23}^f & b_{33}^f \end{pmatrix}\langle H_B^0\rangle. \tag{2.4}$$

In addition to the Z_3 symmetry, we assume a $2\leftrightarrow 3$ symmetry for the matrix $\hat{M}_f$ which is given by Eq. (1.2). (The $2\leftrightarrow 3$ symmetry does not mean the permutation $2\leftrightarrow 3$ symmetry for the fields ψ_{2L} and ψ_{3L}.) Hereafter, for simplicity, we will sometimes drop the index f and denote $a^f\langle H_A^0\rangle$ and $b^f\langle H_B^0\rangle$ as a_f and b_f, respectively. Then we obtain the universal texture (1.1) with Eq. (1.2) for the quark and lepton mass matrices.

Since the present model has two Higgs doublets horizontally, flavor-changing neutral currents (FCNCs) are, in general, caused by the exchange of Higgs scalars. However, this FCNC problem is a common subject to be overcome not only in the present model but also in most models with two Higgs doublets. The conventional mass matrix models based on a grand united theory GUT scenario cannot give realistic mass matrices without assuming more than two Higgs scalars [2]. In addition, if we admit that two such scalars remain until the low energy scale, the well-known beautiful coincidence of the gauge coupling constants at $\mu\sim 10^{16}$ GeV will be spoiled. For these problems, we optimistically consider that only one component of the linear combinations among those Higgs scalars survives at the low energy scale $\mu=m_Z$, while the other component is decoupled at $\mu<M_X$. Such an optimistic scenario in a multi-Higgs-doublet model is indeed possible, and an example can be found, for example, in Ref. [3].

The Hermitian matrix $H=MM^\dagger$ is diagonalized by a unitary matrix U_L as

$$U_L^\dagger H U_L=\mathrm{diag}(m_1^2,m_2^2,m_3^2), \tag{2.5}$$

$$U_L=PR, \tag{2.6}$$

$$R=\begin{pmatrix} c & s & 0 \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2.7}$$

$$s=\sin\theta=\sqrt{\frac{m_1}{m_1+m_2}}, \quad c=\cos\theta=\sqrt{\frac{m_2}{m_1+m_2}}, \tag{2.8}$$

$$\begin{aligned} -m_1&=\frac{1}{2}[b(1+x)-\sqrt{8a^2+b^2(1+x)^2}], \\ m_2&=\frac{1}{2}[b(1+x)+\sqrt{8a^2+b^2(1+x)^2}], \\ m_3&=b(1-x), \end{aligned} \tag{2.9}$$

where a, b, and x are real parameters given in Eq. (1.2) [1]. When we consider $m_3>m_2>m_1$, we can obtain the Cabibbo-Kobayashi-Maskawa (CKM) [4] matrix V,

$$\begin{aligned} V&=U_{uL}^\dagger U_{dL}=R_u^T P R_d \\ &=\begin{pmatrix} c_uc_d+\rho s_us_d & c_us_d-\rho s_uc_d & -\sigma s_u \\ s_uc_d-\rho c_us_d & s_us_d+\rho c_uc_d & \sigma c_u \\ -\sigma s_d & \sigma c_d & \rho \end{pmatrix}, \end{aligned} \tag{2.10}$$

where

$$P=P_u^\dagger P_d=\mathrm{diag}(e^{i\delta_1},e^{i\delta_2},e^{i\delta_3}), \tag{2.11}$$

$$\rho=\frac{1}{2}(e^{i\delta_3}+e^{i\delta_2})=\cos\frac{\delta_3-\delta_2}{2}\exp i\left(\frac{\delta_3+\delta_2}{2}\right), \tag{2.12}$$

$$\sigma=\frac{1}{2}(e^{i\delta_3}-e^{i\delta_2})=\sin\frac{\delta_3-\delta_2}{2}\exp i\left(\frac{\delta_3+\delta_2}{2}+\frac{\pi}{2}\right), \tag{2.13}$$

where we have taken $\delta_1=0$ without losing generality. The result (2.10) leads to the following phase-parameter-independent predictions [5]:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \frac{s_u}{c_u} = \sqrt{\frac{m_u}{m_c}} = 0.0586 \pm 0.0064, \tag{2.14}$$

$$\left|\frac{V_{td}}{V_{ts}}\right| = \frac{s_d}{c_d} = \sqrt{\frac{m_d}{m_s}} = 0.224 \pm 0.014, \tag{2.15}$$

where we used the values [6] at $\mu = m_Z$ as the quark mass values. Although the prediction (2.14) is somewhat small compared with the observed value [7] $|V_{ub}/V_{cd}| = (3.6 \pm 0.7) \times 10^{-3}/(4.12 \pm 2.0) \times 10^{-2} \simeq 0.087$, the prediction (2.10) is satisfactory, roughly speaking. For the neutrino mass matrix M_ν, by taking $\delta_3 - \delta_2 = \pi/2$, we can obtain [1] a satisfactory prediction of the lepton mixing matrix $V_\ell = U_e^\dagger U_\nu$ with a nearly bimaximal mixing.

On the other hand, very recently, it was pointed out by Matsuda and Nishiura [8] that if we assign the up-quark masses as $(m_{u1}, m_{u2}, m_{u3}) = (m_u, m_t, m_c)$ (they called it type B) in contrast to the assignment $(m_{d1}, m_{d2}, m_{d3}) = (m_d, m_s, m_b)$ (type A) in the mass eigenvalues (2.9), then we can obtain the phase-parameter-independent relations

$$\left|\frac{V_{ub}}{V_{tb}}\right| = \frac{s_u}{c_u} = \sqrt{\frac{m_u}{m_t}} = (3.6 \pm 0.5) \times 10^{-3}, \tag{2.16}$$

$$\left|\frac{V_{cd}}{V_{cs}}\right| = \frac{s_d}{c_d} = \sqrt{\frac{m_d}{m_s}} = 0.224 \pm 0.014, \tag{2.17}$$

instead of the relations (2.14) and (2.15). (We will refer to this model as model II.) The relation (2.16) is in excellent agreement with the observed value [7] $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, because we know that $|V_{tb}| \simeq 1$. The relation (2.17) is consistent with the well-known relation [9] $|V_{us}| \simeq \sqrt{m_d/m_s}$, because we know that $|V_{cs}| \simeq 1$ and $|V_{cd}| \simeq |V_{us}|$.

Thus, the new assignment of the quark masses by Matsuda and Nishiura seems to be favorable phenomenologically. However, what causes this different assignment between the up- and down-quark masses?

When this inverse assignment is caused in the up-quark sector, the up-quark mixing matrix U_{uL} is given by

$$U_{uL} = P_u R_u T_{23}, \tag{2.18}$$

where

$$T_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{2.19}$$

so that the CKM mixing matrix V is given by

$$V = T_{23} R_u^T P R_d = \begin{pmatrix} c_u c_d + \rho s_u s_d & c_u s_d - \rho s_u c_d & -\sigma s_u \\ -\sigma s_d & \sigma c_d & \rho \\ s_u c_d - \rho c_u s_d & s_u s_d + \rho c_u c_d & \sigma c_u \end{pmatrix}, \tag{2.20}$$

instead of the relation (2.10). The new CKM matrix (2.20) predicts

$$|V_{cb}| = |\rho| = \cos\frac{\delta_3 - \delta_2}{2}, \tag{2.21}$$

instead of the old prediction $|V_{cb}| \simeq \sin(\delta_3 - \delta_2)/2$. In order to give the observed value $|V_{cb}| = 0.0412$, we must take $\delta_3 - \delta_2 = \pi - \varepsilon$ with $\varepsilon = 4.27°$. In the mass matrix form (1.1), the phase matrix P_f has been introduced as a measure of the phenomenological $2 \leftrightarrow 3$ symmetry breaking. (We have assumed that the $2 \leftrightarrow 3$ symmetry is broken only by the phase parameters.) Therefore, it is natural to consider that these phase parameters δ_i^f show that $|\delta_i^f| \ll 1$. What is the origin of such a large value $\delta_3 - \delta_2 \simeq \pi$?

III. UNIVERSAL TEXTURE OF QUARK AND LEPTON MASS MATRICES

Stimulated by model II [8], in the present section, let us speculate on a new universal texture of the quark and lepton mass matrices.

We consider that the different assignment between the up- and down-quark masses in model II is caused by the difference of the initial values of the parameters a_f, b_f, and c_f between the up-and down-quark sectors in the texture (1.2). In fact, the mass hierarchies $m_3 > m_2 > m_1$ or $m_2 > m_3 > m_1$ occur according as $x_f < 0$ or $x_f > 0$, respectively, for $b_f \gg a_f > 0$. Therefore, in order to give the assignment $(m_1, m_2, m_3) = (m_u, m_t, m_c)$, we take the up-quark mass matrix M_u as

$$\hat{M}_u = a_u \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + b_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1-\xi_u \\ 0 & 1-\xi_u & 1 \end{pmatrix}, \tag{3.1}$$

where we have put $x_f = 1 - \xi_f$ (ξ_f is a small positive parameter). Similarly, if we want to give $(m_1, m_2, m_3) = (m_d, m_s, m_b)$, we should take

$$\hat{M}_d = a_d \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + b_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -(1-\xi_d) \\ 0 & -(1-\xi_d) & 1 \end{pmatrix}. \tag{3.2}$$

From Eqs. (3.1) and (3.2), the following general form of $\hat{M}_f$ is suggested:

$$\hat{M}_f = \begin{pmatrix} 0 & a_f & a_f \\ a_f & b_f & (1-\xi_f) b_f e^{i\beta_f} \\ a_f & (1-\xi_f) b_f e^{i\beta_f} & b_f \end{pmatrix}. \tag{3.3}$$

On the other hand, we must consider the origin of $\delta_3 - \delta_2 \simeq \pi$, which is required in Eq. (2.21) in model II. Therefore, we extend the flavor $2 \leftrightarrow 3$ symmetry which is generated by the operator T_{23} [Eq. (2.19)] to a generalized $2 \leftrightarrow 3$ symmetry which is generated by

$$T_{23}^{\phi}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{-i\phi} \\ 0 & e^{i\phi} & 0 \end{pmatrix}, \tag{3.4}$$

as

$$T_{23}^{\phi}M(T_{23}^{\phi})^T=M. \tag{3.5}$$

(In the present model, too, we assume that the mass matrix M is symmetric, i.e., $M^T=M$.) The requirement (3.5) leads to the relations

$$M_{12}=M_{13}e^{-i\phi}, \quad M_{22}=M_{33}e^{-2i\phi}, \tag{3.6}$$

but the relative phases among M_{13}, M_{23}, and M_{33} are free. Therefore, as a trial, we assume that M_{13}, M_{23}, and M_{33} have the same phases, i.e., we assume the mass matrix form

$$M=\begin{pmatrix} 0 & ae^{-i\phi} & a \\ ae^{-i\phi} & be^{-2i\phi} & (1-\xi)b \\ a & (1-\xi)b & b \end{pmatrix}, \tag{3.7}$$

where a and b are positive parameters of the model. This mass matrix (3.7) can give a nearly bimaximal mixing as we show in the Appendix. However, we still have four parameters in the mass matrix (3.7). We would like to seek for a model with a more concise structure. As we see in the Appendix [Eq. (A5)], in order to give three different mass eigenvalues, we may consider either a model with $\phi=0$ or one with $\xi=0$. However, if we take a model with $\phi=0$, we must introduce an alternative phase factor in order to explain the observed CP violation in the quark sectors.

In the present model, we simply assume a texture with $\xi=0$ in the texture (3.7):

$$M=a\begin{pmatrix} 0 & e^{-i\phi} & 1 \\ e^{-i\phi} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}+b\begin{pmatrix} 0 & 0 & 0 \\ 0 & e^{-2i\phi} & 1 \\ 0 & 1 & 1 \end{pmatrix}$$
$$=\begin{pmatrix} 0 & ae^{-i\phi} & a \\ ae^{-i\phi} & be^{-2i\phi} & b \\ a & b & b \end{pmatrix}, \tag{3.8}$$

i.e., we have assumed a democratic form except for the phases. It is convenient to rewrite the texture (3.8) as

$$M=P(0,-\phi,0)\cdot\hat{M}\cdot P(0,-\phi,0), \tag{3.9}$$

where

$$\hat{M}=\begin{pmatrix} 0 & a & a \\ a & b & be^{i\phi} \\ a & be^{i\phi} & b \end{pmatrix}, \tag{3.10}$$

$$P(\delta_1,\delta_2,\delta_3)=\mathrm{diag}(e^{i\delta_1},e^{i\delta_2},e^{i\delta_3}). \tag{3.11}$$

The matrix $\hat{M}$ (also M) has the following eigenvalues:

$$m_1=b\left(\sqrt{\cos^2\frac{\phi}{2}+2k^2}-\cos\frac{\phi}{2}\right),$$
$$m_2=b\left(\sqrt{\cos^2\frac{\phi}{2}+2k^2}+\cos\frac{\phi}{2}\right), \tag{3.12}$$
$$m_3=2b\sin\frac{\phi}{2},$$

where $k=a/b$. Inversely, from Eq. (3.12), we can evaluate the parameters a, b, and ϕ as follows:

$$a=\sqrt{\frac{m_1m_2}{2}}, \tag{3.13}$$

$$b=\frac{1}{2}\sqrt{m_3^2+(m_2-m_1)^2}, \tag{3.14}$$

$$\tan\frac{\phi}{2}=\frac{m_3}{m_2-m_1}. \tag{3.15}$$

The mixing matrix $\hat{U}$ for the matrix $\hat{M}$ is given by

$$\hat{U}=\hat{R}\cdot P\left(\frac{\phi}{4},\frac{\phi}{4},-\frac{\phi}{4}\right), \tag{3.16}$$

$$\hat{R}=\begin{pmatrix} ce^{-i\phi/2} & se^{-i\phi/2} & 0 \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{3.17}$$

where the mixing matrix $\hat{U}$ has been defined by

$$\hat{U}^\dagger\hat{M}\hat{U}^*=D\equiv\mathrm{diag}(-m_1,m_2,m_3). \tag{3.18}$$

Therefore, the mixing matrix U for the matrix M is given by

$$U=P(0,-\phi,0)\cdot\hat{U}=P(0,-\phi,0)\cdot\hat{R}\cdot P\left(\frac{\phi}{4},\frac{\phi}{4},-\frac{\phi}{4}\right). \tag{3.19}$$

In order to give the phenomenological value $\delta_3-\delta_2=\pi-\varepsilon$ in model II, we assume

$$\phi_u=\varepsilon_u, \quad \phi_d=\pi-\varepsilon_d, \tag{3.20}$$

which lead to the mass assignments $(m_{u1},m_{u2},m_{u3})=(m_u,m_t,m_c)$ and $(m_{d1},m_{d2},m_{d3})=(m_d,m_s,m_b)$, respectively. Then we obtain

$$U_u=P(0,-\varepsilon_u,0)\cdot\hat{R}_u\cdot P\left(\frac{\varepsilon_u}{4},\frac{\varepsilon_u}{4},-\frac{\varepsilon_u}{4}\right), \tag{3.21}$$

TABLE I. Parameter values evaluated from the quark mass values at $\mu=m_Z$. For reference, in addition to those in the present model with $\xi=0$, those in model II (with $\varepsilon=0$) are listed.

	(m_1,m_2,m_3)	M_u (m_u,m_t,m_c)	M_d (m_d,m_s,m_b)	M_e (m_e,m_μ,m_τ)
Model	a/b	0.00507	0.00986	0.00572
with	ε	$\varepsilon_u=\phi_u$	$\varepsilon_d=\pi-\phi_d$	$\varepsilon_e=\pi-\phi_e$
$\xi=0$		$=0.00748$ (0.429°)	$=0.0591$ (3.39°)	$=0.117$ (6.70°)
	b	90.5 GeV	1.50 GeV	0.875 GeV
Model	a/b	0.00506	0.00958	0.00541
with	ξ	0.00745	0.0574	0.111
$\varepsilon=0$	b	90.2 GeV	1.54 GeV	0.924 GeV

$$U_d=P(0,\ \pi-\varepsilon_d,0)\cdot\hat{R}_d\cdot P\left(\frac{\pi}{4}-\frac{\varepsilon_d}{4},\ \frac{\pi}{4}-\frac{\varepsilon_d}{4},\ -\frac{\pi}{4}+\frac{\varepsilon_d}{4}\right), \tag{3.22}$$

so that we obtain the CKM matrix

$$\begin{aligned}V&=T_{23}U_u^\dagger U_d\\&=T_{23}P\left(-\frac{\varepsilon_u}{4},\ -\frac{\varepsilon_u}{4},\ \frac{\varepsilon_u}{4}\right)\cdot\hat{R}_u^\dagger\cdot P(0,\ \pi-\varepsilon_d+\varepsilon_u,\ 0)\\&\quad\cdot\hat{R}_d\cdot P\left(\frac{\pi}{4}-\frac{\varepsilon_d}{4},\frac{\pi}{4}-\frac{\varepsilon_d}{4},\ -\frac{\pi}{4}+\frac{\varepsilon_d}{4}\right),\end{aligned} \tag{3.23}$$

which essentially gives the same results as the mixing matrix (2.20) in model II (but with $\delta_2-\delta_3=\pi-\varepsilon_d+\varepsilon_u$) as far as $|V_{ij}|$ are concerned. In addition to those predictions, we can obtain the new prediction

$$|V_{cd}|=\cos\frac{\phi_d-\phi_u}{2}=\sin\frac{\varepsilon_d+\varepsilon_u}{2}. \tag{3.24}$$

Since, from the formula (3.15), we obtain

$$\tan\frac{\varepsilon_u}{2}=\frac{m_c}{m_t-m_u}\simeq\frac{m_c}{m_t}, \tag{3.25}$$

$$\tan\frac{\varepsilon_d}{2}=\frac{m_s-m_d}{m_b}\simeq\frac{m_s}{m_b}, \tag{3.26}$$

we can predict

$$|V_{cb}|\simeq\frac{m_s}{m_b}+\frac{m_c}{m_t}=0.033 \tag{3.27}$$

by using the quark mass values [6] at $\mu=m_Z$. The value (3.27) is somewhat smaller compared with the observed value [7] $|V_{cd}|=0.0412\pm0.0020$, but it is roughly in agreement with the experimental value. For reference, the parameter values of a, b, and ε that are estimated from the observed quark masses at $\mu=m_Z$ are listed in Table I.

IV. NEUTRINO MASS MATRIX

Now let us investigate the lepton sectors under the ansatz (1.4). We again consider that the charged lepton mass matrix M_e is given by the texture (1.4) with $\phi_e=\pi-\varepsilon_e$ as well as M_d. In model I, the phenomenological parameter $\delta_3-\delta_2$ in the lepton sector was required as $\delta_3-\delta_2=\pi/2$. This suggests $\phi_\nu=\pi/2$ in the present model. At present, there is no reason that we should take $\phi_\nu=\pi/2$. However, from the phenomenological point of view, it is worth investigating the possibility $\phi_\nu=\pi/2$.

When we consider that the neutrino masses are generated by the seesaw mechanism [10], the neutrino mass matrix M_ν is given by $M_\nu=M_DM_R^{-1}M_D^T$, where M_D is a Dirac neutrino mass matrix and M_R is a Majorana mass matrix of right-handed neutrinos ν_{Ri}. Although the origin of the mass generation of M_R is different from that of M_u, M_d, M_e, and M_D, which are generated by Higgs scalars of $SU(2)_L$ doublet, since the texture (1.4) is based on the properties of flavors of $u_{L/R}$, $d_{L/R}$, $e_{L/R}$, and $\nu_{L/R}$, it is likely that the Majorana mass matrix M_R also has the same texture as the Dirac mass matrix M_D. However, note that, even if we assume that the mass matrices M_D and M_R are given by the texture (1.4), in general, the matrix $M_DM_R^{-1}M_D^T$ does not take the texture (1.4). Only when we consider $\phi_D=\phi_R$ does the expression of $M_DM_R^{-1}M_D^T$ become a little simpler. [In order that $M_DM_R^{-1}M_D^T$ has the texture (1.4) completely, the parameter values have to satisfy the relations $a_D/a_R=b_D/b_R$ and $\phi_D=\phi_R$.] In the present paper, we assume only that $\phi_D=\phi_R\equiv\phi_\nu$ and we do not assume $a_D/a_R=b_D/b_R$. Then we obtain

$$\begin{aligned}M_\nu&=M_DM_R^{-1}M_D^T\\&=e^{i(\phi_\nu/2-\pi/2+\varepsilon)}P\left(-\frac{1}{2}\phi_\nu+\frac{1}{2}\pi-\varepsilon,-\phi_\nu,0\right)\\&\quad\cdot\hat{M}\cdot P\left(-\frac{1}{2}\phi_\nu+\frac{1}{2}\pi-\varepsilon,-\phi_\nu,0\right),\end{aligned} \tag{4.1}$$

$$\hat{M}=\begin{pmatrix}0&a&a\\a&b&be^{i\hat{\phi}}\\a&be^{i\hat{\phi}}&b\end{pmatrix}, \tag{4.2}$$

$$a\equiv\frac{a_D^2}{a_R}, \tag{4.3}$$

$$b \equiv \frac{b_D^2}{b_R} \sin\frac{\phi_\nu}{2} \sqrt{1+r^2\cot^2\frac{\phi_\nu}{2}}, \tag{4.4}$$

$$r \equiv 2\frac{a_D b_R}{b_D a_R} - \left(\frac{a_D b_R}{b_D a_R}\right)^2, \tag{4.5}$$

$$\tan\varepsilon = r\cot\frac{\phi_\nu}{2}, \tag{4.6}$$

$$\hat{\phi} = \pi - 2\varepsilon. \tag{4.7}$$

Now, we assume $\phi_\nu = \pi/2$ and $a_D/b_D \ll a_R/b_R$ (i.e., $r \ll 1$), so that we obtain

$$\tan\varepsilon = r. \tag{4.8}$$

Since the mixing matrices U_e and U_ν are given by

$$U_{eL} = P(0, -\phi_e, 0)\cdot \hat{R}_e \cdot P\left(\frac{1}{4}\phi_e, \frac{1}{4}\phi_e, -\frac{1}{4}\phi_e\right), \tag{4.9}$$

$$U_{\nu L} = e^{i(\varepsilon-\pi/4)/2} P\left(\frac{1}{4}\pi - \varepsilon, -\frac{\pi}{2}, 0\right)\cdot \hat{R}_\nu \cdot P\left(\frac{1}{4}\hat{\phi}, \frac{1}{4}\hat{\phi}, -\frac{1}{4}\hat{\phi}\right), \tag{4.10}$$

where $\phi_e = \pi - \varepsilon_e$, $\hat{\phi} = \pi - 2\varepsilon$, and $\hat{R}_e$ and $\hat{R}_\nu$ are given by Eq. (3.17), the lepton mixing matrix $V_\ell = U_{eL}^\dagger U_{\nu L}$ is expressed as follows:

$$V_\ell = \begin{pmatrix} c_e c_\nu e^{i\delta_1} + \rho s_e s_\nu & c_e s_\nu e^{i\delta_1} - \rho s_e c_\nu & -\sigma s_e \\ s_e c_\nu e^{i\delta_1} - \rho c_e s_\nu & s_e s_\nu e^{i\delta_1} + \rho c_e c_\nu & \sigma c_e \\ -\sigma s_\nu & \sigma c_\nu & \rho \end{pmatrix}, \tag{4.11}$$

where

$$s_e = \sqrt{\frac{m_e}{m_\mu + m_e}}, \quad c_e = \sqrt{\frac{m_\mu}{m_\mu + m_e}}, \tag{4.12}$$

$$s_\nu = \sqrt{\frac{m_{\nu 1}}{m_{\nu 2} + m_{\nu 1}}}, \quad c_\nu = \sqrt{\frac{m_{\nu 2}}{m_{\nu 2} + m_{\nu 1}}}, \tag{4.13}$$

$$\delta_1 = \frac{\pi}{4} - \varepsilon + \frac{\hat{\phi}}{2} - \frac{\phi_e}{2}, \tag{4.14}$$

and ρ and σ are defined by Eqs. (2.12) and (2.13) with $\delta_2 = \phi_e - \pi = \varepsilon_e - \pi/2$ and $\delta_3 = 0$. Exactly speaking, the mixing matrix V_ℓ is given by

$$V_\ell = e^{i(\varepsilon-\pi/4)/2} P\left(-\frac{1}{4}\phi_e, -\frac{1}{4}\phi_e, \frac{1}{4}\phi_e\right) \cdot \hat{V}_\ell \cdot P\left(\frac{1}{4}\hat{\phi}, \frac{1}{4}\hat{\phi}, -\frac{1}{4}\hat{\phi}\right), \tag{4.15}$$

where $\widehat{V_\ell}$ in the expression (4.15) is defined by V_ℓ in Eq. (4.11). As far as the magnitudes of $(V_\ell)_{ij}$ are concerned, we can drop the phase factors $e^{i(\varepsilon-\pi/4)/2}P(-\frac{1}{4}\phi_e, -\frac{1}{4}\phi_e, \frac{1}{4}\phi_e)$ and $P(\frac{1}{4}\hat{\phi}, \frac{1}{4}\hat{\phi}, -\frac{1}{4}\hat{\phi})$. Of course, when we deal with neutrinoless double beta decay, a CP violation process, and so on, we must take exactly those phase factors into consideration. For a time, since we discuss $\sin^2 2\theta_{atm}$ and $\tan^2\theta_{solar}$, we neglect these phase factors.

For $\sin^2 2\theta_{atm}$, we obtain

$$\begin{aligned}\sin^2 2\theta_{atm} &= 4|(V_\ell)_{23}|^2|(V_\ell)_{33}|^2 = 4|\sigma|^2|\rho|^2 c_e^2 \\ &= \frac{m_\mu}{m_\mu + m_e}\sin^2\left(\frac{\pi}{2} - \varepsilon_e\right) \\ &\simeq 1 - \frac{m_e}{m_\mu} - 4\left(\frac{m_\mu}{m_\tau}\right)^2 = 0.98.\end{aligned} \tag{4.16}$$

We also obtain

$$|(V_\ell)_{13}|^2 = |\sigma|^2 s_e^2 \simeq \frac{m_e}{2m_\mu}\left(1 - 2\frac{m_\mu}{m_\tau}\right) = 0.0021. \tag{4.17}$$

These values (4.16) and (4.17) are consistent with the observed values [11,12].

For $\tan^2\theta_{solar}$, we obtain results as follows. Note that in the expression $(V_\ell)_{ij}$ $(i,j=1,2)$ given by Eq. (4.11), the phase of the first term relative to the second term, $\delta_1 - (\delta_3 + \delta_2)/2$, is given by

$$\delta_1 - \frac{\delta_3 + \delta_2}{2} = \left(\frac{\pi}{4} - \varepsilon - \frac{1}{2}\hat{\phi} + \frac{1}{2}\phi_e\right) - \frac{1}{2}\left(-\frac{\pi}{2} + \phi_e\right) = 0, \tag{4.18}$$

so that we can write $(V_\ell)_{ij}$ as

$$(V_\ell)_{11} = (c_e c_\nu + |\rho| s_e s_\nu) e^{i\delta_1}, \tag{4.19}$$

$$(V_\ell)_{12} = (c_e s_\nu - |\rho| s_e c_\nu) e^{i\delta_1}, \tag{4.20}$$

and so on. Therefore, we obtain

$$\begin{aligned}\tan^2\theta_{solar} &= \frac{|(V_\ell)_{12}|^2}{|(V_\ell)_{11}|^2} = \left(\frac{s_\nu/c_\nu - |\rho| s_e/c_e}{1 + |\rho|(s_e/c_e)(s_\nu/c_\nu)}\right)^2 \simeq \left(\frac{s_\nu}{c_\nu}\right)^2 \\ &= \frac{m_{\nu 1}}{m_{\nu 2}},\end{aligned} \tag{4.21}$$

for $s_\nu/c_\nu = \sqrt{m_{\nu 1}/m_{\nu 2}} \gg s_e/c_e = \sqrt{m_e/m_\mu}$. (The alternative case $s_\nu/c_\nu \ll s_e/c_e$ is ruled out because it leads to a very small value of $\tan^2\theta_{solar}$.)

For Δm^2_{solar} and Δm^2_{atm}, from Eq. (3.12) with $\phi = \hat{\phi} = \pi - 2\varepsilon$, we can obtain

$$\Delta m^2_{21} \equiv m_2^2 - m_1^2 = 4b^2 \sin\varepsilon\sqrt{\sin^2\varepsilon + 2k^2}, \tag{4.22}$$

$$\Delta m^2_{32} \equiv m_3^2 - m_2^2 = 4b^2\left(1 - \frac{3}{2}\sin^2\varepsilon - \frac{1}{2}k^2 - 2\sin\varepsilon\sqrt{\sin^2\varepsilon + 2k^2}\right), \tag{4.23}$$

TABLE II. Values $\tan^2\theta_{solar}$ and $m_{\nu i}$ $(i=1,2,3)$ for typical value of $x\equiv k/\sin\varepsilon$.

x	$m_{\nu1}/m_{\nu2}\simeq\tan^2\theta_{solar}$	$r_{23}=m_{\nu2}/m_{\nu3}$	$m_{\nu1}$ (eV)	$m\nu2$ (eV)	$m\nu3$ (eV)	$\tan\varepsilon$
2	$1/2=0.5$	0.188	0.0048	0.0096	0.0509	0.094
$\sqrt{2}$	$(3-\sqrt{5})/2=0.382$	0.177	0.0034	0.0090	0.0508	0.109
$\sqrt{3/2}$	$1/3=0.333$	0.174	0.0029	0.0088	0.0508	0.116

where

$$k\equiv\frac{a}{b}=\sqrt{2}\frac{a_D^2 b_R}{b_R^2 a_R}\cos\varepsilon, \tag{4.24}$$

so that we predict

$$R\equiv\frac{\Delta m_{21}^2}{\Delta m_{32}^2}\simeq\sin\varepsilon\sqrt{\sin^2\varepsilon+2k^2}, \tag{4.25}$$

for small ε^2 and k^2. On the other hand, from Eqs. (3.13) and (3.14) [i.e., $\tan\varepsilon=(m_{\nu2}-m_{\nu1})/m_{\nu3}$], we obtain

$$k\equiv\frac{a}{b}=\sqrt{\frac{2m_{\nu1}m_{\nu2}}{m_{\nu3}^2+(m_{\nu2}-m_{\nu1})^2}}=\sqrt{2}\,\frac{\sqrt{m_{\nu1}/m_{\nu2}}}{1-m_{\nu1}/m_{\nu2}}\sin\varepsilon. \tag{4.26}$$

Therefore, if we give a value

$$x\equiv\frac{k}{\sin\varepsilon}, \tag{4.27}$$

we can obtain the value of $m_{\nu1}/m_{\nu2}$ as follows:

$$\tan^2\theta_{solar}\simeq\frac{m_{\nu1}}{m_{\nu2}}=\frac{1}{x^2}(1+x^2-\sqrt{1+2x^2}). \tag{4.28}$$

In the present model, since the value of $R\equiv\Delta m_{21}^2/\Delta m_{32}^2$ depends on the parameters ε and x, the value of $\tan^2\theta_{solar}$ cannot be predicted from the charged lepton masses only. In other words, if we give the values R and $\tan^2\theta_{solar}$, we can determine the values x and ε (k and ε), so that we can also determine the value of $m_{\nu1}$, $m_{\nu2}$, and $m_{\nu3}$. In Table II, we list the numerical predictions $m_{\nu1}/m_{\nu2}=\tan^2\theta_{solar}$ and $(m_{\nu1},m_{\nu2},m_{\nu3})$ for typical values of $x=k/\sin\varepsilon$, where we have used the input values [12–14]

$$R_{obs}=\frac{6.9\times10^{-5}\ \text{eV}^2}{2.5\times10^{-3}\ \text{eV}^2}=2.76\times10^{-2}, \tag{4.29}$$

and $\Delta m_{atm}^2=2.5\times10^{-3}$ eV^2. The value $m_{\nu1}/m_{\nu2}$ in Table II has been evaluated from Eq. (4.26). From the relation

$$R=\left(\frac{m_{\nu2}}{m_{\nu3}}\right)^2\frac{1-(\tan^2\theta_{solar})^2}{1-(m_{\nu2}/m_{\nu3})^2}, \tag{4.30}$$

we obtain

$$r_{23}\equiv\frac{m_{\nu2}}{m_{\nu3}}=\sqrt{\frac{R}{1-\tan^4\theta_{solar}+R}}. \tag{4.31}$$

The mass values $m_{\nu i}$ in Table II were obtained from

$$m_{\nu3}=\sqrt{\frac{\Delta m_{atm}^2}{1-r_{23}^2}}, \tag{4.32}$$

$m_{\nu2}=r_{23}m_{\nu3}$, and $m_{\nu1}=m_{\nu2}\tan^2\theta_{solar}$. The value of $\tan\varepsilon$ in Table II has been estimated, not from the approximate relation (4.25), but from the exact relation

$$\tan\varepsilon=\frac{m_{\nu2}}{m_{\nu3}}\left(1-\frac{m_{\nu1}}{m_{\nu2}}\right)=r_{23}(1-\tan^2\theta_{solar}). \tag{4.33}$$

V. SUMMARY

In conclusion, we have proposed a universal texture (1.4) for quark and lepton mass matrices. The mass matrix M is invariant under the extended flavor $2\leftrightarrow3$ permutation T_{23}^{ϕ} [Eq. (3.4)] as $T_{23}^{\phi}M(T_{23}^{\phi})^T=M$. In addition, the matrix elements M_{ij} $(i=2,3)$ are exactly democratic apart from their phases, i.e.,

$$M_f=a_f\begin{pmatrix}0 & e^{-i\phi_f} & 1\\ e^{-i\phi_f} & 0 & 0\\ 1 & 0 & 0\end{pmatrix}\langle H_A^0\rangle+b_f\begin{pmatrix}0 & 0 & 0\\ 0 & e^{-2i\phi_f} & 1\\ 0 & 1 & 1\end{pmatrix}\times\langle H_B^0\rangle. \tag{5.1}$$

The mass matrix M is described by two parameters ϕ and a/b, as far as the mass ratios and mixings are concerned.

For quark sectors M_u and M_d, we take the parameter ϕ as $\phi_u=\varepsilon_u$ and $\phi_d=\pi-\varepsilon_d$, respectively, where ε_u and ε_d are small positive parameters. Then, we can obtain successful relations for the CKM mixing parameters in terms of quark masses. [Note that, in models I and II, the value of $|V_{cb}|$ is given by a phenomenological parameter $(\delta_3-\delta_2)$ independently of the quark mass ratios, while, in the present model, $|V_{cb}|$ is given in terms of quark mass ratios as shown in Eq. (3.27).] For the charged lepton mass matrix M_e, we take $\phi_e=\pi-\varepsilon_e$ as well as $\phi_d=\pi-\varepsilon_d$, while, for the neutrino mass matrix M_ν, we take $\phi_\nu=\pi/2$ in order to give a nearly bimaximal mixing. The neutrino mass matrix M_ν is described by two parameters ε and $k\equiv a/b$. The predictions $\sin^2 2\theta_{atm}$ and $|(V_\ell)_{13}|^2$ are given only in terms of charged lepton masses independently of the parameters ε and k, as shown in Eqs. (4.16) and (4.17). These predictions are favorable to the data. On the other hand, the quantities $\tan^2\theta_{solar}$,

$R \equiv \Delta m^2_{solar}/\Delta m^2_{atm}$, and $m_{\nu i}$ $(i=1,2,3)$ are dependent on the parameters ε and k in the neutrino mass matrix M_ν. In Table II, we have listed the predictions for typical values of $x \equiv k/\sin\varepsilon$.

Although we have taken $\phi_e = \pi - \varepsilon_e$ for the charged lepton sector, it is not essential. If we take $\phi_e = \varepsilon_e$ as well as ϕ_u, the results for the neutrino mixing are substantially uncharged [e.g., Eqs. (4.16) and (4.17) become merely $\sin^2 2\theta_{atm} \simeq 1 - m_e/m_\tau$ and $|(V_\ell)_{13}|^2 \simeq m_e/2m_\tau$, respectively]. However, the choice $\phi_\nu = \pi/2$ is essential. If we choose another value of ϕ_ν, we cannot obtain $\sin^2 2\theta_{atm} \simeq 1$. It is an open question why we must choose $\phi = \pi/2$ only for the neutrino sector.

Since as seen in Table I the parameter values of ϕ (and a/b) that are determined from the observed fermion masses are different from each other (i.e., $a_e \neq d_d$, $b_e \neq b_d$, and $\phi_e \neq \phi_d$), the present model cannot be applied to a grand unification theory model straightforwardly. However, for example, in an SU(5) GUT model, with matter fields $\overline{5}+10+\overline{5}'+5'$ [15], we can consider a $\overline{5} \leftrightarrow \overline{5}'$ mixing. Therefore, the problem that the parameter values are not universal is not a very serious defect even for a GUT model. Because of the simplicity of the texture (1.4) with few parameters, it is worthwhile taking the present universal texture seriously.

APPENDIX

It is convenient to rewrite the mass matrix

$$M = \begin{pmatrix} 0 & ae^{-i\phi} & a \\ ae^{-i\phi} & be^{-2i\phi} & (1-\xi)b \\ a & (1-\xi)b & b \end{pmatrix} \tag{A1}$$

as

$$M = P(0,-\phi,0)\cdot \hat{M} \cdot P(0,-\phi,0), \tag{A2}$$

where

$$P(\delta_1,\delta_2,\delta_3) = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}), \tag{A3}$$

$$\hat{M} = \begin{pmatrix} 0 & a & a \\ a & b & (1-\xi)be^{i\phi} \\ a & (1-\xi)be^{i\phi} & b \end{pmatrix}. \tag{A4}$$

The eigenvalues m_i and mixing matrix $\hat{U}$ of the mass matrix $\hat{M}$ are as follows:

$$m_1 = \frac{1}{2}\left(\sqrt{p^2+8k^2} - p\right) b,$$

$$m_2 = \frac{1}{2}\left(\sqrt{p^2+8k^2} + p\right) b, \tag{A5}$$

$$m_3 = qb,$$

$$\hat{U} = \hat{R}\cdot P\left(-\frac{\delta}{2}, -\frac{\delta}{2}, \frac{\delta}{2}\right), \tag{A6}$$

$$\hat{R} = \begin{pmatrix} ce^{i\delta} & se^{i\delta} & 0 \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{A7}$$

where $k = a/b$,

$$p^2 = \xi^2 + 4(1-\xi)\cos^2\frac{\phi}{2}, \tag{A8}$$

$$q^2 = \xi^2 + 4(1-\xi)\sin^2\frac{\phi}{2}, \tag{A9}$$

$$\tan\delta = -\frac{(1-\xi)\sin\phi}{1+(1-\xi)\cos\phi}, \tag{A10}$$

and the mixing matrix $\hat{U}$ is defined by

$$\hat{U}^\dagger \hat{M} \hat{U}^* = D \equiv \mathrm{diag}(-m_1, m_2, m_3). \tag{A11}$$

Therefore, the mixing matrix U for the matrix M is given by

$$U = P(0,-\phi,0)\cdot\hat{U} = P(0,-\phi,0)\cdot\hat{R}\cdot P\left(-\frac{\delta}{2}, -\frac{\delta}{2}, \frac{\delta}{2}\right). \tag{A12}$$

If we put $\phi = 0$, then the results (A5)–(A10) become those in models I and II. The model (1.4) in the present paper corresponds to one with $\xi = 0$.

[1] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi, and T. Fukuyama, Phys. Rev. D **66**, 093006 (2002).

[2] For instance, K. Oda, E. Takasugi, M. Tanaka, and M. Yoshimura, Phys. Rev. D **59**, 055001 (1999); K. Matsuda, Y. Koide, and T. Fukuyama, *ibid.* **64**, 053015 (2001).

[3] Y. Koide and J. Sato, Phys. Rev. D **68**, 056004 (2003).

[4] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[5] G.C. Branco and L. Lavoura, Phys. Rev. D **44**, R582 (1991). See also H. Fritzsch and Z.Z. Xing, Phys. Lett. B **413**, 396 (1997); Phys. Rev. D **57**, 594 (1998).

[6] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).

[7] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[8] K. Matsuda and H. Nishiura, Phys. Rev. D **69**, 053005 (2004).
[9] S. Weinberg, Ann. N.Y. Acad. Sci. **38**, 185 (1977); H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); H. Georgi and D.V. Nanopoulos, *ibid.* **B155**, 52 (1979).
[10] T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; G. Senjanović and R.N. Mohapatra, Phys. Rev. Lett. **44**, 912 (1980).
[11] M. Apollonio *et al.*, Phys. Lett. B **466**, 415 (1999).
[12] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); M. Shiozawa, talk presented at ‘‘Neutrino 2002,’’ Munich, Germany, 2002, http://neutrino.t30.physik.tu-muenchen.de/
[13] J.N. Bahcall, M.C. Gonzalez-Garcia, and C. Peña-Garay, J. High Energy Phys. **08**, 014 (2001); G.L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D **64**, 093007 (2001); V. Barger, D. Marfatia, and K. Whisnant, Phys. Rev. Lett. **88**, 011302 (2002); P.I. Krastev and A.Yu. Smirnov, Phys. Rev. D **65**, 073022 (2002); SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011302 (2002).
[14] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
[15] M. Bando and T. Kugo, Prog. Theor. Phys. **101**, 1313 (1999). See also J. Sato and T. Yanagida, Nucl. Phys. B (Proc. Suppl.) **77**, 293 (1999). For a recent model with $\overline{5}$-$\overline{5}'$ mixing see, for example, Y. Koide, Phys. Lett. B **574**, 82 (2003).