Complementarity of the Maldacena and Karch-Randall pictures

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We perform a one-loop test of the holographic interpretation of the Karch-Randall model, whereby a massive graviton appears on an AdS_4 brane in an AdS_5 bulk. Within the AdS/CFT framework, we examine the quantum corrections to the graviton propagator on the brane, and demonstrate that they induce a graviton mass in exact agreement with the Karch-Randall result. Interestingly enough, at one loop order, the spin-0, spin-1/2, and spin-1 loops contribute to the dynamically generated $(\text{mass})^2$ in the same 1:3:12 ratio as enter the Weyl anomaly and the $1/r^3$ corrections to the Newtonian gravitational potential.

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I. INTRODUCTION

An old question is whether the graviton could have a small but nonzero rest mass. If so, it is unlikely to be described by the explicit breaking of general covariance that results from the addition of a Pauli-Fierz mass term to the Einstein Lagrangian. This gives rise to the well-known van Dam–Veltman–Zakharov [1,2] discontinuity problems in the massless limit that come about by jumping from five degrees of freedom to two. Moreover, recent attempts [3,4] to circumvent the discontinuity in the presence of a nonzero cosmological constant work only at the tree level and the discontinuity resurfaces[1] at one loop [6]. On the other hand, in analogy with spontaneously broken gauge theories, one might therefore prefer a dynamical breaking of general covariance, which would be expected to yield a smooth limit. However, a conventional Higgs mechanism, in which a scalar field acquires a nonzero expectation value, does not yield a mass for the graviton. The remaining possibility is that the graviton acquires a mass dynamically and that the would-be Goldstone boson is a *spin-one bound state*. Just such a possibility was suggested in 1975 [7].

Interestingly enough, the idea of a massive graviton arising from a spin-one bound state Goldstone boson has recently been revived by Porrati [8] in the context of the Karch-Randall brane-world [9] whereby our universe is an AdS_4 brane embedded in an AdS_5 bulk. This model predicts a small but finite four-dimensional graviton mass

$$M^2=\frac{3L_5^2}{2L_4^4}, \tag{1}$$

in the limit $L_4\to\infty$, where L_4 and L_5 are the "radii" of AdS_4 and AdS_5, respectively. From the Karch-Randall point of view, the massive graviton bound to the brane arises from solving the classical $D=5$ linearized gravity equations in the brane background [9]. Furthermore, holography of the Karch-Randall model [10,11] consistently predicts an identical graviton mass.

In a previous paper [12], the complementarity between the Maldacena AdS/conformal field theory (CFT) correspondence [13–15] and the Randall-Sundrum [16] Minkowski braneworld picture was put to the test by calculating the $1/r^3$ corrections to the Newtonian gravitational potential arising from the CFT loop corrections to the graviton propagator. At one loop we have [17]

$$V(r)=\frac{G_4m_1m_2}{r}\left(1+\frac{\alpha G_4}{r^2}\right), \tag{2}$$

where G_4 is the four-dimensional Newton's constant,

$$\alpha=\frac{1}{45\pi}(12n_1+3n_{1/2}+n_0), \tag{3}$$

and where n_0, $n_{1/2}$, and n_1 count the number of (real) scalars, (Majorana) spinors, and vectors in the multiplet. The coefficient α is the same one that determines that part of the Weyl anomaly involving the square of the Weyl tensor [18]. The fields on the brane are given by $\mathcal{N}=4$ supergravity coupled to a $\mathcal{N}=4$ super-Yang-Mills CFT with gauge group $U(N)$, for which $(n_1,n_{1/2},n_0)=(N^2,4N^2,6N^2)$. Using both the AdS/CFT relation, $N^2=\pi L_5{}^3/2G_5$, and the brane world relation, $G_4=2G_5/L_5$, we find

$$G_4\alpha=\frac{G_4L_5{}^3}{3G_5}=\frac{2L_5{}^2}{3}, \tag{4}$$

where G_5 is the five-dimensional Newton constant. Hence

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[1]A similar quantum discontinuity arises in the "partially massless" limit as a result of jumping from five degrees of freedom to four [5].

$$V(r)=\frac{G_4 m_1 m_2}{r}\left(1+\frac{2L_5{}^2}{3r^2}\right), \tag{5}$$

which agrees exactly with the Randall-Sundrum bulk result.

This complementarity can be generalized to the Karch-Randall AdS braneworld picture. From an AdS/CFT point of view, one may equally well foliate a Poincaré patch of AdS_5 in AdS_4 slices. The Karch-Randall brane is then such a slice that cuts off the AdS_5 bulk. However, unlike for the Minkowski braneworld, this cutoff is not complete, and part of the original AdS_5 boundary remains [9,11]. Starting with a maximally supersymmetric gauged $\mathcal{N}=8$ supergravity in the five-dimensional bulk, the result is a gauged $\mathcal{N}=4$ supergravity on the brane coupled to a $\mathcal{N}=4$ super-Yang-Mills CFT with gauge group $U(N)$, however, with unusual boundary conditions on the CFT fields [8,10,11,19,20].

As was demonstrated in Ref. [8], the CFT on AdS_4 provides a natural origin for the bound state Goldstone boson, which turns out to correspond to a *massive* representation of $SO(3,2)$. However, while Ref. [8] considers the case of coupling to a single conformal scalar, in this paper we provide a crucial test of the complementarity by computing the dynamically generated graviton mass induced by a complete $\mathcal{N}=4$ super-Yang-Mills CFT on the brane and showing that this quantum computation correctly reproduces the Karch-Randall result, Eq. (1).

We begin in Sec. II by discussing properties of the graviton propagator and providing a general framework for the dynamical generation of graviton mass. In Sec. III, we introduce homogeneous coordinates, and set up the loop computation, which we carry out in Sec. IV. Finally, in Sec. V, we recover the Karch-Randall graviton mass, Eq. (1), from the quantum CFT perspective.

II. TRANSVERSALITY AND THE GRAVITON MASS

We are mainly interested in the properties of the one-loop graviton self-energy, $\Sigma_{\mu\nu,\alpha\beta}(x,y)$. As emphasized in Refs. [7,8], mass generation is compatible with the gravitational Ward identity arising from diffeomorphism invariance. Thus the self-energy remains transverse, $\nabla_x^\mu \Sigma_{\mu\nu,\alpha\beta}=\nabla_y^\alpha \Sigma_{\mu\nu,\alpha\beta}=0$. One is then able to write Σ as a nonlocal expression evaluated at point x^μ, compatible with transversality

$$\Sigma_{\mu\nu,\alpha\beta}(x)=\beta(\Delta)\Pi_{\mu\nu,\alpha\beta}(\Delta)+\gamma(\Delta)K_{\mu\nu,\alpha\beta}(\Delta), \tag{6}$$

where [8]

$$\Pi_{\mu\nu}{}^{\alpha\beta}=\delta_\mu^\alpha\delta_\nu^\beta-\frac{1}{3}g_{\mu\nu}g^{\alpha\beta}+2\nabla_\mu\left(\frac{\delta_\nu^\beta+\nabla_\nu\nabla^\beta/2\Lambda}{\Delta-2\Lambda}\right)\nabla^\alpha - \frac{\Lambda}{3}\left(g_{\mu\nu}+\frac{3}{\Lambda}\nabla_\mu\nabla_\nu\right)\frac{1}{3\Delta-4\Lambda}\left(g^{\alpha\beta}+\frac{3}{\Lambda}\nabla^\alpha\nabla^\beta\right) \tag{7}$$

is the transverse-traceless projection and

$$K_{\mu\nu}{}^{\alpha\beta}=\frac{\Delta-\Lambda}{3\Delta-4\Lambda}d_{\mu\nu}d^{\alpha\beta};\quad d_{\mu\nu}=g_{\mu\nu}+\frac{1}{\Delta-\Lambda}\nabla_\mu\nabla_\nu \tag{8}$$

is the transverse but trace projection. More generally,

$$(\Pi+K)_{\mu\nu}{}^{\alpha\beta}=\delta_\mu^\alpha\delta_\nu^\beta+\frac{2}{\Delta-2\Lambda}\delta_\mu^\alpha\nabla_\nu\nabla^\beta+\frac{1}{(\Delta-2\Lambda)(\Delta-\Lambda)}\nabla_\mu\nabla_\nu\nabla^\alpha\nabla^\beta \tag{9}$$

is an overall transverse projection, regardless of trace. Here, $\Lambda=-3/L_4^2$ is the four-dimensional cosmological constant and Δ is the general Lichnerowicz operator which commutes with covariant derivatives. Symmetrization on $(\mu\nu)$ and $(\alpha\beta)$ is implied throughout. In flat space, these expressions reduce simply to the familiar

$$\Pi_{\mu\nu}{}^{\alpha\beta}=d_\mu^\alpha d_\nu^\beta-\frac{1}{3}d_{\mu\nu}d^{\alpha\beta},\quad K_{\mu\nu}^{\alpha\beta}=\frac{1}{3}d_{\mu\nu}d^{\alpha\beta} \tag{10}$$

where

$$d_{\mu\nu}=\eta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}. \tag{11}$$

In Feynman gauge, the tree-level massless graviton propagator in AdS takes the form

$$D_{\mu\nu}{}^{\alpha\beta}=\frac{1}{\Delta-2\Lambda}\left(\delta_\mu^\alpha\delta_\nu^\beta-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\right). \tag{12}$$

Using the self-energy written in the form Eq. (6), the quantum corrected propagator may be summed to yield

$$\tilde{D}_{\mu\nu}{}^{\alpha\beta}=\frac{1}{\Delta-2\Lambda-\beta}\left(\delta_\mu^\alpha\delta_\nu^\beta-\frac{\Delta-\Lambda}{3\Delta-4\Lambda}g_{\mu\nu}g^{\alpha\beta}\right)-\frac{1}{\Delta-\Lambda+\gamma/2}\left(\frac{1}{2}\frac{\Delta-\Lambda}{3\Delta-4\Lambda}g_{\mu\nu}g^{\alpha\beta}\right) \tag{13}$$

when evaluated between conserved sources. This indicates that a constant piece in the traceless self-energy, $\beta=-M^2$, will shift the spin-2 pole in the propagator, thus yielding a nonzero graviton mass. The second term, involving the trace, may combine with the scalar part of the first. However, a potentially dangerous scalar ghost pole at $3\Delta=4\Lambda$ may appear. This ghost is absent whenever the residue of the pole vanishes, i.e. provided $\gamma=\beta|_{4\Delta=3\Lambda}$. This is in fact the case, as may be seen by explicit computation below. Although the field theory is conformal, the presence of K is demanded by the Weyl anomaly [18]. However, this trace piece is entirely contained in the local part of Σ, and does not contribute directly to the mass. The net result is a pure massive spin-2 propagator

$$\tilde{D}_{\mu\nu}{}^{\alpha\beta}=\frac{1}{\Delta-2\Lambda+M^2}\left[\delta_\mu^\alpha\delta_\nu^\beta-\frac{1}{2}\left(\frac{2\Lambda-2M^2}{2\Lambda-3M^2}\right)g_{\mu\nu}g^{\alpha\beta}\right], \tag{14}$$

where we have taken $\beta=-M^2$ and the Pauli-Fierz combination, $\gamma=(3/\Lambda)(\Delta-\Lambda)\beta$.

Thus the procedure we follow in determining the graviton mass is to compute the one-loop self-energy in an AdS background, and to identify the appropriate constant piece β. Viewed in coordinate space, this is a nonlocal contribution to Σ. But this is precisely what is necessary to induce a graviton mass.

III. HOMOGENEOUS COORDINATES AND BI-TENSORS

Before turning to an explicit calculation of the graviton self-energy, we consider some preliminaries for studying quantum fields in homogeneous spaces. In particular, we establish our notation and review some useful facts about manipulating tensors in homogeneous space. Many of these techniques are by now standard; further details may be found in, e.g., Refs. [21–25].

We find it convenient to work in homogeneous coordinates, which corresponds to the embedding of AdS_4 in R^5 with the pseudo-Euclidean metric, $\eta_{MN}=\mathrm{diag}(-,+,+,+,-)$. AdS_4 is then given by the restriction to the hyperboloid $X^M X^N \eta_{MN}=-L_4^2$. Note that we denote homogeneous coordinates as $X^M, Y^M, \ldots$ $(M,N=0,\ldots,4)$ and intrinsic coordinates as $x^\mu, y^\mu, \ldots$ $(\mu,\nu=0,\ldots,3)$.

Tensor fields $\phi_{MNP\ldots}(X)$ restricted to the hyperboloid must satisfy $X^M\phi_{MNP\ldots}(X)=0$. In addition, we take them to be homogeneous of (arbitrary) degree n, $\phi_{MNP\ldots}(\lambda X)=\lambda^n\phi_{MNP\ldots}(X)$. An important point to note in transforming from intrinsic coordinates to homogeneous coordinates is that all tensor indices must be restricted to lie on the hyperboloid, namely $X^M\phi_{MNP\ldots}(X)=0$. Projecting into the tangent direction at a point X^M is accomplished by the operator

$$G_{MN}(X)=\eta_{MN}+X_M X_N/L^2 \tag{15}$$

which also serves as a metric tensor where $\mathrm{Tr}(G)\equiv G_{MN}G^{MN}=4$ (recall that $X^2=-L^2$).

Two-point functions in coordinate space are in general bi-tensor functions of the points X^M and Y^P. Maximally symmetric scalar functions, $\phi(X,Y)$, are simple and can only depend on the invariant $|X-Y|^2/L^2=-2(Z+1)$ where $Z=X\cdot Y/L^2$. However, in general, we must also consider bi-tensors of the form, $\phi_{MN\ldots,PQ\ldots}(X,Y)$, where the first (second) set of indices refer to point X^M (Y^P). To construct such bi-tensors, we define the unit vectors

$$N_M(X)=\frac{Y_M+ZX_M}{L\sqrt{Z^2-1}},\quad N_P(Y)=\frac{X_P+ZY_P}{L\sqrt{Z^2-1}} \tag{16}$$

where, as before, $Z=X\cdot Y/L^2$. These serve the same purpose as the unit tangent vectors of Ref. [22], except that here they are given in homogeneous coordinates. In addition, we also make use of the mixed tensor

$$\begin{aligned}\hat{G}_{MP}(X,Y)&=G_{MN}(X)\,\eta^{NQ}G_{PQ}(Y)\\&=\eta_{MP}+(X_M X_P+Y_M Y_P+ZX_M Y_P)/L^2.\end{aligned} \tag{17}$$

This serves the same function as the "parallel propagator" of Ref. [22]. However, when converted from intrinsic coordinates, the parallel propagator has the form $g_{MP}=\hat{G}_{MP}-(Z+1)N_M N_P$, which differs at large separations. We choose to use $\hat{G}_{MP}$ because it is symmetric under the antipodal map $Y\to -Y$ in the covering space of AdS, while g_{MP} is not.

It is clear from the condition of maximal symmetry that all bi-tensors may be expressed in terms of the metrics $G_{MN}(X)$, $G_{PQ}(Y)$, unit vectors $N_M(X)$, $N_P(Y)$, and mixed tensor $\hat{G}_{MP}(X,Y)$. For the graviton self-energy, we are interested in the two-point function, $\langle T_{MN}(X)T_{PQ}(Y)\rangle$. Since this is symmetric under either $M\leftrightarrow N$ or $P\leftrightarrow Q$ or the simultaneous interchange of $MN\leftrightarrow PQ$ and $X\leftrightarrow Y$, it may always be decomposed in terms of a set of five basis bi-tensors, which we take to be [23]

$$\mathcal{O}_1=G_{MN}G_{PQ},\quad \mathcal{O}_2=N_M N_N N_P N_Q,\quad \mathcal{O}_3=2\hat{G}_M{}^{(P}\hat{G}_N{}^{Q)},$$
$$\mathcal{O}_4=G_{MN}N_P N_Q+N_M N_N G_{PQ},\quad \mathcal{O}_5=4\hat{G}_{(M}{}^{(P}N_{N)}N^{Q)}. \tag{18}$$

To avoid lengthening the notation, we do not include the X or Y dependence explicitly; indices M and N always refer to X, and P and Q always refer to Y. With all indices contracted against proper homogeneous tensors, these operators may be represented simply by

$$\begin{aligned}&\mathcal{O}_1=\eta_{MN}\eta^{PQ},\quad \mathcal{O}_2=Y_M Y_N X_P X_Q/L^4(Z^2-1)^2,\\&\mathcal{O}_3=2\,\delta_{(M}^{(P}\delta_{N)}^{Q)}\\&\mathcal{O}_4=(\eta_{MN}X^P X^Q+Y_M Y_N\eta^{PQ})/L^2(Z^2-1),\\&\mathcal{O}_5=4\,\delta_{(M}^{(P}Y_{N)}X^{Q)}/L^2(Z^2-1).\end{aligned} \tag{19}$$

These expressions are sufficient for determining the appropriate linear combinations of the operators without having to keep track of complete projections. The complete operators, Eq. (18), may be recovered by projecting all external indices with Eq. (15).

Note that this decomposition follows the notation of Ref. [22] (with tensor quantities converted to homogeneous coordinates), except that we use the mixed tensor $\hat{G}_{MP}$ instead of the parallel propagator g_{MP}. This choice leads to more symmetric expressions, and highlights the interplay between boundary conditions and the use of image charges below. In terms of the parallel propagator, Ref. [23] would define instead

$$\tilde{\mathcal{O}}_3=2g_M{}^{(P}g_N{}^{Q)},\quad \tilde{\mathcal{O}}_5=4g_{(M}{}^{(P}N_{N)}N^{Q)}. \tag{20}$$

The relation between the two bases is given by

$$\begin{aligned}&\mathcal{O}_3=\tilde{\mathcal{O}}_3+(Z+1)\tilde{\mathcal{O}}_5+2(Z+1)^2\mathcal{O}_2,\\&\mathcal{O}_5=\tilde{\mathcal{O}}_5+(Z+1)\mathcal{O}_2\end{aligned} \tag{21}$$

(with the remainder unchanged). This is a straightforward identification at short distances ($Z \to -1$), and only differs at long distances.

In order to investigate the graviton self-energy, it is useful to obtain a basis of transverse traceless bi-tensors. Although, in the flat space limit, transversality is easily expressed in momentum space, this is not the case when working in homogeneous coordinates. We first define the three traceless combinations

$$T_1 = \frac{1}{3(3Z^2+1)}[\mathcal{O}_1 + 16\mathcal{O}_2 - 4\mathcal{O}_4],$$

$$T_2 = -\frac{1}{3}\mathcal{O}_1 + \frac{2}{3}\mathcal{O}_2 + \frac{1}{2}\tilde{\mathcal{O}}_3 + \frac{1}{3}\mathcal{O}_4 + \frac{1}{2}\tilde{\mathcal{O}}_5,$$

$$T_3 = \frac{1}{2Z}[4\mathcal{O}_2 + \tilde{\mathcal{O}}_5], \tag{22}$$

where T_1, T_2, and T_3 are traceless in the sense $G^{MN}T_{MN,PQ} = T_{MN,PQ}G^{PQ} = 0$. For completeness, there is also a pure trace combination

$$P_R = \frac{1}{Z^2(3Z^2+1)}[Z^4\mathcal{O}_1 + (Z^2-1)^2\mathcal{O}_2 - Z^2(Z^2-1)\mathcal{O}_4]. \tag{23}$$

While there is some arbitrariness in the definition of T_1, T_2, and T_3, the above definitions (including normalization) were chosen to have a natural reduction in the flat space (or short distance) limit.

This limit corresponds to taking $Y \to X$, so that $Z \to 1$, and both G and $\hat{G}$ reduce to the (four-dimensional) flat space metric η. In addition, the tangent vectors, Eq. (16), reduce according to

$$N_M \to \hat{r}_\mu, \quad N_P \to -\hat{r}_\rho \tag{24}$$

where $\hat{r} = (\vec{y} - \vec{x})/|\vec{y} - \vec{x}|$. The resulting traceless Eq. (22) and trace Eq. (23) combinations take on the projection form

$$T_1 \to \frac{1}{12}(\eta_{\mu\nu} - 4\hat{r}_\mu\hat{r}_\nu)(\eta^{\rho\sigma} - 4\hat{r}^\rho\hat{r}^\sigma),$$

$$T_2 \to (\delta^{(\rho}_{(\mu} - \hat{r}_\mu\hat{r}^\rho)(\delta^{\sigma}_{\nu} - \hat{r}_{\nu)}\hat{r}^{\sigma)}) - \frac{1}{3}(\eta_{\mu\nu} - \hat{r}_\mu\hat{r}_\nu)(\eta^{\rho\sigma} - \hat{r}^\rho\hat{r}^\sigma),$$

$$T_3 \to (\delta^{(\rho}_{(\mu} - \hat{r}_\mu\hat{r}^\rho)\hat{r}_{\nu)}\hat{r}^{\sigma)},$$

$$P_R \to \frac{1}{4}\eta_{\mu\nu}\eta^{\rho\sigma}. \tag{25}$$

These projections are essentially onto longitudinal, transverse traceless, transverse, and pure trace components, with rank 1, 5, 3, and 1, respectively.

Returning to AdS, it should be noted that, while traceless, T_1, T_2, and T_3 are not in themselves transverse. However, any transverse traceless bi-tensor must be able to be written as a combination

$$\mathcal{T} = a_1(Z)(3Z^2+1)T_1 + a_2(Z)T_2 + a_3(Z)T_3 \tag{26}$$

where transversality imposes two conditions on the three functions a_1, a_2, and a_3. The details are carried out in Appendix B; the result is that to highlight the large separation behavior of $\mathcal{T}$, we construct a basic of transverse traceless bi-tensors $\{\mathcal{T}_{(n)}\}$. Below, when examining the graviton self-energy, we will make use of this basis for extracting the nonlocal quantity responsible for graviton mass generation.

IV. COMPUTATION OF THE GRAVITON SELF-ENERGY

Before addressing the one-loop computation, we start by examining the scalar, fermion, and vector two-point functions, paying attention to necessary boundary conditions [8,26]. Details are provided in the Appendix; here we simply summarize the results. A normalized scalar propagator necessarily has short-distance behavior

$$\Delta_0(X,Y) \sim -\frac{1}{4\pi^2}\frac{1}{|X-Y|^2} \sim \frac{1}{8\pi^2 L_4^2}\frac{1}{Z+1}, \tag{27}$$

so that it reduces properly in the flat space limit. However, boundary conditions must still be satisfied by the addition of an appropriate solution to the homogeneous equation. For AdS energy $E_0 = 1$ or 2, and for mixed boundary conditions encoded by parameters α_+, α_-, the scalar propagator takes the form [26]

$$\Delta_0^{(\alpha)} = \frac{1}{8\pi^2 L_4^2}\left(\frac{\alpha_+}{Z+1} + \frac{\alpha_-}{Z-1}\right). \tag{28}$$

Although normalization demands $\alpha_+ = 1$, we nevertheless find it illuminating to keep α_+ arbitrary, as it highlights the symmetries in the latter expressions for the graviton self-energy computation. Note that $\alpha_- = 0$ corresponds to transparent boundary conditions, while $\alpha = \pm 1$ corresponds to ordinary reflecting ones.

Similarly, the appropriate fermion propagator has the form

$$\Delta_{1/2}^{(\alpha)} = \frac{1}{8\pi^2 L^4}\left(\alpha_+\frac{\Gamma^M(X_M - Y_M)}{(Z+1)^2} + \alpha_-\frac{\Gamma^M(X_M + Y_M)}{(Z-1)^2}\right). \tag{29}$$

The vector propagator is the first case where we have to worry about bi-tensor structures as well as gauge fixing. However, for correlators of the stress tensor, we only need the expression for the gauge invariant two-point function $\langle F_{MN}(X)F_{PQ}(Y)\rangle$. The result is

$$\langle F_{MN}(X)F^{PQ}(Y)\rangle^{(\alpha)} = \frac{1}{2\pi^2L^4} \times\left[\frac{\alpha_+}{(Z+1)^2}[\hat{G}_{[M}{}^{[P}\hat{G}_{N]}{}^{Q]}-2(Z-1)N_{[M}\hat{G}_{N]}{}^{[Q}N^{P]}] + \frac{\alpha_-}{(Z-1)^2}[\hat{G}_{[M}{}^{[P}\hat{G}_{N]}{}^{Q]}-2(Z+1)N_{[M}\hat{G}_{N]}{}^{[Q}N^{P]}]\right]. \tag{30}$$

A. The scalar contribution

The scalar loop contribution to the graviton self-energy was partially computed in Ref. [8], where the proper role of boundary conditions was highlighted. The relevant Lagrangian for a conformally coupled scalar is given by

$$e^{-1}\mathcal{L}=-\tfrac{1}{2}\partial\phi^2-\frac{1}{12}R\phi^2. \tag{31}$$

This gives rise to the equation of motion, $(\Box - R)\phi=0$, as well as the improved stress tensor

$$T_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi-\tfrac{1}{2}g_{\mu\nu}(\partial\phi)^2 -\frac{1}{6}[\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box-(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R)]\phi^2. \tag{32}$$

This stress tensor is both conserved and traceless (on the equations of motion), as expected for a conformal scalar. For computational purposes, we find it convenient to reexpress Eq. (32) as

$$T_{\mu\nu}=\frac{2}{3}\partial_\mu\phi\partial_\nu\phi-\frac{1}{6}g_{\mu\nu}[(\partial\phi)^2+\phi^2/L^2]-\frac{1}{3}\phi\nabla_\mu\nabla_\nu\phi \tag{33}$$

where we have fixed the background AdS metric and made use of the scalar equation of motion.[2]

Evaluation of $\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle$ follows from Wick's theorem

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle=\frac{4}{9}\langle\partial_\mu\phi(x)\partial_\nu\phi(x)\partial_\rho\phi(y)\partial_\sigma\phi(y)\rangle+\cdots =\frac{4}{9}[\partial_\mu\partial_\rho\Delta_0(x-y)\partial_\nu\partial_\sigma\Delta_0(x-y) +\partial_\mu\partial_\sigma\Delta_0(x-y)\partial_\nu\partial_\rho\Delta_0(x-y)]+\cdots, \tag{34}$$

[2]Since the induced graviton mass is a long-distance effect, we are unconcerned with any contact terms that may be discarded by evaluation of the equation of motion on the Green's functions. In any case, such issues may be avoided by, e.g., use of a point splitting regulator.

where Δ_0 is the scalar propagator. Working in homogeneous coordinates, after considerable manipulation, we obtain the self-energy as a bi-local tensor

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle &=\mathcal{O}_1[\tfrac{1}{18}(Z\Delta_0'+\Delta_0)^2-\tfrac{11}{18}\Delta_0'^2+\tfrac{1}{9}\Delta_0\Delta_0''] \\ &+\mathcal{O}_2(Z^2-1)^2[\Delta_0''^2+\tfrac{1}{19}\Delta_0\Delta_0''''-\tfrac{8}{9}\Delta_0'\Delta_0'''] \\ &+\mathcal{O}_3[\tfrac{4}{9}\Delta_0'^2+\tfrac{1}{9}\Delta_0\Delta_0''] \\ &+\mathcal{O}_4(Z^2-1)[-\tfrac{14}{9}\Delta_0'^2+\tfrac{7}{9}\Delta_0\Delta_0''+\tfrac{1}{9}Z\Delta_0\Delta_0''' \\ &-\tfrac{1}{3}Z\Delta_0'\Delta_0'']+\mathcal{O}_5(Z^2-1)[\tfrac{1}{9}\Delta_0\Delta_0'''],\end{aligned} \tag{35}$$

where primes denote differentiation with respect to Z. Note that, to simplify the expression, we have used the scalar equation of motion, $(Z^2-1)\Delta_0''=-2(\Delta_0+2Z\Delta_0')$, where we have dropped the short-distance term $\delta(X-Y)$. Substituting in the explicit form of the scalar propagator, Eq. (28), we find

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle_0 &=\frac{1}{48\pi^4L_4^8}\Bigg[\frac{\alpha_+^2}{(Z+1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2+ZT_3\right) \\ &+\frac{\alpha_-^2}{(Z-1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2-ZT_3\right) \\ &+\frac{2}{3}\frac{\alpha_+\alpha_-}{(Z^2-1)^3}(5(3Z^2+1)T_1+(3Z^2-1)T_2-10Z^2T_3)\Bigg]\end{aligned} \tag{36}$$

(up to contact terms, which we drop).

B. The fermion contribution

Turning next to spin 1/2, we take for simplicity a massless Dirac fermion with Lagrangian

$$e^{-1}\mathcal{L}=\tfrac{1}{2}\bar{\psi}(\overrightarrow{\nabla\!\!\!/}-\overleftarrow{\nabla\!\!\!/})\psi \tag{37}$$

and stress tensor

$$T_{\mu\nu}=\tfrac{1}{2}\bar{\psi}\gamma_{(\mu}(\overrightarrow{\nabla}_{\nu}-\overleftarrow{\nabla}_{\nu)})\psi-\tfrac{1}{2}g_{\mu\nu}\bar{\psi}(\overrightarrow{\nabla\!\!\!/}-\overleftarrow{\nabla\!\!\!/})\psi. \tag{38}$$

Note that this is both traceless and covariantly conserved on the equations of motion, $\nabla\!\!\!/\psi=\bar{\psi}\overleftarrow{\nabla\!\!\!/}=0$.

As in the scalar case, use of the equations of motion on the external vertices allows us to ignore the second term in Eq. (38) when evaluating $\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle$. Promoting this expression to homogeneous coordinates, and using Wick's theorem, we find

$$\langle T_{MN}(X)T_{PQ}(Y)\rangle=-\frac{1}{4}\mathrm{Tr}[\Gamma_{(M}(\overrightarrow{\partial}_N-\overleftarrow{\partial}_{N)})\Delta_{1/2}(X,Y)\times\Gamma_{(P}(\overrightarrow{\partial}_Q-\overleftarrow{\partial}_{Q)})\Delta_{1/2}(Y,X)] \tag{39}$$

(the $-$ sign is for a fermion loop) where $\Delta_{1/2}(X,Y)$ is the spin-1/2 propagator given in Eq. (29). The Dirac trace may be evaluated by writing $\Delta_{1/2}(X,Y)=\Gamma_A\Delta^A_{1/2}(X,Y)$, so that

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle=&-(\delta^A_M\delta^B_P+\delta^A_P\delta^B_M-\eta_{MP}\eta^{AB})\\&\times\{\partial_N\Delta^A_{1/2}(X,Y)\partial_Q\Delta^B_{1/2}(Y,X)\\&+\partial_Q\Delta^A_{1/2}(X,Y)\partial_N\Delta^B_{1/2}(Y,X)\\&-\Delta^A_{1/2}(X,Y)\partial_N\partial_Q\Delta^B_{1/2}(Y,X)\\&-[\partial_N\partial_Q\Delta^A_{1/2}(X,Y)]\Delta^B_{1/2}(Y,X)\}\end{aligned} \tag{40}$$

where a further symmetrization on (MN) and (PQ) is implied. This expression is symmetric under interchange of $X\leftrightarrow Y$ in the propagators. Since this corresponds to taking $\alpha_+\leftrightarrow-\alpha_+$ [as is evident from Eq. (29)], the overall result is to project onto terms even in α_+. In particular, this kills any possible terms proportional to $\alpha_+\alpha_-$ in the two-point function.

A straightforward computation results in the expression

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle&\\=-\frac{1}{32\pi^4L^8}\Bigg[&\frac{\alpha_+^2}{(Z+1)^4}(\mathcal{O}_1-2\mathcal{O}_3+2(Z-1)\mathcal{O}_5\\&-4(Z-1)^2\mathcal{O}_2)+\frac{\alpha_-^2}{(Z-1)^4}(\mathcal{O}_1-2\mathcal{O}_3\\&+2(Z+1)\mathcal{O}_5-4(Z+1)^2\mathcal{O}_2)\Bigg]\end{aligned} \tag{41}$$

which may be rewritten in terms of the traceless T tensors of Eq. (22) as

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle_{1/2}&\\=\frac{1}{8\pi^4L^8}\Bigg[&\frac{\alpha_+^2}{(Z+1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2+ZT_3\right)\\&+\frac{\alpha_-^2}{(Z-1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2-ZT_3\right)\Bigg].\end{aligned} \tag{42}$$

Other than for the absence of the mixed $\alpha_-\alpha_+$ term, this contribution for a Dirac fermion is identical to that of a scalar loop, Eq. (36), but six times larger. For a Majorana fermion, this should be halved, so that the contribution is three times that of a scalar.

C. The vector contribution

The remaining contribution to the graviton self-energy arises from vector loops. For a massless gauge boson with Lagrangian

$$e^{-1}\mathcal{L}=-\frac{1}{4}F^2_{\mu\nu}, \tag{43}$$

the stress tensor is simply

$$T_{\mu\nu}=F_{\mu\lambda}F_\nu{}^\lambda-\frac{1}{4}g_{\mu\nu}F^2. \tag{44}$$

Converting all expressions to homogeneous coordinates, we need to evaluate

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle=&2\langle F_M{}^AF_P{}^B\rangle\langle F_N{}^AF_Q{}^B\rangle\\&-\frac{1}{2}\eta_{MN}\langle F_C{}^AF_P{}^B\rangle\langle F^{CA}F_Q{}^B\rangle\\&-\frac{1}{2}\eta_{PQ}\langle F_M{}^AF_C{}^B\rangle\langle F_N{}^AF^{CB}\rangle\\&+\frac{1}{8}\eta_{MN}\eta_{PQ}\langle F_C{}^AF_D{}^B\rangle\langle F^{CA}F^{DB}\rangle,\end{aligned} \tag{45}$$

where again symmetrization in (MN) and (PQ) is assumed. Note that all contractions are performed with either $G_{MN}(X)$ or $G_{PQ}(Y)$. The main difficulty is in evaluating the first term in this expression; the remaining ones follow simply from tracing over the appropriate indices. Using the explicit form for the vector propagator, Eq. (30), we obtain

$$\begin{aligned}\langle F_M{}^AF_P{}^B\rangle\langle F_N{}^AF_Q{}^B\rangle&\\=\frac{1}{16\pi^4L^8}\Bigg[&\frac{\alpha_+^2}{(Z+1)^4}[\mathcal{O}_1+\mathcal{O}_3-(Z-1)\mathcal{O}_5\\&+2(Z-1)^2\mathcal{O}_2]+\frac{\alpha_-^2}{(Z-1)^4}[\mathcal{O}_1+\mathcal{O}_3-(Z+1)\mathcal{O}_5\\&+2(Z+1)^2\mathcal{O}_2]+\frac{\alpha_+\alpha_-}{(Z^2-1)^2}(\mathcal{O}_1-2\mathcal{O}_4)\Bigg].\end{aligned} \tag{46}$$

Substituting this into Eq. (45), we find that the mixed $\alpha_+\alpha_-$ term vanishes. The result is identical to the fermion case, Eq. (41), except that it is twice as large (as that for the Dirac fermion). Explicitly, this is given by

$$\begin{aligned}\langle T_{MN}(X)T_{PQ}(Y)\rangle_1&\\=\frac{1}{4\pi^4L^8}\Bigg[&\frac{\alpha_+^2}{(Z+1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2+ZT_3\right)\\&+\frac{\alpha_-^2}{(Z-1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2-ZT_3\right)\Bigg].\end{aligned} \tag{47}$$

The complete supermultiplet

Until now, we have treated spins 0, $\frac{1}{2}$, and 1 separately. However, to preserve supersymmetry, the boundary conditions on all fields in the multiplet have to be chosen consistently [27]. This means a single set of α_+ (actually always 1) and α_- suffices for specifying the boundary conditions. Furthermore, for a complex scalar in a Wess-Zumino multiplet, the scalar and pseudoscalar transform with opposite boundary conditions (even when the parity condition is relaxed). Since this corresponds to opposite signs for α_- between the scalar and pseudoscalar, we see that the mixed term in Eq. (36) always drops out when considering pairs of spin-0 states as members of supermultiplets. As a result, we find the simple universal structure for the graviton self-energy

$$\Sigma_{MN,PQ}(X,Y)=8\pi G_4\langle T_{MN}(X)T_{PQ}(Y)\rangle$$
$$=8\pi G_4\frac{n_0+3n_{1/2}+12n_1}{48\pi^4L_4^8}\left[\frac{\alpha_+^2}{(Z+1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2+ZT_3\right)+\frac{\alpha_-^2}{(Z-1)^4}\left(\frac{3Z^2+1}{4}T_1+T_2-ZT_3\right)\right]. \quad (48)$$

V. EXTRACTION OF THE GRAVITON MASS

We now extract the induced graviton mass from the long distance behavior of the self-energy Eq. (48). We first note that the three terms of Π in Eq. (7) correspond to local tensor, nonlocal spin-1 and spin-0 exchange, respectively. Following Ref. [8], the mass can be read off by identifying in Σ a piece proportional to the spin-1 Goldstone boson exchange, given by the second term in Eq. (7):

$$\Pi^{(\text{spin-1})}_{\mu\nu\ \alpha\beta}=2\nabla_\nu\left(\frac{g_{\nu\beta}+\nabla_\nu\nabla_\beta/2\Lambda}{\Delta-2\Lambda}\right)\nabla_\alpha=2\nabla_\mu D_{\nu\beta}\nabla_\alpha. \quad (49)$$

Here, $D_{\mu\nu}$ is the spin-1, $E_0=4$, propagator.

Working in coordinate space, we now rewrite $\Pi^{(\text{spin-1})}$ as a bi-local tensor. To accomplish this, we start with the homogeneous space $E_0=4$ vector propagator, which was worked out in Ref. [22]:

$$D_{MP}=\frac{1}{48\pi^2L^2}\left[\frac{2(8-15Z^2+9Z^4)}{(Z^2-1)^2}-9Z\log\frac{Z+1}{Z-1}\right]\hat{G}_{MP}+\frac{1}{48\pi^2L^2}\left[\frac{2Z(-23+24Z^2-9Z^4)}{(Z^2-1)^2}+9(Z^2-1)\log\frac{Z+1}{Z-1}\right]N_MN_P. \quad (50)$$

We now freely integrate by parts to obtain

$$\Pi^{(\text{spin-1})}_{MNPQ}=2\nabla_{X^M}D_{NQ}\nabla_{Y^P}=-2(\nabla_{X^M}\nabla_{Y^P}D_{NQ})=-\frac{2Z}{3\pi^2L_4^4(Z^2-1)^3}[5(3Z^2+1)T_1+2T_2-5(Z^2+1)T_3]. \quad (51)$$

Note that this nonlocal $\Pi^{(\text{spin-1})}$ is transverse and traceless in itself, while the original expression, Eq. (7), requires an interplay among all terms to ensure transversality. This discrepancy arises only through local terms that we have ignored throughout.

While the one-loop self-energies we have computed all satisfy the homogeneous coordinate transversality condition, Eq. (B6), this condition still allows an undetermined Z-dependent form factor. To read off the correctly induced graviton mass, we essentially need to obtain the constant piece of $\beta(\Delta)$ in Eq. (6), which may be determined by matching the large Z behavior of Eq. (48) with that of the spin-1 part of Π, given by Eq. (51). To do so, we expand both expressions for large Z and match the asymptotic behavior. For the self-energy, we find

$$\Sigma=8\pi G_4\frac{n_0+3n_{1/2}+12n_1}{48\pi^4L^8}\times\left[(\alpha_+^2+\alpha_-^2)\left(\frac{1}{4}\mathcal{T}_{(4)}+\frac{5}{2}\mathcal{T}_{(6)}+\frac{35}{4}\mathcal{T}_{(8)}+\cdots\right)+(\alpha_+^2-\alpha_-^2)(\mathcal{T}_{(5)}+5\mathcal{T}_{(7)}+\cdots)\right], \quad (52)$$

while

$$\Pi^{spin\text{-}1}=\frac{10}{3\pi^2L^4}[\mathcal{T}_{(5)}+3\mathcal{T}_{(7)}+\cdots], \quad (53)$$

where the basis forms, $\mathcal{T}_{(n)}$, are given in Appendix B. Matching the leading $\mathcal{T}_{(5)}$ term gives

$$M^2=8\pi G_4\frac{n_0+3n_{1/2}+12n_1}{160\pi^2L^4}(\alpha_+^2-\alpha_-^2). \quad (54)$$

This expression is our main result, and generalizes that obtained in Ref. [8]. Note, however, that this result differs by a factor of 160 from that of Ref. [8]. We believe that this discrepancy arises from three sources. Firstly, normalization of the $E_0=4$ scalar propagator is determined by demanding the proper strength of the short-distance singularity in the flat space limit [22]. This yields

$$\Delta_0(E_0=4) = -\frac{1}{4\pi^2 L^2}\left[\frac{3Z^2-2}{Z^2-1} - \frac{3}{2}Z\log\frac{Z+1}{Z-1}\right] \to \frac{1}{8\pi^2L^2}\frac{1}{Z+1} \quad \text{as} \quad Z\to -1 \tag{55}$$

[compare with Eq. (27)]. Taking the large separation limit, $Z\to\infty$, then gives

$$\Delta_0(E_0=4) \sim -\frac{1}{10\pi^2L^2}\frac{1}{Z^4} \quad \text{as} \quad Z\to\infty \tag{56}$$

which accounts for a factor of four. Secondly, without examining the tensor structure in detail, there is an ambiguity in attributing the long range structure of the self-energy Σ to the propagation of a spin-1 Goldstone boson in Π. In particular, both $\mathcal{T}_{(5)}$ and $\mathcal{T}_{(7)}$ of Eq. (52) and Eq. (53) have the requisite long range falloff upon integration by parts

$$h\cdot\mathcal{T}_{(5)}\cdot h \sim \frac{3}{10}\partial_M h_{MN}\frac{\eta_{NQ}}{Z^4}\partial_P h_{PQ},$$

$$h\cdot\mathcal{T}_{(7)}\cdot h \sim -\frac{3}{25}\partial_M h_{MN}\frac{\eta_{NQ}}{Z^4}\partial_P h_{PQ}. \tag{57}$$

As a result, both terms would contribute to the coefficient of the $1/Z^4$ piece, while only the actual combination $\mathcal{T}_{(5)}+3\mathcal{T}_{(7)}$ of Eq. (53) may be attributed to the induced graviton mass. In other words, it is important to match only the leading $\mathcal{T}_{(5)}$ behavior between Σ and Π of Eqs. (52) and (53). Of course, this would have been immaterial if the asymptotic expansions had been identical. However, in this case they are not, and this accounts for another factor of five between our expression and that of Ref. [8]. Finally, the remaining factor of eight comes in when determining the mass via the shift in the pole of the resummed propagator, Eq. (13). We find the mass squared to be simply the constant multiplying $\Pi^{spin\text{-}1}$ (up to a sign). Since a canonically normalized graviton couples to the stress tensor with strength $\kappa h^{\mu\nu}T_{\mu\nu}$, and since we do not include symmetry factors in our coordinate space Feynman rules, we have simply

$$\Sigma_{MNPQ}(X,Y) = \kappa^2\langle T_{MN}(X)T_{PQ}(Y)\rangle = 8\pi G_4\langle T_{MN}(X)T_{PQ}(Y)\rangle. \tag{58}$$

We believe this provides a proper accounting for Newton's constant in the self-energy. Comparing with Ref. [8], this appears to be the origin of the remaining factor discrepancy.

Note that the spin-0 term in Π has a different structure. However this term is canceled by the nonlocal part of K. The absence of spin-0 exchange in Σ is in agreement with the AdS Higgs mechanism [8], and yields the massive spin-2 propagator (14) without ghosts.

While we have focused on the dynamical breaking of general covariance, as evidenced by a mass for the graviton, in a supersymmetric Karch-Randall model, a dynamical breaking of local supersymmetry and local gauge invariance also occurs, as evidenced by a mass for the gravitinos and the gauge bosons. For the Karch-Randall braneworld [9], where the CFT fields are that of $\mathcal{N}=4$ $U(N)$ super-Yang-Mills, we substitute transparent boundary conditions ($\alpha_+=1$, $\alpha_-=0$) into the expression for the graviton mass, Eq. (54), and find simply

$$M^2 = \frac{9G_4}{4L_4^4}\alpha, \tag{59}$$

which reproduces exactly the Karch-Randall result of Eq. (1) upon using Eq. (4). Although we focused on the $\mathcal{N}=4$ SCFT to relate the coefficient α to the central charge, the result Eq. Eq. (4) is universal, being independent of which particular CFT appears in the AdS/CFT correspondence. This suggests that α plays a universal role in both the Minkowski and AdS braneworlds, as indicated in Eqs. (59) and (5), and that our result is robust at strong coupling. This presumably explains why our one-loop computation gives the exact Karch-Randall result. However, we do not know for certain whether this persists beyond one loop.

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APPENDIX A: PROPAGATORS IN ADS

Here we collect some information on spin-0, -1/2, and -1 propagators in homogeneous coordinates. First recall that the quadratic Casimir invariant of $SO(2,3)$ is $Q=\frac{1}{2}L_{MN}^2 = E_0(E_0-3)+s(s+1)$ where E_0 and s label the representation $D(E_0,s)$. Acting on scalars $\phi(X)$, the operator Q (corresponding to the Casimir invariant) has the form

$$Q = \tfrac{1}{2}L_{MN}^2 = -\tfrac{1}{2}(X_M\partial_N - X_N\partial_M)^2 = \hat{N}(\hat{N}+3) - X^2\partial^2 \tag{A1}$$

where $\hat{N}=X\cdot\partial$. As a result, the scalar Klein-Gordon equation is simply

$$[\hat{N}(\hat{N}+3) - X^2\partial^2 - E_0(E_0-3)]\phi(X) = 0. \tag{A2}$$

To obtain the scalar Green's function between points X and Y, we note that $\partial^2 = -\partial_Z^2/L^2$ and $\hat{N}=X\cdot\partial = Z\partial_Z$. In this case, we find that $\Delta_0(Z)\equiv\Delta_0(X,Y)$ satisfies the equation

$$[(1-Z^2)\partial_Z^2 - 4Z\partial_Z + E_0(E_0-3)]\Delta_0(Z) = 0. \tag{A3}$$

For either $E_0=1$ or 2, this has a simple pair of solutions, $\Delta_0 \sim 1/(Z\pm 1)$. However, in order to reproduce a short distance behavior $\Delta_0 \sim 1/|X-Y|^2$, we must take the one with the positive sign. As a result, we obtain

$$\Delta_0 = \frac{1}{8\pi^2 L^2}\frac{1}{Z+1} = -\frac{1}{4\pi^2}\frac{1}{|X-Y|^2}. \tag{A4}$$

The normalization is fixed by demanding that Δ_0 reduces properly in the flat space limit.

The propagator of Eq. (A4) in fact corresponds to imposing transparent boundary conditions on the scalar. This is seen by recalling that while (the covering space of) AdS_4 may be conformally mapped into half of the Einstein static universe, with topology $R\times S_3$, Eq. (A4) is in fact well defined on the complete S_3 (so that the boundary is in effect invisible) [26]. Reflective boundary conditions may be imposed by a method of images so that

$$\begin{aligned}\Delta_0^{\pm} &= -\frac{1}{4\pi^2}\left(\frac{1}{|X-Y|^2} \pm \frac{1}{|X+Y|^2}\right)\\ &= \frac{1}{8\pi^2 L^2}\left(\frac{1}{Z+1} \mp \frac{1}{Z-1}\right).\end{aligned} \tag{A5}$$

It is now evident that mixed boundary conditions may be encoded by parameters α_+ , α_- where

$$\Delta_0^{(\alpha)} = \frac{1}{8\pi^2 L^2}\left(\frac{\alpha_+}{Z+1} + \frac{\alpha_-}{Z-1}\right). \tag{A6}$$

While the residue of the short distance pole must be fixed (i.e. $\alpha_+ = 1$), we find it illuminating to keep α_+ arbitrary, as it highlights the symmetries in the latter expressions for the graviton self-energy computation. In terms of Porrati's α and β coefficients, defined[3] by [8]

$$\Delta_0 = -\frac{1}{4\pi^2 L^2}\left(\alpha\frac{1}{Z^2-1} - \beta\frac{Z}{Z^2-1}\right), \tag{A7}$$

we find $\alpha_+ = (\alpha+\beta)$ and $\alpha_- = -(\alpha-\beta)$.

For the fermion propagator, we consider the Dirac equation in homogeneous coordinates. Start by defining the Dirac operator $K=\Gamma^{MN}X_M\partial_N$ where $\{\Gamma^M,\Gamma^N\}=2\eta^{MN}$. By squaring this operator, it is easy to show that $K(K-3)=\hat{N}(\hat{N}+3)-X^2\partial^2$. On the other hand, by squaring the $SO(2,3)$ generators acting on a spin-$\frac{1}{2}$ state, $L_{MN}=i(X_M\partial_N - X_n\partial_M)+(i/2)\Gamma_{MN}$, we may show that $Q=\hat{N}(\hat{N}+3)-X^2\partial^2+\frac{5}{2}-K=K(K-4)+\frac{5}{2}$. When acting on $\Psi(X)$, this must reproduce the quadratic Casimir invariant $Q=E_0(E_0-3)+\frac{3}{4}$. Equating these expressions, we find the factorized relation $(K-\frac{1}{2})(K-\frac{7}{2})=E_0(E_0-3)$, so that either $E_0=K-\frac{1}{2}$ or $E_0=\frac{7}{2}-K$. This gives two possible Dirac equations

$$[K-(E_0+\tfrac{1}{2})]\Psi(X)=0 \quad \text{or} \quad [K+(E_0-\tfrac{7}{2})]\Psi(X)=0. \tag{A8}$$

For the massless case ($E_0=\frac{3}{2}$), both equations degenerate to $(K-2)\Psi(X)=0$.

Next, we note the factorization $(K-\lambda)(K+\lambda-3)=\hat{N}(\hat{N}+3)-X^2\partial^2-\lambda(\lambda-3)$, which holds for arbitrary λ. Since the right hand side is simply the scalar Klein-Gordon operator, Eq. (A2), this provides the AdS equivalent of the relation $(\partial\!\!\!/-m)(\partial\!\!\!/+m)=\Box-m^2$. Denoting either λ or $3-\lambda$ by $E_0^{(0)}$ (indicating the canonical value of E_0 in the scalar equation), this may be rewritten in the suggestive manner [28]

$$\begin{aligned}&[K-(E_0+\tfrac{1}{2})][K+(E_0-\tfrac{5}{2})]\\ &\qquad=\hat{N}(\hat{N}+3)-X^2\partial^2-E_0^{(0)}(E_0^{(0)}-3), \quad E_0^{(0)}=E_0+\tfrac{1}{2}\\ &[K+(E_0-\tfrac{7}{2})][K-(E_0-\tfrac{1}{2})]\\ &\qquad=\hat{N}(\hat{N}+3)-X^2\partial^2-E_0^{(0)}(E_0^{(0)}-3), \quad E_0^{(0)}=E_0-\tfrac{1}{2}\end{aligned} \tag{A9}$$

so that solutions to the Dirac equation, Eq. (A8), are easily obtained from solutions to the scalar equation, Eq. (A2), by taking

$$\Psi(X)=[K+(E_0-\tfrac{5}{2})]\Psi_0\phi(X;E_0^{(0)}=E_0+\tfrac{1}{2})$$

or

$$\Psi(X)=[K-(E_0-\tfrac{1}{2})]\Psi_0\phi(X;E_0^{(0)}=E_0-\tfrac{1}{2}) \tag{A10}$$

with Ψ_0 a constant spinor. This result allows us to immediately determine the fermion propagator in terms of the scalar one in much the same way as one would compute $1/\partial\!\!\!/=\partial\!\!\!/ /\Box$ in the flat limit.

For $E_0=\frac{3}{2}$ (corresponding to $E_0^{(0)}=1$ or 2), we use the form of the scalar propagator, Eq. (A4), and the relation Eq. (A10) to obtain

$$\Delta_{1/2}=\frac{1}{8\pi^2 L^4}\frac{\Gamma^M(X_M-Y_M)}{(Z+1)^2}=\frac{1}{2\pi^2}\frac{\Gamma^M(X_M-Y_M)}{|X-Y|^4}. \tag{A11}$$

This is the massless fermion propagator corresponding to transparent boundary conditions. Similar to Eq. (A6), general boundary conditions may be imposed by introducing parameters α_+ , α_- and taking

[3]Some signs have been changed to conform to our conventions.

TABLE I. First few elements of the transverse traceless bi-tensor basis $\mathcal{T}_{(n)}$. The coefficients a_1, a_2, and a_3 correspond to the decomposition $\mathcal{T}=a_1(3Z^2+1)T_1+a_2T_2+a_3T_3$.

	a_1	a_2	a_3
$\mathcal{T}_{(4)}$	Z^{-4}	$-2Z^{-4}+6Z^{-6}$	$4Z^{-4}$
$\mathcal{T}_{(5)}$	Z^{-5}	$-5Z^{-5}+9Z^{-7}$	$-Z^{-3}+5Z^{-5}$
$\mathcal{T}_{(6)}$	Z^{-6}	$\frac{3}{5}Z^{-4}-\frac{46}{5}Z^{-6}+\frac{63}{5}Z^{-8}$	$-2Z^{-4}+6Z^{-6}$
$\mathcal{T}_{(7)}$	Z^{-7}	$\frac{9}{5}Z^{-5}-\frac{73}{5}Z^{-7}+\frac{84}{5}Z^{-9}$	$-3Z^{-5}+7Z^{-7}$

$$\Delta_{1/2}^{(\alpha)}=\frac{1}{8\pi^2L^4}\left(\alpha_+\frac{\Gamma^M(X_M-Y_M)}{(Z+1)^2}+\alpha_-\frac{\Gamma^M(X_M+Y_M)}{(Z-1)^2}\right). \tag{A12}$$

Turning next to the vector propagator, we use the results of Ref. [22], converted to homogeneous coordinates. The vector propagator is the first case where we have to worry about bi-tensor structures as well as gauge fixing. However, fortunately, for correlators of the stress tensor, we only need the expression for the gauge invariant two-point function $\langle F_{MN}(X)F_{PQ}(Y)\rangle$. Based on symmetry, this expression can be written as

$$\langle F_{MN}(X)F^{PQ}(Y)\rangle=\sigma(Z)\hat{G}_{[M}{}^{[P}\hat{G}_{N]}{}^{Q]}+\tau(Z)N_{[M}\hat{G}_{N]}{}^{[Q}N^{P]} \tag{A13}$$

where $\sigma(Z)$ and $\tau(Z)$ may be determined as in Ref. [22]. Taking into account mixed boundary conditions as well as normalization of the short distance behavior, we find

$$\langle F_{MN}(X)F^{PQ}(Y)\rangle^{(\alpha)}=\frac{1}{2\pi^2L^4}\left[\frac{\alpha_+}{(Z+1)^2}[\hat{G}_{[M}{}^{[P}\hat{G}_{N]}{}^{Q]}-2(Z-1)N_{[M}\hat{G}_{N]}{}^{[Q}N^{P]}]+\frac{\alpha_-}{(Z-1)^2}[\hat{G}_{[M}{}^{[P}\hat{G}_{N]}{}^{Q]}-2(Z+1)N_{[M}\hat{G}_{N]}{}^{[Q}N^{P]}]\right]. \tag{A14}$$

These mixed boundary condition propagators, Eq. (A6), Eq. (A12), and Eq. (A14), are the ones used in the one-loop computation.

APPENDIX B: A TRANSVERSE-TRACELESS BI-TENSOR BASIS

In this appendix, we present a convenient basis into which any transverse-traceless bi-local tensor may be decomposed. Since any traceless tensor, $\mathcal{T}$, may be decomposed in terms of the three T tensors defined in Eq. (22), we start by writing $\mathcal{T}=a_1(3Z^2+1)T_1+a_2T_2+a_3T_3$. The factor $(3Z^2+1)$ is introduced for convenience. We now impose transversality on $\mathcal{T}$. In particular, taking the divergence of $\mathcal{T}$ on the first index gives

$$\nabla^M\mathcal{T}_{MNQP}=\sqrt{\frac{Z^2-1}{L}}\{[(3Z^2+1)a_1'+6Za_1]N\cdot T_1+a_2'N\cdot T_2+a_3'N\cdot T_3\}+a_1(3Z^2+1)\nabla^MT_1+a_2\nabla^MT_2+a_3\nabla^MT_3. \tag{B1}$$

We compute

$$N\cdot T_1=\frac{1}{3Z^2+1}A,\quad N\cdot T_2=0,\quad N\cdot T_3=\frac{1}{2Z}B \tag{B2}$$

and

$$\sqrt{Z^2-1}\nabla\cdot T_1=\frac{2Z(3Z^2+5)}{(3Z^2+1)^2}A-\frac{4}{3(3Z^2+1)}B,$$

$$\sqrt{Z^2-1}\nabla\cdot T_2=-\frac{5}{3}B, \tag{B3}$$

$$\sqrt{Z^2-1}\nabla\cdot T_3=\frac{3Z^2+1}{2Z^2}B-\frac{1}{Z}A,$$

where the tensors A and B are given by

$$A_{NPQ}=(4N_NN_PN_Q-N_NG_{PQ}),$$

$$B_{NPQ}=[(\hat{G}_{NP}N_Q+\hat{G}_{NQ}N_P)-2ZN_NN_PN_Q]. \tag{B4}$$

Thus the vanishing of the divergence in Eq. (B1) leads to two conditions on the three functions,

$$(Z^2-1)Za_1'=-4Z^2a_1+a_3,$$

$$(Z^2-1)Za_3'=\frac{8}{3}Z^2a_1+\frac{10}{3}Z^2a_2-(3Z^2+1)a_3. \tag{B5}$$

These equations may be solved to give a_2 and a_3 in terms of a_1 and its derivatives. As a result, any transverse traceless bi-tensor must take the form

$$\mathcal{T}=a(3Z^2+1)T_1 + \left[\frac{3}{10}(Z^2-1)^2a''+3(Z^2-1)Za'+2(3Z^2-1)a\right]T_2 + [(Z^2-1)Za'+4Z^2a]T_3 \quad \text{(B6)}$$

and is fully specified by the function $a(Z)$.

By choosing a complete set of functions $a(Z)$, we may obtain a basis of transverse traceless bi-tensors. A convenient choice is to take $a_{(n)}=1/Z^n$, whereupon the resulting expression of Eq. (B6) may be denoted $\mathcal{T}_{(n)}$. The first few basis bi-tensors with sufficiently fast large distance falloff are shown in Table I. Note the absence of leading order $1/Z^2$ and $1/Z^3$ behavior in the a_2 and a_3 coefficients of $\mathcal{T}_{(4)}$ and $\mathcal{T}_{(5)}$, respectively.

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