Vacuum stability in heterotic M theory

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The problem of the stabilization of moduli is discussed within the context of compactified strongly coupled heterotic string theory. It is shown that all geometric, vector bundle, and five-brane moduli are completely fixed, within a phenomenologically acceptable range, by nonperturbative physics. This result requires, in addition to the full space of moduli, nonvanishing Neveu-Schwarz flux, gaugino condensation with threshold corrections, and the explicit form of the Pfaffians in string instanton superpotentials. The stable vacuum presented here has a negative cosmological constant. The possibility of ''lifting'' this to a metastable vacuum with positive cosmological constant is briefly discussed.

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I. INTRODUCTION

One of the most important problems in finding realistic four-dimensional vacua in superstrings and M theory is the problem of moduli stabilization. The four-dimensional fundamental constants, such as the Newton and unification gauge parameters, depend on the compactification moduli. Therefore, in any realistic compactification scenario, all the moduli have to be fixed, or very slowly rolling, in a phenomenologically acceptable range. However, string theory moduli do not have a perturbative potential energy. Hence, if their values are to be fixed, it must be by nonperturbative physics. The first attempts to do this [1,2] indicated that nonperturbative superpotentials can lead to runaway behavior. That is, the radius of the compactification manifold was found to run to large values, leading to decompactification. However, this work was very preliminary, involving only a subset of possible moduli and nonperturbative superpotentials.

Over the years, there have been many attempts to prove the stability of moduli in different types of string theory. Recently, progress in this direction was achieved in type IIB string theory in Ref. [3], emphasizing, among other things, the necessity of considering flux compactifications [4–19]. The moduli stabilization in Ref. [3] was demonstrated in two steps. First, all moduli were stabilized at a fixed minimum with a negative cosmological constant. This was achieved by combining fluxes with nonperturbative effects. Second, the minimum was lifted to a metastable vacuum with a positive cosmological constant. This was accomplished by adding anti-D-branes and using previous results, obtained in Ref. [6], that the flux–anti-D-brane system can form a metastable bound state with positive energy. In Ref. [3], it was also shown that one can fine tune various parameters to make the value of the cosmological constant consistent with the observed amount of dark energy.

In this paper, we consider the problem of moduli stabilization in strongly coupled heterotic string theory [20,21] compactified on Calabi-Yau threefolds. Such compactifications are called heterotic M theory and have a number of phenomenologically attractive features (see Ref. [22] for a recent review of the phenomenological aspects of M theory). In Refs. [23–26], a specific set of vacua were constructed consisting of appropriate $SU(5)$ vector bundles over Calabi-Yau threefolds with $\mathbb{Z}_2$ fundamental group. These lead to four-dimensional theories with the standard model $SU(3)\times SU(2)\times U(1)$ gauge group and three families of charged chiral matter. Recently, in Refs. [27–29], these theories were generalized to vacua involving $SU(4)$ bundles over Calabi-Yau threefolds with $\mathbb{Z}_2\times\mathbb{Z}_2$ fundamental group. Such vacua correspond to standard model-like physics in four dimensions with potentially suppressed nucleon decay. In this paper, for simplicity, we consider only vector bundles over simply connected Calabi-Yau threefolds, compactifications which are easier to analyze. However, we see no reason why our results should not apply to more realistic heterotic vacua on nonsimply connected manifolds. Within this context, we will consider all geometric and vector bundle moduli. In addition, we include the translational moduli of M five-branes. One of the features of strongly coupled heterotic string compactifications is the presence of five-branes. In Refs. [30,31], it was argued that vacua with five-branes are more natural since, for example, it is much easier to satisfy the anomaly cancellation condition in their presence. Since, in order to obtain phenomenologically acceptable values for the fundamental constants one has to take the size of the eleventh dimension to be larger than the Calabi-Yau scale [32,33], the translational modes of the five-branes will appear as moduli in the four-dimensional low-energy effective action.

In this paper, we show that all heterotic M-theory moduli, that is, the complex structure, Kahler, vector bundle and five-brane moduli, can be stabilized by nonperturbative superpotentials. Recent discussions of this issue [34], in models with a restricted number of moduli, indicated instabilities caused by membrane instantons [35,36]. However, these models did not include all compactification and vector bundle moduli, as well as all possible sources for superpotentials. The analysis of Ref. [34] was refined in Ref. [37], where it was shown that stabilization of certain moduli can be achieved. Nevertheless, again, not all moduli were taken into account. In addition, the authors in Ref. [37] chose various parameters outside their natural range. In the present paper, we show that including all moduli and all superpotentials does lead to complete moduli stabilization in a phenomenologically accepted range with a negative cosmological constant. Furthermore, this is achieved within the natural values of the parameters. Our stabilization procedure uses various tools such as form flux, nonperturbative superpotentials including their bundle moduli dependent Pfaffians, and gaugino condensation on the hidden brane with threshold corrections. Even though supersymmetry is not broken in the moduli sector, it is softly broken in the gravity and matter sector at the TeV scale by the gaugino condensate [30,38–45].

The paper is organized as follows. In Sec. II, we review the Kahler potentials for all heterotic string moduli with a detailed discussion of their relative scales, including the vector bundle moduli Kahler potential. Even though the latter cannot be evaluated explicitly for vector bundles on a threefold,[1] it is possible to derive the relevant properties we will need in later sections. In Sec. III, we give a careful analysis of the superpotentials with a detailed discussion of their scales. In Sec. III A, we discuss the flux-induced superpotential derived in Refs. [47,48]. This superpotential depends on the complex structure moduli. In Sec. III B, we introduce the superpotential induced by a gaugino condensate on the hidden brane [30,38–42,44,45]. This superpotential depends on the Calabi-Yau volume as well as on the size of the eleventh dimension and the five-brane moduli through the threshold corrections. In Sec. III C, we discuss various nonperturbative superpotentials induced by membrane instantons [35,36]. They depend on the (1,1) moduli, the five-brane moduli, and the vector bundle moduli. Various pieces of these superpotentials were calculated in Refs. [34,50–53]. In order to obtain the total superpotential, one has to sum over contributions coming from all genus zero holomorphic curves in a given Calabi-Yau threefold. We give arguments that, in the models under consideration, the superpotential does not vanish after summation. In Sec. IV, we show, in detail, that all heterotic M theory moduli can be fixed at a stable anti–de Sitter (AdS) minimum. In Sec. IV A, we set up the model. Since the Kahler potentials and the superpotentials are very complicated when the number of (1,1) and vector bundle moduli is large, we have to introduce some simplification in order to obtain an analytic solution. We argue that if we restrict ourselves to consider only one (1,1) modulus, which coincides with the size of the eleventh dimension, and make some further restrictions on the number of the vector bundle moduli, we do not actually lose generality. Finally, in Sec. IV B, we prove the stabilization of the moduli. The complex structure moduli and the Calabi-Yau volume are stabilized by a mechanism similar to that considered in Refs. [3] and [11]. In addition, we show that the vector bundle and five-brane moduli are also fixed. We then analyze how our equations would be modified if we had many (1,1) and vector bundle moduli. We conclude that we would still find a stable solution, and hence that the restriction to a single (1,1) modulus and one vector bundle modulus was without loss of generality. There is only one mild constraint that we have to impose on a single coefficient to make sure that the five-brane modulus is stabilized in an acceptable range. In the Conclusion, we summarize our results. We also discuss the possibility of lifting our minimum to a metastable vacuum with a positive cosmological constant, as was done in Ref. [3]. These results will appear elsewhere.

[1]In fact, in Ref. [46], the vector bundle moduli Kahler potential was approximately computed for special types of Calabi-Yau threefolds and very special types of bundles. The bundles considered in Ref. [46] were taken to be the pullback of vector bundles on a surface. Such bundles admit a gauge connection that is approximately ADHM, provided the instanton is sufficiently small. To a generic bundle on a threefold that does not come from a bundle on lower-dimensional space, the method of Ref. [46] cannot be applied.

II. THE KAHLER POTENTIALS

We consider $E_8\times E_8$ strongly coupled heterotic superstring theory [20,21] on the space

$$M=\mathbb{R}^4\times X\times S^1/Z_2, \tag{2.1}$$

where X is a Calabi-Yau threefold. Let us list the complex moduli fields arising from such a compactification. They are the $h^{1,1}$ moduli T^I, the volume modulus S, the $h^{1,2}$ moduli Z_α and the vector bundle moduli, which we denote by Φ_u. In addition, we will assume that anomaly cancellation requires the existence of a nontrivial five-brane class. Furthermore, for simplicity, we will work in the region of its moduli space corresponding to a single five-brane [30,54]. The five-brane translational complex modulus will be denoted by $\mathbf{Y}$. In this section, we review the Kahler potentials for the T^I, S, Z_α, and $\mathbf{Y}$ moduli and derive some general properties of the vector bundle moduli Kahler potential.

The moduli T^I are defined as

$$T^I=Ra^IV^{-1/3}+\frac{\mathrm{i}}{6}p^I, \tag{2.2}$$

where R is the orbifold plane separation modulus, V is the Calabi-Yau breathing modulus, a^I are the (1,1) moduli of the Calabi-Yau space, and the imaginary parts p^I arise from the eleventh component of the graviphotons. The Calabi-Yau breathing modulus V also appears as the real part of the four-dimensional dilaton multiplet

$$S=V+\mathrm{i}\sqrt{2}\sigma, \tag{2.3}$$

where the imaginary part σ originates from dualizing the four-dimensional B field. The moduli a^I and V are not independent. It can be shown that

$$V=\frac{1}{6}\sum_{I,J,K=1}^{h^{1,1}} d_{IJK}a^Ia^Ja^K, \tag{2.4}$$

where d_{IJK} are the intersection numbers of the Calabi-Yau threefold. Note that the moduli V and R are dimensionless and defined as

$$V=\frac{1}{v_{CY}}\int_{CY}\sqrt{g_{CY}} \tag{2.5}$$

and

$$R=\frac{1}{\pi\rho}\int dx^{11}, \tag{2.6}$$

respectively. Here v_{CY} is the reference volume of the Calabi-Yau threefold, $\pi\rho$ is the reference length of the eleventh dimension, and x^{11} is the coordinate along the interval S^1/Z_2. The actual volume of the threefold and the actual size of the eleventh dimension are $v_{CY}V$ and $\pi\rho R$, respectively. See Refs. [55–57] for more details on the compactification of strongly coupled heterotic string theory to five and four dimensions and the structure of the chiral multiplets. To achieve the correct phenomenological values for the four-dimensional Newton and gauge coupling parameters,

$$M_{Pl}\sim 10^{19}\ \text{GeV},\quad \alpha_{GUT}\sim\frac{1}{25}, \tag{2.7}$$

we assume [32,33] that the inverse reference radius of the Calabi-Yau threefold and the inverse reference length of the eleventh dimension are

$$v_{CY}^{-1/6}\sim 10^{16}\ \text{GeV},\quad (\pi\rho)^{-1}\sim 10^{14}\ \text{GeV}, \tag{2.8}$$

respectively. This implies that, at the present time, the dimensionless moduli V and R have to be stabilized at, or be very slowly rolling near, the values

$$V\sim 1,\quad T\sim 1. \tag{2.9}$$

The Kahler potential for S and T^I moduli was computed in Ref. [57]. It is given by

$$K_{S,T}=-M_{Pl}^2\ln(S+\bar{S}) -M_{Pl}^2\ln\left(\frac{1}{6}\sum_{I,J,K=1}^{h^{1,1}} d_{IJK}(T+\bar{T})^I(T+\bar{T})^J(T+\bar{T})^K\right). \tag{2.10}$$

The Kahler potential for the complex structure moduli Z_α was found in Ref. [58] to be

$$K_Z=-M_{Pl}^2\ln\left(-i\int_X\Omega\wedge\bar{\Omega}\right), \tag{2.11}$$

where Ω is the holomorphic (3,0) form.

The next supermultiplet to discuss is the one associated with the five-brane modulus $\mathbf{Y}$. It was shown in Ref. [30] that, when a five-brane is compactified to four dimensions on a holomorphic curve z of genus g, there are two types of zero-mode supermultiplets that arise. First, there are g Abelian vector superfields, which are not of our interest in this paper. The second multiplet that arises is associated with the translational scalar mode Y. Geometrically Y corresponds to the position of the five-brane in the fifth dimension.[2] It was shown in Ref. [59] that the translational multiplet of the five-brane is a chiral supermultiplet whose bosonic component $\mathbf{Y}$ is given by

$$\mathbf{Y}=\frac{Y}{\pi\rho}\,\text{Re}\,\mathcal{T}+i\left(a+\frac{Y}{\pi\rho}\,\text{Im}\,\mathcal{T}\right). \tag{2.12}$$

Here a is the axion arising from dualizing the three-form field strength propagating on the five-brane world-volume and $\mathcal{T}$ is related to the (1,1)-moduli T^I as follows. Let ω_I, $I=1,\dots,h^{1,1}$ be a basis of harmonic (1,1) forms on our Calabi-Yau threefold. These are naturally dual to a basis z^I, $I=1,\dots,h^{1,1}$ of curves in $H_{(1,1)}(X)$ where

$$\frac{1}{v_5}\int_{z^I}\omega_J=\delta_{IJ}. \tag{2.13}$$

Parameter v_5 is the volume of the curve z on which the five-brane is wrapped. Any holomorphic curve can be expressed as a linear combination of the z^I curves. The curve on which the five-brane is wrapped can be written as

$$z=\sum_{I=1}^{h^{(1,1)}} c_I z^I \tag{2.14}$$

for some coefficients c_I. The modulus $\mathcal{T}$ which appears in (2.12) is defined as

$$\mathcal{T}=\sum_{I=1}^{h^{(1,1)}} c_I T^I. \tag{2.15}$$

The Kahler potential for the $\mathbf{Y}$ modulus was calculated in Ref. [59] and found to be

$$K_5=2M_{Pl}^2\tau_5\frac{(\mathbf{Y}+\bar{\mathbf{Y}})^2}{(S+\bar{S})(\mathcal{T}+\bar{\mathcal{T}})}, \tag{2.16}$$

with the coefficient τ_5 given by

$$\tau_5=\frac{T_5 v_5(\pi\rho)^2}{M_{Pl}^2} \tag{2.17}$$

and T_5 is

$$T_5=(2\pi)^{1/3}\left(\frac{1}{2\kappa_{11}^2}\right)^{2/3}, \tag{2.18}$$

where κ_{11} is the eleven-dimensional gravitational coupling constant. It is related to the four-dimensional Planck mass as

[2] Note that since $v_{CY}^{1/6}\ll\pi\rho$, the eleventh-dimensional coordinate x^{11} parametrizes the fifth dimension of the effective five-dimensional theory.

$$\kappa_{11}^2 = \frac{\pi\rho v_{CY}}{M_{Pl}^2}. \tag{2.19}$$

If we substitute Eq. (2.19) into (2.18) using (2.7) and (2.8), we obtain

$$\tau_5 \approx \pi \frac{v_5}{v_{CY}^{1/3}}. \tag{2.20}$$

Now let us move on to the vector bundle moduli Kahler potential. Its general expression can be obtained from the dimensional reduction of the term in the ten-dimensional action

$$\frac{-1}{4g_{10}^2}\,\mathrm{tr}\int d^{10}x\sqrt{-g}F_{MN}^2, \tag{2.21}$$

where $M,N=0,1,\ldots,9$ and g_{MN} and F_{MN} are the ten-dimensional metric and Yang-Mills field strength, respectively. Upon dimensional reduction, the ten-dimensional metric and gauge field split as follows:

$$ds_{10}^2 = v_{CY}^{-2/3} g_{\mu\nu}dx^\mu dx^\nu + v_{CY}^{1/3} g_{CYm\bar{m}} dX^m d\bar{X}^{\bar{m}},$$

$$A_M = (A_\mu, A_m, \bar{A}_{\bar{m}}), \tag{2.22}$$

where $\mu,\nu=0,1,2,3$ and $m,\bar{m}=1,2,3$. The fields $g_{\mu\nu}$ and A_μ are the four-dimensional metric and the gauge field, respectively, whereas $g_{m\bar{m}}$ and A_m represent the metric and the gauge connection on the Calabi-Yau threefold. Substituting (2.22) into the action (2.21), we obtain the following expression for the vector bundle moduli Kahler potential:

$$\tilde{K}_{bundle} = \frac{1}{2g_{10}^2}\,\mathrm{tr}\int d^6X\sqrt{g_{CY}}\,g^{m\bar{m}}A_m\bar{A}_{\bar{m}}. \tag{2.23}$$

Let us find the scale that controls the strength of the Kahler potential. To do this, introduce the dimensionless quantities

$$\tilde{X}^m = \frac{X^m}{v_{CY}^{1/6}}, \quad \tilde{A}_m = A_m v_{CY}^{1/6}, \tag{2.24}$$

where v_{CY} is the Calabi-Yau reference volume. We also normalize all vector bundle moduli associated with A_m with respect to the Calabi-Yau reference volume so that they too are dimensionless. The Kahler potential then becomes

$$\tilde{K}_{bundle} = \frac{v_{CY}^{2/3}}{2g_{10}^2}\,\mathrm{tr}\int d^6\tilde{X}\sqrt{g_{CY}}\,g^{m\bar{m}}\tilde{A}_m\bar{\tilde{A}}_{\bar{m}}. \tag{2.25}$$

The ten-dimensional gauge coupling parameter is related to the eleven-dimensional Planck scale as [20,21]

$$\frac{1}{g_{10}^2} = \frac{1}{2\pi\kappa_{11}^2}\left(\frac{\kappa_{11}^2}{4\pi}\right)^{2/3}. \tag{2.26}$$

From Eqs. (2.19) and (2.26) we obtain

$$\frac{v_{CY}^{2/3}}{2g_{10}^2} = \frac{1}{(4\pi)^{5/3}}\frac{M_{Pl}^2}{[(\pi\rho)^2M_{Pl}^2]^{1/3}}. \tag{2.27}$$

We can then write the Kahler potential $\tilde{K}_{bundle}$ as

$$\tilde{K}_{bundle} = kM_{Pl}^2K_{bundle}, \tag{2.28}$$

where

$$k = \frac{1}{(4\pi)^{5/3}[(\pi\rho)^2M_{Pl}^2]^{1/3}} \tag{2.29}$$

and

$$K_{bundle} = \mathrm{tr}\int d^6\tilde{X}\sqrt{g_{CY}}\,g^{m\bar{m}}\tilde{A}_m\bar{\tilde{A}}_{\bar{m}}. \tag{2.30}$$

Note that K_{bundle} is dimensionless since it depends on dimensionless vector bundle moduli. The parameter k is also dimensionless. Substituting (2.7) and (2.8) into (2.29), we obtain

$$k \sim 10^{-5}. \tag{2.31}$$

The reason that the strength of the vector bundle moduli Kahler potential is smaller by several orders of magnitude than the strength of the T, S, and Z Kahler potentials is that the F^2 term appears to the next order in α' in the ten-dimensional action as compared to the supergravity multiplet. Unfortunately, to the same order in α' in the ten-dimensional action and, as a consequence, to the same order in k in the four-dimensional action, there is a cross term between the T moduli and the vector bundle moduli. This cross term comes from the

$$\int d^{10}x\sqrt{-g}H\wedge H^* \tag{2.32}$$

term in the ten-dimensional action, where H is given by Ref. [21] (see also Ref. [57])

$$H = dB - \frac{1}{4\sqrt{2}\pi^2\rho}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}(\omega_{YM}-\omega_L) \tag{2.33}$$

and ω_{YM} and ω_L are the Yang-Mills and gravitational Chern-Simons forms, respectively. The term (2.32) leads to the following contribution to the four-dimensional effective action,

$$\sim kM_{Pl}^2\int d^4x\sqrt{-g_4}\,g^{\mu\nu}\sum_{I=1}^{h^{1,1}}(\mathrm{Im}\,T^I)$$
$$\times\int d^6\tilde{X}\sqrt{g_{CY}}\,\omega_I^{m\bar{m}}\partial_\mu\tilde{A}_m\partial_\nu\bar{\tilde{A}}_{\bar{m}}. \tag{2.34}$$

In this expression, $\tilde{X}$ and $\tilde{A}$ are the rescaled Calabi-Yau coordinates and gauge connection (2.24), $\omega_{Im\bar{m}}$ are the basis of the harmonic (1,1) forms on the Calabi-Yau threefold, and the coefficient k is given precisely by (2.29). This cross term does not significantly effect the Kahler potential for the T moduli, since it appears at a lower scale. However, it does effect the vector bundle moduli Kahler potential (2.28)–(2.30). Schematically, the pure vector bundle moduli Kahler metric can be written as

$$\int_X \partial\tilde{A}\overline{\partial\bar{\tilde{A}}}, \tag{2.35}$$

whereas the cross term can be written as

$$\sum_I (\mathrm{Im}\, T^I)\int_X \omega_I \partial\tilde{A}\overline{\partial\bar{\tilde{A}}}. \tag{2.36}$$

It is clear that the cross term can be ignored as long as the values of the imaginary parts of the T moduli are sufficiently smaller than one. For now, we will simply assume that this is the case and discard the cross term. Later, when studying stabilization issues, we will see that one can indeed stabilize the imaginary parts of the T moduli at values sufficiently less than one, thus justifying our assumption.

It is difficult to calculate the vector bundle moduli part of the Kahler potential explicitly without knowing a solution to the Hermitian Yang-Mills equations. Nevertheless, some properties of K_{bundle} can be determined. These properties will be sufficient to allow one to study the issues of moduli stabilization in later sections. At this point, we have to be more specific about the type of Calabi-Yau threefold we choose and the type of vector bundle we put over it. In this paper, the Calabi-Yau threefold will be taken to be elliptically fibered. For such Calabi-Yau spaces, there exists a rather explicit spectral cover construction of stable holomorphic vector bundles [60,61]. The moduli of such vector bundles were discussed in Ref. [62]. In the present paper, we will restrict our discussion to such vector bundles. Geometrically, their moduli space is just a complex projective space $\mathbb{CP}^N$, where N is the total number of the vector bundle moduli [62]. The moduli of vector bundles on elliptically fibered Calabi-Yau manifolds will be reviewed in more detail in the next section. For now, we will only need the fact that the moduli parametrize a complex projective space. Strictly speaking, the moduli space of bundles $\mathcal{M}$ is an open subset in $\mathbb{CP}^N$. The projective space is actually the compactification of $\mathcal{M}$ with respect to certain singular objects known as torsion-free sheaves. The gauge connection becomes singular on these sheaves. However, for simplicity, we will view $\mathbb{CP}^N$ as the moduli space of vector bundles, keeping in mind that it also contains singular points. At these points, the Kahler potential should blow up since the associated gauge connections do. As some of the vector bundle moduli approach certain critical values, the corresponding gauge connection represents a delta-function peak over some holomorphic curve in the Calabi-Yau threefold. These moduli are called the transition moduli associated with this curve [62]. We will cover our $\mathbb{CP}^N$ manifold with standard open sets isomorphic to $\mathbb{C}^N$ by introducing $N+1$ homogeneous coordinates and setting one of them to unity on one of the open sets. Let us consider any open patch $U_\alpha \subset \mathbb{CP}^N$ containing the transition moduli associated with some holomorphic curve. Denote this curve by z and let the number of the transition moduli be M. Let the N local coordinates on this open set be $\Phi_u = (\phi_i, \psi_a)$, where ϕ_i represent the transition moduli of the curve z and ψ_a the remaining moduli. The total number of parameters is, of course, N. One can always choose the coordinate system in such a way that the critical values of the transition moduli are

$$\phi_i = 0, \quad i = 1, \dots, M. \tag{2.37}$$

The codimension $N-M$ subset of $\mathbb{CP}^N$ defined by these equations represents a singularity of the type described above. When all of the ϕ_i go to zero, the bundle becomes a singular torsion-free sheaf. This corresponds to the gauge connection being a distribution that is infinitely peaked about the z curve and smooth everywhere else. As one turns the moduli ϕ_i on, the torsion-free sheaf smears out to produce a smooth vector bundle with an everywhere smooth Hermitian connection. It is clear that at the torsion-free sheaf, where the gauge connection has an infinite peak centered at z, the Kahler potential (2.30) diverges. Note that this is generically true at any singular point in moduli space. The above analysis allows us to say that for values of ϕ_i sufficiently small, we can approximately split the Kahler potential K_{bundle} as

$$K_{bundle} = K_{bundle}(\phi) + K_{bundle}(\psi). \tag{2.38}$$

The reason is that for ϕ_i's small enough, the gauge connection can approximately be written as

$$A = A(\phi) + A(\psi), \tag{2.39}$$

where $A(\phi)$ is strongly centered around the curve z and $A(\psi)$ is smooth everywhere. In the limit of small ϕ_i, the overlap integral of the product of these two pieces of the gauge connection is small. Then (2.38) follows from (2.39) and (2.30). We will also need to know what happens to K_{bundle} as the moduli (either ϕ_i or ψ_a) become large in the sense of coordinates on $\mathbb{C}^N$. Since

$$h^{1,1}(\mathbb{CP}^N) = 1, \tag{2.40}$$

there exists a unique cohomology class of Kahler forms. The Kahler form associated with the well-known Fubini-Study Kahler metric on $\mathbb{CP}^N$ is contained in this nontrivial cohomology class. This means that every Kahler potential on $\mathbb{CP}^N$ can be written as

$$K_{\mathbb{CP}^N} = K_{FS} + f, \tag{2.41}$$

where K_{FS} is the Fubini-Study Kahler potential and f is any global function. The only restriction on f is that the corresponding Kahler metric has to be positive definite. On the coordinate patch U_α with local coordinates Φ_u, we have

$$K_{\mathbb{CP}^N}|_{U_\alpha} = K_{FS}|_{U_\alpha} + f\rho_\alpha, \tag{2.42}$$

where

$$K_{FS}|_{U_\alpha}=\ln\left(1+\sum_{u=1}^{N}|\Phi_u|^2\right) \tag{2.43}$$

and $\{\rho_\alpha\}$ is the partition of unity. As we approach the boundary of the open set,

$$\rho_\alpha\rightarrow 0. \tag{2.44}$$

Furthermore, from (2.43) we find that

$$K_{\mathbb{CP}^N}|_{U_\alpha}\rightarrow\infty \tag{2.45}$$

in this limit. From this analysis, we conclude that K_{bundle} grows as one increases either one of the ϕ_i's or one of the ψ_a's keeping the other variables fixed. These properties of K_{bundle} will be important in the next sections.

III. SUPERPOTENTIALS

In this section, we discuss the superpotentials that will be used to achieve the stabilization of all moduli considered above.

A. The flux-induced superpotential

We want to turn on a nonzero flux of the Neveu-Schwarz three-form H on the Calabi-Yau threefold. The presence of this nonzero flux generates a superpotential for the $h^{1,2}$ moduli of the form [47,48]

$$W_f\sim\int_X H\wedge\Omega. \tag{3.1}$$

This is the heterotic analog of the type IIB superpotential [10,12,13]

$$W_{IIB}\sim\int_X G_3\wedge\Omega, \tag{3.2}$$

where $G_3=F_3-\tau H_3$. Expression (3.1) can be obtained by considering the variation of the ten-dimensional gravitino, dimensionally reducing this to four dimensions and matching it against the well-known gravitino transformation law in four-dimensional supergravity. See Ref. [48] for a detailed derivation.

For later use, we need to find the scale that controls W_f. Since the components of H have dimension one, we find that

$$W_f=\frac{M_{Pl}^2}{v_{CY}}\int_X H\wedge\Omega. \tag{3.3}$$

As before, introduce dimensionless coordinates $\tilde{X}^m$

$$\tilde{X}^m=\frac{X^m}{v_{CY}^{1/6}} \tag{3.4}$$

and dimensionless components for the three-form. Since H is quantized in units of [49]

$$\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\frac{1}{\pi\rho}, \tag{3.5}$$

the components of H and the dimensionless three-form $\tilde{H}$ are related by

$$H_{mnp}=\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\frac{1}{\pi\rho v_{CY}^{1/2}}\tilde{H}_{mnp}. \tag{3.6}$$

As a consequence, W_f can be written as

$$W_f=\frac{M_{Pl}^2}{v_{CY}^{1/2}}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\frac{1}{\pi\rho}\int_X\tilde{H}\wedge\tilde{\Omega}=M_{Pl}^3h_1\int_X\tilde{H}\wedge\tilde{\Omega}, \tag{3.7}$$

where, using Eqs. (2.7) and (2.8), we find

$$h_1=\frac{1}{M_{Pl}^3v_{CY}^{1/2}}\sim 2\times 10^{-8} \tag{3.8}$$

and $\tilde{H}$ and $\tilde{\Omega}$ are both dimensionless.

Note that turning on a nonvanishing flux warps the compactification space away from a pure Calabi-Yau threefold. The strength of this warping is determined by the dimensionless parameter

$$\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\frac{1}{\pi\rho v_{CY}^{1/2}}\int_C\tilde{H}, \tag{3.9}$$

where C is an appropriate three cycle. Since $(\kappa_{11}/4\pi)^{2/3}(1/\pi\rho v_{CY}^{1/2})\sim 2\times 10^{-5}$, it follows that for

$$\int_C\tilde{H}\ll\frac{1}{2}10^5 \tag{3.10}$$

the warping away from a Calabi-Yau threefold is negligibly small. Henceforth, we will always choose the flux to satisfy condition (3.10).

B. Gaugino condensation induced superpotential

We also turn on a gaugino condensate on the hidden brane [30,38–42,44,45]. A nonvanishing gaugino condensate has important phenomenological consequences. Among other things, it is responsible for supersymmetry breaking in the hidden sector. When that symmetry breaking is transported to the observable brane, it leads to soft supersymmetry breaking terms for the gravitino, gaugino, and matter fields on the order of the electroweak scale. A gaugino condensate is also relevant to the discussion in this paper, since it produces a superpotential for S, T, and $\mathbf{Y}$ moduli of the form

$$W_g=M_{pl}^3h_2\exp\left(-\epsilon S+\epsilon\alpha_I^{(2)}T^I-\epsilon\beta\frac{\mathbf{Y}^2}{\mathcal{T}^2}\right). \tag{3.11}$$

Here [44]

$$h_2 \sim \frac{1}{M_{Pl}\sqrt{v_{CY}}(\pi\rho)}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3} \sim 10^{-6}, \tag{3.12}$$

and the coefficient ϵ is related to the coefficient b of the one-loop beta function and is given by

$$\epsilon = \frac{6\pi}{b_0 \alpha_{GUT}}. \tag{3.13}$$

For example, for the E_8 gauge group $b_0 = 90$. Taking α_{GUT} to have its phenomenological value given in (2.7), we obtain

$$\epsilon \sim 5. \tag{3.14}$$

The coefficients $\alpha_I^{(2)}$ represent the tension of the hidden brane measured with respect to the Kahler form ω_I [57]

$$\alpha_I^{(2)} \sim \frac{\pi\rho}{16\pi v_{CY}}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\int_X \omega_I \wedge\left(\operatorname{Tr} F^{(2)}\wedge F^{(2)} - \frac{1}{2}\operatorname{Tr} R\wedge R\right), \tag{3.15}$$

where $F^{(2)}$ is the curvature of the gauge bundle on the hidden brane. One can estimate the order of magnitude of $\alpha_I^{(2)}$ by evaluating the right-hand side of Eq. (3.15). We find that

$$\alpha_I^{(2)} \approx \frac{v_I}{v_{CY}^{1/3}}, \tag{3.16}$$

where v_I is the volume (measured with respect to the Kahler form ω_I) of the two-cycle, which is Poincaré dual to the four-form $\operatorname{Tr} R\wedge R - \frac{1}{2}\operatorname{Tr} F^{(2)}\wedge F^{(2)}$. Similarly, the coefficient β is the tension of the five-brane and given by [63]

$$\beta = \frac{2\pi^2\rho}{v_{CY}^{2/3}}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}\int_X \sum_{I=1}^{h^{1,1}} c_I \omega_I \wedge \mathcal{W}, \tag{3.17}$$

where $\mathcal{W}$ is the four-form Poincaré dual to the holomorphic curve z on which the five-brane is wrapped. Evaluation of the right-hand-side of Eq. (3.17) gives

$$\beta \approx \pi \frac{v_5}{v_{CY}^{1/3}}, \tag{3.18}$$

where v_5 is the volume of the holomorphic curve the five-brane is wrapped on. Note that β is always positive and, from (2.20), is of the same order of magnitude as τ_5. The real part of the combination

$$S - \alpha_I^{(2)} T^I + \beta \frac{\mathbf{Y}^2}{\mathcal{T}^2} \tag{3.19}$$

represents the inverse square of the gauge coupling constant on the hidden brane, with the last two terms being the threshold corrections [30,44].

Note that it is essential that expression (3.19) be strictly positive at the vacuum of the theory. This prevents the effective gauge coupling from diverging, or being undefined, on the hidden orbifold plane. For this to be the case, we must have

$$\operatorname{Re}(\alpha_I^{(2)} T^I) < \operatorname{Re}\left(S + \beta \frac{\mathbf{Y}^2}{\mathcal{T}^2}\right). \tag{3.20}$$

In this paper, we want to work in the strong coupling regime of the heterotic string. It follows that one of the T^I moduli, corresponding to the size of the fifth dimension, must be at least of order unity. Hence, for (3.20) to be satisfied, typically, we must choose the associated $\alpha^{(2)} < 1$. We find that this can always be arranged by the appropriate choice of the vector bundle on the hidden orbifold plane. In fact, in Ref. [19] it was argued that this assumption may be unnecessary if one includes higher-order field-theory corrections that are protected by supersymmetry. This might provide generalizations of the results obtained in this paper as well.

C. Nonperturbative superpotentials

In this section, we will review the structure of nonperturbative superpotentials generated by strings wrapped on holomorphic curves. To be more precise, the nonperturbative contributions to the superpotential come from membrane instantons. As was shown in Ref. [50], to preserve supersymmetry a membrane has to be transverse to the end-of-the-world branes and wrap a holomorphic curve in the Calabi-Yau threefold. In addition, only curves of genus zero contribute [2,36]. At energy scales smaller that the brane separation scale, the membrane configuration reduces to that of a string wrapped on a holomorphic curve. We will refer to such a configuration as a heterotic string instanton. We should point out that there can be three different membrane configurations leading to different nonperturbative contributions to the superpotential.

(1) A membrane can stretch between the two orbifold fixed planes.
(2) A membrane can begin on the visible brane and end on the five-brane in the bulk. Recall that, in this paper, we are assuming that there is only one five-brane in the bulk.
(3) A membrane can begin on the five-brane and end on the hidden brane.

We will discuss the first configuration in detail and then comment on the configurations (2) and (3). It was shown in Ref. [64] that the nonperturbative contribution to the superpotential of a string wrapped on an isolated curve z has the structure

$$W_{np}[z] \propto \operatorname{Pfaff}(\mathcal{D}_-)\exp\left(-\tau \sum_{I=1}^{h^{1,1}} \tilde{\omega}_I T^I\right). \tag{3.21}$$

Let us first discuss the exponential factor

$$\exp\left(-\tau \sum_{I=1}^{h^{1,1}} \tilde{\omega}_I T^I\right) \tag{3.22}$$

which was calculated in Ref. [50]. The coefficient τ in (3.22) is defined as

$$\tau=\frac{1}{2}T_M(\pi\rho)v_z, \tag{3.23}$$

where T_M is the membrane tension given by

$$T_M=(2\pi)^{1/3}\left(\frac{1}{2\kappa_{11}^2}\right)^{1/3} \tag{3.24}$$

and v_z is the volume of the holomorphic curve z. By using Eqs. (2.19), (2.7), and (2.8), we get

$$\tau\sim 500\,\frac{v_z}{v_{CY}^{1/3}}. \tag{3.25}$$

Everywhere in the paper, τ will be taken to be much greater than one which is naturally the case. Furthermore, the $\tilde{\omega}_I$ appearing in (3.22) are the integrals of the pullbacks to the holomorphic curve z of the Ith harmonic (1,1) form on Calabi-Yau threefold. See Ref. [50] for details. Note, that the exponential factor (3.22) gives the nonperturbative contribution to the superpotential for the T moduli, but not for the Calabi-Yau volume modulus S. For example, when $h^{1,1}=1$, the factor (3.22) becomes

$$\exp(-\tau T), \tag{3.26}$$

where

$$T=R+\frac{i}{6}p. \tag{3.27}$$

This shows that the superpotential associated with (3.26) depends on the size of the eleventh dimension only.

Now let us move on to the first factor in (3.21). This factor,

$$\mathrm{Pfaff}(\mathcal{D}_-), \tag{3.28}$$

represents the Pfaffian of the chiral Dirac operator constructed using the Hermitian Yang-Mills connection pulled back to the curve z [52,53,64]. It is clear that it depends on the vector bundle moduli. So far, our discussion has been basically generic. The only restriction on the Calabi-Yau geometry that we have made so far was to assume, in the second half of Sec. II, that it is elliptically fibered. At this point, for specificity, we will choose the Calabi-Yau threefold X to be elliptically fibered over a Hirzebruch surface

$$B=\mathbb{F}_r. \tag{3.29}$$

Let us mention some basic properties of Hirzebruch surfaces that we will need. The second homology group $H_2(\mathbb{F}_r,\mathbb{Z})$ is spanned by two effective classes of curves, denoted by $\mathcal{S}$ and $\mathcal{E}$, with intersection numbers

$$\mathcal{S}\cdot\mathcal{S}=-r,\quad \mathcal{S}\cdot\mathcal{E}=1,\quad \mathcal{E}\cdot\mathcal{E}=0. \tag{3.30}$$

The first Chern class of $\mathbb{F}_r$ is given by

$$c_1(\mathbb{F}_r)=2\mathcal{S}+(r+2)\mathcal{E}. \tag{3.31}$$

Finally, we will assume that X admits a global section σ and that it is unique, which is generically the case.

A Yang-Mills vacuum consists of a stable, holomorphic vector bundle V on the observable end-of-the-world brane with the structure group

$$G\subseteq E_8. \tag{3.32}$$

In general, there can be a vector bundle on the hidden brane. However, in this paper, we will assume that this bundle is trivial. It follows from Refs. [65,66] that each such bundle admits a unique connection satisfying the hermitian Yang-Mills equations. Over X we will construct a stable, holomorphic vector bundle V with structure group

$$G=SU(n). \tag{3.33}$$

This is accomplished [60,61] by specifying a spectral cover

$$\mathcal{C}=n\sigma+\pi^*\eta, \tag{3.34}$$

where

$$\eta=(a+1)\mathcal{S}+b\mathcal{E} \tag{3.35}$$

with $a+1$ and b being non-negative integers, as well as a holomorphic line bundle

$$\mathcal{N}=\mathcal{O}_X[n(\lambda+\tfrac{1}{2})\sigma-(\lambda-\tfrac{1}{2})\pi^*\eta+(n\lambda+\tfrac{1}{2})\pi^*c_1(\mathbb{F}_r)], \tag{3.36}$$

where $\lambda\in\mathbb{Z}+\frac{1}{2}$. In Eqs. (3.34) and (3.36), π is the projection map $\pi:X\rightarrow\mathbb{F}_r$. Note that we use $a+1$, rather than a, as the coefficient of $\mathcal{S}$ in (3.35) to conform with our conventions in Ref. [62]. We will also assume that the variables $a+1$ and b satisfy the positivity conditions [62]

$$a+1>2n,\quad b>ar-n(r-2). \tag{3.37}$$

These conditions ensure that the spectral cover $\mathcal{C}$ is an ample, or positive, divisor. The vector bundle V is then determined via a Fourier-Mukai transformation

$$(\mathcal{C},\mathcal{N})\leftrightarrow V. \tag{3.38}$$

The moduli of the bundle V come from parameters of the spectral cover $\mathcal{C}$. Since the parameters of a divisor form a complex projective space, the moduli space of vector bundles is $\mathbb{CP}^N$, where N is the number of the vector bundle moduli. This fact was already used in Section 1 in our discussion of the properties of the vector bundle moduli Kahler potential. In Refs. [52,53], the Pfaffian $\mathrm{Pfaff}(\mathcal{D}_-)$ was computed in a number of examples for the case of a superstring wrapped on the isolated sphere $\sigma\cdot\pi^*\mathcal{S}$. The Pfaffian was found to be a high-degree polynomial of the vector bundle moduli. In fact, it turned out that it depends only on a subset of the vector bundle moduli, the transition moduli, which are responsible for smoothing out the torsion-free sheaf localized at the curve $\sigma\cdot\pi^*\mathcal{S}$ [62].

In order to find the total nonperturbative superpotential, one has to sum up the contributions from all holomorphic genus zero curves, both isolated and nonisolated. As argued in Refs. [67–69], in certain cases one can actually get zero after the summation. This makes it necessary to discuss the genus zero holomorphic curves in Calabi-Yau threefolds of the type introduced above. After this discussion, we will be able to argue that, in these models, the superpotential does not vanish after the summation. The first class of genus zero holomorphic curves are of the form

$$z=\sigma\cdot\pi^*z', \tag{3.39}$$

where z' is a genus zero holomorphic curve in the base $\mathbb{F}_r$. Below, we will often identify z with z' for notation simplicity. For specificity, let us take the base of the Calabi-Yau threefold to be $\mathbb{F}_2$. Our results will, however, remain true for other Hirzebruch surfaces as well. The Hirzebruch surface $\mathbb{F}_2$, being a rationally ruled surface, contains one isolated genus zero curve $\mathcal{S}$ and infinitely many nonisolated curves. These can be shown to be

$$\mathcal{E} \text{ and } \mathcal{S}+\kappa\mathcal{E}, \tag{3.40}$$

where κ is an integer number greater than one. Let us consider a concrete example. In Ref. [53], it was shown that for the following choice of parameters,

$$n=3,\quad b-2a=3,\quad \lambda=\frac{3}{2}, \tag{3.41}$$

there are nine transition moduli, denoted by $\alpha_i,\beta_i,\gamma_i$, for $i=1,2,3$, associated with the curve $\sigma\cdot\pi^*\mathcal{S}$. The Pfaffian generated by a string wrapped on the curve $\sigma\cdot\pi^*\mathcal{S}$ is nonzero and given by the expression

$$\mathrm{Pfaff}(\mathcal{D}_-)_{\mathcal{S}}=\mathcal{R}^4, \tag{3.42}$$

where $\mathcal{R}$ is the polynomial

$$\mathcal{R}=\alpha_1\beta_2\gamma_3-\alpha_1\beta_3\gamma_2+\alpha_2\beta_3\gamma_1 -\alpha_2\beta_1\gamma_3+\alpha_3\beta_1\gamma_2-\alpha_3\beta_2\gamma_1. \tag{3.43}$$

We will now show, in the context of this example, that one can further restrict the coefficient a in such a way that the vector bundle moduli contribution to the superpotential, that is, the Pfaffian, vanishes on all nonisolated curves of the type (3.40).

As discussed in detail in Refs. [52,64,70], given a holomorphic genus zero curve z, the Pfaffian will vanish if and only if the restriction of the bundle V to the curve z is nontrivial or, equivalently, that

$$h^0(z,V|_z\otimes\mathcal{O}_z(-1))>0. \tag{3.44}$$

It was shown in Refs. [52,53] that

$$h^0(z,V|_z\otimes\mathcal{O}_z(-1))=h^0(C,N(-F)|_C), \tag{3.45}$$

where

$$C=\mathcal{C}|_{\pi^*z},\quad N=\mathcal{N}|_{\pi^*z},\quad N(-F)=N\otimes\mathcal{O}_{\pi^*z}(-F) \tag{3.46}$$

and F is the fiber class. We will show that for nonisolated curves of the form (3.40), $h^0(C,N(-F)|_C)$ does not vanish for any value of the vector bundle moduli. Therefore, the Pfaffian and hence the superpotential generated on such curves will vanish identically. The proof goes as follows. The vector space $H^0(C,N(-F)|_C)$ lies in the exact sequence

$$0\to H^0(\pi^*z,N(-F-C))\to H^0(\pi^*z,N(-F)) \to H^0(C,N(-F)|_C)\to\dots. \tag{3.47}$$

It is easy to see that

$$h^0(\pi^*z,N(-F))\geqslant h^0(\pi^*z,N(-F-C)), \tag{3.48}$$

with the equality holding if and only if

$$h^0(\pi^*z,N(-F))=0. \tag{3.49}$$

On the other hand, it follows from the exact sequence (3.47) that if

$$h^0(\pi^*z,N(-F))>h^0(\pi^*z,N(-F-C)), \tag{3.50}$$

the dimension of the space $H^0(C,N(-F)|_C)$ cannot be zero and, therefore, the Pfaffian will vanish. So, it is enough to show that for the curves of the form (3.40) the following inequality is fulfilled:

$$h^0(\pi^*z,N(-F))>0. \tag{3.51}$$

Slightly abusing notation, we will denote the curves $\sigma\cdot\pi^*\mathcal{E}$ and $\sigma\cdot\pi^*(\mathcal{S}+\kappa\mathcal{E})$ in the threefold by $\mathcal{E}$ and $\mathcal{S}+\kappa\mathcal{E}$, respectively. Using Eqs. (3.30)–(3.35) and (3.41), one can show that

$$N(-F)|_{\pi^*\mathcal{E}}=\mathcal{O}_{\pi^*\mathcal{E}}(6\sigma|_{\pi^*\mathcal{E}}-(a-8)F)). \tag{3.52}$$

If we demand that a satisfy the positivity conditions (3.37), it follows from (3.52) that condition (3.51) is fulfilled for

$$a=6,7,8. \tag{3.53}$$

This means that, for these choices of a, the superpotential of a string wrapped on a nonisolated curve in the homology class of $\mathcal{E}$ will vanish for every representative in this class. Similarly, one finds that

$$N(-F)|_{\pi^*(\mathcal{S}+\kappa\mathcal{E})} =\mathcal{O}_{\pi^*(\mathcal{S}+\kappa\mathcal{E})}(6\sigma|_{\pi^*(\mathcal{S}+\kappa\mathcal{E})}-[(a-9)\kappa+3]F). \tag{3.54}$$

We see that condition (3.51) is fulfilled if and only if

$$(a-9)\kappa+3\leqslant 0,\quad \kappa>1. \tag{3.55}$$

Equation (3.55) is satisfied for

$$a=6,7. \tag{3.56}$$

We conclude that, in the examples specified by

$$n=3, \quad r=2, \quad \lambda=\frac{3}{2}, \quad b-2a=3, \quad a=6,7 \tag{3.57}$$

of the curves of type (3.39), only the isolated curve $\mathcal{S}$ gives a contribution to the superpotential. The contributions of all nonisolated curves vanish identically due to the vanishing of the Pfaffian. Even though these results have been proven within the context of a specific example, they are, in fact, generic, occuring for different values of n, r, λ, a, and b.

Unfortunately, the lifts of $\mathcal{S}$, $\mathcal{E}$, and $\mathcal{S}+\kappa\mathcal{E}$ are not the only genus zero, holomorphic curves in X. There may exist (perhaps infinitely many) such curves contained in multisections of X.[3] These curves are regular in X, but project onto singular curves in the base. They can also be divided into two types, curves that are isolated in X and those that are not isolated. We will denote by $\{I_x\}$ the set of such isolated curves, where x indexes these curves. Similarly, let $\{N_y\}$ be the set of nonisolated curves indexed by y. To continue our analysis, we must consider the Pfaffian on each of these curves as well. Let us begin with the nonisolated curves $\{N_y\}$. In general, we have no reason to believe that the Pfaffian must vanish on each of these curves, as it did on the nonisolated curves $\mathcal{E}$ and $\mathcal{S}+\kappa\mathcal{E}$ in the zero section. Therefore, these nonisolated curves may contribute to the superpotential. However, since each such curve is nonisolated, one must ‘‘integrate’’ over the moduli of the curve. To perform such an ‘‘integration,’’ even to define it properly, is a difficult open problem. However, it has been conjectured by Witten [71] that every nonisolated curve gives zero contribution to the superpotential. In this paper, we will henceforth assume that this conjecture is indeed correct and there is no further contribution to the superpotential arising from nonisolated curves $\{N_y\}$.

What about the isolated curves $\{I_x\}$? Generically, we expect strings wrapped around each curve I_x to produce a nonvanishing superpotential $W_{np}[I_x]$. The whole nonperturbative superpotential generated by membranes stretched between the two orbifold planes can then be written as

$$W_{np}=W_{np}[\mathcal{S}]+\sum_x W_{np}[I_x]. \tag{3.58}$$

We now want to make a very important point. For a generic Calabi-Yau threefold of the type considered here, one can show that none of the curves I_x intersects $\mathcal{S}$. That is,

$$\mathcal{S}\cdot I_x=0 \tag{3.59}$$

for all values of x. This leads to the following conclusion. That is, the superpotentials $W_{np}[\mathcal{S}]$ and $\Sigma_x W_{np}[I_x]$ depend on different vector bundle moduli. Let ϕ_i be the transition moduli associated with the curve $\mathcal{S}$. Since $\mathcal{S}$ and all I_x do not intersect, they do not share transition moduli. Therefore, the sum $\Sigma_x W_{np}[I_x]$ does not depend on ϕ_i. Similarly, $W_{np}[\mathcal{S}]$ does not depend on the vector bundle moduli associated with any of the curves I_x. Let us now split the vector bundle moduli Φ_u as

$$\Phi_u=\{\phi_i,\chi_{\tilde{i}},\psi_a\}, \tag{3.60}$$

where ϕ_i are the transition moduli associated with the curve $\mathcal{S}$, $\chi_{\tilde{i}}$ are the transition moduli associated with all the curves I_x, and ψ_a are the remaining moduli. Since we do not expect that the whole second Chern class of the bundle V is localized on the isolated curves, the moduli ϕ_i and $\chi_{\tilde{i}}$ do not span the entire moduli space. We can now rewrite the superpotential (3.58) as

$$W_{np}=W_{np}(\phi)+W_{np}(\chi), \tag{3.61}$$

where the vector bundle moduli ϕ_i and $\chi_{\tilde{i}}$ do not overlap. Note that W_{np} is independent of the moduli ψ_a. This, in particular, shows that the nonperturbative superpotential is not zero if at least one of the terms is not zero. The term $W_{np}(\phi)$ was calculated and found to be nonvanishing in a number of examples in Refs. [52,53]. For example, the Pfaffian on $\mathcal{S}$ was computed to be (3.42) in our $B=\mathbb{F}_2$ example above. This means that, in such examples, the nonperturbative superpotential is not zero provided the conjecture about the vanishing of the superpotential on nonisolated curves is indeed correct.

Let us now give the generalization of the above discussion to the case when a membrane stretches between one of the orbifold planes and a five-brane. As we have said, in this paper we will assume that there is a single five-brane in the bulk. The nonperturbative superpotential for such a membrane configuration was calculated in Refs. [51,34]. The contribution has a form very similar to (3.21). When a membrane begins on the observable brane and ends on the five-brane wrapped on an isolated genus zero holomorphic curve z, the superpotential is

$$W_5^{(1)}\propto \mathrm{Pfaff}(\mathcal{D}_-)e^{-\tau\mathbf{Y}}, \tag{3.62}$$

where $\mathcal{D}_-$ is the chiral Dirac operator associated with the bundle V on the observable brane restricted to z and the coefficient τ is given in (3.23). When the membrane stretches between the five-brane and the hidden brane, the superpotential will be

$$W_5^{(2)}\propto \mathrm{Pfaff}(\mathcal{D}_-^{hidden})e^{-\tau(\mathcal{T}-\mathbf{Y})}. \tag{3.63}$$

By $\mathrm{Pfaff}(\mathcal{D}_-^{hidden})$, we denote the Pfaffian of the Dirac operator constructed using the pullback to z of the hermitian Yang-Mills connection on the hidden brane. If the vector bundle on the hidden brane is trivial, as we are assuming in this paper, the Pfaffian is simply a constant, independent of moduli, and the corresponding contribution to the superpotential becomes

$$W_5^{(2)}\propto e^{-\tau(\mathcal{T}-\mathbf{Y})}. \tag{3.64}$$

[3]The authors are very grateful to R. Donagi and T. Pantev for discussions on this issue.

Before closing this section, we want to be a little more explicit about what we mean by the assumption that there is a single five-brane in the bulk. For a trivial vector bundle on the hidden brane, the anomaly cancellation condition determines the five-brane class to be

$$\mathcal{W} = c_2(TX) - c_2(V). \tag{3.65}$$

It was shown in Ref. [54] that the moduli space of the homology class $\mathcal{W}$ always contains an irreducible representative curve. Physically, this corresponds to a single five-brane in the bulk space. In this paper, we always take the five-brane to be wrapped on an irreducible curve.

IV. MODULI STABILIZATION

A. Setting up a model

In this section, we will provide a stabilization of the moduli considered above. Unfortunately, the Kahler potentials (2.10) and (2.16) and the superpotentials (3.21), (3.61), (3.62), and (3.64) are very complicated when the number of the (1,1) moduli T^I and the vector bundle moduli $(\phi_i, \chi_{\tilde{i}})$ is large. For the case of Calabi-Yau threefolds elliptically fibered over the Hirzebruch surfaces, the number of the T moduli is three and the number of the vector bundle moduli is of order 100 or larger [62]. The number of transition moduli associated with the curve $\mathcal{S}$ is also quite large, of order ten [62]. Therefore, to give an explicit analytic solution, we have to simplify the model without losing its essential properties. Our first step in this direction will be to assume that we have only one (1,1) modulus. We can do this without loss of generality since, as will become clear in our analysis, any number of the T moduli can be stabilized by the same mechanism. Let us emphasize that the reason for doing this is purely technical. We just want to simplify the equations. We will comment on this further at the appropriate place. Henceforth, we will take only one T modulus, which is associated with the size of the eleventh dimension. We now have to make some simplifications concerning the vector bundle moduli. In the preceding section, we split the vector bundle moduli Φ_u into three categories, the transition moduli ϕ_i, associated with the curve $\mathcal{S}$, the transition moduli $\chi_{\tilde{i}}$, associated with the curves $\{I_x\}$, and the remaining moduli ψ_a. Clearly, the equations of motion for the ϕ_i- and $\chi_{\tilde{i}}$-moduli are very similar. Therefore, we may assume that there are no $\chi_{\tilde{I}}$ moduli at all without any loss of generality. If we manage to stabilize the ϕ_i moduli, the moduli $\chi_{\tilde{i}}$ will be stabilized by precisely the same procedure. Ignoring the χ moduli does not produce conceptual changes in the structure of the T and Y superpotentials either. Specifically, the existence of the χ terms in (3.61), in addition to the first term, can at most produce a racetrack potential energy for the T and Y moduli. This would only strengthen the vacuum stability. However, we must continue to keep the ψ_a moduli, since they do not appear in any of the superpotentials discussed in the preceding section and ignoring them can conceptually alter the potential energy. As a result, we will assume that ϕ_i and ψ_a are all of the vector bundle moduli. We now want to introduce simplifications concerning the number of ϕ_i moduli. In all of the examples studied in Refs. [52,53], it was found that the number of transition moduli associated with the curve $\mathcal{S}$ is large and that the corresponding Pfaffian is a complicated homogeneous polynomial of high degree. Again, for simplicity, we will pretend that there is only one ϕ modulus. From the discussion of its stabilization, it will be obvious that any number of ϕ moduli can be stabilized by the same mechanism. Therefore, we can restrict ourselves to a single ϕ modulus without any loss of generality. To conclude, we consider a model containing the following moduli. We have one T modulus geometrically corresponding to the separation of the orbifold planes, the S modulus corresponding to the Calabi-Yau volume, $h^{1,2}$ moduli Z_α whose precise number is irrelevant, one transition vector bundle modulus ϕ, the remaining vector bundle moduli ψ_a, whose precise number is also irrelevant, and one five-brane modulus $\mathbf{Y}$. We emphasize, once again, that these simplifications are made for purely technical reasons to simplify the equations. Any number of the T, ϕ, and χ moduli can be stabilized by a similar method.

Let us write the simplified Kahler potential and the superpotential relevant for our model. We have

$$K = K_{S,T} + K_Z + K_5 + kK_{bundle}. \tag{4.1}$$

In this expression, $K_{S,T}$ is given by [see Eq. (2.10)]

$$K_{S,T} = -M_{Pl}^2 \ln(S+\bar{S}) - 3M_{Pl}^2(T+\bar{T}), \tag{4.2}$$

where Eq. (2.4) has been used. In order to ignore the cross term (2.36), we must always work in a region of moduli space where

$$|\mathrm{Im}\, T| \ll 1. \tag{4.3}$$

The $h^{2,1}$ moduli Kahler potential K_Z is given in (2.11) by

$$K_Z = -M_{Pl}^2 \ln\left(-i\int_X \Omega\wedge\bar{\Omega}\right). \tag{4.4}$$

The five-brane Kahler potential K_5 (2.16) now becomes

$$K_5 = 2M_{Pl}^2 \tau_5 \frac{(\mathbf{Y}+\bar{\mathbf{Y}})^2}{(S+\bar{S})(T+\bar{T})}. \tag{4.5}$$

By definition [see Eq. (2.12)]

$$0 \leq \mathrm{Re}\, \mathbf{Y} \leq \mathrm{Re}\, T, \tag{4.6}$$

since the five-brane must be between the orbifold planes. The vector bundle moduli Kahler potential is not known explicitly. However, from our discussion at the end of Sec. II, we concluded that K_{bundle} can be split as follows:

$$K_{bundle} = K_{bundle}(\phi) + K_{bundle}(\psi_a). \tag{4.7}$$

We also know that for small values of ϕ, K_{bundle} must diverge. For concreteness, when ϕ is sufficiently less than one, we take $K_{bundle}(\phi)$ to be

$$K_{bundle}(\phi) = -p\ln(\phi+\bar{\phi}), \tag{4.8}$$

where p is some dimensionless, positive constant. Expression (4.8) is the function that diverges most softly at zero. However, one can choose $K_{bundle}(\phi)$ to be any other function that diverges at zero, for example, an inverse polynomial in ϕ. We can show that ϕ can be stabilized for any such functions.

Let us now summarize the superpotential. The total superpotential is given by

$$W = W_f + W_g + W_{np} + W_5^{(1)} + W_5^{(2)}. \tag{4.9}$$

Here, W_f is the flux-induced superpotential [see Eq. (3.7)]

$$W_f = M_{Pl}^3 h_1 \int_X \tilde{H} \wedge \tilde{\Omega}, \tag{4.10}$$

where

$$h_1 \sim 2 \times 10^{-8}. \tag{4.11}$$

W_g is the superpotential induced by the gaugino condensation on the hidden wall. In our model, it follows from (3.11) that

$$W_g = M_{pl}^3 h_2 \exp\left(-\epsilon S + \epsilon \alpha^{(2)} T - \epsilon \beta \frac{\mathbf{Y}^2}{T^2} \right), \tag{4.12}$$

where [see Eqs. (3.12), (3.16), and (3.18)]

$$h_2 \sim 10^{-6}, \quad \alpha^{(2)} \sim \frac{v}{v_{CY}^{1/3}}, \quad \beta \sim \pi \frac{v_5}{v_{CY}^{1/3}}. \tag{4.13}$$

According to our discussion in the preceding section, to make sure that the combination

$$\mathrm{Re}\left(S - \alpha^{(2)} T + \beta \frac{\mathbf{Y}^2}{T^2} \right) \tag{4.14}$$

is positive, we have to take $\alpha^{(2)}$ to be less than one. The nonperturbative superpotential W_{np} [Eq. (3.61)] is now given by

$$W_{np} = c_1 M_{Pl}^3 \phi^{d+1} e^{-\tau T}, \tag{4.15}$$

where we have restored its natural scale and c_1 is some dimensionless coefficient of order unity. The Pfaffian, which must be a homogeneous polynomial, is represented by the factor ϕ^{d+1}. We will assume that $d+1$ is sufficiently large. This is naturally the case in explicit examples [52,53].

To discuss the five-brane superpotentials, we must first specify the holomorphic curve over which the five-brane is wrapped. As emphasized above, that curve can always be chosen to be irreducible corresponding to a single five-brane. However, in general the homology class $\mathcal{W}$ of the curve can contain both horizontal and components, involving $\mathcal{S}$, $\mathcal{E}$ and the fiber F, respectively. We find it easiest to chose $\mathcal{W}$ to be simply at least one copy of the curve $\mathcal{S}$. This can always be accomplished by adjusting the bundle V on the observable brane. Henceforth, in this paper, we will assume that this is the case. The more general case is more difficult to analyze and will be presented elsewhere. The five-brane nonperturbative superpotentials $W_5^{(1)}$ and $W_5^{(2)}$ [Eqs. (3.62)–(3.64)] are then given by

$$W_5^{(1)} = c_2 M_{Pl}^3 \phi^{d+1} e^{-\tau \mathbf{Y}} \tag{4.16}$$

and

$$W_5^{(2)} = c_3 M_{Pl}^3 e^{-\tau(T-\mathbf{Y})}, \tag{4.17}$$

with c_2 and c_3 being dimensionless coefficients of order unity. In Eq. (4.17), we have assumed that the bundle is trivial on the hidden brane. Note that the Pfaffian in $W_5^{(1)}$ is identical to the one in (4.15) since both arise from the Dirac operator restricted to the curve $\mathcal{S}$.

B. Moduli stabilization

In this section, we will show that the system of the equations

$$D_{all\ fields} W = 0, \tag{4.18}$$

where DW is the Kahler covariant derivative

$$DW = \partial W + \frac{1}{M_{Pl}^2} (\partial K) W, \tag{4.19}$$

has a solution in the correct phenomenological range for all fields. In other words, we will show that all moduli described earlier can be stabilized in an AdS vacuum.

We start with the system of equations

$$D_{\psi_a} W = 0. \tag{4.20}$$

Since the superpotential W does not depend on ψ_a, the above equations are reduced to

$$\frac{\partial K_{bundle}(\psi)}{\partial \psi_a} = 0, \tag{4.21}$$

where Eq. (4.7) has been used. We will now argue that this equation always has a solution. From Sec. II, we know that as ψ goes to positive infinity along either its real or imaginary directions, the Kahler potential K_{bundle} grows. On the other hand, as ψ goes to zero, K_{bundle} can either stay regular or diverge. K_{bundle} will diverge if the locus $\psi = 0$ corresponds to a torsion-free sheaf supported on a holomorphic curve different from that associated with the vanishing of ϕ. If K_{bundle} diverges at zero, then Eq. (4.21) must have a solution for positive ψ corresponding to a minimum of the function K_{bundle}. If K_{bundle} is a regular function of ψ at zero, we can ask what happens as ψ grows in its negative real or imaginary directions. From Eqs. (2.42) and (2.43) and the properties of the partition of unity, it follows that K_{bundle} must also grow in these negative directions. Therefore, again, K_{bundle} must have a minimum. Thus the properties of $\mathbb{CP}^N$ guarantee the existence of a solution to Eq. (4.21).

As the second step, consider the equations involving the Kahler covariant derivative with respect to the complex structure moduli Z_α,

$$D_{Z_\alpha} W = 0, \tag{4.22}$$

which is equivalent to

$$\partial_{Z_\alpha} W_f + \frac{1}{M_{Pl}^2}(\partial_{Z_\alpha} K_Z) W = 0. \tag{4.23}$$

Let us make an assumption that the absolute value of W_f is sufficiently larger than all other contributions to the superpotential near the vacuum, that is

$$|W_f| \gg |W_g|, |W_{np}|, |W_5^{(1)}|, |W_5^{(2)}|. \tag{4.24}$$

Later, we will see that our solution is completely consistent with this assumption. Then Eq. (4.22) becomes

$$\partial_{Z_\alpha} W_f + \frac{1}{M_{Pl}^2}(\partial_{Z_\alpha} K_Z) W_f = 0. \tag{4.25}$$

All terms in this equation depend only on the complex structure moduli. It is not known how to find either K_Z or W_f for complicated Calabi-Yau geometries. Both K_Z and W_f are expected to be complicated functions of Z_α. Nevertheless, there is evidence that this system of equations has a nontrivial solution. First, the number of equations is equal to the number of the unknowns, so one can expect a solution. A second piece of evidence comes from Ref. [7], where type IIB flux compactifications on the space of rather simple geometry T^6/Z_2 were considered. In that paper, it was shown that an equation analogous to (4.25) indeed has a solution fixing all the moduli Z_α. Thus, we simply assume that Eqs. (4.25) fix all the complex structure moduli Z_α and that the value of W_f at the minimum is nonzero. Exactly the same assumption was crucial for the moduli stabilization in the type IIB theory discussed in Ref. [3].

Before moving on to the other equations, let us introduce some notation. Let

$$T = T_1 + iT_2, \quad S = S_1 + iS_2, \quad \mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2, \quad \phi = re^{i\theta}. \tag{4.26}$$

Also, write the value of W_f in the minimum as

$$W_f = |W_f| e^{if}, \tag{4.27}$$

that is, we write the complex number W_f in terms of its absolute value and its phase. Now consider the equation

$$D_S W = 0. \tag{4.28}$$

By using Eqs. (4.2) and (4.12), we obtain

$$(2\epsilon S_1) W_g = -W_f. \tag{4.29}$$

This complex equation is equivalent to two real equations, one relating the phases of the left- and right-hand sides, the other relating the absolute values. The phase equation is as follows:

$$-\epsilon S_2 + \epsilon \alpha^{(2)} T_2 - \epsilon \beta \,\mathrm{Im}\left(\frac{\mathbf{Y}^2}{T^2}\right) = f + \pi(2n_1 + 1), \tag{4.30}$$

where n_1 is any integer. Here we have used the notation introduced in (4.26) and (4.27). The absolute value equation, on the other hand, is

$$(2\epsilon S_1)|W_g| = |W_f|. \tag{4.31}$$

From this equation we obtain a solution for S_1 as a function of T and $\mathbf{Y}$. Taking, for simplicity,

$$\mathrm{Re}\left(\alpha^{(2)} T - \beta \frac{\mathbf{Y}^2}{T^2}\right) \ll S_1 \tag{4.32}$$

we find

$$(2\epsilon S_1) e^{-\epsilon S_1} = \frac{|W_f|}{h_2}. \tag{4.33}$$

This equation provides a solution for S_1. This mechanism is similar to that considered in Ref. [3]. The scale that controls W_f was found in Sec. III to be $h_1 \sim 2\times 10^{-8}$, whereas h_2 was found to be $h_2 \sim 10^{-6}$. Therefore, we find that

$$(\epsilon S_1) e^{-\epsilon S_1} \sim \frac{h_1}{h_2} \sim 10^{-2}. \tag{4.34}$$

From here we obtain

$$\epsilon S_1 \approx 7.5. \tag{4.35}$$

Recalling from (3.14) that for the E_8 gauge group ϵ is of order 5, it follows that

$$S_1 \approx 1.5, \tag{4.36}$$

a phenomenologically accepted solution for S_1. We also see from (4.33) that, by turning on a larger flux, we can reduce the value of S_1 to make it closer to one. It is clear that one can find a solution for S_1 of order unity for generic values of

$$\mathrm{Re}\left(\alpha^{(2)} T - \beta \frac{\mathbf{Y}^2}{T^2}\right) \tag{4.37}$$

less than S_1. We should point out that, in principle, the absolute value of the flux superpotential in the minimum can be less, and even much less (in units of M_{Pl}^3), than the order of h_1. Then there are two possibilities. First, we can turn on a larger amount of the flux, thus increasing $|W_f|$, to keep ϵS_1 at the same value as in (4.35), that is, of order ten. The other possibility is that we can put a nontrivial bundle on the hidden brane. It will break the low-energy gauge group on the hidden wall from E_8 down to some proper subgroup. All this will reduce b_0 and, therefore, from (3.13) we see that this will increase ϵ. Hence, we still can solve Eq. (4.33) and find $\epsilon S_1 \sim 10$. Note that, since ϵS_1 is of order ten, we see from (4.31) that $|W_f| \gg |W_g|$. This is in agreement with the relevant part of assumption (4.24) which we made to justify (4.25).

Now let us consider the equation

$$D_\phi W = 0. \tag{4.38}$$

By using Eqs. (4.7), (4.8), (4.15), (4.16), and (4.24), we obtain the following expression:

$$(d+1)\phi^d (c_1 e^{-\tau T} + c_2 e^{-\tau \mathbf{Y}}) = \frac{pk}{\phi + \bar{\phi}} W_f. \tag{4.39}$$

Since τ given in (3.23) is always much larger than one, the first term on the left-hand side of (4.39) is much smaller than the second as long as $\mathbf{Y}_1 < T_1$. We will assume here that this is the case, justifying this assumption later on. Then, approximately, we have

$$W_5^{(1)} e^{-i\theta} = \frac{pk}{2(d+1)\cos\theta} W_f, \tag{4.40}$$

where we have used (4.16) and (4.26). As before, this complex equation is equivalent to two real equations, one relating the phases of W_5^1 and W_f and one relating their absolute values. The phase equation reads

$$d\theta - \tau \mathbf{Y}_2 = f + 2\pi n_2, \tag{4.41}$$

where n_2 is any integer. The equation for the absolute value is

$$|W_5^{(1)}| = \frac{pk}{2(d+1)\cos\theta} |W_f|. \tag{4.42}$$

Equations (4.41) and (4.42) stabilize the vector bundle moduli r and θ provided the five-brane moduli $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are stabilized. Note that, since $k \sim 10^{-5}$ and $(d+1)$ is large, we have $|W_5^{(1)}| \ll |W_f|$ for generic values of c_2 and $\cos\theta$. Similarly, since the first term in (4.39) is proportional to W_{np}, it follows that $|W_{np}| \ll |W_f|$. This is consistent with our assumption (4.24). At this point, we would like to discuss what would happen if we took an arbitrary number, say M, of ϕ moduli. Equations (4.41) and (4.42) would be two sets of M equations, one for M phases θ_i and one for M radii r_i. For the phases, we would have M equations of the type (4.41) that would determine all θ_i's as functions of $\mathbf{Y}_2$. Similarly, we would have M inhomogeneous equations for r_i of the type (4.42). Clearly, for a generic Pfaffian one expects to find a solution. Furthermore, a generic Kahler potential $K_{bundle}(\phi)$ would not drastically modify Eq. (4.42). It would still be an inhomogeneous equation for r (or r_i's in case there are several) and one still expects a solution. It is also clear that the omitted moduli $\chi_{\tilde{i}}$ can be stabilized by the same mechanism.

Let us move on to the equation

$$D_T W = 0. \tag{4.43}$$

By using Eqs. (4.2), (4.5), (4.15)–(4.17), and (4.24) and the fact that $(\mathbf{Y}_1/T_1)^2$ is sufficiently less than one, we obtain

$$2\tau T_1 W_5^{(2)} = -3W_f. \tag{4.44}$$

This equation is very similar to Eq. (4.29). Relating the phases of the left- and right-hand sides of (4.44) gives

$$-\tau T_2 + \tau \mathbf{Y}_2 = f + \pi(2n_3 + 1), \tag{4.45}$$

where n_3 is any integer. Relating the absolute value yields

$$2\tau T_1 |W_5^{(2)}| = 3|W_f|, \tag{4.46}$$

or, more precisely,

$$2c_3 \tau T_1 e^{-\tau(T_1 - \mathbf{Y}_1)} = 3W_f. \tag{4.47}$$

Since we take τ to be much greater than one and $|W_f|$ is much less than one, we can always find a solution for T_1 in the correct phenomenological range of order one by adjusting the parameter τ, provided $\mathbf{Y}_1$ is stabilized. It is clear that a similar consideration would hold in the case of several T moduli, though the equations would be more complicated. Note that, since $\tau \gg 1$, it follows that $|W_f| \gg |W_5^{(1)}|$. Therefore, all conditions in assumption (4.24) are satisfied.

The last equation to consider is

$$D_{\mathbf{Y}} W = 0. \tag{4.48}$$

By using Eqs. (4.5), (4.12), (4.16), (4.17), (4.24), (4.29), (4.31), (4.44), and (4.46), we get the following equation:

$$\frac{\beta \mathbf{Y}}{S_1 T^2} - \frac{3}{2T_1} + 2\tau_5 \frac{\mathbf{Y}_1}{T_1 S_1} = 0. \tag{4.49}$$

Since, to justify dropping the cross term (2.36), we are looking for a solution with $|T_2| \ll 1$, we have

$$\frac{\mathbf{Y}}{T^2} \approx \frac{\mathbf{Y}_1 + i\mathbf{Y}_2}{T_1^2}. \tag{4.50}$$

Then the imaginary part of Eq. (4.49) becomes

$$\mathbf{Y}_2 \approx 0. \tag{4.51}$$

This provides the stabilization of $\mathbf{Y}_2$. The real part of Eq. (4.49) reads

$$\frac{\beta \mathbf{Y}_1}{S_1 T_1} - \frac{3}{2} + 2\tau_5 \frac{\mathbf{Y}_1}{S_1} = 0. \tag{4.52}$$

From here we get

$$\mathbf{Y}_1 = \frac{3S_1}{2\beta/T_1 + 4\tau_5}. \tag{4.53}$$

This is the solution for $\mathbf{Y}_1$, provided it satisfies

$$\mathbf{Y}_1 < T_1, \tag{4.54}$$

to justify our previous assumption. Since both β and τ_5 are of the same order of magnitude [from Eqs. (2.20) and (3.18) we see that they are both of order $v_5/v_{CY}^{1/3}$], (4.54) leads to the following condition on β:

$$\beta > \frac{3S_1}{2+4T_1}. \tag{4.55}$$

This is the condition on the coefficient β in order to make sure that $\mathbf{Y}_1$ is stabilized in the correct range. Taking, for example,

$$S_1 \sim 1, \quad T_1 \sim 1, \tag{4.56}$$

we obtain

$$\beta > 0.5, \tag{4.57}$$

which is a rather mild condition since β is generically of order one. If the condition (4.55) is not satisfied then, at least in the low-energy field theory approximation, the five-brane is pushed all the way to the hidden brane. Let us make sure that we have indeed stabilized the absolute value of the imaginary part T_2 at a value much less than one. From Eqs. (4.45) and (4.51), we get

$$T_2 = -\frac{f+\pi(2n_3+1)}{\tau}. \tag{4.58}$$

Since τ is much greater than one, we can use our freedom to adjust the integer n_3 to make $|T_2| \ll 1$, which justifies dropping the cross term (2.36) in the Kahler potential. Since the imaginary part of the five-brane modulus $\mathbf{Y}_2$ was found in (4.51) to be approximately zero, we can write the solution (4.41) and (4.42) for the phase θ and the absolute value r as follows:

$$\theta = \frac{f+2\pi n_2}{d} \tag{4.59}$$

and

$$r = \left(\frac{pk|W_f|e^{\tau \mathbf{Y}_1}}{2(d+1)c_2 \cos\theta} \right)^{1/d}. \tag{4.60}$$

Similarly, the imaginary part of the S modulus can be easily found from Eq. (4.30) to be

$$S_2 \sim -\frac{f+2\pi n_1}{\epsilon}, \tag{4.61}$$

where we have used Eq. (4.51) and the fact that $|T_2|$ is much less than one. Thus, when (4.55) is satisfied, we have found a stable solution for all of the heterotic M theory moduli.

Let us summarize our solution. In this section, we found a stable AdS minimum for all heterotic M theory moduli, namely, the complex structure moduli Z_α, the dilaton S, the $h^{1,1}$ modulus T, the vector bundle moduli ϕ and ψ_a, and the five-brane modulus $\mathbf{Y}$. The complex structure moduli are fixed by the fluxes. The corresponding equations have the standard form (4.25). The real part of the dilaton, S_1, is obtained by solving Eq. (4.33). As explained below Eq. (4.33), one can stabilize S_1 near its phenomenological value of order one. The imaginary part of S is given by Eq. (4.61). The real part of the T modulus is stabilized in a similar way by solving Eq. (4.47) together with Eq. (4.53) for the five-brane modulus $\mathbf{Y}_1$. Clearly, one can stabilize T_1 at a value near its phenomenological value of order one. For example, if we take

$$c_3 \approx 1, \quad S \approx 1, \quad \beta \approx 0.8, \tag{4.62}$$

from Eqs. (2.20), (3.18), (3.25), (4.47), and (4.53) we find

$$T_1 \approx 0.7, \quad \mathbf{Y}_1 \approx 0.5. \tag{4.63}$$

The imaginary parts of both the T modulus and the $\mathbf{Y}$ modulus are stabilized at values close to zero. The phase and the absolute value of the vector bundle modulus ϕ are given in Eqs. (4.59) and (4.60), respectively. The vector bundle moduli ψ_a are stabilized by the properties of $\mathbb{CP}^N$, as explained below Eq. (4.21). Remarkably, the only constraint that we have to impose on the various coefficients is given in Eq. (4.55), which is easily satisfied.

Finally, it is straightforward to write the value of the potential energy at the minimum. It is given by the equation

$$V_{min} = -3e^{K/M_{Pl}^2} \frac{|W|^2}{M_{Pl}^2} \sim -\frac{|W_f|^2}{M_{Pl}^2}, \tag{4.64}$$

where Eq. (4.24) has been used. The size of the potential energy is determined by the value of the flux-induced superpotential. Since the scale that controls W_f is of order 10^{-8} (in units of M_{Pl}^3), we expect V_{min} to be

$$V_{min} \sim -10^{-16} M_{Pl}^4 \sim -10^{60}\ \mathrm{GeV}^4. \tag{4.65}$$

Clearly, the masses of the excitations around this minimum are also determined by the fluxes.

V. CONCLUSION

In this paper, we have shown that all moduli of strongly coupled heterotic string theory can be stabilized with vacuum expectation values in a phenomenologically accepted range. This vacuum preserves $\mathcal{N}=1$ supersymmetry in the moduli sector, but has a rather deep negative cosmological constant whose scale is set by the compactification mass. Supersymmetry is, however, softly broken in the gravity and matter sectors at the TeV scale by the gaugino condensate. Our result is the heterotic string analog of the supersymmetry preserving part of the stabilization procedure presented in the type IIB context in Ref. [3]. There are, however, a number of new, nontrivial elements in the heterotic discussion. These include the vector bundle moduli and their nonperturbative superpotentials, the gaugino condensate superpotential with threshold corrections, and the inclusion of a bulk five-brane and its nonperturbative dynamics.

It is natural to ask whether, by appropriate modification of our heterotic theory, the value of the potential energy at the local minimum can be lifted from its large, negative value to a small, positive cosmological constant of the order, say, of dark energy. This was accomplished in the type IIB context in Ref. [3] by adding anti-D-branes. It would be interesting to try to find a heterotic analog of this mechanism involving anti-M-five-branes. Alternatively, one could try to use the mechanisms recently proposed in Ref. [72] to lift the vacuum to a positive value. We will discuss this elsewhere.

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