# $D=4$ supergravity dynamically coupled to superstring in a superfield Lagrangian approach

Igor A. Bandos*
*Departamento de Física Teórica and IFIC (CSIC–UVEG), Universidad de Valencia, 46100-Burjassot (Valencia), Spain and Institute for Theoretical Physics, NSC KIPT, UA61108, Kharkov, Ukraine*

José M. Isidro†
*Instituto de Física Corpuscular (CSIC–UVEG), Apartado de Correos 22085, Valencia 46071, Spain*
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We elaborate a full superfield description of the interacting system of dynamical $D=4$, $N=1$ supergravity and a dynamical superstring. As far as a minimal formulation of simple supergravity is used, such a system should contain as well the tensor (real linear) multiplet which describes the dilaton and the two-superform gauge field whose pullback provides the Wess-Zumino term for the superstring. The superfield action is given by the sum of the Wess-Zumino action for $D=4$, $N=1$ superfield supergravity, the superfield action for the tensor multiplet in curved superspace, and the Green-Schwarz superstring action. The latter includes the coupling to the tensor multiplet both in the Nambu-Goto and in the Wess-Zumino terms. We derive superfield equations of motion including, besides the superfield supergravity equations with the source, the source-full superfield equations for the linear multiplet. The superstring equations keep the same form as for the superstring in supergravity and 2-superform background. The analysis of gauge symmetries shows that the superfield description of the interacting system is gauge equivalent to the dynamical system described by the sum of the spacetime, component action for supergravity interacting with the tensor multiplet, and of the purely bosonic string action.

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## I. INTRODUCTION

Recently, there has been renewed interest in the superfield description of supergravity [1–4]. It is motivated, in particular, by the search for a superfield formulation of 10-dimensional supergravity incorporating superstring corrections (see [5–7] for early studies and [8,9] for discussions). Supergravity was known to appear at the pointlike limit of the superstring which corresponds to the $\alpha'\rightarrow 0$ limit—i.e., to zeroth order in the decomposition in the Regge slope parameter $\alpha'$. Already at first order in $\alpha'$, the string corrections modify the supergravity equations of motion. On the other hand, the known superfield formulations of $D=10$ supergravity provide its on-shell description; it is given by the *on-shell* constraints on superspace torsion, which imply the dynamical equations for the physical fields. These are just the equations which correspond to the $\alpha'\rightarrow 0$ limit of the superstring. Thus the incorporation of $\alpha'$ corrections requires modification of the standard superspace constraints: a search for a possibility to replace the on-shell constraints by a set of off-shell constraints or, at least, by a set of "on any shell" constraints [1] including some parameters which specify the right-hand side of the supergravity equations and which can be chosen to describe the superstring corrections to such equations.

Basically the same problem appears when one searches for a superfield description of a superbrane interacting with higher-dimensional supergravity. The superbrane is defined as a brane moving in superspace. It is well known [10,11] that the requirement of a smooth flat superspace limit for the superbrane in curved superspace (which implies that the superbrane action in a curved superspace background should possess the same number of gauge symmetries, including fermionic $\kappa$ symmetries [12]) results in the standard on-shell supergravity constraints. However, as said above, such constraints imply "free" supergravity equations of motion without any superbrane source. On the other hand, as is clear from the purely bosonic limit (gravity interacting with a bosonic brane), the brane should provide a source in the Einstein equation.[1] So one could get the (mistaken) impression that supersymmetry forbids interaction with an extended object, at least at the classical level. Certainly this is not the case. The resolution of such a paradox and the search for a consistent (quasi)classical description of the *supergravity-superbrane* interacting systems is of interest in its own, as

*Email address: bandos@ific.uv.es; bandos@kipt.kharkov.ua
†Email address: jmisidro@ific.uv.es

[1]Note that a similar problem appeared for the heterotic superstring [13] in $D=10$, $N=1$ supergravity and $E_8\otimes E_8$ [or SO(32)] super Yang-Mills (SUGRA-SYM) background [7]. Namely, the requirement of $\kappa$ symmetry of the classical heterotic superstring model results in constraints which describe decoupled SUGRA and SYM systems, while the Green-Schwarz anomaly cancellation mechanism required their nontrivial interaction. This problem had motivated the study [7] of (one-loop) quantum anomalies in the $\kappa$ symmetry transformations. As was shown in [7], such anomalies occur, but may be absorbed by consistent quantum corrections to the classical (tree-level) expressions for superspace torsion and curvature. The new (one-loop) torsion constraints lead to the desired coupled equations for the SUGRA-SYM system.

 

well as in relation with its possible applications to the study of quantum gauge theories in the language of classical supergravity-superbrane models along the line of the AdS conformal field theory (CFT) correspondence [14–16].

The study of the complete Lagrangian superfield description of the supergravity-superbrane interacting system, when it is possible, e.g., in $D=4$, $N=1$ curved superspace, might provide new insights into the search for a modification of higher-dimensional ($D=10$, 11) supergravity constraints in such a way that they would produce dynamical equations with sources, including singular sources from superbranes and nonsingular sources describing the stringy corrections.

Such a study has been carried out in [17] for the interacting system of dynamical $D=4$, $N=1$ supergravity and a massless superparticle source. Here we elaborate the superfield description of the next more complex system which includes, besides dynamical $D=4$, $N=1$ supergravity, the dynamical superstring. It has some specific features in comparison with the system already studied in [17]. First, the source is provided by a supersymmetric *extended* object. Second, as far as the minimal formulation of $D=4$, $N=1$ supergravity is considered, one finds (see [18]) that the Wess-Zumino term of the superstring describes a coupling to an additional, dynamical tensor (or real linear) multiplet [19,20] which can be used to formulate the nontrivial 2-superform gauge theory in superspace [21]. Moreover, superstring $\kappa$ symmetry requires the identification of the tensor multiplet with a dilaton superfield and its coupling to the kinetic, Nambu-Goto term of the superstring action. Thus the superfield action for the interacting system which we will study in this paper includes, in addition to the Wess-Zumino action for supergravity [22] and the $D=4$, $N=1$ Green-Schwarz superstring action [23], also a superfield action [24] for the tensor multiplet [19] in a curved superspace of minimal supergravity.[2] It has the form (see the main text for the notation)

$$S=\int d^8Z\,\mathrm{sdet}(E_M{}^A)\left(1+s\frac{\Phi}{2}e^{\Phi/2}\right)+\frac{1}{2\pi\alpha'}S_{sstr}, \tag{1.1}$$

$$S_{sstr}=\frac{1}{2}\int_{W^2}d^2\xi e^{\hat{\Phi}/2}\sqrt{|\det(\hat{E}_m^a\hat{E}_n^b\eta_{ab})|}-\int_{W^2}\hat{B}_2, \tag{1.2}$$

where $s$ is the coupling constant for the tensor multiplet, $1/2\pi\alpha'$ is the superstring tension, which we set equal to unity in the main text of the present article, $\xi^m=(\tau,\sigma)$ are local coordinates of the string world sheet $\mathcal{W}^2$, $\hat{E}_m^a:=\partial_m\hat{Z}^M(\xi)E_M{}^A(\hat{Z}(\xi))$, the supervielbein $E_M{}^A(Z)$ and 2-superform $B_2:=\frac{1}{2}dZ^M\wedge dZ^N B_{NM}(Z)$ are subject to the constraints given in Sec. II, and, finally, the superfield $e^{\Phi/2}$ satisfies the defining constraints of a tensor multiplet in curved superspace,

$$(\mathcal{D}\mathcal{D}-\bar{R})e^{\Phi/2}=0,\quad (\overline{\mathcal{D}\mathcal{D}}-R)e^{\Phi/2}=0. \tag{1.3}$$

One of the main results of this paper is the complete set of superfield equations of motion for the interacting dynamical system (1.1), including the superfield supergravity equations and the dynamical equations for the tensor multiplet (1.3) with the superstring source.[3] Although the original hope was that these might provide some insight in a search for sourcefull superfield equations for more complicated interacting systems in $D=10,11$ superspaces, one might find the obtained superfield equations quite complicated and rather showing difficulties which should appear in the construction of their higher-dimensional generalizations. However, as we hope, these superfield equations, supplemented by a proper supersymmetric ansatz, can be used to search for superfield solutions of the superfield supergravity equations;[4] this might open completely new possibilities.

On the other hand, as far as the multidimensional generalization of the supergravity-superbrane interacting system is concerned, we also discuss a more pragmatic approach to their investigation, using the $D=4$, $N=1$ system as a simplified model. A recent study of supergravity-superbrane interactions indicates the gauge equivalence of the superfield description of the dynamical supergravity interacting with a superbrane source with a simpler system which is described by the sum of a standard (component) supergravity action and of the action of a purely bosonic brane (a purely bosonic limit of the associated superbrane). Although the proof of such a gauge equivalence [17,32] is quite general and does not need a detailed superfield supergravity formulation (still hypothetical and, probably, even not existing for the $D=10,11$ cases), one may find it necessary to check it explicitly by comparing the equations of motion derived from the gauge-fixed action with the gauge-fixed version of the dynamical equations derived from the complete superfield action, at least for the cases when the latter exists. The gauge equivalence and completeness of the gauge-fixed description, shown by general arguments in [17,32], implies that these two set of equations should coincide. Until now this had

[2]The sum of the superfield minimal supergravity action and a superfield tensor multiplet action was motivated by being a low-energy limit of a $D=4$, $N=1$ compactification of the heterotic string [18,25]. Nevertheless, this limit, as well as its $N>1$ generalizations, which was intensive studied in the 1980s and 1990s [26–29], is not a subject of the present paper. We rather consider the $D=4$ supergravity-superstring interacting system as a relatively simple model for a more complicated $D=10,11$ supergravity-superbrane systems (see [41]). An interesting alternative possibility is to consider the new minimal formulation of superfield supergravity [30], where the supergravity *auxiliary* fields are provided by a tensor multiplet, interacting with the Green-Schwarz superstring. This, however, is beyond the score of the present paper.

[3]Note that superfield equations of motion for the field theoretical part of our interacting system—i.e., following from the action $S=\int d^8Z\,\mathrm{sdet}(E_M{}^A)[1+s(\Phi/2)e^{\Phi/2}]$, but without the $S_{sstr}$ term—were considered in [27,29].

[4]Note that a toy model of the superfield interacting system—the coupled system of $D=2$ supergravity and a superparticle—was studied in [31]. We thank W. Kummer and A. Nurmagambetov for pointing out this reference.

been checked by a straightforward study of the $D=4$ supergravity-superparticle interacting system [17]. In this paper we show the coincidence of the two sets of equations for the $D=4$, $N=1$ supergravity-superstring and thus check the gauge equivalence for the dynamical system including an extended object. This should convince one that the above-mentioned gauge equivalence is not an artifact of the system including material superparticles in addition to dynamical supergravity, but rather a general property of the supergravity-superbrane interacting system.

Notice that the above-discussed gauge-equivalent description of the supergravity-superbrane system promises to be useful to search for new solitonic solutions of superfield supergravity, with nontrivial fermionic fields. (Very few such solutions are known; an example is the *pp*-wave solution of [33].)

This paper is organized as follows. In Sec. II we review the properties of the Green-Schwarz superstring in curved $D=4$, $N=1$ superspace, Eq. (1.2), which are necessary for the study of the interacting system. The superstring action (1.2) involves, in addition to (the pullbacks of) the supervielbein, the scalar superfield $\Phi$ and the 2-superform $B_2$, neither of which is involved in the superfield description of minimal $D=4$, $N=1$ superfield supergravity. In Sec. II A we show (in a way close to [18]) that the requirement of preservation of the superstring $\kappa$ symmetry in the minimal curved $D=4$, $N=1$ superspace results in constraints for the field strengths $H_3=dB_2$ and find that these constraints express $H_3$ in terms of the dilaton superfield $\Phi$. Furthermore, a study of the Bianchi identities $dH_3\equiv 0$ in a way close to the one of Refs. [18,21] (Sec. II B) concludes that the superfield $e^{\Phi/2}$ obeys the defining constraints of the tensor multiplet, Eqs. (1.3).

Section III collects the necessary information about the superfield supergravity action [22,37] and its variation [17,22]. Section IV describes admissible variations of the tensor multiplet (the dilaton superfield $\Phi$) and of the $B_2$ superform. In Sec. V we present the complete superfield action for the *supergravity–tensor-multiplet–superstring* interacting system and derive the superfield equations of motion by its variation (Secs. V A, V B, V C). (Superfield equations which follow from the sum of the superfield action of the minimal supergravity and the tensor multiplet, without including the superstring action, were considered in [27,29] in connection with the $D=4$, $N=1$ limit and compactification of the heterotic superstring.) The superfield generalizations of the Einstein and Rarita-Schwinger equations *with sources* are presented in Sec. V D and those of the Kalb-Ramond equations for tensor gauge fields in Sec. V E.

Then, in Sec. VI A, by studying the gauge symmetries and using known results about fixing the Wess-Zumino gauge (see [17] and references therein) we show that the superfield description of the supergravity–tensor-multiplet–superstring interacting system is gauge equivalent to a supergravity–tensor-multiplet–bosonic string dynamical system described by the sum of the *component* (spacetime) action for supergravity interacting with tensor multiplet [24] and the action for the purely bosonic string, the purely bosonic limit of the Green-Schwarz superstring. In Sec. VI B we check this gauge equivalence at the level of equations of motion by proving that the dynamical equations of motion which follow from the complete superfield description, when considered in the special "fermionic unitary gauge" $[\hat{Z}^M(\xi) := (\hat{x}^\mu(\xi), \hat{\theta}^{\check{\alpha}}(\xi)) = (\hat{x}^\mu(\xi), 0)]$ have the same properties as the equations derived from the gauge-fixed action (the action in the "fermionic unitary gauge"). Namely, we show that all dynamical equations for fermions become sourceless in this gauge. Our conclusions are collected in Sec. VII.

The flat superspace limit of the superstring action is reviewed in Appendix A. Appendix B collects some useful formulas.

## II. GREEN-SCHWARZ SUPERSTRING IN A $D=4$, $N=1$ SUPERGRAVITY BACKGROUND

The Green-Schwarz superstring spans a two-dimensional world sheet $\mathcal{W}^2$ in superspace $\Sigma^{(4|4)}$:

$$\mathcal{W}^2 \subset \Sigma^{(4|4)}, \quad Z^M = \hat{Z}^M(\xi^m). \tag{2.1}$$

In Eqs. (2.1) $Z^M = (x^\mu, \theta^{\check{\alpha}})$ are coordinates of $D=4$, $N=1$ superspace ($\mu=0,1,2,3$, $\check{\alpha}=1,2,3,4$), $\xi^m=(\tau,\sigma)$ are local world sheet coordinates ($m=0,1$), and $\hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^{\check{\alpha}}(\xi))$ are *supercoordinate functions* that define the surface $\mathcal{W}^2$ in $\Sigma^{(4|4)}$. One can also say that $\hat{Z}^M(\xi)$ are defined by the map

$$\hat{\phi}: W^2 \to \Sigma^{(4|4)}, \quad \xi^m \mapsto \hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^{\check{\alpha}}(\xi)), \tag{2.2}$$

of the coordinate chart $W^2$ into $\Sigma^{(4|4)}$.

The $D=4$, $N=1$ version of the Green-Schwarz superstring action reads

$$S_{sstr} = \int_{W^2} \hat{\mathcal{L}}_2 = \int_{W^2} \left[ \frac{1}{4} e^{\hat{\Phi}/2} * \hat{E}^a \wedge \hat{E}^b \eta_{ab} - \hat{B}_2 \right]$$

$$\equiv \int d^2\xi \sqrt{\det|g|} - \int_{W^2} \hat{B}_2, \tag{2.3}$$

$$g = \det(g_{mn}), \quad g_{mn} = \hat{E}^a_m \hat{E}_{na}. \tag{2.4}$$

It involves the pullbacks of the forms on superspace to $\mathcal{W}^2$,

$$\hat{E}^a \equiv \hat{\phi}^*(E^a) = d\hat{Z}^M(\xi) E_M^{\ a}(\hat{Z}) \equiv d\xi^m \hat{E}^a_m,$$

$$\hat{E}^a_m := \partial_m \hat{Z}^M E_M^{\ a}(\hat{Z}(\xi)), \tag{2.5}$$

for the bosonic supervielbein form $E^a$ on $\Sigma^{(4|4)}$,

$$E^A \equiv (E^a, E^{\underline{\alpha}}) = (E^a, E^\alpha, \bar{E}^{\dot{\alpha}}), \tag{2.6}$$

$$E^a = dZ^M E_M^a(Z), \tag{2.7}$$

$$E^{\underline{\alpha}} = dZ^M E_M^{\underline{\alpha}}(Z) \quad \leftrightarrow \quad \begin{cases} E^\alpha = dZ^M E_M^\alpha(Z), \\ \bar{E}^{\dot{\alpha}} = dZ^M \bar{E}_M^{\dot{\alpha}}(Z). \end{cases} \tag{2.8}$$

In Eqs. (2.6)–(2.8), $a=0,1,2,3$ is a tangent space vector index, $\alpha=1,2,3,4$ is a Majorana spinor index, and $\alpha=1,2$, $\dot{\alpha}=1,\bar{2}$ are Weyl spinor indices.

The action (2.3) involves also the pullback $\hat{\Phi}\equiv\hat{\phi}^*(\Phi)=\Phi(\hat{Z})$ of a dilaton superfield $\Phi(Z)$ and the pullback

$$\hat{B}_2\equiv\hat{\phi}^*(B_2)=B_2(\hat{Z}(\xi))$$
$$=\frac{1}{2}\hat{E}^B\wedge\hat{E}^A B_{AB}(\hat{Z}(\xi))$$
$$\equiv\frac{1}{2}d\xi^m\wedge d\xi^n\hat{B}_{nm}(\xi),$$

$$\hat{B}_{nm}(\xi):=\partial_m\hat{Z}^M\partial_n\hat{Z}^N B_{NM}(\hat{Z}), \tag{2.9}$$

of a two-form on $\Sigma^{(4|4)}$,

$$B_2=\frac{1}{2}E^B\wedge E^A B_{AB}(Z). \tag{2.10}$$

The world sheet Hodge star operator $*$,

$$*\hat{E}^a=d\xi^n\sqrt{|g|}\epsilon_{nk}g^{km}\hat{E}^a_m$$
$$\Rightarrow\quad *\hat{E}^a\wedge\hat{E}^b=d^2\xi\sqrt{|g|}g^{mn}\hat{E}^a_m\hat{E}^b_n, \tag{2.11}$$

can be defined using the induced world sheet metric (2.4). Then

$$\frac{1}{4}*\hat{E}_a\wedge\hat{E}^a=\frac{1}{2}d^2\xi\sqrt{|g|}, \tag{2.12}$$

$$\delta(*\hat{E}_a\wedge\hat{E}^a)=2*\hat{E}_a\wedge\delta\hat{E}^a. \tag{2.13}$$

Substituting Eq. (2.12) into Eq. (2.3) one arrives at the more familiar form of the Green-Schwarz superstring action (1.2).

The Green-Schwarz superstring action in flat superspace and its $\kappa$ symmetry are discussed in Appendix A.

### A. Superstring $\kappa$ symmetry and superspace constraints for supergravity and the tensor multiplet

When the Green-Schwarz superstring is considered in curved superspace—i.e., in the presence of superfield supergravity—the natural self-consistency condition is the existence of a smooth flat superspace limit. This implies, in particular, that the number of local symmetries of the action in *a superfield supergravity background* should be the same as in the case of flat superspace (see Appendix A). This means, again in particular, that $\kappa$ symmetry should be present in the superstring model in a supergravity background.

However, it is well known that $\kappa$ symmetry occurs only when the background satisfies certain constraints [10,11]. This occurs also with the $D=4$, $N=1$ Green-Schwarz superstring [18] (see also [25,27,28]). For the sake of completeness and to establish the notation used in next sections we present here some details of the constraint derivation.

The variation of the Green-Schwarz superstring action (2.3) in a general curved superspace looks like

$$\delta S_{sstr}=\int_{W^2}\left[\frac{1}{2}e^{\hat{\Phi}/2}*\hat{E}_a\wedge\delta\hat{E}^a+\frac{1}{8}e^{\hat{\Phi}/2}*\hat{E}_a\wedge\hat{E}^a\delta\Phi-\delta\hat{B}_2\right]. \tag{2.14}$$

As the only dynamical variables in a curved superspace *background* are the supercoordinate functions $\hat{Z}^M(\xi)$, one only considers, in this case,

$$\delta_{\hat{Z}}S_{sstr}=\int_{W^2}\left[\frac{1}{2}e^{\hat{\Phi}/2}*\hat{E}_a\wedge\delta_{\hat{Z}}\hat{E}^a+\frac{1}{8}e^{\hat{\Phi}/2}*\hat{E}_a\wedge\hat{E}^a\delta_{\hat{Z}}\Phi-\delta_{\hat{Z}}\hat{B}_2\right], \tag{2.15}$$

and the variations $\delta_{\hat{Z}}$ of the pullbacks of differential forms are given by Lie derivatives,

$$\delta_{\hat{Z}}\hat{E}^a\equiv\delta_{\hat{Z}}E^a(\hat{Z}):=E^a(\hat{Z}+\delta\hat{Z})-E^a(\hat{Z})$$
$$=i_{\delta\hat{Z}}(d\hat{E}^a)+d(i_{\delta\hat{Z}}\hat{E}^a)$$
$$=i_{\delta\hat{Z}}\hat{T}^a+\mathcal{D}(i_{\delta\hat{Z}}\hat{E}^a)+\hat{E}^b i_{\delta\hat{Z}}w_b{}^a, \tag{2.16}$$

$$\delta_{\hat{Z}}\hat{B}_2=i_{\delta\hat{Z}}\hat{H}_3+d i_{\delta\hat{Z}}\hat{B}_2,\quad H_3:=dB_2, \tag{2.17}$$

where

$$i_{\delta\hat{Z}}E^a(\hat{Z}):=\delta\hat{Z}^M E^a_M(\hat{Z}), \tag{2.18}$$

$$i_{\delta\hat{Z}}w^{ab}:=\delta\hat{Z}^M w^{ab}_M(\hat{Z}), \tag{2.19}$$

$$i_{\delta\hat{Z}}\hat{T}^a:=\hat{E}^C i_{\delta\hat{Z}}\hat{E}^B T_{BC}{}^a(\hat{Z}),\quad \text{etc.}, \tag{2.20}$$

the torsion $T^a$ and the covariant exterior derivative $\mathcal{D}$ are defined below [Eqs. (2.21)–(2.23)] and $w^{ab}=dZ^M w_M{}^{ab}=-w^{ba}$ is the spin connection.

Now it is clear that $\kappa$ symmetry is not present in a general curved superspace. As we are interested in the superstring interaction with supergravity, we have to impose first the constraints on the torsion of curved superspace,

$$T^a:=\mathcal{D}E^a=dE^a-E^b\wedge w_b{}^a$$
$$\equiv\frac{1}{2}E^B\wedge E^C T_{CB}{}^a, \tag{2.21}$$

$$T^\alpha:=\mathcal{D}E^\alpha=dE^\alpha-E^\beta\wedge w_\beta{}^\alpha$$
$$\equiv\frac{1}{2}E^B\wedge E^C T^\alpha_{CB}, \tag{2.22}$$

$$T^{\dot{\alpha}}:=\mathcal{D}\bar{E}^{\dot{\alpha}}=d\bar{E}^{\dot{\alpha}}-\bar{E}^{\dot{\beta}}\wedge w_{\dot{\beta}}{}^{\dot{\alpha}}$$
$$\equiv\frac{1}{2}E^B\wedge E^C T^{\dot{\alpha}}_{CB}, \tag{2.23}$$

and on the Riemann curvature 2-form

$$R^{ab}:=dw^{ab}-w^{ac}\wedge w_c{}^b \equiv \frac{1}{2}E^C\wedge E^D R_{DC}{}^{ab}. \quad (2.24)$$

Notice that

$$R^{ab}=\frac{1}{2}R^{\alpha\beta}(\sigma^a\tilde{\sigma}^b)_{\alpha\beta}-\frac{1}{2}R^{\dot\alpha\dot\beta}(\tilde{\sigma}^a\sigma^b)_{\dot\alpha\dot\beta} \quad (2.25)$$

and it is convenient to formulate constraints in terms of

$$R^{\alpha\beta}:=\frac{1}{4}R^{ab}(\sigma_a\tilde{\sigma}_b)^{\alpha\beta}=dw^{\alpha\beta}-w^{\alpha\gamma}\wedge w_\gamma{}^\beta, \quad (2.26)$$

and $R^{\dot\alpha\dot\beta}=-\frac{1}{4}R^{ab}(\tilde{\sigma}^a\sigma^b)^{\dot\alpha\dot\beta}=(R^{\alpha\beta})^*$.

The *minimal* off-shell formulation of $D=4$, $N=1$ supergravity is described by the set of constraints (see [34] and references therein)

$$T_{\alpha\dot\beta}{}^a=-2i\sigma^a_{\alpha\dot\beta},$$

$$T_{\alpha\beta}{}^A=0=T_{\dot\alpha\dot\beta}{}^A, \quad T_{\alpha\dot\beta}{}^{\dot\gamma}=0, \quad T_{\alpha b}{}^c=0, \quad (2.27)$$

$$R_{\alpha\dot\beta}{}^{ab}=0. \quad (2.28)$$

These constraints and their consequences (derived from the Bianchi identities) can be collected in the following expressions for the torsion 2-forms [22,34],

$$T^a=-2i\sigma^a_{\alpha\dot\alpha}E^\alpha\wedge\bar{E}^{\dot\alpha}+\frac{1}{16}E^b\wedge E^c\varepsilon^a{}_{bcd}G^d, \quad (2.29)$$

$$T^\alpha=\frac{i}{8}E^c\wedge E^\beta(\sigma_c\tilde{\sigma}_d)_\beta{}^\alpha G^d-\frac{i}{8}E^c\wedge\bar{E}^{\dot\beta}\epsilon^{\alpha\beta}\sigma_{c\beta\dot\beta}R$$
$$+\frac{1}{2}E^c\wedge E^b T_{bc}{}^\alpha, \quad (2.30)$$

$$T^{\dot\alpha}=\frac{i}{8}E^c\wedge E^\beta\epsilon^{\dot\alpha\dot\beta}\sigma_{c\beta\dot\beta}\bar{R}-\frac{i}{8}E^c\wedge\bar{E}^{\dot\beta}(\tilde{\sigma}_d\sigma_c)^{\dot\alpha}{}_{\dot\beta}G^d$$
$$+\frac{1}{2}E^c\wedge E^b T_{bc}{}^{\dot\alpha}, \quad (2.31)$$

and in the expression for the superspace Riemann curvature 2-form (2.26) which can be found in Appendix B.

The right-hand sides (RHS) of Eqs. (2.29), (2.30) [and Eq. (B1)] include the so-called "main superfields" $R$, $\bar{R}=R^*$, $G_a=(G_a)^*$ and the symmetric spin tensor $W_{\alpha\beta\gamma}=W_{(\alpha\beta\gamma)}=(\bar{W}_{\dot\alpha\dot\beta\dot\gamma})^*$, which obey (also as a result of Bianchi identities)

$$\mathcal{D}_\alpha\bar{R}=0, \quad \bar{\mathcal{D}}_{\dot\alpha}R=0, \quad (2.32)$$

$$\bar{\mathcal{D}}_{\dot\alpha}W^{\alpha\beta\gamma}=0, \quad \mathcal{D}_\alpha\bar{W}^{\dot\alpha\dot\beta\dot\gamma}=0, \quad (2.33)$$

$$\bar{\mathcal{D}}^{\dot\alpha}G_{\alpha\dot\alpha}=\mathcal{D}_\alpha R, \quad \mathcal{D}^\alpha G_{\alpha\dot\alpha}=\bar{\mathcal{D}}_{\dot\alpha}\bar{R}, \quad (2.34)$$

$$\mathcal{D}_\gamma W^{\alpha\beta\gamma}=\bar{\mathcal{D}}_{\dot\gamma}\mathcal{D}^{(\alpha}G^{\beta)\dot\gamma},$$

$$\bar{\mathcal{D}}_{\dot\gamma}\bar{W}^{\dot\alpha\dot\beta\dot\gamma}=\mathcal{D}_\gamma\bar{\mathcal{D}}^{(\dot\alpha|}G^{\gamma|\dot\beta)}. \quad (2.35)$$

Here and below the brackets (square brackets) denote symmetrization (antisymmetrization) with unit weight—e.g., $\mathcal{D}^{(\alpha}G^{\beta)\dot\gamma}:=\frac{1}{2}(\mathcal{D}^\alpha G^{\beta\dot\gamma}+\mathcal{D}^\beta G^{\alpha\dot\gamma})$.

After the minimal $D=4$, $N=1$ supergravity constraints (2.29), (2.30) are taken into account, one finds that $\kappa$ symmetry occurs in the action (2.3) when also the constraints

$$H_{\alpha\beta\gamma}=0, \quad H_{\alpha\beta\dot\gamma}=0, \quad \text{and c.c.},$$

$$H_{\alpha\beta c}=0, \quad H_{\dot\alpha\beta c}=0, \quad (2.36)$$

$$H_{\alpha\dot\alpha a}=-ie^{\hat\Phi/2}\sigma_{a\alpha\dot\alpha} \quad (2.37)$$

are imposed on the field strength $H_3=dB_2$ of the 2-superform $B_2$ (2.10).

### B. Gauge superform and tensor multiplet

The study of the Bianchi identities $dH_3\equiv 0$ in the superspace restricted by the supergravity constraints (2.29), (2.30), (2.31) shows that the field strength $H_3$ is completely determined by the constraints (2.36), (2.37). It is expressed in terms of the dilaton superfield $\Phi(Z)$ and the main superfields of the minimal supergravity by

$$H_3\equiv dB_2=-iE^a\wedge E^\alpha\wedge E^{\dot\alpha}\sigma_{a\alpha\dot\alpha}e^{\Phi/2}$$
$$+\frac{1}{8}E^a\wedge E^b\wedge E^\alpha(\sigma_{[a}\tilde{\sigma}_{b]})_\alpha{}^\beta e^{\Phi/2}\nabla_\beta\Phi+\text{c.c.}$$
$$+\frac{1}{3!}E^a\wedge E^b\wedge E^c H_{cba}, \quad (2.38)$$

$$H_{abc}=\frac{5}{32}e^{\Phi/2}\epsilon_{abcd}G^d+\frac{1}{8}\epsilon_{abcd}\tilde{\sigma}^{d\dot\alpha\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\alpha}]e^{\Phi/2}. \quad (2.39)$$

Thus the 2-form field strength is expressed, essentially, through one real dilaton superfield $\Phi(Z)$.

Moreover, the same study of Bianchi identities brings also the equations which, on first sight, seem to be relations between the dilaton superfield and the chiral main superfield of minimal supergravity:

$$\bar{R}=e^{-\Phi/2}\mathcal{D}\nabla e^{\Phi/2}, \quad R=e^{-\Phi/2}\bar{\mathcal{D}}\bar{\nabla}e^{\Phi/2}. \quad (2.40)$$

However, one notes that Eqs. (2.40) can be written as

$$(\mathcal{D}\mathcal{D}-\bar{R})e^{\Phi/2}=0, \quad (\overline{\mathcal{D}\mathcal{D}}-R)e^{\Phi/2}=0, \quad (2.41)$$

and they just imply that the $D=4$, $N=1$ dilaton superfield describes the real linear multiplet.[5]

The fact that a 2-form in $D=4$, $N=1$ superspace is described by a real linear multiplet (tensor multiplet) is known from the study in [21].

Thus we conclude that the complete superfield action for the $D=4$, $N=1$ interacting system including the superstring should involve, in addition to the superstring action (2.3) and the Wess-Zumino action for the *minimal* supergravity multiplet, $\int d^8ZE$ [see Eq. (3.1) below] also an action for the tensor multiplet, described by the real superfield $e^{\Phi/2}$ which obeys the constraints (2.41). The kinetic term of the latter action should involve, in particular, the kinetic term for the two-index antisymmetric tensor gauge field (or Kalb-Ramond field [35], first introduced in $D=4$ under the name *notoph* [36]) which interacts naturally with a string. Such a kinetic term can be written as [24]

$$\int d^8ZE\frac{\Phi}{2}e^{\Phi/2}. \tag{2.42}$$

Note that the first proposal for the tensor multiplet action was different, $\int d^8ZEe^{\Phi}\equiv\int d^8ZE(e^{\Phi/2})^2$ (see [21] and references therein). The tensor multiplet with the action (2.42) was referred to as the "improved tensor multiplet" [24]. Its distinguishing property is invariance under the Weyl transformations acting also on the dilaton superfield, $E^a\rightarrow e^{\Lambda}E^a$, $E^{\underline{\alpha}}\rightarrow e^{\Lambda/2}E^{\underline{\alpha}}$, $\Phi(z)\rightarrow\Phi(Z)-4\Lambda$, when $\Lambda$ is given by a sum of chiral and antichiral superfields, $\Lambda=i(\phi-\bar{\phi})$, $\mathcal{D}_\alpha\bar{\phi}=0=\bar{\mathcal{D}}_{\dot{\alpha}}\phi$. The fact that the superstring action (2.3) also possesses such a symmetry makes the improved tensor multiplet action preferable for a description of the tensor multiplet-superstring interacting system.

Actually in the study of $D=4$, $N=1$ limit and compactification of the heterotic string [18,25–29] it was argued that such a limit rather provided the minimal supergravity–tensor-multiplet action [18,25]. We are not addressing this problem here but rather considering the $D=4$, $N=1$ interacting system (including the $D=4$, $N=1$ superstring) as a relatively simple model for the (quasi)classical description of a more complicated, higher-dimensional ($D=10,11$) supergravity-superbrane interacting system [in particular, the $D=10$, $N=1$ supergravity–super Yang-Mills–heterotic-string interacting system described by a hypothetical superfield action also including, in addition to the supergravity and the super-YM parts, the heterotic string action [13], as well as the $D=10$ type-II-supergravity–super-*Dp*-brane and $D=11$ supergravity–super-*Mp*-brane systems]. As such, an interesting alternative possibility is to consider the so-called *new minimal* formulation of simple supergravity [30], where the *auxiliary* fields can be collected into a real linear multiplet. Here we will not consider this possibility, but proceed with the description of superstring interacting with dynamical real linear multiplet and minimal $N=1$ supergravity.[6]

### C. Superstring $\kappa$ symmetry in the supergravity and tensor multiplet background

With the constraints (2.29), (2.30), (2.31), and (2.38) variation of the superstring action (2.3) [see Eq. (B3) in Appendix B] becomes

$$\begin{aligned}\delta_{\hat{Z}}S_{sstr}=&-\frac{1}{2}\int_{W^2}\left[\mathcal{D}(e^{\hat{\Phi}/2}*\hat{E}_a)-\frac{1}{4}e^{\hat{\Phi}/2}*\hat{E}_b\wedge\hat{E}^b\hat{\nabla}_a\hat{\Phi}\right]i_{\delta\hat{Z}}E^a\\&+i\int_{W^2}e^{\hat{\Phi}/2}(\hat{E}_a-*\hat{E}_a)\wedge\sigma_{a\alpha\dot{\alpha}}\hat{\bar{E}}^{\dot{\alpha}}i_{\delta\hat{Z}}E^{\alpha}+\text{c.c.}\\&+\frac{i}{8}\int_{W^2}e^{\hat{\Phi}/2}(\hat{E}_b-*\hat{E}_b)\wedge\hat{E}^b\hat{\nabla}_\alpha\hat{\Phi}i_{\delta\hat{Z}}E^{\alpha}+\text{c.c.}\end{aligned} \tag{2.43}$$

Equation (2.43) makes evident the presence of the local fermionic $\kappa$ symmetry defined by Eq. (A4), but now with curved space supervielbein:

$$i_\kappa\hat{E}^a=0,\quad i_\kappa\hat{E}^\alpha\sigma_{a\alpha\dot{\alpha}}(*\hat{E}^a-\hat{E}^a)=0. \tag{2.44}$$

The solution of Eqs. (2.44) [cf. Eq. (A5) and above] provides us with an explicit form of the $\kappa$-symmetry transformations,

$$\delta_\kappa\hat{Z}^M(\xi)=\bar{\kappa}^{\,n}_{\dot{\alpha}}(\delta^m_n-\sqrt{|g|}\epsilon_{nk}g^{km})\hat{E}^a_m\tilde{\sigma}_a^{\dot{\alpha}\alpha}E^M_\alpha(\hat{Z})+\text{c.c.}, \tag{2.45}$$

where $g^{nm}(\xi)$ is the matrix inverse to the induced metric:

$$\begin{aligned}g_{mn}(\xi)&=E^a_m(\hat{Z})E^b_m(\hat{Z})\eta_{ab}\\&=\partial_m\hat{Z}^M(\xi)\partial_n\hat{Z}^N(\xi)E_{Na}(\hat{Z})E^a_M(\hat{Z}).\end{aligned} \tag{2.46}$$

The standard flat superspace Green-Schwarz $\kappa$-symmetry transformations [Eqs. (A5) in Appendix A] can be derived from Eq. (2.45) by substitution of the flat superspace expressions for the (inverse) supervielbein coefficients $E^M_\alpha(\hat{Z})$ and for $E^a_M(\hat{Z})$ in Eq. (2.46).

[5]One should keep in mind that $(\mathcal{D}\mathcal{D}-\bar{R})$ is a chiral projector—i.e., that $\mathcal{D}_\alpha(\mathcal{D}\mathcal{D}-\bar{R})U\equiv0$ for any superfield $U=U(Z)$.

[6]In the framework of the conformal tensor calculus, to arrive at the *component* form of the action of the new minimal (off-shell) supergravity one starts from the improved tensor multiplet action in flat superspace, $\int d^8ZL\ln L$ with $DDL=0=\bar{D}\bar{D}L$, performs the Grassmann integration in it, then one introduces the coupling to the conformal supergravity and makes a gauge choice for the special superconformal transformation. As a result, one arrives at the action [30] $\int d^4x[e\mathcal{R}+\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma+A_\mu\epsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma}+\frac{1}{2}(\epsilon^{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma})^2]$, which includes both the antisymmetric tensor $B_{\mu\nu}$ and the vector gauge field $A_\mu$ as *auxiliary fields* (see [24]). In contrast, in our case the interacting action (1.1)) involves the improved tensor multiplet action coupled to minimal supergravity in superfield formulation, $\int d^8ZEL\ln L$ with $\ln L=\Phi/2$. In this case the antisymmetric tensor field $B_{\mu\nu}$ is dynamical and its equations of motion contain a source from the superstring.

## III. $D=4$, $N=1$ SUPERFIELD SUPERGRAVITY ACTION

The action of $D=4$, $N=1$ supergravity is given by the invariant supervolume of $D=4$, $N=1$ superspace [22]:

$$S_{SG}=\int d^4x\tilde{d}^4\theta\,\mathrm{sdet}(E_M^A)\equiv\int d^8ZE, \tag{3.1}$$

where $E:=\mathrm{sdet}(E_M^A)$ is Berezinian (superdeterminant) of the supervielbein $E_M^A(Z)$, Eq. (2.7), and $E_M^A(Z)$ are assumed to be subject to the constraints (2.27), (2.28).

### A. Admissible variations of supervielbein

As the supervielbein is considered to be restricted by the constraints, its variation cannot be treated as independent.[7] To find its admissible variations one can, following [22], denote the general variation of the supervielbein and spin connections by

$$\delta E_M^A(Z)=E_M^B\mathcal{K}_B^{\ A}(\delta),\qquad \delta w_M^{ab}(Z)=E_M^C u_C^{ab}(\delta), \tag{3.2}$$

and obtain the equations to be satisfied by $\mathcal{K}_B^{\ A}(\delta)$, $u_C^{ab}(\delta)$ from the requirement that the constraints (2.27), (2.28) be preserved under Eq. (3.2).

Quite complicated but straightforward calculations result in the following expression [17] for admissible variations of the supervielbein:

$$\delta E^a=E^a(\Lambda(\delta)+\bar{\Lambda}(\delta))-\frac{1}{4}E^b\tilde{\sigma}_b^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a$$
$$+iE^\alpha\mathcal{D}_\alpha\delta H^a-i\bar{E}^{\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a, \tag{3.3}$$

$$\delta E^\alpha=E^a\Xi_a^{\ \alpha}(\delta)+E^\alpha\Lambda(\delta)+\frac{1}{8}\bar{E}^{\dot{\alpha}}R\sigma_{a\dot{\alpha}}^{\ \ \alpha}\delta H^a. \tag{3.4}$$

In Eqs. (3.3), (3.4), $\Lambda(\delta)$, $\bar{\Lambda}(\delta)$ are given by

$$\Lambda(\delta)=\frac{1}{24}\tilde{\sigma}_a^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a+\frac{i}{4}\mathcal{D}_a\delta H^a+\frac{1}{24}G_a\delta H^a$$
$$+2(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}-(\overline{\mathcal{D}\mathcal{D}}-R)\delta\bar{\mathcal{U}}, \tag{3.5}$$

$$\Lambda(\delta)+\bar{\Lambda}(\delta)=\frac{1}{12}\tilde{\sigma}_a^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a+\frac{1}{12}G_a\delta H^a$$
$$+(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}+(\overline{\mathcal{D}\mathcal{D}}-R)\delta\bar{\mathcal{U}}; \tag{3.6}$$

the explicit expression for $\Xi_a^{\ \alpha}(\delta)$ in Eq. (3.4) as well as the expression for the basic variations of the spin connection, $u_C^{ab}(\delta)$ in Eq. (3.2), will not be needed below (they can be found in [17]).

Note that the variations representing the manifest gauge symmetries of supergravity are factored out from the above expressions. These are the superspace local Lorentz transformations and the variational version of the superspace general coordinate transformations (see [22]).

For the free supergravity action (3.1) the nontrivial dynamical equations of motion should follow from the variations (3.3), (3.4) with (3.5), (3.6) only. The variation of the superdeterminant $E=\mathrm{sdet}(E_M^A)$ under Eqs. (3.3), (3.4) has the form (see [22])

$$\delta E=E\left[-\frac{1}{12}\tilde{\sigma}_a^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a+\frac{1}{6}G_a\delta H^a\right.$$
$$\left.+2(\overline{\mathcal{D}\mathcal{D}}-R)\delta\bar{\mathcal{U}}+2(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}\right]. \tag{3.7}$$

In the light of the identity

$$\int d^8ZE\mathcal{D}_A\xi^A(-1)^A=\int d^8ZE(\mathcal{D}_A\xi^A+\xi^BT_{BA}{}^A)(-1)^A$$
$$\equiv0, \tag{3.8}$$

all the terms with derivatives can be omitted in Eq. (3.7) when one considers the variation of the action (3.1). [The first equation in Eqs. (3.8) uses the minimal supergravity constraints which imply $T_{BA}{}^A(-1)^A=0$.] Hence,

$$\delta S_{SG}=\int d^8Z\,\delta E=\int d^8ZE\left[\frac{1}{6}G_a\delta H^a-2R\,\delta\bar{\mathcal{U}}-2\bar{R}\,\delta\mathcal{U}\right] \tag{3.9}$$

and one arrives at the following superfield equations of motion for "free," simple $D=4$, $N=1$ supergravity:

$$\frac{\delta S_{SG}}{\delta H^a}=0\quad\Rightarrow\quad G_a=0, \tag{3.10}$$

$$\frac{\delta S_{SG}}{\delta\bar{\mathcal{U}}}=0\quad\Rightarrow\quad R=0, \tag{3.11}$$

$$\frac{\delta S_{SG}}{\delta\mathcal{U}}=0\quad\Rightarrow\quad \bar{R}=0. \tag{3.12}$$

## IV. TENSOR MULTIPLET IN CURVED SUPERSPACE

### A. "Improved" action for the tensor multiplet

As was argued in Sec. II D (see also [18,25,27]), the most suitable action for the description of a tensor multiplet interacting with a superstring is provided by the Weyl invariant de Wit–Roček action [24], Eq. (2.42),

$$S_\Phi=s\int d^8ZE\,\frac{\Phi}{2}e^{\Phi/2},$$

$$(\mathcal{D}\mathcal{D}-\bar{R})e^{\Phi/2}=0,\quad(\overline{\mathcal{D}\mathcal{D}}-R)e^{\Phi/2}=0. \tag{4.1}$$

[7] Note that with an independent variation of the supervielbein, the action (3.1) would lead to the equation stating the vanishing of the superdeterminant $E$, which contradicts the original assumption about its nondegeneracy.

The variation of such an action with respect to the superfield $\Phi$ constrained by Eq. (2.41) and with respect to the supergravity multiplet is technically quite involved. However, the following observation helps. The 2-form satisfying the constraints (2.38) is expressed essentially in terms of the tensor multiplet superfield $e^{\Phi/2}$. Thus the problem of varying the real linear multiplet is equivalent to the problem of finding admissible variations of the 2-form $B_2$ satisfying the constraints (2.38). Such a task appears to be more algorithmic. Moreover, for the variation of the interacting action we will, anyway, need the form of the admissible variations of $B_2$ superform, as its pullback defines the Wess-Zumino term of the Green-Schwarz superstring (2.3).

### B. Varying the tensor multiplet: Admissible variations of the 2-form gauge superfield

Clearly the constraints (2.38) make it impossible to consider the variations of the 2-form $B_2$ as independent. One rather has to define (cf. Sec. III A)

$$\delta B_2=\frac{1}{2}E^A\wedge E^B b_{BA}(\delta) \tag{4.2}$$

and find the expressions for $b_{BA}(\delta)=-(-1)^{BA}b_{AB}(\delta)$ from the conditions of conservation of the constraints (2.38). Factoring out the gauge transformations $\delta^{gauge}B_2=d\alpha_1$, one finds, after tedious calculations,

$$b_{\alpha\beta}(\delta)=0,\quad b_{\alpha\dot\beta}(\delta)=0,\quad b_{\dot\alpha\dot\beta}(\delta)=0, \tag{4.3}$$

$$b_{\beta b}(\delta)=\sigma_{b\beta\dot\beta}(\mathcal{D}\mathcal{D}-\bar{R})\delta\bar{\nu}^{\dot\beta}+\mathcal{O}(\delta H^a), \tag{4.4}$$

$$b_{\dot\beta b}(\delta)=-\sigma_{b\beta\dot\beta}(\overline{\mathcal{D}\mathcal{D}}-R)\delta\nu^{\beta}+\mathcal{O}(\delta H^a), \tag{4.5}$$

$$b_{ab}(\delta)=-\frac{i}{4}(\tilde\sigma_{[a}\sigma_{b]})^{\dot\beta}{}_{\dot\alpha}\bar{\mathcal{D}}_{\dot\beta}(\mathcal{D}\mathcal{D}-\bar{R})\delta\bar{\nu}^{\dot\alpha} -\frac{i}{4}(\sigma_{[a}\tilde\sigma_{b]})_{\alpha}{}^{\beta}\mathcal{D}_{\beta}(\overline{\mathcal{D}\mathcal{D}}-R)\delta\nu^{\alpha}+\mathcal{O}(\delta H^a), \tag{4.6}$$

where $\mathcal{O}(\delta H^a)$ denotes terms containing the $\delta H^a$ variation [see Appendix B, Eqs. (B4)–(B7) for the complete expressions].

As the components of the superfield strength of the 2-form $B_2$ are expressed through the dilaton superfield, it should not be a surprise that the preservation of the constraints (2.38) defines as well the variation of the dilaton superfield,

$$\delta e^{\Phi/2}=\frac{i}{2}\bar{\mathcal{D}}_{\dot\alpha}(\mathcal{D}\mathcal{D}-\bar{R})\delta\bar{\nu}^{\dot\alpha}-\frac{i}{2}\mathcal{D}_{\alpha}(\overline{\mathcal{D}\mathcal{D}}-R)\delta\nu^{\alpha} -\frac{1}{4}\tilde\sigma_a^{\dot\beta\beta}[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot\beta}]e^{\Phi/2}\delta H^a-2e^{\Phi/2}[\Lambda(\delta)+\bar\Lambda(\delta)], \tag{4.7}$$

where $[\Lambda(\delta)+\bar\Lambda(\delta)]$ is defined in Eq. (3.6).

## V. INTERACTING ACTION AND SUPERFIELD EQUATIONS OF MOTION

Now that we have found all the necessary basic variations, we may turn to varying the coupled action

$$S=\int d^8Z\,\mathrm{sdet}(E_M{}^A)\left(1+s\frac{\Phi}{2}e^{\Phi/2}\right)+S_{sstr}, \tag{5.1}$$

$$S_{sstr}=\int_{W^2}\left[\frac{1}{4}{*}\hat{E}_a\wedge\hat{E}^a e^{\hat\Phi/2}-\hat{B}_2\right], \tag{5.2}$$

to derive the equations of motion.

### A. Superstring equations

Clearly, the superstring equations of motion for the interacting system keep the same form as the superstring equations in the superspace background of the superfield supergravity and tensor multiplet:

$$({*}\hat{E}^a-\hat{E}^a)\wedge\left(\sigma_{a\alpha\dot\alpha}\hat{\bar{E}}{}^{\dot\alpha}-\frac{i}{8}\hat{E}^b(\sigma_a\tilde\sigma_b)_\alpha{}^\beta\hat\nabla_\beta\hat\Phi\right)=0, \quad \text{and c.c.,} \tag{5.3}$$

$$\mathcal{D}(e^{\hat\Phi/2}{*}\hat{E}^a)+\hat{E}^c\wedge\hat{E}^b\hat{H}_{abc}-\frac{1}{2}{*}\hat{E}_b\wedge\hat{E}^b\hat\nabla_a e^{\hat\Phi/2}+\hat{E}^c\wedge\hat{E}^\alpha(\sigma_{[a}\tilde\sigma_{b]})_\alpha{}^\beta\hat\nabla_\beta e^{\hat\Phi/2}-2i\hat{E}^\alpha\wedge\hat{\bar{E}}{}^{\dot\alpha}\sigma_{a\alpha\dot\alpha}e^{\hat\Phi/2}=0. \tag{5.4}$$

### B. Superfield equations for tensor multiplet

The equations of motion for the tensor multiplet appear as a result of the $\delta\nu^\alpha$ and $\delta\bar\nu^{\dot\alpha}$ variations of the dilaton superfield, Eq. (4.7), and the 2-superform $B_2$, Eqs. (4.3)–(4.6). They are

$$s(\overline{\mathcal{D}\mathcal{D}}-R)(e^{-\Phi/2}\mathcal{D}_\alpha e^{\Phi/2}) =-(\overline{\mathcal{D}\mathcal{D}}-R)\mathcal{D}_\alpha K_a{}^a+4i\sigma_{b\alpha\dot\beta}(\overline{\mathcal{D}\mathcal{D}}-R)W^{b\dot\beta} -\frac{1}{2}(\sigma_{[a}\tilde\sigma_{b]})_\alpha{}^\beta(\overline{\mathcal{D}\mathcal{D}}-R)\mathcal{D}_\beta W^{ab}, \tag{5.5}$$

$$s(\mathcal{D}\mathcal{D}-\bar{R})(e^{-\Phi/2}\bar{\mathcal{D}}_{\dot\alpha}e^{\Phi/2}) =-(\mathcal{D}\mathcal{D}-\bar{R})\bar{\mathcal{D}}_{\dot\alpha}K_a{}^a+4i\sigma_{b\beta\dot\alpha}(\mathcal{D}\mathcal{D}-\bar{R})W^{b\beta} +\frac{1}{2}(\tilde\sigma_{[a}\sigma_{b]})^{\dot\beta}{}_{\dot\alpha}(\mathcal{D}\mathcal{D}-\bar{R})\bar{\mathcal{D}}_{\dot\beta}W^{ab}, \tag{5.6}$$

where

$$W^{BA}:=\frac{1}{2}\int_{W^2}\frac{1}{\hat{E}}\hat{E}^B\wedge\hat{E}^A\,\delta^8(Z-\hat{Z}) \qquad (5.7)$$

are current prepotentials which appear naturally in any variation of the Wess-Zumino term of the superstring action. In the same manner, any variation of the Nambu-Goto terms of the superstring action will be expressed through the current prepotential

$$K_a^B:=\frac{1}{4}\int_{W^2}\frac{e^{\hat{\Phi}/2}}{\hat{E}}*\hat{E}_a\wedge\hat{E}^B\,\delta^8(Z-\hat{Z}) \qquad (5.8)$$

(cf. with the superparticle current prepotentials in [17]).

### C. Superfield supergravity equations

Now let us turn to the supergravity equations for the coupled system, which appear as a result of the $\delta\mathcal{H}^a$, $\delta\mathcal{U}$, $\delta\bar{\mathcal{U}}$ variations (see Sec. III for free supergravity).

The first observation is that, in accordance with Eqs. (4.7) and (3.3), the variation of the Nambu-Goto terms of the superstring action with respect to supergravity superfields does not contain an input from $\Lambda(\delta)$, $\bar{\Lambda}(\delta)$, defined in Eqs. (3.5), (3.6):

$$\delta_{\mathcal{U}}S_{sstr}=\int_{W^2}\left[\frac{1}{2}*\hat{E}_a\wedge\delta_{\mathcal{U}}\hat{E}^a e^{\hat{\Phi}/2}+\frac{1}{4}*\hat{E}_a\wedge\hat{E}^a\,\delta_{\mathcal{U}}e^{\hat{\Phi}/2}\right]$$
$$=\frac{1}{2}\int_{W^2}*\hat{E}_a\wedge\hat{E}^a e^{\hat{\Phi}/2}(\mathcal{D}\mathcal{D}-\bar{R})\,\delta\mathcal{U}$$
$$-\frac{1}{4}\int_{W^2}*\hat{E}_a\wedge\hat{E}^a e^{\hat{\Phi}/2}2(\mathcal{D}\mathcal{D}-\bar{R})\,\delta\mathcal{U}=0. \qquad (5.9)$$

Clearly, no such inputs come from the variations of the pullback $\hat{B}_2$ of the 2-form $B_2$, as Eqs. (4.3)–(4.6) does not contain $\Lambda(\delta)$, $\bar{\Lambda}(\delta)$ at all. As the chiral variations $(\mathcal{D}\mathcal{D}-\bar{R})\,\delta\mathcal{U}$ and c.c. are involved in the variations of the supervielbein only inside the $\Lambda(\delta)$, $\bar{\Lambda}(\delta)$ combinations, this means that the equations $\delta S/\delta\mathcal{U}=0$ and $\delta S/\delta\bar{\mathcal{U}}=0$ do not possess an input from the superbrane source (as was also the case with the supergravity-superparticle system; see [17]). These equations can acquire an input from the action of the real linear multiplet, which, in general, has the form $s\int d^8ZEf(\Phi/2)$ with an arbitrary function $f$. However, one can check that an input in the $\delta\mathcal{U}$ variation of the improved kinetic term, $s\int d^8ZE\frac{\Phi}{2}e^{\Phi/2}$, also vanishes

$$\delta_{\mathcal{U}}\int d^8ZE\tfrac{\Phi}{2}e^{\Phi/2}=0. \qquad (5.10)$$

Thus for the coupled action (5.1) one finds that the chiral superfield equation $\delta S/\delta\mathcal{U}=0$ remains the same as in the case of "free" supergravity:

$$\frac{\delta S}{\delta\mathcal{U}}=0 \quad\Rightarrow\quad \bar{R}=0, \qquad (5.11)$$

$$\frac{\delta S}{\delta\bar{\mathcal{U}}}=0 \quad\Rightarrow\quad R=0. \qquad (5.12)$$

Thus, in this case, as in the case of supergravity-massless superparticle [17], only the vector superfield supergravity equation $\delta S/\delta H^a=0$ acquires a source term from the superstring. However, in the coupled system under consideration, these equations are more complicated due to the supergravity interaction with the tensor multiplet:

$$\frac{\delta S}{\delta H^a}=0 \quad\Rightarrow G_a(1-se^{\Phi/2})$$
$$=\mathcal{J}_a+s\left(5+3\frac{\Phi}{2}\right)\tilde{\sigma}_a^{\dot{\beta}\beta}[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}]e^{\Phi/2}$$
$$+3s\tilde{\sigma}_a^{\dot{\beta}\beta}e^{-\Phi/2}\mathcal{D}_\beta e^{\Phi/2}\bar{\mathcal{D}}_{\dot{\beta}}e^{\Phi/2}. \qquad (5.13)$$

The superstring current potential

$$\mathcal{J}_a=-6\frac{\delta S_{sstr}}{\delta H^a} \qquad (5.14)$$

entering the RHS of Eq. (5.13), can be expressed through *two types* of current prepotentials, Eqs. (5.8) and (5.7), as follows

$$-\frac{1}{6}\mathcal{J}_a=2i\mathcal{D}_\beta K_a^\beta-2i\bar{\mathcal{D}}_{\dot{\beta}}K_a^{\dot{\beta}}-\tilde{\sigma}_b^{\dot{\beta}\beta}[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}]K_a{}^b$$
$$-\frac{1}{4}K_b{}^b\tilde{\sigma}_a^{\dot{\beta}\beta}[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}]e^{\Phi/2}$$
$$+iW^{b\alpha}(\eta_{ba}\delta+\sigma_b\tilde{\sigma}_a)_\alpha{}^\beta\nabla_\beta e^{\Phi/2}$$
$$-iW^{b\dot{\alpha}}(\eta_{ba}\delta+\tilde{\sigma}_a\sigma_b)^{\dot{\beta}}{}_{\dot{\alpha}}\bar{\nabla}_{\dot{\beta}}e^{\Phi/2}$$
$$+\frac{1}{2}\tilde{\sigma}^{b\dot{\beta}\beta}[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}](e^{\Phi/2}W_{ab})$$
$$+\frac{i}{2}\epsilon_{abcd}W^{bc}\nabla^d e^{\Phi/2}. \qquad (5.15)$$

The first line of the RHS of Eq. (5.15) has exactly the same form as the expression for the current through the superparticle current potential in the supergravity-superparticle coupled system [17]; the second line contains the trace $K_b{}^b$ of the bosonic current potential (5.8) which vanishes in the superparticle case but is nonzero for the superstring; the remaining part of the RHS of Eq. (5.15) contains the current prepotentials (5.7) which come from the variation of the superstring Wess-Zumino term.

Note that on the shell of Eqs. (5.11), (5.12), $R=0=\bar{R}$, the equations for the tensor multiplet (5.5), (5.6) simplify to

$$\bar{\mathcal{D}}\bar{\mathcal{D}}\Big[se^{-\Phi/2}\mathcal{D}_\alpha e^{\Phi/2}+\mathcal{D}_\alpha K_a{}^a-4i\sigma_{b\alpha\dot{\beta}}W^{b\dot{\beta}}$$
$$+\frac{1}{2}(\sigma_{[a}\tilde{\sigma}_{b]})_\alpha{}^\beta\mathcal{D}_\beta W^{ab}\Big]=0, \qquad (5.16)$$

$$\mathcal{D}\mathcal{D}[se^{-\Phi/2}\bar{\mathcal{D}}_{\dot\alpha}e^{\Phi/2}+\bar{\mathcal{D}}_{\dot\alpha}K_a{}^a-4i\sigma_{b\beta\dot\alpha}W^{b\beta}$$
$$-\tfrac{1}{2}(\tilde\sigma_{[a}\sigma_{b]})^{\dot\beta}{}_{\dot\alpha}\bar{\mathcal{D}}_{\dot\beta}W^{ab}]=0. \quad (5.17)$$

### D. Superfield generalization of the Einstein and Rarita-Schwinger equations with sources

In the minimal (off-shell) supergravity the superfield generalization of the Ricci tensor and of the Rarita-Schwinger spin tensor are expressed through the vector and chiral scalar superfields as follows [see Appendix B, Eq. (B1), as well as [34,17] and references therein]:

$$R_{bc}{}^{ac}=\frac{1}{32}(\mathcal{D}^{\beta}\bar{\mathcal{D}}^{(\dot\alpha|}G^{\alpha|\dot\beta)}-\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{(\beta}G^{\alpha)\dot\alpha})\sigma^{a}_{\alpha\dot\alpha}\sigma_{b\beta\dot\beta}$$
$$-\frac{3}{64}(\overline{\mathcal{D}\mathcal{D}R}+\mathcal{D}\mathcal{D}R-4R\bar{R})\delta^a_b, \quad (5.18)$$

$$\Psi^{a}_{\dot\alpha}:=\epsilon^{abcd}T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha}=\frac{i}{8}\tilde\sigma^{a\dot\beta\beta}\bar{\mathcal{D}}_{(\dot\beta|}G_{\beta|\dot\alpha)}+\frac{3i}{8}\sigma^{a}_{\beta\dot\alpha}\mathcal{D}^{\beta}R. \quad (5.19)$$

On the mass shell of the interacting system, taking into account the superfield equations of motion (5.12), (5.11), one finds that the scalar curvature vanishes,

$$R_{ab}{}^{ab}=0, \quad (5.20)$$

and the Ricci tensor (5.18) and the Rarita-Schwinger spin tensor (5.19) simplify. Then, to obtain the superfield generalization of the Einstein and Rarita-Schwinger equations for the interacting system one substitutes in Eqs. (5.18), (5.19) with $R=0=\bar{R}$ the formal solution

$$G_a=\frac{1}{1-se^{\Phi/2}}\mathcal{J}_a+\mathcal{G}_a(\Phi) \quad (5.21)$$

of the superfield equation (5.13). In Eq. (5.21), $\mathcal{G}_a(\Phi)$ denotes the on-shell "value" of the $G_a$ superfield in the system of supergravity interacting with a dynamical tensor multiplet; i.e., in the absence of the superstring,

$$\mathcal{G}_a(\Phi):=s\frac{5+3\frac{\Phi}{2}}{1-se^{\Phi/2}}\tilde\sigma_a^{\dot\beta\beta}[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot\beta}]e^{\Phi/2}$$
$$+\frac{3s}{1-se^{\Phi/2}}\tilde\sigma_a^{\dot\beta\beta}e^{-\Phi/2}\mathcal{D}_{\beta}e^{\Phi/2}\bar{\mathcal{D}}_{\dot\beta}e^{\Phi/2}. \quad (5.22)$$

Thus the superfield generalizations of the Rarita-Schwinger and Einstein equations in the supergravity–tensor-multiplet–superstring interacting system read

$$\Psi^{a}_{\dot\alpha}:=\epsilon^{abcd}T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha}=\frac{i}{8}\tilde\sigma^{a\dot\beta\beta}\bar{\mathcal{D}}_{(\dot\beta|}\mathcal{G}_{\beta|\dot\alpha)}(\Phi)$$
$$+\frac{i}{8}\tilde\sigma^{a\dot\beta\beta}\bar{\mathcal{D}}_{(\dot\beta|}(\mathcal{J}_{\beta|\dot\alpha)}/(1-se^{\Phi/2})) \quad (5.23)$$

and

$$R_{bc}{}^{ac}=\frac{1}{32}\Big[(\mathcal{D}^{\beta}\bar{\mathcal{D}}^{(\dot\alpha|}\mathcal{G}^{\alpha|\dot\beta)}(\Phi)-\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{(\beta}\mathcal{G}^{\alpha)\dot\alpha}(\Phi)]\sigma^{a}_{\alpha\dot\alpha}\sigma_{b\beta\dot\beta}$$
$$+\frac{1}{32}\{\mathcal{D}^{\beta}\bar{\mathcal{D}}^{(\dot\alpha|}[\mathcal{J}^{\alpha|\dot\beta)}/(1-se^{\Phi/2})]$$
$$-\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{(\beta}[\mathcal{J}^{\alpha)\dot\alpha}/(1-se^{\Phi/2})]\}\sigma^{a}_{\alpha\dot\alpha}\sigma_{b\beta\dot\beta}. \quad (5.24)$$

The spacetime Einstein and Rarita-Schwinger equations can be obtained as the leading ($\theta=0$) components of the superfield equations (5.24), (5.23) in the Wess-Zumino gauge (see [17,22,34], and references therein and also Sec. VI A below). One should note that

$$T_{ab}{}^{\alpha}|_{\theta=0}=2e^{\mu}_{a}e^{\nu}_{b}\mathcal{D}_{[\mu}\psi^{\alpha}_{\nu]}(x)-\frac{i}{4}(\psi_{[a}\sigma_{b]})_{\dot\beta}G^{\alpha\dot\beta}\Big|_{\theta=0}$$
$$-\frac{i}{4}(\tilde\sigma_{[a}\bar\psi_{b]})^{\alpha}R\Big|_{\theta=0} \quad (5.25)$$

differs from the standard definition of the gravitino field strength, $\mathcal{D}_{[\mu}\psi^{\alpha}_{\nu]}(x)=\partial_{[\mu}\psi^{\alpha}_{\nu]}(x)-\psi^{\beta}_{[\nu}(x)w_{\mu]\beta}{}^{\alpha}|_{\theta=0}$, by the leading components of the main superfields $G^a|_{\theta=0}$ and $R|_{\theta=0}$ only. In our case on the mass shell $R|_{\theta=0}=0$ [see Eq. (5.12)] and the $G^a$ superfield is determined by Eqs. (5.21), (5.22). Thus

$$T_{ab}{}^{\alpha}|_{\theta=0}=2e^{\mu}_{a}e^{\nu}_{b}\mathcal{D}_{[\mu}\psi^{\alpha}_{\nu]}(x)-\frac{i}{4}(\psi_{[a}\sigma_{b]})_{\dot\beta}\mathcal{G}^{\alpha\dot\beta}(\Phi)\Big|_{\theta=0}$$
$$-\frac{i}{4(1-se^{\phi(x)/2})}(\psi_{[a}\sigma_{b]})_{\dot\beta}\mathcal{J}^{\alpha\dot\beta}(\Phi)\Big|_{\theta=0}, \quad (5.26)$$

where $\phi(x)=\Phi|_{\theta=0}$.

### E. Superfield generalization of the Kalb-Ramond gauge field equations with source for the interacting system

Taking the vector covariant derivative of the expression (2.39) for $H_{abc}$, one finds the off-shell expression for the LHS of the 2-superform gauge field equation:

$$\mathcal{D}^c H_{abc} = \frac{1}{32}(\sigma_{[a}\tilde{\sigma}_{b]})^{\alpha\beta}\mathcal{D}_{(\alpha}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_{\beta)}e^{\Phi/2} + \text{c.c.} - \frac{1}{2}[\mathcal{D}_a,\mathcal{D}_b]e^{\Phi/2} + \frac{1}{2}G_{[a}\mathcal{D}_{b]}e^{\Phi/2} + \frac{1}{2}H_{abc}G^c + \frac{5}{32}\epsilon_{abcd}\mathcal{D}^c(e^{\Phi/2}G^d) - \frac{i}{64}(\sigma_{[a}\tilde{\sigma}_{b]})^{\alpha\beta}W_{\alpha\beta\gamma}\mathcal{D}^\gamma e^{\Phi/2} + \text{c.c.} \tag{5.27}$$

To arrive from Eq. (5.27) at the (superfield generalization) of the antisymmetric tensor gauge field equations (Kalb-Ramond equations), we shall substitute the expression for $\mathcal{D}_{(\alpha}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_{\beta)}e^{\Phi/2}$ which follows from acting by the spinor covariant derivative $\mathcal{D}_\beta$ on the superfield equations of motion (5.16) for the tensor multiplet and, then, substitute Eq. (5.21) for $G^a$. The equations thus obtained have quite a complicated form. Writing explicitly only the terms with the maximal number of the spinor covariant derivatives acting on the current prepotentials (see below for a special role of such terms) one gets

$$s\mathcal{D}^c H_{abc} = -\frac{1}{32}e^{-\Phi/2}(\sigma_{[a}\tilde{\sigma}_{b]})^{\alpha\beta}\mathcal{D}_{(\alpha}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_{\beta)}K_a{}^a + \frac{i}{8}e^{-\Phi/2}(\sigma_{[a}\tilde{\sigma}_{b]}\sigma_c)_{\alpha\dot{\alpha}}\mathcal{D}^\alpha\overline{\mathcal{D}\mathcal{D}}W^{\dot{\alpha}c} - \frac{1}{64}e^{-\Phi/2}(\sigma_{[a}\tilde{\sigma}_{b]}\sigma_{[c}\tilde{\sigma}_{d]})_\alpha{}^\beta\mathcal{D}_\beta\overline{\mathcal{D}\mathcal{D}}\mathcal{D}^\alpha W^{cd} + \text{c.c.} + \cdots. \tag{5.28}$$

A further study of the complete form of the superfield equations for the tensor multiplet and for the Kalb-Ramond gauge field entering that multiplet will be the subject of a separate paper.

Below we will show that the knowledge of the general form of the tensor multiplet superfield equations with the source, Eq. (5.16), and of its relation to the gauge field equation [through Eq. (5.28)] already allow one to make interesting conclusions that provide a shortcut in the study of the interacting system.

## VI. GAUGE-EQUIVALENT DESCRIPTION OF THE SUPERFIELD INTERACTING SYSTEM

### A. Superdiffeomorphism symmetry gauge fixing

The interacting action (5.1) is manifestly invariant under the local Lorentz symmetry and under superdiffeomorphisms

$$Z'^M = Z^M + b^M(Z): \quad \begin{cases} x'^\mu = x^\mu + b^\mu(x,\theta), \\ \theta'^{\check{\alpha}} = \theta^{\check{\alpha}} + \varepsilon^{\check{\alpha}}(x,\theta), \end{cases} \tag{6.1}$$

$$E'^A(Z') = E^A(Z), \quad \Phi'(Z') = \Phi(Z), \quad w'^{ab}(Z') = w^{ab}(Z), \quad \text{etc.}, \tag{6.2}$$

which act on the superstring variables, coordinate functions $\hat{Z}^M = \hat{Z}^M(\xi) \equiv (\hat{x}^\mu(\xi), \hat{\theta}^{\check{\alpha}}(\xi))$, by the pullback of the transformations (6.1):

$$\hat{Z}'^M = \hat{Z}^M + b^M(\hat{Z}): \begin{cases} \hat{x}'^\mu(\xi) = \hat{x}^\mu + b^\mu(\hat{x},\hat{\theta}), \\ \hat{\theta}'^{\check{\alpha}}(\xi) = \hat{\theta}^{\check{\alpha}} + \varepsilon^{\check{\alpha}}(\hat{x},\hat{\theta}). \end{cases} \tag{6.3}$$

The action (5.1) is also invariant under the world sheet reparametrizations and under the $\kappa$ symmetry (2.45), which act on the coordinate functions only.

Thus, omitting the world volume reparametrization for simplicity, the complete variation of the superstring coordinate function under the local symmetries of the interacting action (5.1) is given by

$$\delta\hat{Z}^M(\xi) = b^M(\hat{Z}(\xi)) + \delta_\kappa\hat{Z}^M(\xi), \tag{6.4}$$

where $\delta_\kappa\hat{Z}^M(\xi)$ is defined in Eq. (2.45).

Now we observe [17,32] that the superdiffeomorphism symmetry can be used to fix the "fermionic unitary gauge"

$$\hat{\theta}^{\check{\alpha}}(\xi) = 0 \quad \Leftrightarrow \quad \hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), 0). \tag{6.5}$$

Moreover, in the same manner as in [17] one can show that *this gauge can be fixed simultaneously with the Wess-Zumino gauge for supergravity* (see [17,37] and references therein),

$$\theta^{\check{\alpha}}E^a_{\check{\alpha}}(x,\theta) = 0, \quad \theta^{\check{\alpha}}(E^{\beta}_{\check{\alpha}}(x,\theta) - \delta^{\beta}_{\check{\alpha}}) = 0, \quad \theta^{\check{\alpha}}w^{ab}_{\check{\alpha}}(x,\theta) = 0, \tag{6.6}$$

where, in particular [22,34],

$$E^a_\mu|_{\theta=0} \propto e^a_\mu(x), \quad E^{\alpha}_\mu|_{\theta=0} \propto \psi^{\alpha}_\mu, \tag{6.7}$$

$$E^a_{\check{\beta}}|_{\theta=0} = 0, \quad E^{\alpha}_{\check{\beta}}|_{\theta=0} = \delta^{\alpha}_{\check{\beta}}, \tag{6.8}$$

$$w^{ab}_\mu|_{\theta=0} \propto \omega^{ab}_\mu(x). \tag{6.9}$$

This can be understood by observing that, although both gauges, Eqs. (6.6) and (6.5), are fixed with the use of the same superdiffeomorphism symmetry with the parameter $b^M(Z) = b^M(x,\theta)$, the transformation rules of the supergravity superfields involve only derivatives of $b^M(x,\theta)$ (characteristic property of the gauge field transformations), while the transformation rules of the coordinate functions $\hat{Z}^M(\xi)$, Eq. (6.4), contain the additive contribution of $b^M(\hat{Z}(\xi)) = b^M(\hat{x}(\xi),\hat{\theta}(\xi))$ (characteristic property of the Goldstone field transformations, but for Goldstone fields defined on a surface in superspace, see Sec. VII B of [17] and [32,38]).

### B. Gauge-fixed action

Let us discuss what happens with the interacting action (5.1), (5.2) in the gauge (6.5), (6.6). After integration over the Grassmann coordinates, the Wess-Zumino supergravity action (3.1) becomes the standard supergravity action with the minimal set of auxiliary fields [39],

$$S_{SG}=\int d^4x\tilde{d}^4\theta\,\mathrm{sdet}(E_M^A)\propto S_{sg}$$
$$=\int d^4x(e\mathcal{R}+\epsilon^{\mu\nu\rho\sigma}\overline{\psi}_\mu\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma)$$
$$+\mathcal{O}(g_a(x),r(x),\overline{r}(x)), \tag{6.10}$$

where $\mathcal{R}=R_{\mu\nu}{}^{ab}(x)e_a^\mu(x)e_b^\nu(x)$ is the scalar curvature of spacetime, $e=\det(e_\mu^a)$, and $\mathcal{O}(g_a(x),r(x),\overline{r}(x))$ denotes terms with auxiliary fields,

$$G_a|_{\theta=0}\propto g_a(x),\quad R|_{\theta=0}\propto r(x),\quad \overline{R}|_{\theta=0}\propto\overline{r}(x), \tag{6.11}$$

which are not essential for the consideration below. The improved tensor multiplet action (2.42) also entering Eq. (5.1), becomes, schematically (see [24]),

$$S_{TM}=s\int d^8ZE\frac{\Phi(Z)}{2}e^{\Phi(Z)/2}\propto S_{tm}$$
$$=\int d^4xe\left[\frac{1}{2}e^{\phi(x)/2}g^{\mu\nu}\mathcal{D}_\mu\phi\mathcal{D}_\nu\phi+\frac{1}{3!}H^{\mu\nu\rho}H_{\mu\nu\rho}\right.$$
$$\left.+ie^{\phi/2}(\mathcal{D}_a\chi^\alpha\sigma^a_{\alpha\dot\alpha}\overline{\chi}^{\dot\alpha}+\text{c.c.})+\cdots\right], \tag{6.12}$$

where $\mathcal{D}_\mu$ denotes the spacetime covariant derivatives and the component fields of the tensor multiplet are defined by

$$\Phi|_{\theta=0}\propto\phi(x),\quad \mathcal{D}_\alpha e^{\Phi/2}|_{\theta=0}\propto\chi_\alpha(x), \tag{6.13}$$

and [cf. Eq. (2.38)]

$$[\mathcal{D}_\alpha,\overline{\mathcal{D}}_{\dot\alpha}]e^{\Phi/2}|_{\theta=0}\propto\sigma_{a\alpha\dot\alpha}e_\mu^a(x)\epsilon^{\mu\nu\rho\sigma}H_{\nu\rho\sigma}, \tag{6.14}$$

$H_{\mu\nu\rho}(x)=3\partial_{[\mu}B_{\nu\rho]}(x)$. In Eq. (6.12) we have written explicitly only the kinetic terms; the remaining ones may be extracted from the formulas in [24] and are not essential for what follows.

Finally, the superstring action (5.2) in the gauge (6.5) reduces to the *bosonic string action*

$$S_{sstr}|_{\hat\theta=0}\propto S_{bstr}=\int_{W^2}\left[\frac{1}{4}{}^*\hat{e}_a\wedge\hat{e}^ae^{\hat\phi/2}-B_2(\hat{x})\right]$$
$$=\int_{W^2}\left[\frac{1}{2}d^2\xi\sqrt{|\det(\hat{e}_m^a\hat{e}_{an})|}e^{\phi(\hat{x})/2}-B_2(\hat{x})\right], \tag{6.15}$$

where

$$B_2(\hat{x})=\frac{1}{2}d\hat{x}^\mu\wedge d\hat{x}^\nu B_{\mu\nu}(\hat{x})=d^2\xi\partial_\tau\hat{x}^\mu\partial_\sigma\hat{x}^\nu B_{\mu\nu}(\hat{x}).$$

Thus the complete gauge-fixed action for the interacting system reads

$$S_{intGF}=S_{sg}(e,\psi,g_a,r,\overline{r})+S_{tm}(\phi,\chi,B;e,\psi,g_a)$$
$$+S_{bstr}(\hat{x};e,B), \tag{6.16}$$

with $S_{sg}$, $S_{tm}$, and $S_{bstr}$ defined in Eqs. (6.10), (6.12), and (6.15), respectively.

### C. Supersymmetry of the gauge-fixed action

Note that, although the gauge-fixed action (6.16) includes the action for the purely bosonic string (6.15), it possesses 1/2 of the local supersymmetry characteristic for the supergravity action. Actually, the direct proof of this fact can be found in [40]. Here we will show this in a different way which is based on the observation that the symmetries of the gauge-fixed action (6.16) can be identified as a subset of the symmetries of the complete (superfield) action (5.1) which preserve the gauge (6.5) and the Wess-Zumino gauge (see [17] for the supergravity-superparticle interacting system).

First, note that in the Wess-Zumino gauge (6.6) the index of the superspace Grassmann coordinate is identified with the Lorentz group spinor index. Indeed, due to the second equation in Eqs. (6.6),

$$\theta^{\underline{\beta}}\equiv(\theta^\beta,\overline{\theta}_{\dot\beta}):=\theta^{\check\alpha}E_{\check\alpha}^{\underline{\beta}}(Z)=\theta^{\check\alpha}\delta_{\check\alpha}^{\underline{\beta}}. \tag{6.17}$$

Clearly, the same is true for the superstring spinorial Grassmann coordinate functions. Their transformation rules can be read off Eq. (6.4),

$$\delta\hat\theta^\alpha(\xi)=b^M(\hat{x},\hat\theta)E_M^\alpha(\hat{x},\hat\theta)+\delta_\kappa\hat\theta^\alpha(\xi), \tag{6.18}$$

where $\delta_\kappa\hat\theta^\alpha(\xi)$ reads [see Eqs. (2.45) and (6.17)]

$$\delta_\kappa\hat\theta^\alpha(\xi)=\overline{\kappa}^n_{\dot\alpha}(\delta_n^m-\sqrt{|g|}\epsilon_{nk}g^{km})\hat{E}_m^a(\hat{x},\hat\theta)\tilde\sigma_a^{\dot\alpha\alpha},$$
$$\delta_\kappa\hat{\overline{\theta}}{}^{\dot\alpha}(\xi)=(\delta_\kappa\hat\theta^\alpha(\xi))^*. \tag{6.19}$$

Clearly, the superdiffeomorphism parameter $b^M(x,\theta)$ in Eq. (6.18) is to be restricted by the conditions of the preservation of the Wess-Zumino gauge. However (see [17,22,37] and references therein) the parameter $\epsilon^\alpha(x)=b^M(x,0)E_M^\alpha(x,0)$ remains unrestricted and is identified with the parameter of the local supersymmetry of the component (spacetime) formulation of supergravity.

Now, the preservation of the gauge (6.5), $\hat\theta^\alpha(\xi)=0$, imposes the condition $\delta\hat\theta^\alpha(\xi)|_{\hat\theta^\alpha(\xi)=0}=0$—i.e.,

$$\epsilon^\alpha(\hat{x})=-\delta_\kappa\hat\theta^\alpha(\xi)|_{\hat\theta^\alpha(\xi)=0}$$
$$=-\overline{\kappa}^n_{\dot\alpha}(\xi)(\delta_n^m-\sqrt{|g|}\epsilon_{nk}g^{km})\hat{e}_m^a(\hat{x})\tilde\sigma_a^{\dot\alpha\alpha}, \tag{6.20}$$

on the parameter of the local supersymmetry. This restriction appears only on the string world sheet and expresses the pullback $\epsilon^{\alpha}(\hat{x})$ of the supersymmetry parameter $\epsilon^{\alpha}(x)$ through a world sheet parameter $\overline{\kappa}^{n}_{\dot{\alpha}}(\xi)$ contracted (on both indices) with the expression $(\delta_n^m - \sqrt{|g|}\epsilon_{nk}g^{km})\hat{e}_m^a(\hat{x})\tilde{\sigma}_a^{\dot{\alpha}\alpha}$. The latter is the $\hat{\theta}=0$ "value" of the Green-Schwarz $\kappa$-symmetry "projector" and makes only one parameter included in $\overline{\kappa}^{n}_{\dot{\alpha}}(\xi)$ being involved *effectively* in the expression.

Thus on the world sheet $W^2$ of a (dynamical) string the four parameters of local supersymmetry of the free supergravity are reduced by the condition of the invariance of the gauge fixed interacting action (6.16), to the two effective parameters of the $\kappa$-symmetry-like transformations, while out of $W^2$ these four parameters, $\epsilon^{\alpha}(x), \overline{\epsilon}^{\dot{\alpha}}(x)$, remain unrestricted. This can be characterized by stating the preservation of the 1/2 of the local supersymmetry of the free supergravity by the gauge fixed interacting action (6.16).

### D. Equations of motion following from the gauge-fixed action

An important observation is that the gauge fixed version of the superstring action (6.15) involves only the *physical bosonic fields* of the supergravity and tensor multiplet, the graviton $e_{\mu}^{a}(x)$, the antisymmetric tensor $B_{\mu\nu}(x)$, and the scalar $\phi(x)$. Neither auxiliary fields nor fermions appear in the string action (6.15). As a result, both the equations for the auxiliary fields and for the fermions of the interacting system (6.16) will keep formally the same form as in the absence of superstring. In particular, this means that *in the gauge (6.5), (6.6) neither the Rarita-Schwinger equations nor the equations for the fermionic field of the tensor multiplet will include a source term from the superstring* (although they will be written with covariant derivatives for the spin connections satisfying the sourceful Einstein equations with an input from the superstring energy-momentum tensor).

This is a manifestation of a counterpart of the super-Higgs effect in dynamical supergravity interacting with a superstring or superbrane object, which we will address in a separate paper (see [38] for a spacetime counterpart of the Higgs effect in general relativity interacting with material particles, strings and $p$-branes).

Similar properties have already been found in the dynamical $D=4$, $N=1$ supergravity interacting with a massless superparticle [17]. The present consideration generalizes it for the case of dynamical supergravity interacting with a simplest supersymmetric extended object.

Below we present a simple check of the gauge equivalence described above at the level of equations of motion. Namely, we will show that the dynamical equations for fermions, which follow from the superfield equations (5.11), (5.12), (5.13), (5.16), are indeed sourceless in the gauge (6.5), (6.6).

### E. Superfield equations and the equations for physical fields in the "fermionic unitary" gauge

The superstring contributions to all the superfield equations of motion, Eqs. (5.16), (5.17), and (5.13) [or, equivalently, Eqs. (5.21), (5.22)] with Eq. (5.15), come in the form of current prepotentials (5.8), (5.7), or their derivatives. This is true as well for their consequences like Eqs. (5.23), (5.24), (5.28).

Both current prepotentials (5.8) and (5.7) contain the superspace delta function

$$\delta^8(Z-\hat{Z}) = \delta^4(x-\hat{x})(\theta-\hat{\theta})^4 \tag{6.21}$$

integrated over $W^2$ with the corresponding measure. In the gauge (6.5), $\hat{\theta}^{\check{\alpha}}(\xi)=0$, the delta function becomes proportional to the highest degree of the Grassmann coordinate $\theta$:

$$\hat{\theta}^{\check{\alpha}}=0: \quad \delta^8(Z-\hat{Z}) = \delta^4(x-\hat{x})\,\theta^4. \tag{6.22}$$

As a result, in this gauge both superstring current prepotentials are proportional to the fourth degree of the Grassmann coordinate:

$$K_a^B \propto \theta^4, \quad W^{AB} \propto \theta^4. \tag{6.23}$$

Then a covariant derivative of any of the current prepotentials is proportional to $\theta^3$:

$$\mathcal{D}_C K_a^B \propto \theta^3, \quad \mathcal{D}_C W^{AB} \propto \theta^3. \tag{6.24}$$

The action of two derivatives which is *not* reducible to one derivative, $\mathcal{D}^2_{AB} = \mathcal{D}_A\mathcal{D}_B + (-1)^{AB}\mathcal{D}_A\mathcal{D}_B$ but *not* $[\mathcal{D}_A, \mathcal{D}_B\} = -T_{AB}{}^{C}\mathcal{D}_C + R_{AB}$ (e.g., $\mathcal{D}^2_{\alpha\dot{\beta}} \equiv [\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot{\beta}}]$, but not $\{\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot{\beta}}\} = 2i\sigma^a_{\alpha\dot{\beta}}\mathcal{D}_a$) may result in expressions proportional to $\theta^2$,

$$\mathcal{D}^2_{CD} K_a^B \propto \theta^2, \quad \mathcal{D}_{CD} W^{AB} \propto \theta^2, \tag{6.25}$$

etc.,

$$\mathcal{D}^3_{CDE} K_a^B \propto (\theta)^1, \quad \mathcal{D}^3_{CDE} W^{AB} \propto (\theta)^1. \tag{6.26}$$

Only the action of four derivatives may produce a $\propto \theta^0$ input—i.e., terms which have a nonvanishing $\theta=0$ value:

$$\mathcal{D}^4_{CDEF} K_a^B \propto \theta^0, \quad \mathcal{D}^4_{CDEF} W^{AB} \propto \theta^0. \tag{6.27}$$

This implies, in particular, that the current potential (5.15) is proportional to the second power of the superspace Grassmann coordinate:

$$\mathcal{J}_a \propto \theta^2. \tag{6.28}$$

Then

$$\mathcal{D}_A \mathcal{J}_a \propto \theta^1, \tag{6.29}$$

and only the second derivative of the current potential may produce a term with a nonvanishing leading component ($\theta$-independent part):

$$\mathcal{D}^2_{BC} \mathcal{J}_a \propto \theta^0. \tag{6.30}$$

The spacetime fermionic equations of motion of the interacting system may be obtained as leading ($\theta=0$) components of Eqs. (5.23) and (5.16). Ignoring the inputs with a smaller number of derivatives applied to the current potential, one can write the leading components of Eq. (5.23) as

$$(\Psi^{a}_{\dot\alpha}-\Xi^{a}_{\dot\alpha}(\Phi))|_{\theta=0}\propto\mathcal{D}_B\mathcal{J}_a|_{\theta=0}, \qquad (6.31)$$

where $\Xi^{a}_{\dot\alpha}(\Phi)$ denotes the tensor multiplet contribution to the gravitino equation, which is given by the first term on the RHS of Eq. (5.23). Then Eq. (6.29) implies that the RHS of Eq. (6.32) vanishes in the gauge (6.5)—i.e., that in this gauge, Eq. (6.32), reads

$$\Psi^{a}_{\dot\alpha}|_{\theta=0}=\Xi^{a}_{\dot\alpha}(\Phi)|_{\theta=0} \qquad (6.32)$$

and does not contain an explicit input from the superstring.

In the same manner one finds that the dynamical equation for the fermionic component of the tensor multiplet, given by the leading component of Eq. (5.16),

$$s\overline{\mathcal{D}\mathcal{D}}[e^{-\Phi/2}\mathcal{D}_{\alpha}e^{\Phi/2}]|_{\theta=0}=-\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_{\alpha}K_a{}^{a}|_{\theta=0} -\tfrac{1}{2}(\sigma_{[a}\tilde{\sigma}_{b]})_{\alpha}{}^{\beta}[\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_{\beta}W^{ab}]|_{\theta=0} +4i\sigma_{b\alpha\dot\beta}\overline{\mathcal{D}\mathcal{D}}W^{b\dot\beta}|_{\theta=0}, \qquad (6.33)$$

implies, in the light of Eq. (6.26), that the superstring-produced RHS of this equation vanishes in the gauge (6.5):

$$s\overline{\mathcal{D}\mathcal{D}}[e^{-\Phi/2}\mathcal{D}_{\alpha}e^{\Phi/2}]|_{\theta=0}=0. \qquad (6.34)$$

As far as the bosonic superfield equations are concerned, they preserve the nontrivial input from the superstring source in the gauge (6.5). Indeed, the leading components of Eqs. (5.24), (5.28) contain the second derivatives of the current potential and fourth derivatives of the current prepotentials, which remain nonvanishing in accordance with Eqs. (6.30) and (6.26).

The explicit form of the Einstein equation can be derived in a way close to the one used in [17] for the supergravity-superparticle interacting system (although the presence of both Nambu-Goto and Wess-Zumino terms in the superstring action, as well as of the tensor multiplet in the action of the interacting system, makes the Einstein equation of the supergravity–tensor-multiplet–superstring system a bit more complicated). As far as the Kalb-Ramon field equations are concerned, taking into account Eqs. (6.23)–(6.27) and the conditions of the Wess-Zumino gauge (6.6), (6.7) [which implies, in particular, $\mathcal{D}_{\alpha}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}^{\beta}(\theta)^4\propto\delta_{\alpha}^{\beta}+\mathcal{O}(\theta)$] one finds that, in the gauge (6.5), the leading component of the superfield equation (5.28) becomes

$$s\mathcal{D}^{c}H_{abc}|_{\theta=0}=\frac{1}{16}e^{-\phi/2}w^{cd}+\cdots, \qquad (6.35)$$

where

$$w^{ba}:=\frac{1}{2}\int_{W^2}\frac{1}{\hat{e}}\hat{e}^{b}\wedge\hat{e}^{a}\delta^4(x-\hat{x}). \qquad (6.36)$$

Hence, in the gauge (6.5) the superstring input on the RHS of the Kalb-Ramond gauge field equation for the supergravity–tensor-multiplet–superstring interacting system is nonvanishing and, moreover, coincides with the input of the bosonic string.

Thus we have checked that the spacetime equations of motion for the fermionic fields which follow from the complete superfield action (5.1) of the supergravity–tensor-multiplet–superstring interacting system become sourceless in the gauge (6.5). This is true both for the gravitino equations and for the fermionic field of the tensor multiplet. At the same time, the corresponding bosonic equations are clearly sourceful in any gauge. This is a characteristic property of the equations which follow from the gauge fixed action (6.16).

We should note that the gauge-fixed fermionic equations are not completely decoupled from the superstring. They are written in terms of the spacetime covariant derivatives with (composed) spin connections satisfying the Einstein equation with a source. The same is true for the equations derived directly from the gauge fixed action (6.16).

### F. Possible application of the gauge equivalence

Note that the gauge-fixed description (6.16) of the superfield interacting system (6.16) is complete in the following sense. Along the line of [40] one may check that the gauge-fixed action (6.10), (6.12) reproduces the gauge-fixed version of all the dynamical equations which might be derived from the complete superfield action, including the *fermionic equations for the bosonic string*. This is the manifestation of the purely gauge (or Goldstone) nature of the superstring coordinate functions $\hat{Z}^M(\xi)$ (not to be confused with the supercoordinates $Z^M$; see [32,38] for further discussion).

A counterpart of the gauge-fixed action (6.16) can be written in any dimension, for any supergravity interacting with any superbrane. Hence, using the above-described gauge equivalence one may already proceed with studying the $D=11$ supergravity interacting with super-M2-branes and super-M5-branes, as well as the $D=10$ type II supergravity interacting with super-*Dp*-branes (in spite of the fact the $D=10,11$ superfield supergravity actions are not known).

## VII. CONCLUSIONS

In this paper we studied the full superfield Lagrangian description of the $D=4$ interacting system of dynamical supergravity and a superstring described by the sum $S=S_{SG}+S_{TM}+S_{sstr}$ of the superfield supergravity action $S_{SG}$ [22], the Green-Schwarz superstring action $S_{str}$ [23], and a superfield action for the dynamical tensor multiplet $S_{TM}$ [24].

The superfield theoretical system $S=S_{SG}+S_{TM}$ was argued to be related to the low-energy limit of $D=4$ compactification of the heterotic superstring [18,25](see also [28]) and considered in [27,29]. So our main interest here has been to analyze the influence of the superstring action $S_{sstr}$ on the dynamics of the interacting system—namely, in the superstring-produced source terms both in the superfield and component form of the equations.

We have obtained the complete set of superfield equations with sources provided by the superstring. In the supergravity sector we found that the scalar superfield equation remains the same as for free supergravity, while the vector superfield equation is modified both by the interaction with the tensor multiplet and by the source (current potential) coming from the superstring. The current potential is constructed from the two types of current *prepotentials* coming from the variation of the Nambu-Goto and Wess-Zumino terms of the superstring action, respectively. The superfield equations for the tensor multiplet are also modified by inputs from the above-mentioned current prepotentials. The equations of motion for the superstring variables are the same as in the *background* of supergravity interacting with dilaton and super-2-form superfields.

These superfield equations appeared to be quite complicated, which might indicate that their higher-dimensional $D=10,11$ generalizations, even if they exist, will be quite difficult to deal with. We have considered as well an exit from this problem.

By analyzing the gauge symmetries and taking into account the properties of the Wess-Zumino gauge (see [17] and references therein) we have shown that there exists a complete gauge-equivalent description of the "superfield" interacting system, $S=S_{SG}+S_{TM}+S_{sstr}$, given by the sum $S=S_{sg}+S_{tm}+S_{bstr}$ of the spacetime *component* action for supergravity $S_{sg}$ (without auxiliary fields), the component action for the tensor multiplet $S_{tm}$, and the action for a *bosonic* string $S_{str}$ (which appears as the purely bosonic "limit" of the superstring action $S_{sstr}$). We checked this gauge equivalence by studying the properties of the gauge-fixed version of the equations of motion derived from the complete superfield action. Despite the quite complicated form of the superfield generalizations of the Einstein and Rarita-Schwinger equations, as well as of the Kalb-Ramond equations and the equations for the fermionic fields of the tensor multiplet, it turned out to be quite easy to show that in the above-mentioned "fermionic unitary gauge" all the fermionic equations are sourceless (although they include the covariant derivatives with the spin connections obeying the sourceful Einstein equations) while the bosonic equations, including the Einstein and Kalb-Ramond field equations, acquired a source from the superstring.

This extends the supergravity–massless-superparticle results of [17] to the case of dynamical interacting systems including supergravity and an extended supersymmetric object and strongly supports that the above-mentioned gauge equivalence is not an artifact of the simpler massless superparticle case, but rather is a general property of the above-mentioned interacting systems. As the component actions for supergravity are known in all dimensions, including $D=10,11$ (the most interesting from an M-theoretic perspective), our results allow one to obtain and to study the complete set of equations for dynamical supergravity interacting with dynamical super-$p$-brane, at least in its gauge-fixed version. This promises to be a useful tool in a future search for new solitonic solutions of higher-dimensional supergravity including the ones with nonvanishing fermionic fields.

An analysis of the superfield equations with sources obtained in this paper, as well as an investigation of the $D=10,11$ supergravity-superbrane interacting systems with the use of the gauge equivalence of their complete superfield description, with the description by the sum of spacetime (component) action for supergravity and the action for bosonic brane, will be the subject of future work.

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## APPENDIX A: SUPERSTRING $\kappa$ SYMMETRY IN FLAT SUPERSPACE

*In flat superspace* one may consider a vanishing dilaton superfield, $\Phi(Z)=0$ [although this is not obligatory in $D=4$, $N=1$ superspace, where $\Phi(z)$ is described by a separate multiplet; see Sec. I] and use the standard expression for the supervielbein:

flat superspace:

$$E^a=dX^\mu\delta_\mu^a-i\,d\theta^\alpha\sigma^a_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}+i\,\theta^\alpha\sigma^a_{\alpha\dot\alpha}d\bar\theta^{\dot\alpha}, \tag{A1}$$

$$E^\alpha=d\theta^{\check\beta}\delta_{\check\beta}{}^\alpha\equiv d\theta^\alpha,$$

$$\bar E^{\dot\alpha}=d\theta^{\check\beta}\delta_{\check\beta}{}^{\dot\alpha}\equiv d\bar\theta^{\dot\alpha}, \tag{A2}$$

where $\check\alpha=1,2,3,4$ shall be treated as a Majorana spinor index, $\check\alpha=\alpha$; $\theta^{\check\beta}=\theta^{\underline\beta}=(\theta^\beta,\bar\theta_{\dot\beta})$ (i.e., $\theta^{\check\beta}\equiv\theta^\alpha\delta^{\check\beta}{}_\alpha+\bar\theta_{\dot\alpha}\delta^{\check\beta\dot\alpha}$). The "vac$\bar{\text{u}}$um value" of the 2-form $B_2$ has to be chosen as

flat superspace, $\Phi(Z)=0$:

$$B_2=-\frac{i}{2}dX^\mu\delta_\mu^a\wedge[d\theta^\alpha\sigma_{a\alpha\dot\alpha}\bar\theta^{\dot\alpha}-\theta^\alpha\sigma_{a\alpha\dot\alpha}d\bar\theta^{\dot\alpha}]. \tag{A3}$$

Then the pullback $\hat B_2=\phi^*(B_2)$ of the 2-form $B_2$, Eq. (A3), produces the standard form of the Wess-Zumino term of the $D=4$ Green-Schwarz superstring action [23]. As a result of its presence, the action possesses a local fermionic symmetry, the seminal $\kappa$ symmetry [12,23]. Its transformation rules can be formulated as follows:

$$i_\kappa\hat E^a:=\delta_\kappa\hat Z^M E_M^a(\hat Z)=0,$$

$$i_\kappa\hat E^\alpha\sigma_{a\alpha\dot\alpha}(*\hat E^a-\hat E^a)=0. \tag{A4}$$

Indeed, as $(*\hat{E}^a - \hat{E}^a) = d\xi^n(\delta_n^m + \sqrt{|g|}\epsilon_{nk}g^{km})\hat{E}_m^a$ and, in flat superspace, $E^a$ has the form of Eq. (A1), the solution of Eqs. (A4) with respect to $\delta_\kappa X^\mu$ and $\delta_\kappa \theta^\alpha$ gives the $D=4$ version of the Green-Schwarz expression for the superstring $\kappa$ symmetry [23]:

$$\text{flat superspace,} \quad \Phi(Z)=0:$$

$$\delta_\kappa \hat{X}^\mu = i\,\delta_\kappa \theta^\alpha \sigma^a_{\alpha\dot{\alpha}} \overline{\theta^{\dot{\alpha}}} + \text{c.c.},$$

$$\delta_\kappa \hat{\theta}^\alpha = \overline{\kappa}^n_{\dot{\alpha}}(\delta_n^m - \sqrt{|g|}\epsilon_{nk}g^{km})\hat{E}_m^a \tilde{\sigma}_a^{\dot{\alpha}\alpha},$$

$$\delta_\kappa \hat{\bar{\theta}}^{\dot{\alpha}} = (\delta_\kappa \hat{\theta}^\alpha)^*. \tag{A5}$$

## APPENDIX B: SOME USEFUL FORMULAS

(B1) In our notation the Riemannian curvature 2-form of the minimal $D=4$, $N=1$ supergravity is given by

$$R^{\alpha\beta} \equiv dw^{\alpha\beta} - w^{\alpha\gamma}\wedge w_\gamma{}^\beta \equiv \frac{1}{4}R^{ab}(\sigma_a\tilde{\sigma}_b)^{\alpha\beta}$$

$$= -\frac{1}{2}E^\alpha \wedge E^\beta \overline{R} - \frac{i}{8}E^c \wedge E^{(\alpha}\tilde{\sigma}_c{}^{\dot{\gamma}\beta)}\overline{\mathcal{D}}_{\dot{\gamma}}\overline{R}$$

$$- \frac{i}{8}E^c \wedge E^\gamma(\sigma_c\tilde{\sigma}_d)_\gamma{}^{(\beta}\mathcal{D}^{\alpha)}G^d$$

$$- \frac{i}{8}E^c \wedge \overline{E}^{\dot{\beta}}\sigma_{c\gamma\dot{\beta}}W^{\alpha\beta\gamma} + \frac{1}{2}E^d \wedge E^c R_{cd}{}^{\alpha\beta}, \tag{B1}$$

$$R^{\dot{\alpha}\dot{\beta}} = (R^{\alpha\beta})^*. \tag{B2}$$

(B2) After the minimal $D=4$, $N=1$ supergravity constraints (2.29), (2.30) are taken into account, one finds, for the variation of the superstring action,

$$\delta_{\hat{Z}}S_{sstr} = -\frac{1}{2}\int_{W^2}\left[\mathcal{D}(e^{\hat{\Phi}/2} * \hat{E}_a) - \frac{1}{4}e^{\hat{\Phi}/2} * \hat{E}_b \wedge \hat{E}^b \hat{\nabla}_a \hat{\Phi}\right] i_{\delta\hat{Z}}E^a$$

$$- i\int_{W^2} e^{\hat{\Phi}/2} * \hat{E}_a \wedge \sigma_{a\alpha\dot{\alpha}}\hat{\bar{E}}^{\dot{\alpha}} + \text{c.c.} - \frac{i}{8}\int_{W^2} e^{\hat{\Phi}/2} * \hat{E}_b$$

$$\wedge \hat{E}^b \hat{\nabla}_\alpha \hat{\Phi}\, i_{\delta\hat{Z}}E^\alpha + \text{c.c.} - \int_{W^2} i_{\delta\hat{Z}}H_3, \tag{B3}$$

where we denote $\nabla_A \Phi|_{Z=\hat{Z}(\xi)} := \hat{\nabla}_A \hat{\Phi}$ and ignore the boundary contribution $\int_{W^1=\partial W^2}[\frac{1}{2}e^{\hat{\Phi}/2} * \hat{E}_b i_{\delta\hat{Z}}\hat{E}^b - i_{\delta\hat{Z}}\hat{B}_2]$ (which always vanishes for the case of a closed superstring with $\partial W^2 = \emptyset$). The consideration of the flat superspace (see Appendix A) suggests that $\kappa$ symmetry occurs in the action (2.3) when also the constraints (2.36), (2.37) are imposed on the field strength $H_3 = dB_2$ of the 2-superform $B_2$, Eq. (2.10).

(B3) The admissible variation of the 2-form $B_2$ subject to the superspace constraints (2.38) is defined by Eq. (4.2) with

$$b_{\alpha\beta}(\delta) = 0, \quad b_{\alpha\dot{\beta}}(\delta) = 0, \quad b_{\dot{\alpha}\dot{\beta}}(\delta) = 0, \tag{B4}$$

$$b_{\beta b}(\delta) = \sigma_{b\beta\dot{\beta}}(\mathcal{D}\mathcal{D} - \overline{R})\,\delta\overline{\nu}^{\dot{\beta}}$$
$$- \frac{i}{2}(\eta_{ab}\delta + \sigma_b\tilde{\sigma}_a)_\beta{}^\gamma \nabla_\gamma e^{\Phi/2}\delta H^a, \tag{B5}$$

$$b_{\dot{\beta} b}(\delta) = -\sigma_{b\beta\dot{\beta}}(\overline{\mathcal{D}\mathcal{D}} - R)\,\delta\nu^\beta$$
$$+ \frac{i}{2}(\eta_{ab}\delta + \tilde{\sigma}_a\sigma_b)^{\dot{\gamma}}{}_{\dot{\beta}}\overline{\nabla}_{\dot{\gamma}} e^{\Phi/2}\delta H^a, \tag{B6}$$

$$b_{ab}(\delta) = -\frac{i}{4}(\tilde{\sigma}_{[a}\sigma_{b]})^{\dot{\beta}}{}_{\dot{\alpha}}\overline{\mathcal{D}}_{\dot{\beta}}(\mathcal{D}\mathcal{D} - \overline{R})\,\delta\overline{\nu}^{\dot{\alpha}}$$
$$- \frac{i}{4}(\sigma_{[a}\tilde{\sigma}_{b]})_\alpha{}^\beta \mathcal{D}_\beta(\overline{\mathcal{D}\mathcal{D}} - R)\,\delta\nu^\alpha$$
$$+ \frac{1}{2}e^{\Phi/2}\tilde{\sigma}_{[a}^{\dot{\beta}\beta}[\mathcal{D}_\beta, \overline{\mathcal{D}}_{\dot{\beta}}]\delta H_{b]} - \frac{i}{2}\epsilon_{abcd}\mathcal{D}^c e^{\Phi/2}\delta H^d. \tag{B7}$$


[1] B.E.W. Nilsson, in *Proceedings of the 31st International Ahrenshoop Symposium on the Theory of Elementary Particles*, Buckow, Germany, 1997, edited by H. Dorn, D. Lüst, and G. Weigt, (Wiley-VCH, Berlin, 1998), pp. 52–57. M. Cederwall, U. Gran, M. Nielsen, and B.E.W. Nilsson, J. High Energy Phys. **10**, 041 (2000); D.S. Berman, V.L. Campos, M. Cederwall, U. Gran, H. Larsson, M. Nielsen, B.E.W. Nilsson, and P. Sundell, *ibid.* **05**, 002 (2001); M. Cederwall, B.E.W. Nilsson, and D. Tsimpis, *ibid.* **06**, 034 (2001); **07**, 042 (2001); **02**, 009 (2002); **11**, 023 (2002); M. Cederwall, in *New Developments in Fundamental Interaction Theories*, edited by J. Lukierski and J. Rembielinski, AIP Conf. Proc. No. 589 (AIP, Melville, NY, 2001), pp. 46–60.

[2] S.J. Gates and L. Rana, Russ. Phys. J. **45**, 682 (2002); W.D. Linch, M.A. Luty, and J. Phillips, Phys. Rev. D **68**, 025008 (2003); A. Kiss and E. Loyer, ''$N=8$ Supergravity in Central Charge Superspace,'' hep-th/0209035.

[3] K. Peeters, P. Vanhove, and A. Westerberg, Class. Quantum Grav. **18**, 843 (2001); ''Supersymmetric $R^4$ actions and quantum corrections to superspace torsion constraints,'' hep-th/0010182.

[4] S. de Haro, A. Sinkovics, and K. Skenderis, Phys. Rev. D **67**, 084010 (2003).

[5] S.J. Gates and H. Nishino, Phys. Lett. B **173**, 46 (1986); **173**, 52 (1986); Nucl. Phys. **291**, 205 (1987); S.J. Gates and S. Vashakidze, *ibid.* **B291**, 172 (1987); S. Bellucci and S.J. Gates, *ibid.* **B208**, 456 (1988); S. Bellucci, D.A. Depireux, and S.J. Gates, *ibid.* **B238**, 315 (1990).

[6] R. D'Auria and P. Fré, Phys. Lett. B **200**, 63 (1988); R. D'Auria, P. Fré, M. Raciti, and F. Riva, Int. J. Mod. Phys. A **3**, 953 (1988); L. Bonora, P. Pasti, and M. Tonin, Phys. Lett. B **188**, 112 (1987); L. Bonora, M. Bregola, K. Lechner, P. Pasti, and M. Tonin, Nucl. Phys. **B296**, 877 (1988).

[7] M. Tonin, Int. J. Mod. Phys. A **4**, 1983 (1989); in *Geometrical and Algebraic Aspects of Nonlinear Field Theory, Proceedings of the International Meeting*, Amalfi, Italy, 1988, edited by M. Marianaro *et al.* (North-Holland, Amsterdam, 1989), p. 229.

[8] L. Bonora, M. Bregola, R. D'Auria, P. Fre, K. Lechner, P. Pasti, I. Pesando, M. Raciti, F. Riva, M. Tonin, and D. Zanon, Phys. Lett. B **277**, 306 (1992).

[9] S.J. Gates and H. Nishino, Phys. Lett. B **508**, 155 (2001).

[10] M.T. Grisaru, P.S. Howe, L. Mezincescu, B. Nilsson, and P.K. Townsend, Phys. Lett. **162B**, 116 (1985); J.A. Shapiro and C.C. Taylor, Phys. Rep. **191**, 221 (1990) and references therein.

[11] E. Bergshoeff, E. Sezgin, and P.K. Townsend, Phys. Lett. B **189**, 75 (1987); Ann. Phys. (N.Y.) **185**, 330 (1988).

[12] J.A. de Azcárraga and J. Lukierski, Phys. Lett. **113B**, 170 (1982); Phys. Rev. D **28**, 1337 (1983); W. Siegel, Phys. Lett. **128B**, 397 (1983).

[13] D.J. Gross, J.A. Harvey, E.J. Martinec, and R. Rohm, Nucl. Phys. **B256**, 253 (1985).

[14] J.M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998); O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. **323**, 183 (2000).

[15] E. Witten, Nucl. Phys. **B500**, 3 (1997); **B507**, 658 (1997); J. High Energy Phys. **07**, 003 (1997) and references therein.

[16] P. Claus, R. Kallosh, J. Kumar, P.K. Townsend, and A. Van Proeyen, J. High Energy Phys. **06**, 004 (1998); P. Claus, Phys. Rev. D **59**, 066003 (1999); P. Pasti, D. Sorokin, and M. Tonin, Phys. Lett. B **447**, 251 (1999); and references therein.

[17] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo, and J. Lukierski, Phys. Rev. D **67**, 065003 (2003).

[18] S.J. Gates, P. Majumdar, R.N. Oerter, and A.E. van de Ven, Phys. Lett. B **214**, 26 (1988).

[19] W. Siegel, Phys. Lett. **85B**, 333 (1979).

[20] S.J. Gates, M.T. Grisaru, M. Rocek, and W. Siegel, Front. Phys. **58**, 1 (1983).

[21] S.J. Gates, Jr., Nucl. Phys. **B184**, 381 (1981).

[22] J. Wess and B. Zumino, Phys. Lett. **74B**, 51 (1978).

[23] M.B. Green and J.H. Schwarz, Phys. Lett. **136B**, 367 (1984); Nucl. Phys. **B243**, 285 (1984).

[24] B. de Wit and M. Roček, Phys. Lett. **109B**, 439 (1982).

[25] W. Siegel, Phys. Lett. B **211**, 55 (1988).

[26] S. Cecotti, S. Ferrara, and L. Girardello, Phys. Lett. B **198**, 336 (1987); B. Ovrut, in Proceedings of the Sainte-Adele 1988 Group Theoretical Methods in Physics, p. 733; Phys. Lett. B **205**, 455 (1988); **214**, 177 (1988); Nucl. Phys. **B321**, 163 (1989).

[27] N. Berkovits and W. Siegel, Nucl. Phys. **B462**, 213 (1996); W. Siegel, Phys. Rev. D **53**, 3324 (1996).

[28] S.J. Gates, Phys. Lett. B **390**, 161 (1997); in *Supersymmetry, Supergravity and Supercolliders, TASI 97 Proceedings*, edited by J.A. Bagger (World Scientific, Singapore, 1999), pp. 153–258; S.J. Gates and H. Nishino, Phys. Lett. B **492**, 178 (2000).

[29] J. de Boer and K. Skenderis, Nucl. Phys. **B481**, 129 (1996).

[30] V.P. Akulov, D.V. Volkov, and V.A. Soroka, Theor. Math. Phys. **31**, 285 (1977); M. Sohnius and P.C. West, Phys. Lett. **105B**, 353 (1981); Nucl. Phys. **B198**, 493 (1982).

[31] M.E. Knutt-Wehlau and R.B. Mann, Class. Quantum Grav. **16**, 937 (1999).

[32] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo, and J. Lukierski, "On dynamical supergravity interacting with super-$p$-brane sources," invited talk at the 3rd Sakharov International Conference, Moscow, 2002, to appear in the Proceedings, hep-th/0211065.

[33] C.M. Hull, Phys. Lett. **139B**, 39 (1984).

[34] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, 1992); I.L. Buchbinder and S.M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace* (IOP, Bristol, UK, 1995); W. Siegel, "Fields," hep-th/9912205.

[35] M. Kalb and P. Ramond, Phys. Rev. D **9**, 2273 (1974).

[36] V.I. Ogievetsky and I.V. Polubarinov, Yad. Fiz. **4**, 216 (1966) [Sov. J. Nucl. Phys. **4**, 156 (1967)].

[37] I.A. Bandos and V.G. Zima, "Normal gauge in $N=1$ supergravity," Report No. ITF-85-85R, Kiev, 1985 (in Russian, unpublished); I.A. Bandos, Ph.D. thesis, Kharkov, 1985 (in Russian).

[38] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo, and J. Lukierski, Phys. Rev. D **68**, 046004 (2003).

[39] K.S. Stelle and P.C. West, Phys. Lett. **74B**, 330 (1978); S. Ferrara and P. van Nieuwenhuizen, *ibid.* **74B**, 333 (1978).

[40] I.A. Bandos, J.A. de Azcárraga, and J.M. Izquierdo, Phys. Rev. D **65**, 105010 (2002).

[41] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo, and J. Lukierski, Phys. Rev. D **65**, 021901 (2002).