

Supersymmetric string model with 30 κ symmetries in an extended $D=11$ superspace and $\frac{30}{32}$ BPS states

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A supersymmetric string model in the $D=11$ superspace maximally extended by antisymmetric tensor bosonic coordinates $\Sigma^{(528|32)}$ is proposed. It possesses 30 κ symmetries and 32 target space supersymmetries. The usual preserved supersymmetry- κ -symmetry correspondence suggests that it describes the excitations of a Bogomol'nyi-Prasad-Sommerfield (BPS) state preserving all but two supersymmetries. The model can also be formulated in any $\Sigma^{((n(n+1)/2|n)}$ superspace, $n=32$ corresponding to $D=11$. It may also be treated as a "higher-spin generalization" of the usual Green-Schwarz superstring. Although the global symmetry of the model is a generalization of the super-Poincaré group $\Sigma^{((n(n+1)/2|n)} \bowtie Sp(n)$, it may be formulated in terms of constrained $OSP(2n|1)$ orthosymplectic supertwistors. We work out this supertwistor realization and its Hamiltonian dynamics. We also give the supersymmetric p -brane generalization of the model. In particular, the $\Sigma^{(528|32)}$ supersymmetric membrane model describes excitations of a $\frac{30}{32}$ BPS state, as the $\Sigma^{(528|32)}$ supersymmetric string does, while the supersymmetric 3-brane and 5-brane correspond, respectively, to $\frac{28}{32}$ and $\frac{24}{32}$ BPS states.

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I. MODELS IN NON-STANDARD SUPERSPACES: PARTICLES, STRINGS, BPS PREONS AND HIGHER SPIN THEORIES

In the early period of superstring theory, when it was found that all $D=10$ supergravities appear as low energy limits of superstring models, a question arose: what is the origin of maximally extended $D=11$ supergravity? Its relation with the supermembrane [1] was established by studying the supermembrane action in a supergravity background; however, a straightforward quantization of the supermembrane was fraught with difficulties. An indication was found [2] that the quantum state spectrum of the supermembrane is continuous, a problem now sorted out by treating [3] the supermembrane as an object composed of D0-branes in the framework of the matrix model approach [4]. Another aspect of the same problem was that the membrane was shown to develop stringlike instabilities [2]. The Green-Schwarz superstring is free from these problems, but it is a $D=10$ theory. Thus, it was tempting to search for possible new classical $D=11$ superstring models hoping that, after quantization, their low energy limit would be $D=11$ supergravity. Such a search requires, clearly, going beyond the standard superspace framework: in moving from $D=10$ to $D=11$ one has to add also extra bosonic degrees of freedom, thus arriving to an *enlarged* $D=11$ superspace rather than to the standard one.

A. Curtright supersymmetric string model in the enlarged $D=11$ superspace $\Sigma^{(528|32)}$

A first example of a supersymmetric string action in an enlarged $D=11$ superspace was found in [5]. The model, possessing 32 supersymmetries and 16 κ symmetries, was constructed in the enlarged superspace $\Sigma^{(528|32)}$. This contains 32 fermionic coordinates θ^α and 528 bosonic coordi-

nates $x^\mu, y^{\mu\nu}, y^{\mu_1 \dots \mu_5}$ ($y^{\mu\nu} = -y^{\nu\mu} \equiv y^{[\mu\nu]}$, $y^{\mu_1 \dots \mu_5} = y^{[\mu_1 \dots \mu_5]}$) which may be collected in a symmetric spin tensor $X^{\alpha\beta} = X^{\beta\alpha}$,

$$X^{\alpha\beta} = \frac{1}{32} x^\mu \Gamma_\mu^{\alpha\beta} - \frac{1}{2!} \frac{1}{32} y^{\mu\nu} \Gamma_{\mu\nu}^{\alpha\beta} + \frac{1}{5!} \frac{1}{32} y^{\mu_1 \dots \mu_5} \Gamma_{\mu_1 \dots \mu_5}^{\alpha\beta}, \quad (1)$$

so that the coordinates of $\Sigma^{(528|32)}$ are

$$\mathcal{Z}^{\mathcal{M}} = (X^{\alpha\beta}, \theta^\alpha), \quad X^{\alpha\beta} = X^{\beta\alpha}, \quad \alpha, \beta = 1, 2, \dots, 32. \quad (2)$$

Because of the special properties of the eleven-dimensional gamma matrices, the model of [5] may also be restricted to the superspaces $\Sigma^{(66|32)} [(x^\mu, y^{\mu\nu}, \theta^\alpha)]$, with 66 bosonic coordinates] and $\Sigma^{(462|32)} [(x^\mu, y^{\mu_1 \dots \mu_5}, \theta^\alpha)]$, with 462 bosonic coordinates].¹ For the sake of definiteness, we shall call here *maximal superspaces* to those with bosonic coordinates of symmetric "spin-tensorial" type, like $\Sigma^{(528|32)}$ and its counterparts $\Sigma^{((n(n+1)/2|n)}$,

$$\Sigma^{((n(n+1)/2|n)} = \{ \mathcal{Z}^\Sigma = (X^{\alpha\beta}, \theta^\alpha) \}, \quad X^{\alpha\beta} = X^{\beta\alpha}, \quad \alpha, \beta = 1, 2, \dots, n, \quad (3)$$

where $n=2^l$. This name distinguishes the $\Sigma^{((n(n+1)/2|n)}$ superspaces from other, not maximally (in the bosonic sector) extended superspaces like $\Sigma^{(66|32)}$ and $\Sigma^{(462|32)}$ whose bosonic coordinates may be described by a spin tensor $X^{\alpha\beta}$ only if it satisfies some conditions.

¹All these superspaces $\Sigma^{(528|32)}$, $\Sigma^{(462|32)}$ and $\Sigma^{(66|32)}$, considered as supergroup manifolds, may be seen as central extensions of an Abelian 32 dimensional fermionic group by tensorial [Eq. (1)] bosonic groups [6].

The $\Sigma^{(528|32)}$ superspace has a special interest because it is the supergroup manifold associated with the maximal $D = 11$ supersymmetry algebra [7–9]

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}, \quad P_{\alpha\beta} = P_{\beta\alpha}, \quad [Q_\alpha, P_{\beta\gamma}] = 0, \\ \alpha, \beta, \gamma = 1, 2, \dots, 32, \quad (4)$$

$$P_{\alpha\beta} = P_\mu \Gamma_{\alpha\beta}^\mu + Z_{\mu\nu} \Gamma_{\alpha\beta}^{\mu\nu} + Z_{\mu_1 \dots \mu_5} \Gamma_{\alpha\beta}^{\mu_1 \dots \mu_5}, \quad (5)$$

called M-theory superalgebra or M algebra [9].² This algebra encodes full information about the nonperturbative Bogomol’nyi-Prasad-Sommerfield (BPS) states of the hypothetical underlying M theory: as was shown in [11], the additional bosonic generators $Z_{\mu\nu} = -Z_{\nu\mu} = Z_{[\mu\nu]}$, $Z_{\mu_1 \dots \mu_5} = Z_{[\mu_1 \dots \mu_5]}$ of the M algebra (4) are related to the topological charges of the supermembrane and the super-M5-brane.³ These “one-brane” BPS states can be associated with solitonic solutions [16,17] of the “usual” $D = 11$ supergravity [18] or with fundamental M-theory objects described by their worldvolume actions [1,19].

Note that although the M algebra (4) leads naturally to a $D = 11$ interpretation when the splitting (5) is used, it also allows for a $D = 10$ type IIB treatment when one considers the $\alpha = 1, \dots, 32$ index as a double one $\alpha = \check{\alpha}I$, where $\check{\alpha}$ labels the components of a $D = 10$ Majorana-Weyl spinor, $\check{\alpha} = 1, \dots, 16$, and I is an internal index, $I = 1, 2$. Then one uses the direct product of 16×16 (Majorana-Weyl) ten-dimensional sigma matrices, $\sigma_{\alpha\check{\beta}}^\mu$ (or $\tilde{\sigma}_{\alpha\check{\beta}}^\mu$) and real 2×2 matrices $\delta^{IJ}, \tau_1^{IJ}, i\tau_2^{IJ}, \tau_3^{IJ}$ to write a $D = 10$ type IIB counterpart of Eq. (5) [9,20]. Similarly, a $D = 10$ type IIA treatment is also possible as the $D = 10$ gamma matrices coincide with the $D = 11$ ones. As a result, the information about nonperturbative BPS states of the $D = 10$ superstring theories (including Dirichlet superbranes) can also be extracted from the algebra (4). Moreover, it encodes as well all the duality relations between different $D = 10$ and $D = 11$ superbranes. These facts add further reasons to call (4) the M-theory superalgebra [9].

B. Maximal $\Sigma^{(528|32)}$ superspace, BPS preons and other BPS states with supernumerary supersymmetries

Interestingly enough, an algebraic classification of all BPS states may be achieved by introducing [21] the hypo-

thetical basic constituents of M theory. These are the BPS preons $|\lambda\rangle$, which are characterized by the relation

$$\hat{P}_{\alpha\beta}|\lambda\rangle = \lambda_\alpha \lambda_\beta |\lambda\rangle, \quad (6)$$

where λ_α is a bosonic “spinor” [actually, a $GL(n, \mathbb{R})$ vector; see footnote 10 and below Eq. (24)].⁴

The states $|\lambda\rangle$ preserve all supersymmetries *but one* i.e., they are $\nu = 31/32$ states. To make this transparent one can use the $GL(32, \mathbb{R})$ automorphism symmetry [27,28] of the algebra (4) to write the spinor λ_α that characterizes the BPS preon state $|\lambda\rangle$ in the “preferred frame” $\lambda_\alpha = (1, 0, \dots, 0)$, where Eqs. (6) and (4) imply

$$(\hat{Q}_2)^2|\lambda\rangle = 0, \quad \dots, \quad (\hat{Q}_{32})^2|\lambda\rangle = 0. \quad (7)$$

For Hermitian operators in a positive definite Hilbert space Eqs. (7) imply

$$(\hat{Q}_2)|\lambda\rangle = 0, \quad \dots, \quad (\hat{Q}_{32})|\lambda\rangle = 0, \quad (8)$$

which means that a BPS preon preserves all but one space-time supersymmetries.

BPS states $|k\rangle$ preserving $k \geq 1$ supersymmetries can be treated as composites of a number $\#_p = 32 - k$ of BPS preons [21] (in the same way as e.g., hadrons are composed of quarks). Indeed, for such a state $|k\rangle$ one can always find a set of $32 - k$ bosonic spinors λ_α^r ($r = 1, \dots, k$) such that

$$\hat{P}_{\alpha\beta}|k\rangle = \sum_{r=1}^{\#_p=32-k} \lambda_\alpha^r \lambda_\beta^r |k\rangle; \quad (9)$$

the single preon state $|\lambda\rangle$ corresponds to $|k\rangle \equiv |31\rangle$.

In this perspective, all the one-brane solutions of 11-dimensional supergravity, which preserve 16 out of 32 supersymmetries, correspond to composites of 16 BPS preons. Multibrane solutions usually preserve *fewer* than 16 supersymmetries ($\nu < 1/2$) and thus correspond to composites of *more* than 16 preons. There also exist *pp*-wave solutions with “supernumerary supersymmetries” [29–31], i.e. with $16 < k < 32$. The known solutions preserving $k = 18, 20, 22, 24, 26$ and 28 supersymmetries can be considered as composites of $\#_p = 14, 12, 10, 8, 6$ and 4 BPS preons respectively. Initially, it seemed that solutions preserving all supersymmetries but one, i.e. describing the excitations of a BPS preon, could not exist in the framework of the standard brane *Ansätze* used to solve the usual 11-dimensional supergravity [18] equations. A more general study in the context of standard $D = 11$ su-

²See [6,10] and references therein for further generalizations of the M-theory superalgebra and for their structure.

³This result was extended in [12] by showing that these generators also contain a contribution from the topological charges of the eleven-dimensional Kaluza-Klein monopole ($Z_{0\mu_1 \dots \mu_4} \propto \epsilon_{0\mu_1 \dots \mu_4 \nu_1 \dots \nu_6} \tilde{Z}^{\nu_1 \dots \nu_6}$) and of the M9-brane ($Z_{0\mu} \propto \epsilon_{0\mu \nu_1 \dots \nu_9} \tilde{Z}^{\nu_1 \dots \nu_9}$) which is usually identified with the Hořava-Witten hyperplane [13] (for the Kaluza-Klein monopole and the M9-brane only bosonic actions are known [14,15]).

⁴Note that the expression $p_{\alpha\beta} = \lambda_\alpha \lambda_\beta$ for the eigenvalue of the generalized momentum operator $P_{\alpha\beta}$ may be looked at as a generalization (see [22]) of the Penrose representation $p_{A\check{B}} = \lambda_A \bar{\lambda}_{\check{B}}$ of a $D = 4$ lightlike vector (see [23]). Interestingly enough, its generalizations for tensorial charges ($Z_{\mu\nu\rho} = \lambda \Gamma_{\mu\nu\rho} \lambda$ in $D = 8 = 0 + 8$ and $Z_{\mu\nu} = \lambda \Gamma_{\mu\nu} \lambda$ in $D = 4 = 1 + 3$) were considered, in a completely different context, in [24] and [25]. Recently, the original $D = 4$ Penrose twistor formalism has found an interesting application in the analysis of perturbative scattering amplitudes in Yang-Mills theory [26] which refers to a string in $D = 4$ twistor space CP^3 .

pergravity has shown [32,33] that the existence of such solutions is not ruled out. However, and independently of whether the BPS preons can be associated with solutions of standard supergravity or whether there is a kind of *BPS preon conspiracy* preventing the existence of one BPS preon in *standard* $D=11$ spacetime or superspace, BPS preons do provide an algebraic classification of the M-theory BPS states [21]. Also, dynamical models with the properties of BPS preons are known in the $\Sigma^{([n(n+1)/2]|n)}$ superspace [22,34]. In this perspective such a BPS preon conspiracy, if it exists, would rather indicate the necessity of a wider geometric framework for a suitable description of M theory, such as extended superspaces and supertwistors. If, on the contrary, solitonic solutions with the properties of BPS preons were actually found, the extended superspaces would still provide a useful tool for a description of M-theory.⁵ One is led to expect that the additional tensorial coordinates of these superspaces carry a counterpart of the information which, in the framework of standard $D=10,11$ supergravity, is encoded in the antisymmetric tensor gauge fields entering the supergravity multiplets (cf. [6]). This point of view may be also supported by the observation that in the standard topological charge treatment of the tensorial generators of the M algebra [11], these topological charges are associated just with these gauge fields.

The general results about the treatment of tensorial central charges as topological charges of the corresponding branes are certainly relevant in the more general case

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}, \quad [Q_\alpha, P_{\beta\gamma}] = 0, \quad (10)$$

$$P_{\alpha\beta} = P_{\beta\alpha}, \quad \alpha, \beta, \gamma = 1, 2, \dots, n, \quad (11)$$

with $n=2^l$ for any integer l . The simplest representations of the algebra (10) can be constructed on the maximal superspace $\Sigma^{([n(n+1)/2]|n)}$, Eq. (3).

The $GL(n, \mathbb{R})$ symmetry of (10) becomes broken down to $Spin(t, D-t) \subset GL(n, \mathbb{R})$ when a (Eq. (5)-like) decomposition is introduced using a $n \times n$ realization of the gamma-matrices of a $(t, D-t)$ spacetime with t timelike dimensions (t is not obliged to be one, see [35,36]). On the other side, the $GL(n, \mathbb{R})$ symmetry is a subgroup of $Sp(2n)$, which is a characteristic symmetry of higher spin theories.

C. Models in maximal superspaces and higher spin theories

The main problem of the approach in [5] is how to treat the large number of additional bosonic degrees of freedom⁶ in the coset(s) $\Sigma^{(528|32)}/\Sigma^{(11|32)}$ ($\Sigma^{(462|32)}/\Sigma^{(11|32)}$, $\Sigma^{(66|32)}/\Sigma^{(11|32)}$), where

⁵There are also related reasons to consider more general superspaces, as the ensuing *fields-extended superspace democracy* associated with extended superspaces [6].

⁶See [37] for a later related search based on an attempt to replace the κ -symmetry requirement by a dynamically generated projection constraint on the spinor coordinate functions. This approach also suffers from the problem of additional bosonic degrees of freedom.

$$\Sigma^{(11|32)} : Z^M = (x^\mu, \theta^\alpha), \quad \mu = 0, 1, \dots, 10, \quad (12)$$

is the “standard” $D=11$ superspace. Actually, one has to face this problem in any approach dealing with enlarged superspaces [6,21,22,34,38–44]. Thus, one has to find a mechanism that either suppresses the additional (with respect to the usual spacetime/superspace $\Sigma^{(D|n)}$) degrees of freedom or provides a physical interpretation for them. In this respect $\Sigma^{([n(n+1)/2]|n)}$, despite having a maximal bosonic part, has some advantages with respect to non-maximally extended superspaces (see below (3)). Indeed, the bosonic sector of the maximal superspace (3),

$$\Sigma^{(n(n+1)/2|0)} : X^{\alpha\beta} = X^{\beta\alpha}, \quad \alpha, \beta = 1, 2, \dots, n, \quad (13)$$

was proposed for $n=4$ [45] as a basis for the construction of $D=4$ higher-spin theories [38,39,46]. Moreover, it was shown in [40] that the quantization of a simple superparticle model [22] in $\Sigma^{([n(n+1)/2]|n)}$ for $n=2, 4, 8, 16$ results in a wave function describing a tower of massless fields of all possible spins (helicities). Such an infinite tower of higher spin fields allows for a nontrivial interaction in AdS spacetimes [46,47].⁷

To give an idea of the relation between higher spin theories and maximally extended superspaces, let us consider the free bosonic massless higher-spin equations proposed in [38] (for $n=4$). These can be collected as the following set of equations for a scalar function b on $\Sigma^{([n(n+1)/2]|0)}$

$$\partial_{\alpha[\beta} \partial_{\gamma]\delta} b(X) = 0, \quad (14)$$

where $\partial_{\alpha\beta} = \partial/\partial X^{\alpha\beta}$. Equation (14) states that $\partial_{\alpha\beta} \partial_{\gamma\delta}$ is fully symmetric on a non-trivial solution. In the generalized momentum representation Eq. (14) reads

$$k_{\alpha[\beta} k_{\gamma]\delta} b(k) = 0. \quad (15)$$

This implies that $b(k)$ has support on the $n(n+1)/2 - n(n-1)/2 = n$ -dimensional surface in momentum space $\Sigma^{([n(n+1)/2]|0)}$ (actually, in $\Sigma^{([n(n+1)/2]|0)} \setminus \{0\}$) on which the rank of the matrix $k_{\gamma\delta}$ is equal to unity [39]. This is the surface defined by $k_{\alpha\beta} = \lambda_\alpha \lambda_\beta$ (or $-\lambda_\alpha \lambda_\beta$) characterized by the n components of λ_α . In a “ $GL(n, \mathbb{R})$ -preferred” frame (an analogue of the standard frame for lightlike ordinary momentum), $\lambda_\alpha = (1, 0, \dots, 0)$ and the surface is the $GL(n, \mathbb{R})$ orbit of the point $k_{\alpha\beta} = \delta_{\alpha 1} \delta_{\beta 1}$. Thus, Eq. (15) may also be written as

$$(k_{\alpha\beta} - \lambda_\alpha \lambda_\beta) b = 0, \quad (16)$$

which is equivalent to writing Eq. (14) in the form

⁷A relation between the generalized $n=4$ superparticle wave functions [40] and Vasiliev’s “unfolded” equations for higher spin fields was noted in [38]. This was elaborated in detail in [48], where the quantization of an AdS superspace generalization of the $n=4$ model of [22] was also carried out (see also [49] for a related study of higher spin theories in the maximal generalized AdS₄ superspace).

$$(i\partial_{\alpha\beta} - \lambda_{\alpha}\lambda_{\beta})b = 0. \quad (17)$$

Equations (16) and (17) may be considered as the generalized momentum ($k_{\alpha\beta}$) and coordinate ($X^{\alpha\beta}$) representations of the definition (6) of a BPS preon [21]. The solutions of Eqs. (16), (17) are the momentum and coordinate “wave functions” corresponding to a BPS preon state $|\lambda\rangle$, $b(X) = \langle X|\lambda\rangle$, $b(k) = \langle k|\lambda\rangle$. These equations also appear as a result of the quantization [40] of the superparticle model in [22].⁸

Thus, in contrast with other extended superspaces, the models in the maximal superspaces $\Sigma^{(n(n+1)/2|n)}$ can be regarded as higher spin generalizations of the models in standard superspace $\Sigma^{(D|n)}$.⁹

D. A new supersymmetric string model in $\Sigma^{(528|32)}$: Outlook

In Sec. II we present another action for a supersymmetric string in $\Sigma^{(528|32)}$. In distinction to the model in [5], it does not use $D=11$ gamma matrices, but instead includes two auxiliary bosonic spinor variables, λ_{α}^{+} and λ_{α}^{-} .¹⁰ As a consequence, the resulting $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string action (although it does not include a Wess-Zumino term like that of [5]) possesses 30 local fermionic κ symmetries and provides an extended object model for a state composed of two BPS preons (see above).

The model can be written as well in $\Sigma^{(n(n+1)/2|n)}$ for arbitrary even n (although $n=2^l$ is preferable for a spinor interpretation of the α, β indices). It possesses $(n-2)$ κ symmetries (Sec. III). For $n=2$, our model describes a string in the $D=3$, $N=1$ standard superspace; however, this string does not possess any κ symmetry ($n-2=0$) and, then, the ground state of this string model is not a stable (BPS) state, as such a property is guaranteed by the preservation of a non-zero number of supersymmetries.

For $n \geq 4$ our model possesses $k > 0$ κ symmetries, 2 for $n=4$ ($D=4$), 6 for $n=8$ ($D=6$), 14 for $n=16$ ($D=10$) and 30 for $n=32$ ($D=11$), and hence describes excitations of a two preons BPS state. Moreover, only for $n=4$ is the number of κ symmetries the same as that of the $D=4$ ($N=1$) Green-Schwarz superstring. For $n \geq 8$ the number of κ

symmetries of our model exceeds half of the number of supersymmetries ($\nu > \frac{1}{2}$), while the κ symmetries of the Green-Schwarz superstring are $n/2$ for all $D=3,4,6,10$ cases.

The pointlike models [22,40] in maximal superspace are enlarged superspace generalizations of the Ferber-Shirafuji [51] approach to the Brink-Schwarz superparticle. The tensionless supersymmetric string models in the maximal $\Sigma^{(n(n+1)/2|n)}$ superspaces [34,44] can be treated as generalizations of the $D=4$ null-superstring model [52]. In the same sense our $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string model can be looked at as a generalization to the maximal superspaces $\Sigma^{(n(n+1)/2|n)}$ of the Lorentz-harmonic formulation of the $D=4$ Green-Schwarz superstring in [53].¹¹

In Sec. IV we carry out a Hamiltonian analysis of the $\Sigma^{(n(n+1)/2|n)}$ model and describe its gauge symmetries, including the $(n-2)$ κ symmetries and their “superpartners,” the $(n-1)(n-2)/2$ bosonic gauge b symmetries. We also discuss there the number of degrees of freedom of our model. In Sec. V we show that its action may be formulated in terms of a pair of constrained $OSp(2n|1)$ supertwistors (see [22]) which are invariant under both κ and b symmetries. Note that one of the constraints imposed on the supertwistors breaks the $OSp(2n|1)$ invariance down to the semidirect product $\Sigma^{(n(n+1)/2|n)} \rtimes Sp(n)$ of the symplectic group $Sp(n) \subset Sp(2n)$ and the supergroup associated with the algebra (4), also denoted $\Sigma^{(n(n+1)/2|n)}$ since this superspace is the associated supergroup manifold; we may look at $\Sigma^{(n(n+1)/2|n)} \rtimes Sp(n)$ as a generalization of the super-Poincaré group $\Sigma^{(D|n)} \rtimes SO(t, D-t)$. The $OSp(2n|1)$ supergroup has been considered as a generalization of the superconformal group [8,21,22,27,45,54] (see [21,27,54] for the relevance of $OSp(64|1)$ in M theory). This generalized superconformal group symmetry is present in massless particlelike models [22,40] and in the tensionless superstring [34]; however, it is broken down to $\Sigma^{(n(n+1)/2|n)} \rtimes Sp(n)$ in our tensionful supersymmetric string model (Appendix A). This is natural: the conformal symmetry is broken in the massive superparticle [55] and in the Nambu-Goto string and Green-Schwarz superstring models, while it remains the symmetry of the massless particle and the Brink-Schwarz superparticle, as well as of the tensionless branes and superbranes [56].

The Hamiltonian analysis of the supertwistor formulation is performed in Secs. VI and VII. The generalization of the model to the super- p -brane case is given in Sec. VIII, and conclusions are given in Sec. IX.

⁸In [38,50] Eq. (17) was written as $[\partial_{\alpha\beta} - (\partial/\partial\mu^{\alpha}) \times (\partial/\partial\mu^{\beta})]b(X, \mu) = 0$, which is an equivalent “momentum” representation obtained by a Fourier transformation with respect to λ_{α} ; see [48].

⁹Although the idea of higher spin fields has been discussed at present for $D \leq 7$ only, the results of [40] can be regarded as a first step toward its $D=10$ generalization. Understanding the $D=11$ case is a problem for future study.

¹⁰Actually, the model possesses $Sp(32)$ symmetry in addition to the $SO(1,10)$ one, so that λ_{α}^{\pm} may be considered as symplectic vectors (called “s-vectors” in [38,39]) rather than Lorentz spinors. We, however, keep the “spinors” name for them, keeping in mind the possibility of spacetime treatment, although this is not straightforward and requires additional study (see Sec. I C and also [26] for a very recent spacetime treatment of a CP^3 sigma model i.e., of a string theory in twistor space, through its relation to Yang-Mills amplitudes).

¹¹The $D=4$ version of our supersymmetric string in $\Sigma^{(10|4)}$ differs from the Lorentz harmonic formulation [53] of the $D=4$, $N=1$ Green-Schwarz superstring, by skipping the Wess-Zumino term of the latter and by substituting $\Pi^{\alpha\beta} = dX^{\alpha\beta} - id\theta^{\alpha}\theta^{\beta}$ for $\gamma_{\mu}^{\alpha\beta}\Pi^{\mu} := \gamma_{\mu}^{\alpha\beta}(dx^{\mu} - id\theta^{\gamma}\gamma^{\mu}\theta)$ in the kinetic term of the action in [53]. The first of the above steps clearly breaks the two κ symmetries of the $D=4$, $N=1$ Green-Schwarz superstring, while the second step, which extends the bosonic body of the standard superspace $\Sigma^{(4|4)}$ [$x^{\mu} \rightarrow (x^{\mu}, y^{\mu\nu})$; $X^{\alpha\beta} = x^{\mu}\gamma_{\mu}^{\alpha\beta} + y^{\mu\nu}\gamma_{\mu\nu}^{\alpha\beta}$] to get $\Sigma^{(10|4)}$, restores them.

II. A NEW SUPERSYMMETRIC STRING ACTION IN THE MAXIMALLY ENLARGED SUPERSPACE

A supersymmetric string in $\Sigma^{((n(n+1)/2)|n)}$ is described by worldsheet functions $X^{\alpha\beta}(\xi)$, $\theta^\alpha(\xi)$, where $\xi=(\tau,\sigma)$ are the worldsheet W^2 coordinates. We propose the following action:

$$S = \frac{1}{\alpha'} \int_{W^2} [e^{++} \wedge \Pi^{\alpha\beta} \lambda_\alpha^- \lambda_\beta^- - e^{--} \wedge \Pi^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^+ - e^{+-} \wedge e^{-}], \quad (18)$$

where

$$\begin{aligned} \Pi^{\alpha\beta}(\xi) &= dX^{\alpha\beta}(\xi) - id\theta^{(\alpha}\theta^{\beta)}(\xi) \\ &= d\tau\Pi_\tau^{\alpha\beta} + d\sigma\Pi_\sigma^{\alpha\beta}; \end{aligned} \quad (19)$$

$$\alpha, \beta = 1, \dots, n, \quad m = 0, 1, \quad \xi^m = (\tau, \sigma),$$

and $[1/\alpha'] = ML^{-1}$, $[\Pi^{\alpha\beta}] = L$, $[e^{\pm\pm}] = L(c=1)$. The two auxiliary worldvolume fields, the *bosonic* spinors $\lambda_\alpha^-(\xi), \lambda_\alpha^+(\xi)$, are *dimensionless* and constrained by

$$C^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^- = 1; \quad (20)$$

$e^{\pm\pm}(\xi) = d\xi^m e_m^{\pm\pm}(\xi) = d\tau e_\tau^{\pm\pm}(\xi) + d\sigma e_\sigma^{\pm\pm}(\xi)$ are two auxiliary worldvolume one-forms. The one-forms $e^{\pm\pm}$ are assumed to be linearly independent and, hence, define an auxiliary worldsheet zweibein

$$\begin{aligned} e^a &= (e^0, e^1) = d\xi^m e_m^a(\xi) \\ &= \left(\frac{1}{2}(e^{++} + e^{--}), \frac{1}{2}(e^{++} - e^{--}) \right). \end{aligned} \quad (21)$$

The $C^{\alpha\beta}$ in Eq. (20) is an invertible constant antisymmetric matrix

$$C^{\alpha\beta} = -C^{\beta\alpha}, \quad dC^{\alpha\beta} = 0, \quad (22)$$

which can be used to raise and lower the spinor indices (as the charge conjugation matrix in Minkowski spacetimes). The invertibility of the matrix $C^{\alpha\beta}$ requires n to be even; this is not really a limitation since, after all, we are interested in $n=2^l$ to allow for a spinor treatment of the α, β indices.

For $n=32$ the presence of $C^{\alpha\beta}$ hampers a possible $D=10$, type IIB treatment of our model. This would require a $C^{\check{a}I\check{b}J} = -C^{\check{b}J\check{a}I}$ constructed from the 16×16 Majorana-Weyl sigma matrices and a 2×2 matrix in a Lorentz covariant manner, and there is not a 16×16 charge conjugation matrix in the $D=10$ Majorana-Weyl representation. As a result we shall refer to our $n=32$ model as a *supersymmetric string in the enlarged $D=11$ superspace* $\Sigma^{(528|32)}$, which implies the decomposition of Eq. (5). Nevertheless, the $n=32$ case also admits a $D=10$, type IIA treatment, which uses the same $C^{\alpha\beta}$ as the $D=11$ case, and in which the decomposition (5) is replaced by its $D=10$, IIA counterpart obtained from Eq. (5) by separating the eleventh value of the vector index.

The action (18) is invariant under the supersymmetry transformations

$$\delta_\epsilon X^{\alpha\beta} = i\theta^{(\alpha}\epsilon^{\beta)}, \quad \delta_\epsilon \theta^\alpha = \epsilon^\alpha, \quad (23)$$

$$\delta_\epsilon \lambda_\alpha^\pm = 0, \quad \delta_\epsilon e^{\pm\pm} = 0, \quad (24)$$

as well as under rigid $Sp(n)$ ‘‘rotations’’ acting on the α, β indices.

Note also that, although formally the action (18) possesses a manifest $GL(n, \mathbb{R})$ invariance, the constraint (20) breaks it down to $Sp(n) \subset GL(n, \mathbb{R})$. Under the action of $Sp(n)$, the Grassmann coordinate functions $\theta^\alpha(\xi)$ and the auxiliary fields $\lambda_\alpha^\pm(\xi)$ are transformed as symplectic vectors and $X^{\alpha\beta}(\xi)$ as a symmetric symplectic tensor. Nevertheless, we keep for them the ‘‘spinor’’ and ‘‘spin-tensor’’ terminology, having in mind their transformation properties under the subgroup $Spin(t, D-t) \subset Sp(n)$, which would appear in a ‘‘standard’’ $(t, D-t)$ spacetime treatment.

The above $\Sigma^{((n(n+1)/2)|n)}$ supersymmetric string model may also be described by an action written in terms of *dimensionful* unconstrained spinors $\Lambda_\alpha^\pm(\xi)$, $[\Lambda_\alpha^\pm] = (ML^{-1})^{1/2}$,

$$S = \int_{W^2} [e^{++} \wedge \Pi^{\alpha\beta} \Lambda_\alpha^- \Lambda_\beta^- - e^{--} \wedge \Pi^{\alpha\beta} \Lambda_\alpha^+ \Lambda_\beta^+ - \alpha' e^{+-} \wedge e^{-} (C^{\alpha\beta} \Lambda_\alpha^+ \Lambda_\beta^-)^2]. \quad (25)$$

Indeed, one can see that the action (25) possesses two independent scaling gauge symmetries defined by the transformation rules

$$e^{++}(\xi) \rightarrow e^{2\alpha(\xi)} e^{++}(\xi), \quad \Lambda_\alpha^-(\xi) \rightarrow e^{-\alpha(\xi)} \Lambda_\alpha^-(\xi) \quad (26)$$

and

$$e^{--}(\xi) \rightarrow e^{2\beta(\xi)} e^{--}(\xi), \quad \Lambda_\alpha^+(\xi) \rightarrow e^{-\beta(\xi)} \Lambda_\alpha^+(\xi). \quad (27)$$

This allows one to obtain $C^{\alpha\beta} \Lambda_\alpha^+ \Lambda_\beta^- = 1/\alpha'$ as a gauge fixing condition. Then the gauge fixed version of the action (25) coincides with Eq. (18) up to the trivial redefinition $\Lambda_\alpha^\pm = (\alpha')^{-1/2} \lambda_\alpha^\pm$. The gauge $C^{\alpha\beta} \Lambda_\alpha^+ \Lambda_\beta^- = 1/\alpha'$ [equivalent to Eq. (20)] is preserved by a one-parametric combination of Eqs. (26) and (27) with $\alpha = -\beta$, which is exactly the $SO(1,1)$ gauge symmetry (worldvolume Lorentz symmetry) of the action (18),

$$e^{\pm\pm}(\xi) \rightarrow e^{\pm 2\alpha(\xi)} e^{\pm\pm}(\xi), \quad \lambda_\alpha^\pm(\xi) \rightarrow e^{\pm\alpha(\xi)} \lambda_\alpha^\pm(\xi). \quad (28)$$

The tension parameter $T=1/\alpha'$ enters in the last (‘‘cosmological’’) term of the action (25) only. Setting in it $\alpha' = 0$ one finds that the model is non-trivial only for $e^{++} \neq 0$ and $\Lambda^+ \Lambda^-$ in which case one arrives at the

tionless super- p -brane action of Ref. [34], $S = \int d^2\xi \rho^{++m} \Pi_m^{\alpha\beta} \Lambda_\alpha^- \Lambda_\beta^-$. As we are not interested in this case, we set $\alpha' = 1$ below since the α' factors can be easily restored by dimensional considerations.

The most interesting feature of the model (18), (25) is that, being formulated in the maximal $\Sigma^{(n(n+1)/2|n)}$ superspace with n fermionic coordinates, it possesses $(n-2)$ κ symmetries; we will prove this in the next section. For a supersymmetric extended object in standard superspace, the κ symmetry of its worldvolume action determines the number k of supersymmetries which are preserved by the ground state (which is a $\nu = k/n$ BPS state made out of $\#_p = n - k$ preons if at least one supersymmetry, $k \geq 1$, is preserved). In the present case, we may expect that the ground state of our model should preserve $(n-2)$ out of n supersymmetries, i.e. is a $(n-2)/n$ BPS state ($\#_p = 2, \frac{30}{32}$ BPS state for the $D = 11$ maximal superspace $\Sigma^{(528|32)}$).

For $n=2$, $X^{\alpha\beta}$ provides a representation of the 3-dimensional Minkowski space coordinates, $X^{\alpha\beta\gamma} \gamma_\mu^{\alpha\beta} x^\mu$ ($\alpha, \beta = 1, 2; \mu = 0, 1, 2$). Thus the $n=2$ model (18) describes a string in the $D=3$ standard $\Sigma^{(3|2)}$ superspace. However, in the light of the above discussion, it does not possess any κ symmetry and, hence, its ground state is not a BPS state since it does not preserve any supersymmetry.

The situation becomes different starting with the $n=4$ model (8), which possesses two κ symmetries, the same number as the Green-Schwarz superstring in the standard $D=4$ superspace. For $D \geq 6$, $n \geq 8$ the number of κ symmetries of our model exceeds $n/2$ and thus the model describes the excitations of BPS states with ‘‘supernumerary’’ supersymmetries [29], a $\frac{30}{32}$ BPS state in the $D=11$ $\Sigma^{(528|32)}$ superspace.

The number of *bosonic* degrees of freedom (the number of bosonic chiral fields) of our model is $4n - 6$ (Sec. IV). It is not as large as it might look at first sight due to the ‘‘momentum space dimensional reduction mechanism’’ [40] which occurs due to the presence of auxiliary spinor variables entering the generalized Cartan-Penrose relation [Eq. (72)] generated by our model. However, it is larger than that of the ($D=3, 4, 6, 10$) Green-Schwarz superstring (which has $D [2n = 4(D-2)]$ bosonic [fermionic] configuration space real degrees of freedom, which reduce to $D-2 [2(D-2)]$ after taking into account reparametrization invariance (κ symmetry), thus resulting in $2(D-2)$ bosonic and $2(D-2)$ fermionic phase space degrees of freedom. This, in the light of the above mentioned relation of the models in maximal superspaces with higher spin theories, allows us to consider our model as a higher spin generalization of the Green-Schwarz superstring, containing additional information about the nonperturbative states of string/ M-theory.

The number of *fermionic* degrees of freedom of our model is 2 for any n , less than that of the $D=4, 6, 10$ ($N=2$) Green-Schwarz superstring.

III. PROPERTIES OF THE $\Sigma^{(n(n+1)/2|n)}$ SUPERSYMMETRIC STRING MODEL

A. Equations of motion

Consider the variation of the action (18). Allowing for integration by parts one finds

$$\begin{aligned} \delta S = & \int_{W^2} d(e^{--} \lambda_\alpha^+ \lambda_\beta^+ - e^{++} \lambda_\alpha^- \lambda_\beta^-) i_\delta \Pi^{\alpha\beta} \\ & - 2i \int_{W^2} e^{++} \wedge d\theta^\alpha \lambda_\alpha^- \delta\theta^\beta \lambda_\beta^- \\ & + 2i \int_{W^2} e^{--} \wedge d\theta^\alpha \lambda_\alpha^+ \delta\theta^\beta \lambda_\beta^+ \\ & + \int_{W^2} (\Pi^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^+ - e^{++}) \wedge \delta e^{--} \\ & - \int_{W^2} (\Pi^{\alpha\beta} \lambda_\alpha^- \lambda_\beta^- - e^{--}) \wedge \delta e^{++} + \delta_\lambda S, \end{aligned} \quad (29)$$

where $i_\delta \Pi^{\alpha\beta} \equiv \delta X^{\alpha\beta} - i \delta\theta^{(\alpha} \theta^{\beta)}$ and the last term

$$\begin{aligned} \delta_\lambda S = & + \int_{W^2} 2e^{++} \wedge \Pi^{\alpha\beta} \lambda_\beta^- \delta \lambda_\alpha^- \\ & - \int_{W^2} 2e^{--} \wedge \Pi^{\alpha\beta} \lambda_\beta^+ \delta \lambda_\alpha^+, \end{aligned} \quad (30)$$

collects the variations of the bosonic spinors $\lambda_\alpha^\pm(\xi)$.

One easily finds that the equations of motion for the bosonic coordinate functions, $\delta S / \delta X^{\alpha\beta} (= \delta S / i_\delta \Pi^{\alpha\beta}) = 0$ restrict the auxiliary spinors and auxiliary one-forms,

$$d(e^{--} \lambda_\alpha^+ \lambda_\beta^+ - e^{++} \lambda_\alpha^- \lambda_\beta^-) = 0. \quad (31)$$

The equations for the fermionic coordinate functions, $\delta S / \delta \theta^\alpha = 0$, read

$$e^{++} \wedge d\theta^\alpha \lambda_\alpha^- \lambda_\beta^- - e^{--} \wedge d\theta^\alpha \lambda_\alpha^+ \lambda_\beta^+ = 0, \quad (32)$$

which, due to the linear independence of the spinors λ_α^+ and λ_α^- , imply

$$e^{++} \wedge d\theta^\alpha \lambda_\alpha^- = 0, \quad e^{--} \wedge d\theta^\alpha \lambda_\alpha^+ = 0. \quad (33)$$

The equations for the one-forms $e^{\pm\pm}(\xi)$ express them through the worldsheet covariant bosonic form (19) of the $\Sigma^{(n(n+1)/2|n)}$ superspace and the spinors $\lambda_\alpha^\pm(\xi)$,

$$e^{++} = \Pi^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^+, \quad (34)$$

$$e^{--} = \Pi^{\alpha\beta} \lambda_\alpha^- \lambda_\beta^-. \quad (35)$$

This reflects the auxiliary nature of $e^{\pm\pm}$ and implies that Eqs. (31) and (33) actually restrict $\Pi^{\alpha\beta}$ and $d\theta^\alpha$,

$$d(\Pi^{\gamma\delta} \lambda_\gamma^- \lambda_\delta^- \lambda_\alpha^+ \lambda_\beta^+ - \Pi^{\gamma\delta} \lambda_\gamma^+ \lambda_\delta^+ \lambda_\alpha^- \lambda_\beta^-) = 0, \quad (36)$$

$$\Pi^{\gamma\delta} \lambda_\gamma^+ \lambda_\delta^+ \wedge d\theta^\alpha \lambda_\alpha^- = 0, \quad (37)$$

$$\Pi^{\gamma\delta} \lambda_\gamma^- \lambda_\delta^- \wedge d\theta^\alpha \lambda_\alpha^+ = 0. \quad (38)$$

Moreover, looking at Eqs. (34), (35) one can easily see the necessity of the constraints (20) on the bosonic spinor variables. Indeed, if one ignored these constraints and varied the action with respect to unconstrained λ_α^\pm , one would ar-

rive, from Eq. (30), at $e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} = 0$ and $e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} = 0$. By Eq. (34) [or Eq. (35)] this would imply, in particular, $e^{++} \wedge e^{--} = 0$, contradicting the original assumption of independence of the one-forms e^{++} and e^{--} and, actually, reducing the present model to a $p=1$ version of the tensionless p -brane model [34].

As λ_{α}^{\pm} are restricted by the constraint (20), this constraint has to be taken into account in the variational problem. Instead of applying the Lagrange multiplier technique, one may restrict the variations to those that preserve Eq. (20), i.e. such that

$$C^{\alpha\beta} \delta\lambda_{\alpha}^{+} \lambda_{\beta}^{-} + C^{\alpha\beta} \lambda_{\alpha}^{+} \delta\lambda_{\beta}^{-} = 0. \quad (39)$$

One can solve Eq. (39) by introducing a set of $n-2$ auxiliary spinors u_{α}^I ‘‘orthogonal’’ to the λ^{\pm} (cf. [52,57]),

$$C^{\alpha\beta} u_{\alpha}^I \lambda_{\beta}^{\pm} = 0, \quad I = 1, \dots, n-2, \quad (40)$$

and normalized by

$$C^{\alpha\beta} u_{\alpha}^I u_{\beta}^J = C^{IJ}, \quad C^{IJ} = -C^{JI}, \quad (41)$$

where C^{IJ} is an antisymmetric constant invertible $(n-2) \times (n-2)$ matrix.

The n spinors

$$\{\lambda_{\alpha}^{+}, \lambda_{\alpha}^{-}, u_{\alpha}^I\}, \quad I = 1, \dots, n-2, \quad (42)$$

provide a basis that can be used to decompose an arbitrary spinor worldvolume function (cf. [58]), and in particular the variations $\delta\lambda^{+}$, $\delta\lambda^{-}$. Then one finds that the only consequence of Eq. (39) is that the sum of the coefficient for λ^{+} in the decomposition of $\delta\lambda^{+}$ and that of λ^{-} in the decomposition of $\delta\lambda^{-}$ vanish. In other words, the general solution of Eq. (39) reads

$$\delta\lambda_{\alpha}^{+} = \omega(\delta)\lambda_{\alpha}^{+} + \Omega^{++}(\delta)\lambda_{\alpha}^{-} + \Omega_I^{+}(\delta)u_{\alpha}^I, \quad (43)$$

$$\delta\lambda_{\alpha}^{-} = -\omega(\delta)\lambda_{\alpha}^{-} + \Omega^{--}(\delta)\lambda_{\alpha}^{+} + \Omega_I^{-}(\delta)u_{\alpha}^I, \quad (44)$$

where $\Omega_I^{\pm}(\delta)$, $\Omega^{\pm\pm}(\delta)$ and $\omega(\delta)$ are arbitrary variational parameters.

Substituting Eqs. (43), (44) into Eq. (30), one finds

$$\begin{aligned} \delta_{\lambda} S = & - \int_{W^2} (2e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} \lambda_{\alpha}^{-} + 2e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} \lambda_{\alpha}^{+}) \omega(\delta) \\ & + \int_{W^2} 2e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} \lambda_{\alpha}^{+} \Omega^{--}(\delta) \\ & + \int_{W^2} 2e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} \lambda_{\alpha}^{-} \Omega^{++}(\delta) \\ & + \int_{W^2} 2e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} u_{\alpha}^I \Omega_I^{-}(\delta) \\ & - \int_{W^2} 2e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} u_{\alpha}^I \Omega_I^{+}(\delta). \end{aligned} \quad (45)$$

Now we can easily write the complete set of equations of motion which include, in addition to Eqs. (31), (33), (34), (35), the set of equations for λ_{α}^{\pm} , which follows from $\delta S/\omega(\delta) = 0$, $\delta S/\Omega^{++}(\delta) = 0$, $\delta S/\Omega_I^{+}(\delta) = 0$, $\delta S/\Omega^{--}(\delta) = 0$, and $\delta S/\Omega_I^{-}(\delta) = 0$, namely

$$e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} \lambda_{\alpha}^{-} + e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} \lambda_{\alpha}^{+} = 0, \quad (46)$$

$$e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} \lambda_{\alpha}^{+} = 0, \quad (47)$$

$$e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} \lambda_{\alpha}^{-} = 0, \quad (48)$$

$$e^{++} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{-} u_{\alpha}^I = 0, \quad (49)$$

$$e^{--} \wedge \Pi^{\alpha\beta} \lambda_{\beta}^{+} u_{\alpha}^I = 0. \quad (50)$$

Due to the linear independence of $e^{++} = d\xi^m e_m^{++}(\xi)$ and $e^{--} = d\xi^m e_m^{--}(\xi)$, Eqs. (47), (48) imply

$$\Pi^{\alpha\beta} \lambda_{\beta}^{-} \lambda_{\alpha}^{+} = 0. \quad (51)$$

Decomposing the bosonic invariant one-form $\Pi^{\alpha\beta} = d\xi^m \Pi_m^{\alpha\beta}$ in the ‘‘unholonomic’’ basis provided by $e^{\pm\pm}$,

$$\Pi^{\alpha\beta} = e^{++} \Pi_{++}^{\alpha\beta} + e^{--} \Pi_{--}^{\alpha\beta}, \quad (52)$$

$$\Pi_{\pm\pm}^{\alpha\beta} = \nabla_{\pm\pm} X^{\alpha\beta} - i \nabla_{\pm\pm} \theta^{(\alpha} \theta^{\beta)}, \quad (53)$$

where $\nabla_{\pm\pm}$ is defined by

$$d \equiv e^{\pm\pm} \nabla_{\pm\pm} = e^{++} \nabla_{++} + e^{--} \nabla_{--}, \quad (54)$$

one finds that Eqs. (49) and (50) restrict only the left and right chiral derivatives (∇_{++} , ∇_{--}) of the bosonic coordinate function $X^{\alpha\beta}(\xi)$, respectively,

$$\Pi_{--}^{\alpha\beta} \lambda_{\beta}^{-} u_{\alpha}^I \equiv (\nabla_{--} X^{\alpha\beta} - i \nabla_{--} \theta^{(\alpha} \theta^{\beta)}) \lambda_{\beta}^{-} u_{\alpha}^I = 0, \quad (55)$$

$$\Pi_{++}^{\alpha\beta} \lambda_{\beta}^{+} u_{\alpha}^I \equiv (\nabla_{++} X^{\alpha\beta} - i \nabla_{++} \theta^{(\alpha} \theta^{\beta)}) \lambda_{\beta}^{+} u_{\alpha}^I = 0. \quad (56)$$

In the same manner, Eqs. (33) can be written as

$$\nabla_{--} \theta^{\alpha} \lambda_{\alpha}^{-} = 0, \quad \nabla_{++} \theta^{\alpha} \lambda_{\alpha}^{+} = 0. \quad (57)$$

The analysis of the above set of equations in the maximal superspace, the search for solutions and their reinterpretation in standard D -dimensional spacetime (possibly along the line of the fields-extended superspace democracy of [6], or of the ‘‘two-time physics’’ [36]) is a problem for future study.

B. Gauge symmetries

The expression (29), with (45), for the general variation of the $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string action (18) shows that the model possesses n supersymmetries and $(n-2)$ κ symmetries of the form

$$\delta_{\kappa} \theta^{\alpha}(\xi) = C^{\alpha\beta} u_{\beta}^I(\xi) \kappa_I(\xi), \quad (58)$$

$$\delta_{\kappa} X^{\alpha\beta}(\xi) = i \delta_{\kappa} \theta^{(\alpha}(\xi) \theta^{\beta)}(\xi), \quad (59)$$

$$\delta_\kappa \lambda_\alpha^\pm(\xi) = 0, \quad \delta_\kappa e_m^{\pm\pm}(\xi) = 0, \quad (60)$$

with $(n-2)$ fermionic gauge parameters $\kappa_I(\xi)$ (30 for $\Sigma^{(528|32)}$). In the framework of the second Noether theorem this κ symmetry is reflected by the fact that only 2 of the n fermionic equations (32) are independent. We stress that the $(n-2)$ $GL(n, \mathbb{R})$ vector fields u_α^I defined by Eq. (40) are auxiliary. They allow us to write explicitly the general solution of the equations

$$\delta_\kappa \theta^\alpha(\xi) \lambda_\alpha^\pm(\xi) = 0, \quad (61)$$

which define implicitly the κ -symmetry transformation (58). Note that the dynamical system is κ symmetric despite the fact that it does not contain a Wess-Zumino term. This property seems to be specific to models defined on maximal superspaces.

Our model also possesses $\frac{1}{2}(n-1)(n-2)$ b symmetries, which are the bosonic ‘‘superpartners’’ of the fermionic κ symmetries, defined by

$$\begin{aligned} \delta_b X^{\alpha\beta} &= b_{IJ}(\xi) u^{\alpha I} u^{\beta J}, \\ \delta_b \theta^\alpha &= 0, \quad \delta_b \lambda_\alpha^\pm = 0, \quad \delta_b e^{\pm\pm} = 0, \end{aligned} \quad (62)$$

where $b_{IJ}(\xi)$ is symmetric and $I, J = 1, \dots, n-2$. They are reflected by the $(n-1)(n-2)/2$ Noether identities stating that the contractions of the bosonic equations (31) with the $u^{\alpha I} u^{\beta J}$ bilinears of the $(n-2)$ auxiliary bosonic spinors $u^{\alpha I} (= C^{\alpha\beta} u_\beta^I)$ vanish.¹²

The remaining gauge symmetries of the action (18) are the $SO(1,1)$ worldsheet Lorentz invariance

$$\begin{aligned} \delta X^{\alpha\beta} &= 0, \quad \delta \theta^\alpha = 0, \\ \delta \lambda_\alpha^\pm &= \pm \omega(\delta) \lambda_\alpha^\pm, \quad \delta e^{\pm\pm} = \pm 2\omega(\delta) e^{\pm\pm}, \end{aligned} \quad (63)$$

which is reflected by the fact that Eq. (46) is satisfied identically when Eqs. (34), (35) are taken into account, and the symmetry under worldvolume general coordinate transformations.

As customary in string models, the general coordinate invariance and the $SO(1,1)$ gauge symmetry allow one to fix locally the conformal gauge where $e_m^a(\xi) = e^{\phi(\xi)} \delta_m^a$ or, equivalently

¹²In the massless $\Sigma^{(n(n+1)/2|n)}$ superparticle and tensionless super- p -brane models the b symmetry [22,40,34] (cf. [59]) is $n(n-1)/2$ parametric. This comes from the fact that such models contain a single bosonic spinor λ_α and the nontrivial b -symmetry variation is the general solution of the spinorial equation $\delta_b X^{\alpha\beta} \lambda_\alpha = 0$. In our tensionful $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string model with two bosonic spinors $\lambda_\alpha^\pm(\xi)$, the $(n-1)(n-2)/2$ parametric b -symmetry transformations [Eq. (62)] are the solutions of two equations $\delta_b X^{\alpha\beta} \lambda_\alpha^+ = 0$ and $\delta_b X^{\alpha\beta} \lambda_\alpha^- = 0$.

$$e^{++} = e^{\phi(\xi)}(d\tau + d\sigma), \quad e^{--} = e^{\phi(\xi)}(d\tau - d\sigma), \quad (64)$$

$$\Leftrightarrow e_\sigma^{++} = e_\tau^{++} = e^{\phi(\xi)}, \quad e_\sigma^{--} = -e_\tau^{--} = -e^{\phi(\xi)}. \quad (65)$$

This indicates that it makes sense to consider the fields $e_\sigma^{\pm\pm}(\tau, \sigma)$ as nonsingular ($1/e_\sigma^{\pm\pm} = \pm e^{-\phi(\xi)}$ in the conformal gauge), a fact used in the Hamiltonian analysis below.

There is a correspondence [60,61] between the κ symmetry of the worldvolume action and the supersymmetry preserved by a BPS state (e.g. by a solitonic solution of the supergravity equations of motion). Thus, the action (18) defines a dynamical model for the excitations of a BPS state preserving *all but two* supersymmetries. Such a BPS state can be treated as a composite of two BPS preons ($\#_p = 32 - 30$). This will become especially transparent after the Hamiltonian analysis of the next section.

IV. HAMILTONIAN MECHANICS

The gauge symmetry structure has already been shown in the Lagrangian framework. However, our dynamical system clearly possesses additional, second class, constraints [62], one of which is condition (20). In this section we carry out the Hamiltonian analysis of our $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string model. In particular, this will allow us to find the number of field theoretical degrees of freedom and to establish the relation of our model to the notion of BPS preons [21].

The Lagrangian density \mathcal{L} for the action (18),

$$S = \int_{W^2} d\tau d\sigma \mathcal{L}, \quad (66)$$

is given by

$$\begin{aligned} \mathcal{L} &= (e_\tau^{++} \Pi_\sigma^{\alpha\beta} - e_\sigma^{++} \Pi_\tau^{\alpha\beta}) \lambda_\alpha^- \lambda_\beta^- \\ &\quad - (e_\tau^{--} \Pi_\sigma^{\alpha\beta} - e_\sigma^{--} \Pi_\tau^{\alpha\beta}) \lambda_\alpha^+ \lambda_\beta^+ \\ &\quad - (e_\tau^{++} e_\sigma^{--} - e_\sigma^{++} e_\tau^{--}), \end{aligned} \quad (67)$$

where

$$\begin{aligned} \Pi_\tau^{\alpha\beta} &= \partial_\tau X^{\alpha\beta} - i \partial_\tau \theta^{(\alpha} \theta^{\beta)}, \\ \Pi_\sigma^{\alpha\beta} &= \partial_\sigma X^{\alpha\beta} - i \partial_\sigma \theta^{(\alpha} \theta^{\beta)} \end{aligned} \quad (68)$$

are the worldsheet components of the one-form (19).

The momenta $P_{\mathcal{M}}$ canonically conjugate to the configuration space variables

$$\mathcal{Z}^{\mathcal{M}} \equiv \mathcal{Z}^{\mathcal{M}}(\tau, \sigma) := (X^{\alpha\beta}, \theta^\alpha, \lambda_\alpha^\pm, e_\tau^{\pm\pm}, e_\sigma^{\pm\pm}) \quad (69)$$

are defined as usual:

$$P_{\mathcal{M}} = (P_{\alpha\beta}, \pi_\alpha, P_\pm^{\alpha(\lambda)}, P_{\pm\pm}^\tau, P_{\pm\pm}^\sigma) = \frac{\partial \mathcal{L}}{\partial (\partial_\tau \mathcal{Z}^{\mathcal{M}})}. \quad (70)$$

The canonical equal τ graded Poisson brackets,

$$[\mathcal{Z}^{\mathcal{N}}(\sigma), P_{\mathcal{M}}(\sigma')]_p = -(-1)^{\mathcal{N}\mathcal{M}}[P_{\mathcal{M}}(\sigma'), \mathcal{Z}^{\mathcal{N}}(\sigma)]_p,$$

are defined by

$$[\mathcal{Z}^{\mathcal{N}}(\sigma'), P_{\mathcal{M}}(\sigma)]_p := (-1)^{\mathcal{N}} \delta_{\mathcal{M}}^{\mathcal{N}} \delta(\sigma - \sigma'), \quad (71)$$

where $(-1)^{\mathcal{N}} \equiv (-1)^{\deg(\mathcal{N})}$ and the degree $\deg(\mathcal{N}) \equiv \deg(\mathcal{Z}^{\mathcal{N}})$ is 0 for the bosonic fields, $\mathcal{Z}^{\mathcal{N}} = X^{\alpha\beta}, \lambda_{\alpha}^{\pm}, e_m^{\pm\pm}$ [or for the ‘‘bosonic indices’’ $\mathcal{N} = (\alpha\beta), (\alpha\pm), (\pm\pm), m$], and 1 for the fermionic fields $\mathcal{Z}^{\mathcal{N}} = \theta^{\alpha}$ (or for the ‘‘fermionic indices’’ $\mathcal{N} = \alpha$ and $\mathcal{N} = \pm$ which we will meet below in the supertwistor formulation of the model).

Since the action (18) is clearly of first order type, it is not surprising that the expression of every momentum results in a primary [62] constraint. Explicitly,

$$P_{\alpha\beta} = P_{\alpha\beta} + e_{\sigma}^{++} \lambda_{\alpha}^{-} \lambda_{\beta}^{-} - e_{\sigma}^{--} \lambda_{\alpha}^{+} \lambda_{\beta}^{+} \approx 0, \quad (72)$$

$$\mathcal{D}_{\alpha} = \pi_{\alpha} + i \theta^{\beta} P_{\alpha\beta} \approx 0, \quad (73)$$

$$P_{\pm}^{\alpha(\lambda)} \approx 0, \quad (74)$$

$$P_{\pm\pm}^{\sigma} \approx 0, \quad (75)$$

$$P_{\pm\pm}^{\tau} \approx 0, \quad (76)$$

where only \mathcal{D}_{α} is fermionic. Condition (20),

$$\mathcal{N} := C^{\alpha\beta} \lambda_{\alpha}^{+} \lambda_{\beta}^{-} - 1 \approx 0, \quad (77)$$

imposed on the bosonic spinors from the beginning, is also a primary constraint and has to be treated on the same footing as Eqs. (72)–(76).

The *canonical* Hamiltonian density \mathcal{H}_0 ,

$$\mathcal{H}_0 = \partial_{\tau} \mathcal{Z}^{\mathcal{M}} P_{\mathcal{M}} - \mathcal{L}, \quad (78)$$

calculated on the primary constraints (72)–(76) hypersurface reads

$$\begin{aligned} \mathcal{H}_0 = & e_{\tau}^{--} \Pi_{\sigma}^{\alpha\beta} \lambda_{\alpha}^{+} \lambda_{\beta}^{+} - e_{\tau}^{++} \Pi_{\sigma}^{\alpha\beta} \lambda_{\alpha}^{-} \lambda_{\beta}^{-} \\ & + (e_{\tau}^{++} e_{\sigma}^{--} - e_{\sigma}^{++} e_{\tau}^{--}). \end{aligned} \quad (79)$$

The evolution of any functional $f(\mathcal{Z}^{\mathcal{M}}, P_{\mathcal{N}})$ is defined by $\partial_{\tau} f = [f, \int d\sigma \mathcal{H}']_p$ involving the total Hamiltonian, $\int d\sigma \mathcal{H}'$, where the Hamiltonian density \mathcal{H}' is the sum of \mathcal{H}_0 in Eq. (79) and the terms given by integrals of the primary constraints (72)–(76) multiplied by arbitrary functions (Lagrange multipliers) [62]. Then one has to check that the primary constraints are preserved under the evolution, $\partial_{\tau} P_{\alpha\beta} \approx 0$, etc. At this stage additional, secondary constraints may be obtained. This is the case for our system.

Indeed, since the constraints (76) have zero Poisson brackets with any other primary constraint, their time evolution is just determined by the canonical Hamiltonian \mathcal{H}_0 , $\partial_{\tau} P_{\pm\pm}^{\tau} = [P_{\pm\pm}^{\tau}, \int d\sigma \mathcal{H}_0]_p$. Then one easily sees that $\partial_{\tau} P_{\pm\pm}^{\tau} \approx 0$ produces a pair of secondary constraints,

$$\begin{aligned} \Phi_{\pm\pm} & := \Pi_{\sigma}^{\alpha\beta} \lambda_{\alpha}^{\mp} \lambda_{\beta}^{\mp} - e_{\sigma}^{\mp\mp} \\ & = (\partial_{\sigma} X^{\alpha\beta} - i \partial_{\sigma} \theta^{(\alpha} \theta^{\beta)}) \lambda_{\alpha}^{\mp} \lambda_{\beta}^{\mp} - e_{\sigma}^{\mp\mp} \approx 0. \end{aligned} \quad (80)$$

Slightly more complicated calculations with the total \mathcal{H}' show that we also have the secondary constraint

$$\begin{aligned} \Phi^{(0)} & := \Pi_{\sigma}^{\alpha\beta} \lambda_{\alpha}^{+} \lambda_{\beta}^{-} \\ & = (\partial_{\sigma} X^{\alpha\beta} - i \partial_{\sigma} \theta^{(\alpha} \theta^{\beta)}) \lambda_{\alpha}^{+} \lambda_{\beta}^{-} \approx 0 \end{aligned} \quad (81)$$

[details about its derivation can be found below Eq. (95)]. The appearance of this secondary constraint may be understood as well by comparing with the results of the Lagrangian approach: it is just the σ component of the differential form equation (51).

The secondary constraints (80) imply that the canonical Hamiltonian \mathcal{H}_0 , Eq. (78), vanishes on the surface of constraints (80),

$$\mathcal{H}_0 \approx 0, \quad (82)$$

a characteristic property of theories with general coordinate invariance. Hence the total Hamiltonian reduces to a linear combination of the constraints (72)–(76), (80), (81),

$$\begin{aligned} \mathcal{H} = & -e_{\tau}^{++} \Phi_{++} + e_{\tau}^{--} \Phi_{--} + l^{(0)} \Phi^{(0)} + L^{\alpha\beta} P_{\alpha\beta} + \xi^{\alpha} \mathcal{D}_{\alpha} \\ & + l_{\pm}^{\pm} P_{\pm}^{\alpha(\lambda)} + L^{\pm\pm} P_{\pm\pm}^{\sigma} + h^{\pm\pm} P_{\pm\pm}^{\tau} + L^{(n)} \mathcal{N} \end{aligned} \quad (83)$$

where $l^{(0)}$, $L^{\alpha\beta}$, ξ^{α} , l_{\pm}^{\pm} , $L^{\pm\pm}$, $h^{\pm\pm}$, $L^{(n)}$ and $\pm e_{\tau}^{\pm\pm}$ are Lagrangian multipliers whose form should be fixed from the preservation of all the primary and secondary constraints under τ evolution.

Note that the constraints (76) are trivially first class, since their Poisson brackets with all the other constraints, including Eqs. (80) and (81), vanish. This allows us to state that $e_{\tau}^{\pm\pm}(\xi)$ are not dynamical fields but rather Lagrange multipliers (as the time component of electromagnetic potential A_0 in electrodynamics). Nevertheless, the appearance of these Lagrange multipliers from the τ components of the zweibein $e_m^{\pm\pm}$ put a ‘‘topological’’ restriction on a possible gauge fixing; in particular the gauge $e_{\tau}^{\pm\pm} = 0$ is not allowed. Indeed, the nondegeneracy of the zweibein, assumed from the beginning, reads

$$\det[e_m^a(z)] \equiv \frac{1}{2} (e_{\tau}^{--} e_{\sigma}^{++} - e_{\tau}^{++} e_{\sigma}^{--}) \neq 0. \quad (84)$$

Just due to this restriction, studying the τ preservation of the primary constraints, one finds the secondary constraint (81).

If by checking the (primary and secondary) constraints preservation under τ evolution one finds that some Lagrangian multipliers remain unfixed, then they correspond to *first class constraints* [62] which generate gauge symmetries of the system through the Poisson brackets. In other words, since the canonical Hamiltonian vanishes in the weak sense, the total Hamiltonian is a linear combination of all first class constraints [62].

If some of the equations resulting from the τ evolution of the constraints (or their linear combinations) do not restrict

the Lagrangian multiplier, but imply the vanishing of a combination of the canonical variables, they correspond to new secondary constraints, which have to be added with new Lagrange multipliers to obtain a new total Hamiltonian. In this case the check that all the constraints are preserved under τ evolution has to be repeated.

This does not happen for our dynamical system: a further check of the constraint τ preservation does not result in the appearance of new constraints. Indeed, it leads to the following set of equations for the Lagrange multipliers:

$$\partial_\sigma(e_\tau^- \lambda_\alpha^+ \lambda_\beta^+ - e_\tau^+ \lambda_\alpha^- \lambda_\beta^- + l^{(0)} \lambda_{(\alpha}^+ \lambda_{\beta)}) - 2e_\sigma^- \lambda_{(\alpha}^+ l_{\beta)}^+ + 2e_\sigma^+ \lambda_{(\alpha}^- l_{\beta)}^- + L^{++} \lambda_\alpha^- \lambda_\beta^- - L^{--} \lambda_\alpha^+ \lambda_\beta^+ \approx 0, \quad (85)$$

$$\lambda_\alpha^- [2ie_\tau^+ (\partial_\sigma \theta \lambda^-) - il^{(0)} (\partial_\sigma \theta \lambda^+) + 2ie_\sigma^+ (\xi \lambda^-)] - \lambda_\alpha^+ [2ie_\tau^- (\partial_\sigma \theta \lambda^+) + il^{(0)} (\partial_\sigma \theta \lambda^-) + 2ie_\sigma^- (\xi \lambda^+)] \approx 0, \quad (86)$$

$$-2e_\tau^- \Pi_\sigma^{\alpha\beta} \lambda_\beta^+ - l^{(0)} \Pi_\sigma^{\alpha\beta} \lambda_\beta^- + 2e_\sigma^- L^{\alpha\beta} \lambda_\beta^+ - L^{(n)} C^{\alpha\beta} \lambda_\beta^- \approx 0, \quad (87)$$

$$2e_\tau^+ \Pi_\sigma^{\alpha\beta} \lambda_\beta^- - l^{(0)} \Pi_\sigma^{\alpha\beta} \lambda_\beta^+ - 2e_\sigma^+ L^{\alpha\beta} \lambda_\beta^- - L^{(n)} C^{\alpha\beta} \lambda_\beta^+ \approx 0, \quad (88)$$

$$e_\tau^{--} - L^{\alpha\beta} \lambda_\alpha^- \lambda_\beta^- \approx 0, \quad (89)$$

$$e_\tau^{++} - L^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^+ \approx 0, \quad (90)$$

$$l_\alpha^+ C^{\alpha\beta} \lambda_\beta^- - l_\alpha^- C^{\alpha\beta} \lambda_\beta^+ \approx 0, \quad (91)$$

$$\partial_\sigma L^{\alpha\beta} \lambda_\alpha^- \lambda_\beta^- + 2i(\xi \lambda^-) (\partial_\sigma \theta \lambda^-) + 2l^- \Pi_\sigma \lambda^- - L^{--} \approx 0, \quad (92)$$

$$\partial_\sigma L^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^+ + 2i(\xi \lambda^+) (\partial_\sigma \theta \lambda^+) + 2l^+ \Pi_\sigma \lambda^+ - L^{++} \approx 0, \quad (93)$$

$$\partial_\sigma L^{\alpha\beta} \lambda_\alpha^+ \lambda_\beta^- + i(\xi \lambda^+) (\partial_\sigma \theta \lambda^-) - i(\xi \lambda^-) (\partial_\sigma \theta \lambda^+) + l^+ \Pi_\sigma \lambda^- + l^- \Pi_\sigma \lambda^+ \approx 0, \quad (94)$$

where the weak equality sign is used to stress that one may use the constraints in solving the above system of equations. For brevity, in Eqs. (85)–(94) and below we often omit spinor indices in the contractions

$$(\partial_\sigma \theta \lambda^\pm) \equiv \partial_\sigma \theta^\beta \lambda_\beta^\pm, \quad (\xi \lambda^\pm) \equiv \xi^\beta \lambda_\beta^\pm, \quad (95)$$

$$l^\pm \Pi_\sigma \lambda^\pm \equiv l_\alpha^\pm \Pi_\sigma^{\alpha\beta} \lambda_\beta^\pm, \quad l^\pm L \lambda^\pm \equiv l_\alpha^\pm L_\sigma^{\alpha\beta} \lambda_\beta^\pm.$$

Note that Eqs. (85)–(91) come from the requirement of τ preservation of the primary constraints, while that for the secondary constraints leads to Eqs. (92)–(94). Thus the

above statement about the appearance of the secondary constraint (81) can be checked by studying Eqs. (85)–(91) with $l^{(0)}=0$. In this case the contraction of Eq. (87) with $(-\lambda_\alpha^-)$ and of Eq. (88) with λ_α^+ results, respectively, in the equations $e_\tau^- \lambda^+ \Pi_\sigma \lambda^- - e_\sigma^- \lambda^+ L \lambda^- \approx 0$ and $e_\tau^+ \lambda^+ \Pi_\sigma \lambda^- - e_\sigma^+ \lambda^+ L \lambda^- \approx 0$. Due to the nondegeneracy of the zweibein, Eq. (84), the solution to these two equations is trivial, i.e. it implies $\lambda^+ L \lambda^- \approx 0$ and $\lambda^+ \Pi_\sigma \lambda^- \approx 0$, the last of which is just the secondary constraint (81).

To solve this system of equations for the Lagrange multipliers and thus to describe explicitly the first class constraints, we can use the auxiliary spinor fields $u_\alpha^I(\xi)$ defined as in Eqs. (40), (41). The general solution of Eqs. (85)–(94) obtained in such a framework can be found in Appendix B [Eqs. (B1)–(B7)]. Schematically, it reads

$$L^{\alpha\beta} = b_{IJ} u^{\alpha I} u^{\beta J} + e_\tau^{++}(\dots) + e_\tau^{--}(\dots), \quad (96)$$

$$\xi^\alpha = \kappa_I u^{\alpha I} + \frac{e_\tau^{++}}{e_\sigma^{++}} (\partial_\sigma \theta \lambda^-) \lambda^{+\alpha} - \frac{e_\tau^{--}}{e_\sigma^{--}} (\partial_\sigma \theta \lambda^+) \lambda^{-\alpha}, \quad (97)$$

$$l_\alpha^+ = \omega^{(0)} \lambda_\alpha^+ + e_\tau^{++}(\dots) + e_\tau^{--}(\dots), \quad (98)$$

$$l_\alpha^- = -\omega^{(0)} \lambda_\alpha^- + e_\tau^{++}(\dots) + e_\tau^{--}(\dots), \quad (99)$$

$$L^{\pm\pm} = \partial_\sigma e_\tau^{\pm\pm} \pm 2e_\sigma^{\pm\pm} \omega^{(0)} + e_\tau^{++}(\dots) + e_\tau^{--}(\dots), \quad (100)$$

$$L^{(n)} = -4 \det(e_m^a) \equiv -2(e_\tau^- e_\sigma^{++} - e_\tau^+ e_\sigma^-), \quad (101)$$

$$l^{(0)} = 0. \quad (102)$$

In this solution the parameters

$$\text{bosonic: } b^{IJ} = b^{JI}, \omega^{(0)}, e_\tau^{\pm\pm}, h^{\pm\pm}, \quad (103)$$

$$\text{fermionic: } \kappa_I, \quad (104)$$

are indefinite. They correspond to the first class constraints

$$\mathcal{P}^{IJ} := \mathcal{P}_{\alpha\beta} u^{\alpha I} u^{\beta J} \approx 0, \quad (105)$$

$$\mathcal{D}^I := \mathcal{D}_{\alpha I} u^{\alpha I} \approx 0, \quad (106)$$

$$G^{(0)} := \lambda_\alpha^+ P_+^{\alpha(\lambda)} - \lambda_\alpha^- P_-^{\alpha(\lambda)} + 2e_\sigma^{++} P_{++}^\sigma - 2e_\sigma^{--} P_{--}^\sigma \approx 0, \quad (107)$$

$$\begin{aligned}
 \tilde{\Phi}_{++} := & \Phi_{++} + \partial_\sigma P_{++}^\sigma - 2\Omega_\sigma^{(0)} P_{++}^\sigma - 2e_\sigma^- \mathcal{N} \\
 & - \left[\lambda^{-\alpha} \lambda^{-\beta} + \frac{2}{e_\sigma^{++}} [\lambda_\gamma^- \Pi_\sigma^{\gamma\alpha} \lambda^{+\beta} - (\lambda^- \Pi_\sigma \lambda^+) \lambda^{-\alpha} \lambda^{+\beta} + (\lambda^- \Pi_\sigma \lambda^-) \lambda^{+\alpha} \lambda^{+\beta}] \right] \mathcal{P}_{\alpha\beta} \\
 & - \frac{1}{e_\sigma^{++}} (\partial_\sigma \theta \lambda^-) (\lambda^{+\alpha} \mathcal{D}_\alpha) - \frac{1}{2e_\sigma^{++}} [e_\sigma^- \Omega_\sigma^{++} + e_\sigma^+ \Omega_\sigma^{--} + i \partial_\sigma \theta \lambda^+ \partial_\sigma \theta \lambda^- + \Pi_\sigma^{\alpha\beta} (\partial_\sigma \lambda_\alpha^+ \lambda_\beta^- - \lambda_\alpha^+ \partial_\sigma \lambda_\beta^-)] \\
 & \times \left(\frac{\lambda_\alpha^- P_{++}^{\alpha(\lambda)}}{e_\sigma^{--}} + \frac{\lambda_\alpha^+ P_{++}^{\alpha(\lambda)}}{e_\sigma^{++}} \right) - \frac{1}{e_\sigma^{++}} (\partial_\sigma \lambda_\alpha^- + \Omega_\sigma^{(0)} \lambda_\alpha^-) P_{++}^{\alpha(\lambda)}, \tag{108}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\Phi}_{--} := & \Phi_{--} - \partial_\sigma P_{--}^\sigma + 2\Omega_\sigma^{(0)} P_{--}^\sigma - 2e_\sigma^+ \mathcal{N} \\
 & + \left[\lambda^{+\alpha} \lambda^{+\beta} - \frac{2}{e_\sigma^{--}} [\lambda_\gamma^+ \Pi_\sigma^{\gamma\alpha} \lambda^{-\beta} - (\lambda^+ \Pi_\sigma \lambda^+) \lambda^{-\alpha} \lambda^{-\beta} + (\lambda^+ \Pi_\sigma \lambda^-) \lambda^{+\alpha} \lambda^{-\beta}] \right] \mathcal{P}_{\alpha\beta} \\
 & - \frac{1}{e_\sigma^{--}} (\partial_\sigma \theta \lambda^+) (\lambda^{-\alpha} \mathcal{D}_\alpha) + \frac{1}{2e_\sigma^{--}} [-e_\sigma^- \Omega_\sigma^{++} - e_\sigma^+ \Omega_\sigma^{--} + i \partial_\sigma \theta \lambda^+ \partial_\sigma \theta \lambda^- - \Pi_\sigma^{\alpha\beta} (\partial_\sigma \lambda_\alpha^+ \lambda_\beta^- - \lambda_\alpha^+ \partial_\sigma \lambda_\beta^-)] \\
 & \times \left(\frac{\lambda_\alpha^- P_{--}^{\alpha(\lambda)}}{e_\sigma^{--}} + \frac{\lambda_\alpha^+ P_{--}^{\alpha(\lambda)}}{e_\sigma^{++}} \right) + \frac{1}{e_\sigma^{--}} (\partial_\sigma \lambda_\alpha^+ + \Omega_\sigma^{(0)} \lambda_\alpha^+) P_{--}^{\alpha(\lambda)}, \tag{109}
 \end{aligned}$$

and

$$P_{\pm\pm}^\tau \approx 0. \tag{110}$$

In Eqs. (108),(109) [cf. Eqs. (43)]

$$\Omega_\sigma^{++} := \partial_\sigma \lambda^+ C \lambda^+, \quad \Omega_\sigma^{--} := \partial_\sigma \lambda^- C \lambda^-, \tag{111}$$

$$\Omega_\sigma^{(0)} := \frac{1}{2} (\partial_\sigma \lambda^+ C \lambda^- - \lambda^+ C \partial_\sigma \lambda^-). \tag{112}$$

and the relation

$$\delta_\alpha^\beta \approx \lambda_\alpha^+ \lambda^{-\beta} - \lambda_\alpha^- \lambda^{+\beta} - u_\alpha^I u^{J\beta} C_{IJ}, \tag{113}$$

$$\lambda^{\pm\beta} := C^{\beta\alpha} \lambda_\alpha^\pm, \quad u^{I\beta} := C^{\beta\alpha} u_\alpha^I, \tag{114}$$

is used to remove the auxiliary variables u_α^I in all places where it is possible. Note that Eq. (113) is a consequence of the constraint (77) and of the definition of the u_α^I spinors, Eqs. (40),(41) (see further discussion on the use of u variables below). Thus we are allowed to use them in the solution of the equation for the Lagrange multipliers and, then, in the definition of the first class constraints, as the product of any two constraints is a first class one since its Poisson brackets with any other constraint vanishes weakly.

Using the Poisson brackets (71), the first class constraints generate gauge symmetries. In our dynamical system the fermionic first class constraints (106) are the generators of the $(n-2)$ -parametric κ symmetry (58)–(60). The P^{IJ} in Eq. (105) are the $\frac{1}{2}(n-1)(n-2)$ generators of the b symmetry (62). The constraint $G^{(0)}$ (107) generates the $SO(1,1)$ gauge

symmetry (28). Finally, the constraints $\tilde{\Phi}_{\pm\pm}$, Eqs. (108), (109), generate worldvolume reparametrizations. They provide a counterpart of the Virasoro constraints characteristic of the Green-Schwarz superstring action. Thus, as could be expected, our $\Sigma^{([n(n+1)/2|n]}$ supersymmetric string is a two-dimensional conformal field theory.

As was noted above, the presence of the first class constraints (110) indicates the pure gauge nature of the fields $e_\tau^{\pm\pm}(\xi)$; the freedom of the gauge fixing is, nevertheless, restricted by the ‘‘topological’’ conditions (84).

Note that the κ -symmetry and b -symmetry generators, Eqs. (106) and (105), are the u_α^I and $u_\alpha^I u_\beta^J$ components of Eq. (73) and Eq. (72), respectively, while all other first class constraints can be defined without any reference to auxiliary variables.

The use of the auxiliary spinors $u_\alpha^I(\xi)$ to define the first class constraints requires some discussion. Clearly, any spinor can be decomposed in the basis (42), but the use of u_α^I to define constraints requires us, to be rigorous, to consider them as (auxiliary) dynamical variables, to introduce momenta, and to take into account any additional constraints for them, including Eqs. (41) and the vanishing of the momenta conjugate to u_α^I (cf. [53]). An alternative is to consider these auxiliary spinors as defined by Eqs. (40), (41) and by the gauge symmetries of these constraints, i.e. to treat them as some implicit functions of λ_α^\pm (cf. [63]). Such a description can be obtained rigorously by the successive gauge fixings of all the additional gauge symmetries that act only on u_α^I and by introducing Dirac brackets accounting for all the second class constraints for the u_α^I variables. Nevertheless, with some precautions, the above simpler alternative can be used

from the beginning. In this case, one has to keep in mind, in particular, that the u_α^I 's do not commute with $P_\pm^{\alpha(\lambda)}$. Indeed, as conditions (40) have to be treated in a strong sense, one has to assume $[P_\pm^{\alpha(\lambda)}(\sigma), u_\beta^I(\sigma')]_P \approx \pm \lambda_\beta^\pm C^{\alpha\gamma} u_\gamma^I \delta(\sigma - \sigma')$. However, one notices that this does not change the result of the analysis of the number of first and second class constraints among Eqs. (72)–(77), (80), (81), which do not involve $u_\gamma^I(\xi)$. The reason is that one only uses $u_\gamma^I(\xi)$ as multipliers needed to extract the first and second class constraints from the mixed ones (72), (73). Thus, the Poisson brackets of the projected constraints $\mathcal{P}_{\alpha\beta} u^{\alpha I} u^{\beta J}$, $\mathcal{D}_\alpha u^{\alpha I}$ with other constraints (e.g., $[\mathcal{P}_{\alpha\beta} u^{\alpha I} u^{\beta J}, \dots]_P$) and the projected Poisson brackets of the original constraints $\mathcal{P}_{\alpha\beta}$, \mathcal{D}_α with the same ones (e.g., $u^{\alpha I} u^{\beta J} [\mathcal{P}_{\alpha\beta}, \dots]_P$) are equivalent in the sense that a non-zero difference ($[\mathcal{P}_{\alpha\beta} u^{\alpha I} u^{\beta J}, \dots]_P - u^{\alpha I} u^{\beta J} [\mathcal{P}_{\alpha\beta}, \dots]_P$) will be proportional to $\mathcal{P}_{\alpha\beta}$ or \mathcal{D}_α and, hence, will vanish weakly. This observation allows us to use the basis (42) to solve the equations (85)–(91), that is to say, to decompose the constraints (72)–(77), (80), (81) into first and second class ones, without introducing momenta for the $u_\gamma^I(\xi)$ and without studying the constraints restricting these variables.

The remaining constraints are second class. In particular, these are the λ^\pm components of the fermionic constraints (73),

$$\mathcal{D}^\pm = \mathcal{D}_\alpha \lambda^{\alpha\pm} = \pi_\alpha \lambda^{\alpha\pm} + i e_\sigma^{\pm\pm} \theta^\beta \lambda_\beta^\mp \approx 0 \quad (115)$$

with Poisson brackets

$$\begin{aligned} \{\mathcal{D}^+(\sigma), \mathcal{D}^+(\sigma')\}_P &\approx +2i e_\sigma^{++} \delta(\sigma - \sigma'), \\ \{\mathcal{D}^-(\sigma), \mathcal{D}^-(\sigma')\}_P &\approx -2i e_\sigma^{--} \delta(\sigma - \sigma'), \\ \{\mathcal{D}^+(\sigma), \mathcal{D}^-(\sigma')\}_P &\approx 0 \end{aligned} \quad (116)$$

[recall that, having in mind the possibility of fixing the conformal gauge (64), we assume nondegeneracy of $e_\sigma^{\pm\pm}(\sigma)$, i.e. that the expression $1/e_\sigma^{\pm\pm}(\sigma)$ is well defined]. The selection of the basic second class constraints and the simplification of their Poisson bracket algebra is a technically involved problem.

In the next section we show that the dynamical degrees of freedom of our supersymmetric string in $\Sigma^{(n(n+1)/2|n)}$ may be presented in a more economic way in terms of constrained $OSp(2n|1)$ supertwistors. The Hamiltonian mechanics also simplifies in this symplectic supertwistor formulation. In particular, all the first class constraints can be extracted without using the auxiliary fields u_α^I . The reason is that the supertwistor variables are invariant under both κ and b symmetry. Thus, moving to the twistor form of our action means rewriting it in terms of trivially κ - and b -invariant quantities, effectively removing all variables that transform nontrivially under these gauge symmetries. Since the description of κ and b symmetries is the one requiring the introduction of the $u_\alpha^I(\xi)$ fields, it is natural that these are not needed in the supertwistor Hamiltonian approach.

This consideration already allows us to calculate the number of the (field theoretical worldsheet) degrees of freedom

TABLE I. Supersymmetric string model degrees of freedom.

D	n	No. of bosonic DOFs = $4n - 6$	No. of fermionic DOFs = 2	BPS states
3	2	2	2	No
4	4	10	2	1/2
6	8	26	2	6/8
10	16	58	2	14/16
11	32	122	2	30/32

of our $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string model. The dynamical system described by the action (18) possesses $\frac{1}{2}(n-1)(n-2)+5$ bosonic first class constraints [Eqs. (105), (107), (108), (109) and (110)] out of a total number of $\frac{1}{2}n(n+1)+2n+8$ constraints [Eqs. (72), (74), (75), (76), (77), (80) and (81)]. This leaves $4n+2$ bosonic second class constraints. Since the phase space dimension corresponding to the worldvolume bosonic fields $\mathcal{Z}^{\mathcal{M}}(\tau, \sigma) = (X^{\alpha\beta}, \lambda_\alpha^\pm, e_\sigma^{\pm\pm}, e_\tau^{\pm\pm})$ is $2[n(n+1)/2+2n+4]$, the action (18) turns out to have $(4n-6)$ bosonic degrees of freedom.

Likewise, the $(n-2)$ fermionic first class constraints (106) and the 2 fermionic second class constraints, Eqs. (115), reduce the original $2n$ phase space fermionic degrees of freedom of the action (18) down to 2.

Thus our supersymmetric string model in $\Sigma^{(n(n+1)/2|n)}$ superspace carries $(4n-6)$ bosonic and 2 fermionic worldvolume field theoretical degrees of freedom. Treating the number n as the number of components of an irreducible spinor representation of the D -dimensional Lorentz group $SO(1, D-1)$, one finds the results in Table I, where DOF indicates degrees of freedom. Thus, the number of bosonic degrees of freedom of our $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string model exceeds that of the Green-Schwarz superstring [where it exists, $4n-6 > 2(D-2)$], while the number of fermionic dimensions, 2, is smaller than that of the Green-Schwarz superstring for $D=6, 10$.

The additional bosonic degrees of freedom might be treated as higher spin degrees of freedom and/or as corresponding to the additional ‘‘brane’’ central charges in the maximal supersymmetry algebra (4). The smaller number of physical fermionic degrees of freedom just reflects the presence of supernumerary κ symmetries [$(n-2) > n/2$ for $n > 4$] in our $\Sigma^{(36|8)}$, and $\Sigma^{(528|32)}$, supersymmetric string models. Our $\Sigma^{(n(n+1)/2|n)}$ superstring model describes, as argued, the excitations of a BPS state preserving $k=(n-2)$ supersymmetries (a $\frac{30}{32}$ BPS state for the supersymmetric string in the enlarged $D=11$ superspace $\Sigma^{(528|32)}$).

A search for solitonic solutions of the usual $D=11$ and $D=10$ type II supergravities with such properties is being carried out at present [32,33,67]. If successful, it would be interesting to study how the additional bosonic degrees of freedom of our model are mapped into the moduli of these solutions, presumably related to the gauge fields of the supergravity multiplet (cf. [6]). Nevertheless, if it were shown that such solutions do not appear in the standard $D=11$ supergravity, this could indicate that M theory does require an extension of the usual superspace for its adequate description.

To conclude this section we comment on the BPS preon interpretation of our model. In accordance with [21], it can be treated as a composite of $\#_p = n - k = 2$ BPS preons. To support this conclusion one can have a look at the constraint (72). As we have shown, it is a mixture of first and second class constraints. However, performing a ‘‘conversion’’ of the second class constraints [64] to obtain first class constraints (in a way similar to the one carried out for a pointlike model in [40]), one arrives at the first class constraint

$$\mathcal{P}_{\alpha\beta} = P_{\alpha\beta} + e_{\sigma}^{++} \tilde{\lambda}_{\alpha}^{-} \tilde{\lambda}_{\beta}^{-} - e_{\sigma}^{-} \tilde{\lambda}_{\alpha}^{+} \tilde{\lambda}_{\beta}^{+} \approx 0, \quad (117)$$

where the $\tilde{\lambda}_{\alpha}^{\pm}$ are related, but not just equal, to λ_{α}^{\pm} . In the quantum theory this first class constraint can be imposed on quantum states giving rise to a relation similar to Eq. (9) with $\#_p = 2$.

V. ORTHOSYMPLECTIC TWISTOR FORM OF THE $\Sigma^{([n(n+1)/2]|n)}$ SUPERSYMMETRIC STRING ACTION

A further analysis of the Hamiltonian mechanics of our $\Sigma^{([n(n+1)/2]|n)}$ supersymmetric string model would become quite involved. Instead, we present in this section a more economic description.

The action (18) can be rewritten ($\alpha' = 1$) in the form

$$\begin{aligned} S = \int_{W^2} [& e^{++} \wedge (d\mu^{-\alpha} \lambda_{\alpha}^{-} - \mu^{-\alpha} d\lambda_{\alpha}^{-} - id\eta^{-} \eta^{-}) \\ & - e^{-} \wedge (d\mu^{+\alpha} \lambda_{\alpha}^{+} - \mu^{+\alpha} d\lambda_{\alpha}^{+} - id\eta^{+} \eta^{+}) \\ & - e^{++} \wedge e^{-}], \end{aligned} \quad (118)$$

where the bosonic $\mu^{\pm\alpha}$ and the fermionic η^{\pm} are defined by

$$\mu^{\pm\alpha} = X^{\alpha\beta} \lambda_{\beta}^{\pm} - \frac{i}{2} \theta^{\alpha} \theta^{\beta} \lambda_{\beta}^{\pm}, \quad \eta^{\pm} = \theta^{\beta} \lambda_{\beta}^{\pm}. \quad (119)$$

Equations (119) are reminiscent of the Ferber generalization [51] of the Penrose correspondence relation [23] (see also [21,22]). The two sets of $2n+1$ variables belonging to the same real one-dimensional (Majorana-Weyl spinor) representation of the worldsheet Lorentz group $SO(1,1)$,

$$(\mu^{+\alpha}, \lambda_{\alpha}^{+}, \eta^{+}) := Y^{+\Sigma}, \quad (\mu^{-\alpha}, \lambda_{\alpha}^{-}, \eta^{-}) := Y^{-\Sigma}, \quad (120)$$

can be treated as the components of two $OSp(2n|1)$ supertwistors, $Y^{+\Sigma}$ and $Y^{-\Sigma}$. However, Eqs. (119) considered together imply the following constraint:

$$\lambda_{\alpha}^{+} \mu^{-\alpha} - \lambda_{\alpha}^{-} \mu^{+\alpha} - i \eta^{-} \eta^{+} = 0. \quad (121)$$

One has to consider as well the ‘‘kinematic’’ constraint (20), which breaks $GL(n, \mathbb{R})$ down to $Sp(n)$. In terms of the supertwistors $Y^{\pm\Sigma}$ the action (18) and the constraints (121), (20) can be written as follows:

$$\begin{aligned} S = \int_{W^2} [& e^{++} \wedge dY^{-\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} \\ & - e^{-} \wedge dY^{+\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi} - e^{++} \wedge e^{-}]; \end{aligned} \quad (122)$$

$$Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} = 0, \quad (123)$$

$$Y^{+\Sigma} C_{\Sigma\Pi} Y^{-\Pi} = 1, \quad (124)$$

where the nondegenerate matrix $\Omega_{\Sigma\Pi} = -(-1)^{\deg(\pm\Sigma)\deg(\pm\Pi)} \Omega_{\Pi\Sigma}$ is the orthosymplectic metric,

$$\Omega_{\Sigma\Pi} = \begin{Bmatrix} 0 & \delta_{\alpha}^{\beta} & 0 \\ -\delta_{\beta}^{\alpha} & 0 & 0 \\ 0 & 0 & -i \end{Bmatrix}, \quad (125)$$

preserved by $OSp(2n|1)$. The degenerate matrix $C_{\Sigma\Pi}$ in Eq. (124) has the form

$$C_{\Sigma\Pi} = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & C^{\alpha\beta} & 0 \\ 0 & 0 & 0 \end{Bmatrix} \quad (126)$$

with $C^{\alpha\beta}$ defined in Eq. (22).

One can also find the orthosymplectic twistor form for the action (118) with unconstrained spinors. It reads

$$\begin{aligned} S = \int_{W^2} [& e^{++} \wedge (d\mathcal{M}^{-\alpha} \Lambda_{\alpha}^{-} - \mathcal{M}^{-\alpha} d\Lambda_{\alpha}^{-} - id\chi^{-} \chi^{-}) \\ & - e^{-} \wedge (d\mathcal{M}^{+\alpha} \Lambda_{\alpha}^{+} - \mathcal{M}^{+\alpha} d\Lambda_{\alpha}^{+} - id\chi^{+} \chi^{+}) \\ & - e^{++} \wedge e^{-} (C^{\alpha\beta} \Lambda_{\alpha}^{+} \Lambda_{\beta}^{-})^2], \end{aligned} \quad (127)$$

where

$$\mathcal{M}^{\pm\alpha} = X^{\alpha\beta} \Lambda_{\beta}^{\pm} - \frac{i}{2} \theta^{\alpha} \theta^{\beta} \Lambda_{\beta}^{\pm}, \quad \chi^{\pm} = \theta^{\beta} \Lambda_{\beta}^{\pm}. \quad (128)$$

Equation (128) differs from Eq. (119) only by replacement of the constrained dimensionless λ^{\pm} by the unconstrained dimensionful Λ^{\pm} . But, as a result, the $OSp(2n|1)$ supertwistors

$$Y^{\pm\Sigma} := (\mathcal{M}^{\pm\alpha}, \Lambda_{\alpha}^{\pm}, \chi^{\pm}) \quad (129)$$

are restricted by only one condition similar to Eq. (123),

$$Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} = 0. \quad (130)$$

The action in terms of $Y^{\pm\Sigma}$ includes the degenerate matrix $C_{\Sigma\Pi}$, and reads

$$\begin{aligned} S = \int_{W^2} [& e^{++} \wedge dY^{-\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} - e^{-} \wedge dY^{+\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi} \\ & - e^{++} \wedge e^{-} (Y^{+\Sigma} C_{\Sigma\Pi} Y^{-\Pi})^2]. \end{aligned} \quad (131)$$

The global symmetry of our $\Sigma^{([n(n+1)/2]|n)}$ supersymmetric string is transparent now. The orthosymplectic supertwist-

ors $Y^{\pm\Sigma}$ are both in the fundamental representation of the $OSp(2n|1)$ supergroup. The constraints (123) [or (130)] are also $OSp(2n|1)$ invariant. However, condition (124) [or the last term in the action (131)] breaks the $OSp(2n|1)$ invariance down to the semidirect product $\Sigma^{(ln(n+1)/2|n)} \bowtie Sp(n)$ of $Sp(n) \subset Sp(2n)$ and the maximal superspace group $\Sigma^{(ln(n+1)/2|n)}$ (see Appendix A).

Summarizing, our $\Sigma^{(ln(n+1)/2|n)}$ supersymmetric string model breaks the $OSp(2n|1)$ symmetry down to a generalization $\Sigma^{(ln(n+1)/2|n)} \bowtie Sp(n)$ of the Poincaré supergroup. In contrast, both the pointlike model in [22] and the tensionless superbrane model of [34] possess full $OSp(2n|1)$ symmetry. This is in agreement with treating $OSp(2n|1)$ as a generalized superconformal group, as the standard conformal and superconformal symmetry is broken in any model with mass, tension or another dimensional parameter.

VI. HAMILTONIAN ANALYSIS IN THE $OSp(2n|1)$ SUPERTWISTOR FORMULATION

The Hamiltonian analysis simplifies in the supertwistor formulation (122) of the action (18). This is due to the fact that moving from (18) to (122) reduces essentially the number of fields involved in the model.

The Lagrangian of the action (122) reads

$$\begin{aligned} \mathcal{L} = & (e_{\tau}^{++} \partial_{\sigma} Y^{-\Sigma} - e_{\sigma}^{++} \partial_{\tau} Y^{-\Sigma}) \Omega_{\Sigma\Pi} Y^{-\Pi} \\ & - (e_{\tau}^{--} \partial_{\sigma} Y^{+\Sigma} - e_{\sigma}^{--} \partial_{\tau} Y^{+\Sigma}) \Omega_{\Sigma\Pi} Y^{+\Pi} \\ & - (e_{\tau}^{++} e_{\sigma}^{--} - e_{\sigma}^{++} e_{\tau}^{--}), \end{aligned} \quad (132)$$

and involves the $2(2n+1+2) = 4n+6$ configuration space worldvolume fields

$$\tilde{\mathcal{Z}}^{\tilde{M}} \equiv \tilde{\mathcal{Z}}^{\tilde{M}}(\tau, \sigma) = (Y^{\pm\Sigma}, e_{\tau}^{\pm\pm}, e_{\sigma}^{\pm\pm}). \quad (133)$$

The calculation of their canonical momenta

$$\tilde{P}_{\tilde{M}} = (P_{\pm\Sigma}, P_{\pm\pm}^{\tau}, P_{\pm\pm}^{\sigma}) = \frac{\partial \mathcal{L}}{\partial (\partial_{\tau} \tilde{\mathcal{Z}}^{\tilde{M}})} \quad (134)$$

provides the following set of primary constraints:

$$\mathcal{P}_{\pm\Sigma} = P_{\pm\Sigma} \mp e_{\sigma}^{\mp\mp} \Omega_{\Sigma\Pi} Y^{\pm\Pi} \approx 0, \quad (135)$$

$$P_{\pm\pm}^{\sigma} \approx 0, \quad (136)$$

$$P_{\pm\pm}^{\tau} \approx 0. \quad (137)$$

Conditions (123), (124) should also be taken into account after all the Poisson brackets are calculated and, hence, are also primary constraints,

$$\mathcal{U} := Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} \approx 0, \quad (138)$$

$$\mathcal{N} := Y^{+\Sigma} C_{\Sigma\Pi} Y^{-\Pi} - 1 \approx 0. \quad (139)$$

The *canonical* Hamiltonian density \mathcal{H}_0 corresponding to the action (122) reads

$$\begin{aligned} \mathcal{H}_0 = & [-e_{\tau}^{++} \partial_{\sigma} Y^{-\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} + e_{\tau}^{--} \partial_{\sigma} Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi} \\ & + (e_{\tau}^{++} e_{\sigma}^{--} - e_{\sigma}^{++} e_{\tau}^{--})]. \end{aligned} \quad (140)$$

The preservation of the primary constraints under τ evolution (see Sec. IV) leads to the secondary constraints

$$\Phi_{++} = \partial_{\sigma} Y^{-\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} - e_{\sigma}^{--} \approx 0, \quad (141)$$

$$\Phi_{--} = \partial_{\sigma} Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi} - e_{\sigma}^{++} \approx 0, \quad (142)$$

$$\Phi^{(0)} = \partial_{\sigma} Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} - Y^{+\Sigma} \Omega_{\Sigma\Pi} \partial_{\sigma} Y^{-\Pi} \approx 0. \quad (143)$$

Again (see Sec. IV) the canonical Hamiltonian vanishes on the surface of constraints (141), (142), and thus the τ evolution is defined by the Hamiltonian density [cf. Eq. (83)]

$$\begin{aligned} \mathcal{H}' = & -e_{\tau}^{++} \Phi_{++} + e_{\tau}^{--} \Phi_{--} + l^{(0)} \Phi^{(0)} + L^{\pm\Sigma} \mathcal{P}_{\pm\Sigma} \\ & + L^{(0)} \mathcal{U} + L^{(n)} \mathcal{N} + L^{\pm\pm} P_{\pm\pm}^{\sigma} + h^{\pm\pm} P_{\pm\pm}^{\tau} \end{aligned} \quad (144)$$

and the canonical Poisson brackets

$$[P_{\pm\Lambda}(\sigma), Y^{\pm\Sigma}(\sigma')]_p = -\delta_{\Lambda}^{\Sigma} \delta(\sigma - \sigma'), \quad (145)$$

$$[e_{\sigma}^{\pm\pm}(\sigma), P_{\pm\pm}^{\sigma}(\sigma')]_p = \delta(\sigma - \sigma'), \quad (146)$$

$$[e_{\tau}^{\pm\pm}(\sigma), P_{\pm\pm}^{\tau}(\sigma')]_p = \delta(\sigma - \sigma'). \quad (147)$$

Then the τ -preservation requirement of the primary and secondary constraints results in the following system of equations for the Lagrange multipliers:

$$\begin{aligned} L^{+\Sigma} \approx & \frac{e_{\tau}^{--}}{e_{\sigma}^{--}} \partial_{\sigma} Y^{+\Sigma} + \frac{\partial_{\sigma} e_{\tau}^{--} - L^{--}}{2e_{\sigma}^{--}} Y^{+\Sigma} + \frac{l^{(0)}}{e_{\sigma}^{--}} \partial_{\sigma} Y^{-\Sigma} \\ & + \frac{\partial_{\sigma} l^{(0)} - L^{(0)}}{2e_{\sigma}^{--}} Y^{-\Sigma} - \frac{L^{(n)}}{2e_{\sigma}^{--}} Y^{-\Pi} (C\Omega)_{\Pi}^{\Sigma}, \end{aligned} \quad (148)$$

$$\begin{aligned} L^{-\Sigma} \approx & \frac{e_{\tau}^{++}}{e_{\sigma}^{++}} \partial_{\sigma} Y^{-\Sigma} + \frac{\partial_{\sigma} e_{\tau}^{++} - L^{++}}{2e_{\sigma}^{++}} Y^{-\Sigma} - \frac{l^{(0)}}{e_{\sigma}^{++}} \partial_{\sigma} Y^{+\Sigma} \\ & - \frac{\partial_{\sigma} l^{(0)} + L^{(0)}}{2e_{\sigma}^{++}} Y^{+\Sigma} - \frac{L^{(n)}}{2e_{\sigma}^{++}} Y^{+\Pi} (C\Omega)_{\Pi}^{\Sigma}, \end{aligned} \quad (149)$$

$$L^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} \approx L^{-\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi}, \quad (150)$$

$$L^{+\Sigma} C_{\Sigma\Pi} Y^{-\Pi} \approx L^{-\Sigma} C_{\Sigma\Pi} Y^{+\Pi}, \quad (151)$$

$$L^{-\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} \approx e_{\tau}^{--}, \quad (152)$$

$$L^{+\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi} \approx e_{\tau}^{++}, \quad (153)$$

and

$$L^{--} \approx \partial_\sigma L^{-\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} - L^{-\Sigma} \Omega_{\Sigma\Pi} \partial_\sigma Y^{-\Pi}, \quad (154)$$

$$L^{++} \approx \partial_\sigma L^{+\Sigma} \Omega_{\Sigma\Pi} Y^{+\Pi} - L^{+\Sigma} \Omega_{\Sigma\Pi} \partial_\sigma Y^{+\Pi}, \quad (155)$$

$$\sum_{\pm} (\partial_\sigma L^{\pm\Sigma} \Omega_{\Sigma\Pi} Y^{\mp\Sigma} - L^{\pm\Sigma} \Omega_{\Sigma\Pi} \partial_\sigma Y^{\mp\Sigma}) \approx 0, \quad (156)$$

where $(C\Omega)_{\Pi}^{\Sigma} := C_{\Pi\Lambda} \Omega^{\Lambda\Sigma}$ and $\Omega^{\Sigma\Pi} = -\Omega_{\Sigma\Pi}$ is the inverse of the orthosymplectic metric (125),

$$\Omega_{\Sigma\Lambda} \Omega^{\Lambda\Pi} = \delta_{\Sigma}^{\Pi}, \quad \Omega^{\Sigma\Pi} = \begin{Bmatrix} 0 & -\delta_{\beta}^{\alpha} & 0 \\ \delta_{\alpha}^{\beta} & 0 & 0 \\ 0 & 0 & i \end{Bmatrix}. \quad (157)$$

Equations (148)–(153) come from the preservation of the primary constraints, while Eqs. (154)–(156) from the preservation of the secondary constraints. Again, as in Sec. IV, one can follow the appearance of the secondary constraint (143) by considering Eqs. (148)–(153) with $l^{(0)}=0$.

Denoting

$$A_{\sigma}^{(0)} = \frac{1}{2} (\partial_\sigma Y^{+\Sigma} C_{\Sigma\Pi} Y^{-\Pi} - Y^{+\Sigma} C_{\Sigma\Pi} \partial_\sigma Y^{-\Pi}), \quad (158)$$

$$A_{\sigma}^{++} = \partial_\sigma Y^{+\Sigma} C_{\Sigma\Pi} Y^{+\Pi}, \quad (159)$$

$$A_{\sigma}^{--} = \partial_\sigma Y^{-\Sigma} C_{\Sigma\Pi} Y^{-\Pi}, \quad (160)$$

$$B^{(0)} = \mathcal{S} - \frac{\partial_\sigma Y^{+\Sigma} \Omega \partial_\sigma Y^{-\Sigma}}{2e_{\sigma}^{++} e_{\sigma}^{--}}, \quad (161)$$

$$\mathcal{S} = \frac{1}{2} \left(\frac{A_{\sigma}^{++}}{e_{\sigma}^{++}} + \frac{A_{\sigma}^{--}}{e_{\sigma}^{--}} \right), \quad (162)$$

one can write the general solution of Eqs. (148)–(153) in the form

$$\begin{aligned} L^{+\Sigma} \approx & \omega^{(0)} Y^{+\Sigma} + \frac{e_{\tau}^{--}}{e_{\sigma}^{--}} (\partial_\sigma Y^{+\Sigma} - A_{\sigma}^{(0)} Y^{+\Sigma} - e_{\sigma}^{++} B^{(0)} Y^{-\Sigma} \\ & + e_{\sigma}^{++} (Y^{-} C \Omega)^{\Sigma}) + e_{\tau}^{++} (B^{(0)} Y^{-\Sigma} - (Y^{-} C \Omega)^{\Sigma}), \end{aligned} \quad (163)$$

$$\begin{aligned} L^{-\Sigma} \approx & -\omega^{(0)} Y^{-\Sigma} + \frac{e_{\tau}^{++}}{e_{\sigma}^{++}} (\partial_\sigma Y^{-\Sigma} + A_{\sigma}^{(0)} Y^{-\Sigma} + e_{\sigma}^{--} B^{(0)} Y^{+\Sigma} \\ & - e_{\sigma}^{--} (Y^{+} C \Omega)^{\Sigma}) - e_{\tau}^{--} (B^{(0)} Y^{+\Sigma} - (Y^{+} C \Omega)^{\Sigma}), \end{aligned} \quad (164)$$

$$L^{(0)} = 2(e_{\tau}^{--} e_{\sigma}^{++} - e_{\tau}^{++} e_{\sigma}^{--}) B^{(0)}, \quad (165)$$

$$L^{\pm\pm} = \partial_\sigma e_{\tau}^{\pm\pm} \mp 2e_{\tau}^{\pm\pm} A_{\sigma}^{(0)} \pm 2e_{\sigma}^{\pm\pm} \omega^{(0)}, \quad (166)$$

$$L^{(n)} = -4 \det(e_m^a) \equiv -2(e_{\tau}^{--} e_{\sigma}^{++} - e_{\tau}^{++} e_{\sigma}^{--}), \quad (167)$$

$$l^{(0)} = 0. \quad (168)$$

Note that Eqs. (167),(168) have the same form as Eqs. (B6), (B7) (Appendix B) for the Lagrange multipliers in the original formulation, and Eqs. (166) are similar to Eqs. (B5).

The above solution contains the indefinite worldsheet field parameters $h^{\pm\pm}(\xi)$, $\omega^{(0)}(\xi)$ and $e_{\tau}^{\pm\pm}(\xi)$ which correspond to the five first class constraints which generate the gauge symmetries of the symplectic twistor formulation of our $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string model. They are

$$P_{\pm\pm}^{\tau} \approx 0 \quad (169)$$

and

$$G^{(0)} := Y^{+\Sigma} \mathcal{P}_{+\Sigma} - Y^{-\Sigma} \mathcal{P}_{-\Sigma} + 2e_{\sigma}^{++} P_{++}^{\sigma} - 2e_{\sigma}^{--} P_{--}^{\sigma} \approx 0, \quad (170)$$

$$\begin{aligned} \tilde{\Phi}_{++} := & \Phi_{++} + \partial_\sigma P_{++}^{\sigma} + 2A_{\sigma}^{(0)} P_{++}^{\sigma} + 2e_{\sigma}^{--} B^{(0)} \mathcal{U} \\ & - 2e_{\sigma}^{--} \mathcal{N} + \mathcal{F}_{++}^{+\Sigma} \mathcal{P}_{\pm\Sigma}, \end{aligned} \quad (171)$$

$$\begin{aligned} \tilde{\Phi}_{--} := & \Phi_{--} - \partial_\sigma P_{--}^{\sigma} + 2A_{\sigma}^{(0)} P_{--}^{\sigma} + 2e_{\sigma}^{++} B^{(0)} \mathcal{U} \\ & - 2e_{\sigma}^{++} \mathcal{N} + \mathcal{F}_{--}^{+\Sigma} \mathcal{P}_{\pm\Sigma}, \end{aligned} \quad (172)$$

where

$$\mathcal{F}_{++}^{+\Sigma} = -B^{(0)} Y^{-\Sigma} + (Y^{-} C \Omega)^{\Sigma}, \quad (173)$$

$$\begin{aligned} \mathcal{F}_{++}^{-\Sigma} = & -\frac{1}{e_{\sigma}^{++}} [\partial_\sigma Y^{-\Sigma} + A_{\sigma}^{(0)} Y^{-\Sigma} \\ & + B^{(0)} e_{\sigma}^{--} Y^{+\Sigma} - e_{\sigma}^{--} (Y^{+} C \Omega)^{\Sigma}], \end{aligned} \quad (174)$$

$$\begin{aligned} \mathcal{F}_{--}^{+\Sigma} = & \frac{1}{e_{\sigma}^{--}} [\partial_\sigma Y^{+\Sigma} - A_{\sigma}^{(0)} Y^{+\Sigma} - B^{(0)} e_{\sigma}^{++} Y^{-\Sigma} \\ & + e_{\sigma}^{++} (Y^{-} C \Omega)^{\Sigma}], \end{aligned} \quad (175)$$

$$\mathcal{F}_{--}^{-\Sigma} = -B^{(0)} Y^{+\Sigma} + (Y^{+} C \Omega)^{\Sigma}. \quad (176)$$

Using Poisson brackets, the constraint (170) generates the $SO(1,1)$ worldsheet Lorentz gauge symmetry, Eqs. (171) and (172) are the reparametrization (Virasoro) generators, and the symmetry generated by Eqs. (169) indicates the pure gauge nature of the $e_{\tau}^{\pm\pm}(\xi)$ fields [again, subject to the non-degeneracy condition (84) that restricts the gauge choice freedom for them].

Note that both the b -symmetry and the κ -symmetry generators, Eqs. (105) and (106), are not present in the symplectic supertwistor formulation. Actually, the number of variables in this formulation minus the constraint among them, Eq. (123), is $(4n+6)-1$ and equal to the number of variables in the previous formulation $[n(n+1)/2+n+2n+4]$,

minus the number of b - and κ -symmetry generators $[(n-1)(n-2)/2+(n-2)]$. This clearly indicates that the transition to the supertwistor form of the action corresponds to an implicit gauge fixing of these symmetries and the removal of the additional variables, since the remaining supertwistor ones are invariant under both b and κ symmetry.¹³

Other constraints are second class. Indeed, e.g. the algebra of the constraints $\mathcal{P}_{\pm\Sigma}$, Eq. (135),

$$[\mathcal{P}_{+\Sigma}(\sigma), \mathcal{P}_{+\Lambda}(\sigma')]_p = 2e_{\sigma}^{-} \Omega_{\Lambda\Sigma} \delta(\sigma - \sigma'), \quad (177)$$

$$[\mathcal{P}_{-\Sigma}(\sigma), \mathcal{P}_{-\Lambda}(\sigma')]_p = -2e_{\sigma}^{++} \Omega_{\Lambda\Sigma} \delta(\sigma - \sigma'), \quad (178)$$

$$[\mathcal{P}_{+\Sigma}(\sigma), \mathcal{P}_{-\Lambda}(\sigma')]_p = 0, \quad (179)$$

clearly indicates their second class nature. As such, one can introduce the graded Dirac (or starred [62]) brackets that allow one to put them strongly equal to zero. For any arbitrary two (bosonic or fermionic) functionals f and g of the canonical variables (133), (134) they are defined by

$$\begin{aligned} [f(\sigma_1), g(\sigma_2)]_D &= [f(\sigma_1), g(\sigma_2)]_p - \frac{1}{2} \int d\sigma \left(\frac{1}{e_{\sigma}^{--}(\sigma)} [f(\sigma_1), \mathcal{P}_{+\Sigma}(\sigma)]_p \right. \\ &\quad \times \Omega^{\Pi\Sigma} [\mathcal{P}_{+\Pi}(\sigma), g(\sigma_2)]_p \\ &\quad \left. - \frac{1}{e_{\sigma}^{++}(\sigma)} [f(\sigma_1), \mathcal{P}_{-\Sigma}(\sigma)]_p \Omega^{\Pi\Sigma} [\mathcal{P}_{-\Pi}(\sigma), g(\sigma_2)]_p \right). \end{aligned} \quad (180)$$

Using these and reducing further the number of phase space degrees of freedom by setting $P_{\pm\Sigma} = 0$ strongly, the supertwistor becomes a self-conjugate variable,

$$[Y^{\pm\Sigma}(\sigma), Y^{\pm\Pi}(\sigma')]_D = \mp \frac{1}{2e_{\sigma}^{\mp\mp}} \Omega^{\Sigma\Pi} \delta(\sigma - \sigma'). \quad (181)$$

For the ‘‘components’’ of the supertwistor, Eq. (181) implies

$$[\lambda_{\alpha}^{\pm}(\sigma), \mu^{\pm\beta}(\sigma')]_D = \mp \frac{1}{2e_{\sigma}^{\mp\mp}} \delta_{\alpha}^{\beta} \delta(\sigma - \sigma'), \quad (182)$$

$$\{\eta^{\pm}(\sigma), \eta^{\pm}(\sigma')\}_D = \mp \frac{i}{2e_{\sigma}^{\mp\mp}} \delta(\sigma - \sigma'). \quad (183)$$

The Dirac brackets for $e_{\sigma}^{\pm\pm}$, $e_{\sigma}^{\mp\mp}$ and $P_{\pm\Sigma}^{\tau}$ coincide with the Poisson brackets, while for $P_{\pm\Sigma}^{\sigma}$ one finds

$$\begin{aligned} [P_{++}^{\sigma}(\sigma), \dots]_D &= [P_{++}^{\sigma}(\sigma), \dots]_p \\ &\quad - \frac{1}{2e_{\sigma}^{++}} Y^{-\Sigma}(\sigma) [\mathcal{P}_{-\Sigma}(\sigma), \dots]_p, \end{aligned} \quad (184)$$

$$\begin{aligned} [P_{--}^{\sigma}(\sigma), \dots]_D &= [P_{--}^{\sigma}(\sigma), \dots]_p \\ &\quad - \frac{1}{2e_{\sigma}^{--}} Y^{+\Sigma}(\sigma) [\mathcal{P}_{+\Sigma}(\sigma), \dots]_p. \end{aligned} \quad (185)$$

However, $P_{\pm\Sigma}^{\sigma}(\sigma)$ still commute among themselves, $[P_{\pm\Sigma}^{\sigma}(\sigma), P_{\pm\Sigma}^{\sigma}(\sigma')]_D = 0 = [P_{++}^{\sigma}(\sigma), P_{--}^{\sigma}(\sigma')]_D$.

When the constraints (135) are taken as strong equations, the first class constraints (170)–(172) simplify to

$$G^{(0)} := 2e_{\sigma}^{++} P_{++}^{\sigma} - 2e_{\sigma}^{--} P_{--}^{\sigma} \approx 0, \quad (186)$$

$$\begin{aligned} \tilde{\Phi}_{++} &:= \Phi_{++} + \partial_{\sigma} P_{++}^{\sigma} + 2A_{\sigma}^{(0)} P_{++}^{\sigma} \\ &\quad + 2e_{\sigma}^{--} B^{(0)} \mathcal{U} - 2e_{\sigma}^{--} \mathcal{N}, \end{aligned} \quad (187)$$

$$\begin{aligned} \tilde{\Phi}_{--} &:= \Phi_{--} - \partial_{\sigma} P_{--}^{\sigma} + 2A_{\sigma}^{(0)} P_{--}^{\sigma} \\ &\quad + 2e_{\sigma}^{++} B^{(0)} \mathcal{U} - 2e_{\sigma}^{++} \mathcal{N}, \end{aligned} \quad (188)$$

and the remaining second class constraints can be taken in the form

$$K^{(0)} := e_{\sigma}^{++} P_{++}^{\sigma} + e_{\sigma}^{--} P_{--}^{\sigma} \approx 0, \quad (189)$$

$$\mathcal{N} = Y^{+\Sigma} C_{\Sigma\Pi} Y^{-\Pi} - 1 \approx 0, \quad (190)$$

$$\mathcal{U} = Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} \approx 0, \quad (191)$$

$$\Phi^{(0)} = \partial_{\sigma} Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} - Y^{+\Sigma} \Omega_{\Sigma\Pi} \partial_{\sigma} Y^{-\Pi} \approx 0. \quad (192)$$

One has to take into account that, under the Dirac brackets, $P_{\pm\Sigma}^{\sigma}$ and $Y^{\mp\Sigma}$ do not commute,

$$[P_{++}^{\sigma}(\sigma), Y^{-\Sigma}(\sigma')]_D = \frac{1}{2e_{\sigma}^{++}} Y^{-\Sigma}(\sigma) \delta(\sigma - \sigma'), \quad (193)$$

$$[P_{--}^{\sigma}(\sigma), Y^{+\Sigma}(\sigma')]_D = \frac{1}{2e_{\sigma}^{--}} Y^{+\Sigma}(\sigma) \delta(\sigma - \sigma'). \quad (194)$$

Then one easily checks that, under Dirac brackets, $G^{(0)}$ generates the $SO(1,1)$ transformations of the supertwistors,

$$[G^{(0)}(\sigma), Y^{\pm\Sigma}(\sigma')]_D = \mp Y^{\pm\Sigma}(\sigma) \delta(\sigma - \sigma'). \quad (195)$$

On the other hand, one finds that the second class constraint \mathcal{U} interchanges the $Y^{+\Sigma}$ and $Y^{-\Sigma}$ supertwistors,

$$[\mathcal{U}(\sigma), Y^{+\Sigma}(\sigma')]_D = \frac{1}{2e_{\sigma}^{--}} Y^{-\Sigma}(\sigma) \delta(\sigma - \sigma'),$$

¹³This invariance was known for the massless superparticle and the tensionless superstring cases; see e.g. [22,34,40,65]; cf. [59].

TABLE II. Matrix of Dirac brackets of Eqs. (189)–(192).

$[\dots \downarrow, \dots \rightarrow]_D \approx$	$(\Phi^{(0)}(\sigma') + SK^{(0)}(\sigma'))$	$\mathcal{U}(\sigma')$	$K^{(0)}(\sigma')$	$\mathcal{N}(\sigma')$
$(\Phi^{(0)} + SK^{(0)})(\sigma)$	0	$-\delta_{\sigma\sigma'}$	0	0
$\mathcal{U}(\sigma)$	$\delta_{\sigma\sigma'}$	0	0	0
$K^{(0)}(\sigma)$	0	0	0	$\delta_{\sigma\sigma'}$
$\mathcal{N}(\sigma)$	0	0	$-\delta_{\sigma\sigma'}$	0

$$[\mathcal{U}(\sigma), Y^{-\Sigma}(\sigma')]_D = \frac{1}{2e_{\sigma}^{++}} Y^{+\Sigma}(\sigma) \delta(\sigma - \sigma'). \quad (196)$$

It is interesting to note that in the original supertwistor formulation of the $D=4$, $N=1$ superparticle [51] there exists a counterpart of the \mathcal{U} constraint; however, there it is the first class constraint generating the internal $U(1)$ symmetry.¹⁴

The Dirac brackets of the above second class constraints (189)–(192) can be found in Appendix B, Eqs. (B10)–(B14). They are characterized by the matrix shown in Table II, where $\mathcal{S}(\sigma) = \frac{1}{2}[A_{\sigma}^{++}(\sigma)/e_{\sigma}^{++}(\sigma) + A_{\sigma}^{--}(\sigma)/e_{\sigma}^{--}(\sigma)]$ [Eq. (162)] and $\delta_{\sigma\sigma'} \equiv \delta(\sigma - \sigma')$. This table indicates that the $K^{(0)}$ constraint is canonically conjugate to \mathcal{N} while the second class constraint $\Phi^{(0)} + SK^{(0)}$ is conjugate to \mathcal{U} . One may easily pass to the (doubly starred) Dirac brackets with respect to the above mentioned four second class constraints. However, the new Dirac brackets for the supertwistor variables would have a very complicated form, so that it looks more practical either to apply the formalism using (singly starred) Dirac brackets [Eq. (180)] and simple first and second class constraints, Eqs. (186)–(188) and (189)–(192), or to search for a conversion [64] of the remaining second class constraints into first class ones. Note that a phenomenon similar to conversion occurs when one moves from Eq. (122) to the dynamical system with unnormalized twistors described by the action (131). We discuss this in more detail in the next section.

As the simplest application of the above Hamiltonian analysis let us calculate the number of field theoretical degrees of freedom of the dynamical system (122). In this supertwistor formulation one finds from Eqs. (133) and (120) $(4n+4)$ bosonic and 2 fermionic configuration space variables, which corresponds to a phase space with $2(4n+4)$ and 4 fermionic “dimensions.” The system has 5 bosonic first class constraints, Eqs. (169)–(172), out of a total number of $4n+9$ bosonic constraints [the bosonic components of Eqs. (135) and (136), (137), (141)–(143)]. Thus, in agreement with Sec. IV, one finds that the $\Sigma^{([n(n+1)/2]|n)}$ supersymmetric string described by the action (122) possesses $4n-6$ bosonic degrees of freedom. Likewise, the 2 fermionic constraints of the action [the fermionic components of Eq. (135)] reduce to 2 the fermionic degrees of freedom.

¹⁴See [66] for a detailed study of the Hamiltonian mechanics in the twistorlike formulation of the $D=4$ superparticle, where the possibility of constraint class transmutation was noted.

VII. HAMILTONIAN ANALYSIS IN TERMS OF “UNNORMALIZED” $Y^{\pm\Sigma}$ SUPERTWISTORS

As shown in Sec. V, the action (122) may be considered as a gauge fixed form of the action (131) written in terms of supertwistors (129) restricted by only one Lagrangian constraint (130). The second constraint (124), the “normalization” condition that distinguishes among the $Y^{\pm\Sigma}$ and $Y^{\pm\Sigma}$ supertwistors, may be obtained by gauge fixing the direct product of the two scaling gauge symmetries (26) and (27) down to the $SO(1,1)$ worldsheet Lorentz symmetry (28) of the action (122). As a result, one may expect that the Hamiltonian structure of the model (131) will differ from the one of the model (122) by the absence of one second class constraint (190) and the presence of one additional first class constraint replacing (189).

This is indeed the case. An analysis similar to the one carried in Sec. VI allows one to find the following set of primary,

$$\mathcal{P}_{\pm\Sigma} = P_{\pm\Sigma} \mp e_{\sigma}^{\mp\mp} \Omega_{\Sigma\Pi} Y^{\pm\Pi} \approx 0, \quad (197)$$

$$P_{\pm\pm}^{\sigma} \approx 0, \quad (198)$$

$$P_{\pm\pm}^{\tau} \approx 0, \quad (199)$$

$$\mathcal{U} := Y^{+\Sigma} \Omega_{\Sigma\Pi} Y^{-\Pi} \equiv Y^{+\Omega} Y^{-} \approx 0, \quad (200)$$

and secondary constraints,

$$\Phi_{++} = \partial_{\sigma} Y^{-} \Omega Y^{-} - e_{\sigma}^{--} (Y^{+} C Y^{-})^2 \approx 0, \quad (201)$$

$$\Phi_{--} = \partial_{\sigma} Y^{+} \Omega Y^{+} - e_{\sigma}^{++} (Y^{+} C Y^{-})^2 \approx 0, \quad (202)$$

$$\Phi^{(0)} = \partial_{\sigma} Y^{+} \Omega Y^{-} - Y^{+} \Omega \partial_{\sigma} Y^{-} \approx 0, \quad (203)$$

that restrict the phase space variables

$$\tilde{\mathcal{Z}}^{\tilde{\mathcal{M}}} \equiv \tilde{\mathcal{Z}}^{\tilde{\mathcal{M}}}(\tau, \sigma) = (Y^{\pm\Sigma}, e_{\tau}^{\pm\pm}, e_{\sigma}^{\pm\pm}), \quad (204)$$

$$\tilde{P}_{\tilde{\mathcal{M}}} = (P_{\pm\Sigma}, P_{\pm\pm}^{\tau}, P_{\pm\pm}^{\sigma}) = \frac{\partial \mathcal{L}}{\partial (\partial_{\tau} \tilde{\mathcal{Z}}^{\tilde{\mathcal{M}}})}. \quad (205)$$

The set (197)–(203) contains 6 first class constraints [versus five first class constraints (169)–(172) in the (122) system], namely

$$P_{\pm\pm}^{\tau} \approx 0, \quad (206)$$

$$2e_{\sigma}^{++} P_{++}^{\sigma} - Y^{-\Sigma} \mathcal{P}_{-\Sigma} \approx 0, \quad (207)$$

$$2e_{\sigma}^{--} P_{--}^{\sigma} - Y^{+\Sigma} \mathcal{P}_{+\Sigma} \approx 0, \quad (208)$$

$$\begin{aligned} \tilde{\Phi}_{++} = & \Phi_{++} + \frac{2e_{\sigma}^{--} \mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} \mathcal{U} - \partial_{\sigma} P_{++}^{\sigma} \\ & - \frac{\mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} Y^{-\Sigma} \mathcal{P}_{+\Sigma} + (Y^{+}CY^{-}) Y^{-} C \Omega \mathcal{P}_{+} \\ & - \frac{1}{e_{\sigma}^{++}} \left[\partial_{\sigma} Y^{-\Sigma} \mathcal{P}_{-\Sigma} + \frac{e_{\sigma}^{--} \mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} Y^{+\Sigma} \mathcal{P}_{-\Sigma} \right. \\ & \left. - e_{\sigma}^{--} (Y^{+}CY^{-}) Y^{+} C \Omega \mathcal{P}_{-} \right] \approx 0, \quad (209) \end{aligned}$$

$$\begin{aligned} \tilde{\Phi}_{--} = & \Phi_{--} + \frac{2e_{\sigma}^{++} \mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} \mathcal{U} + \partial_{\sigma} P_{--}^{\sigma} \\ & - \frac{\mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} Y^{+\Sigma} \mathcal{P}_{-\Sigma} + (Y^{+}CY^{-}) Y^{+} C \Omega \mathcal{P}_{-} \\ & + \frac{1}{e_{\sigma}^{--}} \left[\partial_{\sigma} Y^{+\Sigma} \mathcal{P}_{+\Sigma} - \frac{e_{\sigma}^{++} \mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} Y^{-\Sigma} \mathcal{P}_{+\Sigma} \right. \\ & \left. + e_{\sigma}^{++} (Y^{+}CY^{-}) (Y^{-} C \Omega \mathcal{P}_{+}) \right] \approx 0, \quad (210) \end{aligned}$$

where [cf. Eq. (161)]

$$\begin{aligned} \mathcal{B}^{(0)} = & \frac{1}{2} \left(\frac{\partial_{\sigma} Y^{+} C Y^{+} (Y^{+} C Y^{-})}{e_{\sigma}^{++}} + \frac{\partial_{\sigma} Y^{-} C Y^{-} (Y^{+} C Y^{-})}{e_{\sigma}^{--}} \right. \\ & \left. - \frac{\partial_{\sigma} Y^{+} \Omega \partial_{\sigma} Y^{-}}{e_{\sigma}^{++} e_{\sigma}^{--}} \right). \quad (211) \end{aligned}$$

Using Dirac brackets to account for the second class constraints (197), where [cf. Eq. (181)]

$$[Y^{\pm\Sigma}(\sigma), Y^{\pm\Pi}(\sigma')]_D = \mp \frac{1}{2e_{\sigma}^{\mp\mp}} \Omega^{\Sigma\Pi} \delta(\sigma - \sigma'), \quad (212)$$

the first class constraints simplify to

$$P_{\pm\pm}^{\tau} \approx 0, \quad (213)$$

$$P_{++}^{\sigma} \approx 0, \quad (214)$$

$$P_{--}^{\sigma} \approx 0, \quad (215)$$

$$\tilde{\Phi}_{++} = \Phi_{++} + \frac{2e_{\sigma}^{--} \mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} \mathcal{U} \approx 0, \quad (216)$$

$$\tilde{\Phi}_{--} = \Phi_{--} + \frac{2e_{\sigma}^{++} \mathcal{B}^{(0)}}{(Y^{+}CY^{-})^2} \mathcal{U} \approx 0, \quad (217)$$

which clearly corresponds to the set of constraints (186)–(188) of the “normalized” supertwistor description with the addition of the constraint (189), which is now “converted” into a first class one due to disappearance of the normalization constraint (190).

The remaining two bosonic constraints, Eqs. (200) and (203), are second class. Their Dirac bracket

$$\begin{aligned} [\mathcal{U}(\sigma), \Phi^{(0)}(\sigma')]_D = & (Y^{+}CY^{-})^2 \delta(\sigma - \sigma') \\ & + \left(\frac{\Phi_{++}}{2e_{\sigma}^{++}} + \frac{\Phi_{--}}{2e_{\sigma}^{--}} \right) \delta(\sigma - \sigma') \\ \approx & (Y^{+}CY^{-})^2 \delta(\sigma - \sigma') \quad (218) \end{aligned}$$

is nonvanishing due to the linear independence of the $Y^{+\Sigma}$ and $Y^{-\Sigma}$ supertwistors (129) (coming from the linear independence of their Λ_{α}^{+} and Λ_{α}^{-} components, $\Lambda_{\alpha}^{+} C^{\alpha\beta} \Lambda_{\alpha}^{-} \neq 0$). For a further simplification of the Hamiltonian formalism it might be convenient to make a conversion of this pair of second class constraints into first class by adding a pair of canonically conjugate variables, $q(\xi)$ and $P^{(q)}(\xi)$, ($[q(\sigma), P^{(q)}(\sigma')]_p = \delta(\sigma - \sigma')$) to our phase space.

The above Hamiltonian formalism and its further development can be applied to quantize the $\Sigma^{([n(n+1)/2]|n)}$ supersymmetric string model. This should produce a quantum higher spin generalization of the Green-Schwarz superstring for $n=4,8,16$ and, for $n=32$, an exactly solvable quantum description of a conformal field theory carrying, somehow, information about the nonperturbative brane BPS states of M theory.

VIII. SUPERSYMMETRIC p -BRANES IN MAXIMAL SUPERSPACE $\Sigma^{([n(n+1)/2]|n)}$

The model may be generalized to describe higher-dimensional extended objects (supersymmetric p -branes) in $\Sigma^{([n(n+1)/2]|n)}$.

The expression of the supersymmetric p -brane action in terms of dimensionful unconstrained bosonic spinors reads [cf. Eq. (25)]

$$\begin{aligned} S_p = & \int_{W^{p+1}} e_a^{\wedge p} \wedge \Pi^{\alpha\beta} (\Lambda_{\alpha}^r \rho_{rs}^a \Lambda_{\beta}^s) \\ & - (-\alpha')^p \int_{W^{p+1}} e^{\wedge(p+1)} \det(C^{\alpha\beta} \Lambda_{\alpha}^r \Lambda_{\beta}^s), \quad (219) \end{aligned}$$

where $a=0,1,\dots,p$, $r=1,\dots,\tilde{n}(p)$, $\alpha=1,\dots,n$,

$$e_a^{\wedge p} \equiv \frac{1}{p!} \epsilon_{ab_1\dots b_p} e^{b_1} \wedge \dots \wedge e^{b_p}, \quad (220)$$

and $e^{\wedge(p+1)}$ is the W^{p+1} volume element

$$e^{\wedge(p+1)} \equiv \frac{1}{(p+1)!} \epsilon_{b_1 \dots b_{p+1}} e^{b_1} \wedge \dots \wedge e^{b_{p+1}}. \quad (221)$$

In Eq. (219), the $(p+1)$ $e^a = d\xi^m e_m^a(\xi)$ are auxiliary world-volume vielbein fields, $\xi^m = (\tau, \sigma^1, \dots, \sigma^p)$ are the world-volume W^{p+1} local coordinates and $\Lambda_\alpha^r(\xi)$ is a set of $\tilde{n} = \tilde{n}(p)$ unconstrained auxiliary real bosonic fields with a ‘‘spacetime’’ spinorial [actually, an $Sp(n)$ vector] index $\alpha = 1, \dots, n$. The number $\tilde{n}(p)$ of real spinor fields $\Lambda_\alpha^r(\xi)$ as well as the meaning of the symmetric real matrices ρ_{rs}^a depend on the worldvolume dimension $d = p+1$. For $d = 2, 3, 4 \pmod{8}$, where a Majorana spinor representation exists, the ρ_{rs}^a are $Spin(1, p)$ gamma matrices multiplied by the charge conjugation matrix or sigma matrices, provided they are symmetric. If not, it is always possible to find a real symmetric matrix by doubling the index r , $\check{r} = rI$ ($I = 1, 2$), as in the case of $d = 6$ symplectic Majorana spinors. For dimensions with only Dirac spinors (like $d = 5$) $\Lambda_\alpha^r \rho_{rs}^a \Lambda_\beta^s$ should be understood as $\bar{\Lambda}_\alpha \gamma^a \Lambda_\beta + \bar{\Lambda}_\beta \gamma^a \Lambda_\alpha$, etc. For simplicity we present Eq. (219) and other formulas of this section for ‘‘Majorana dimensions’’ d with symmetric $C\gamma$ matrices; the generalization to the other cases is straightforward, although one should be careful determining the value of $\tilde{n}(p)$ for a given $d = p+1$. For $p = 1$, where the irreducible Majorana-Weyl spinor is one dimensional [$Spin(1, 1)$ is Abelian], one needs Λ_α^r to be in a reducible Majorana representation in the worldsheet spinor index r , i.e. $\Lambda_\alpha^r = (\Lambda_\alpha^+, \Lambda_\alpha^-)$; otherwise the second term in Eq. (219) would be zero and the action would become that of a tensionless $\Sigma^{(n(n+1)/2|n)}$ supersymmetric string. Then, the action (219) reduces to (25) using (21).

The fermionic variation δ_f of the action (219), $\delta_f S_p$, comes only from the variation of $\Pi^{\alpha\beta}$. Let us simplify it by taking $\delta_f X^{\alpha\beta} = i \delta_f \theta^{\alpha\beta}$ [cf. below Eq. (29)], so that $i \delta_f \Pi^{\alpha\beta} = 0$ and $\delta_f \Pi^{\alpha\beta} = -2id\theta^{\alpha\beta}$. As $\Pi^{\alpha\beta}$ enters in the action (219) only through its contraction with $\Lambda_\alpha^r \gamma_{rs}^a \Lambda_\beta^s$ we find

$$\delta_f S_p = -2i \int_{W^{p+1}} e_a^{\wedge p} \wedge d\theta^\alpha \Lambda_\alpha^r \rho_{rs}^a \Lambda_\beta^s \delta\theta^\beta. \quad (222)$$

Thus only $\tilde{n}(p)$ fermionic variations $\delta\theta^\beta \Lambda_\beta^s$ out of the n variations $\delta\theta^\beta$ are effectively involved in $\delta_f S_p$.

This reflects the presence of $[n - \tilde{n}(p)]$ κ symmetries in the dynamical system described by the supersymmetric p -brane action (219). They are defined by

$$\delta_\kappa X^{\alpha\beta} = i \delta_\kappa \theta^{\alpha\beta}, \quad \delta_\kappa e^a = 0, \quad (223)$$

and by the following condition on $\delta_\kappa \theta^\alpha$:

$$\delta_\kappa \theta^\alpha \Lambda_\alpha^r = 0, \quad r = 1, \dots, \tilde{n}(p). \quad (224)$$

This can be solved, using the auxiliary spinor fields $u^{\alpha J}$ (where now $J = 1, \dots, [n - \tilde{n}(p)]$) orthogonal to Λ_α^r , as

$$\delta_\kappa \theta^\alpha = \kappa_J(\xi) u^{\alpha J}(\xi), \quad u^{\alpha J}(\xi) \Lambda_\alpha^r(\xi) = 0, \\ J = 1, \dots, [n - \tilde{n}(p)], \quad r = 1, \dots, \tilde{n}(p). \quad (225)$$

The κ symmetry (223), (225) implies the preservation of all but $\tilde{n}(p)$ supersymmetries by the corresponding $\nu = [n - \tilde{n}(p)]/n$ BPS state.

For instance, for $p = 2$, $n = 32$, $Spin(1, 2) \approx SL(2, \mathbb{R})$ and $\tilde{n} = 2$. The action (219) then describes excitations of a membrane BPS state preserving all but 2 supersymmetries, a $\frac{30}{32}$ BPS state. For $p = 5$ and $\tilde{n} = 8$ the action (219) with $n = 32$ describes a $\frac{24}{32}$ supersymmetric 5-brane model in $\Sigma^{(528|32)}$. Both the supermembrane (M2-brane) and the super-5-brane (M5-brane) are known in the standard $D = 11$ superspace, where they correspond to $\frac{16}{32}$ BPS states. It is tempting to speculate that the ‘‘usual’’ M2- and M5-superbranes are related to the generalized $\Sigma^{(528|32)}$ supersymmetric 2-brane and 5-brane described by the action (222) for $p = 2$ and 5. For instance, they might be related to some particular solutions to the equations of motion of the corresponding $\frac{30}{32}$ and $\frac{24}{32}$ $\Sigma^{(528|32)}$ models preserving 16 supersymmetries and/or with the result of a dimensional reduction of them. For the $p = 5$ case a question of special interest would be the role of the M5 self-dual worldvolume gauge field in the $\Sigma^{(528|32)}$ superspace description (see [6] for a related discussion).

For $p = 3$ and $\tilde{n} = 4$ we have a $\frac{28}{32}$ BPS state, a BPS 3-brane. Neither the Green-Schwarz superstring nor the super-3-brane exist in the standard $D = 11$ superspace, but a super-D3-brane does exist in the $D = 10$ type IIB superspace, as the superstring does. As we have already noted, although $\Sigma^{(528|32)}$ also allows a treatment as an enlarged type IIB superspace [9, 20], the $\Sigma^{(528|32)}$ supersymmetric p -brane action (219) explicitly involves the 32×32 matrix $C^{\alpha\beta}$ which cannot be constructed out of type IIB matrices in a $SO(1, 9)$ Lorentz covariant manner. The same problem appears with the $\Sigma^{(528|32)}$ supersymmetric 9-brane described by the $p = 9$ version of the $\Sigma^{(528|32)}$ model (219) with $\tilde{n} = 16$, which corresponds to a $\frac{16}{32}$ BPS state; its possible relation to the space-time filling type IIB super-D9-brane in the usual $D = 10$ superspace is also quite unclear.

IX. CONCLUSIONS AND OUTLOOK

We have presented a supersymmetric string model in the ‘‘maximal’’ superspace $\Sigma^{(n(n+1)/2|n)}$ with additional tensorial central charge coordinates (for $n > 2$). The model possesses n rigid supersymmetries and $n - 2$ local fermionic κ symmetries. This implies that it provides the worldsheet action for the excitations of a BPS state preserving $(n - 2)$ supersymmetries. In particular, for $n = 32$ our model describes a supersymmetric string with 30 κ symmetries in $\Sigma^{(528|32)}$, which corresponds to a BPS state preserving 30 out of 32 supersymmetries. This model can be treated as a composite of two BPS preons [21] and is the second (after the $D = 11$ Curtright model [5]) tensionful extended object model in $\Sigma^{(528|32)}$.

In contrast with the Curtright model [5], our supersymmetric string action in the enlarged $D=11$ superspace $\Sigma^{(528|32)}$ does not involve any gamma matrices, but instead makes use of two constrained bosonic spinor variables, λ_α^+ and λ_α^- , corresponding to the two BPS preons from which the superstring BPS state is composed. As a result, our model preserves the $Sp(32)$ subgroup of the $GL(32, \mathbb{R})$ automorphism symmetry of the $D=11$ M algebra.

Our $\Sigma^{((n(n+1)/2|n)}$ supersymmetric string model can be treated as a higher spin generalization of the classical Green-Schwarz superstring. At the same time, the additional bosonic tensorial coordinate fields of the $n=32$ case might contain information about topological charges corresponding to the higher branes of the superstring or M theory [67].

The $\Sigma^{((n(n+1)/2|n)}$ model may also be formulated in terms of a pair of constrained worldvolume $OSp(2n|1)$ supertwistors. The transition to the supertwistor formulation is similar to that for the massless superparticle and the tensionless $\Sigma^{((n(n+1)/2|n)}$ supersymmetric p -branes [22,34]. In our case, however, the supertwistors are restricted by a constraint that breaks the generalized superconformal $OSp(64|1)$ symmetry down to a generalization of the super-Poincaré group, $\Sigma^{(528|32)} \otimes Sp(32)$. Such a breaking is characteristic of tensionful models. We note that this constrained $OSp(2n|1)$ supertwistor framework might also be useful for massive higher spin theories.

We have developed the Hamiltonian formalism, both in the original and in the symplectic supertwistor representation, and found that, while the Hamiltonian analysis in the original formulation requires the use of the additional auxiliary spinor variables u_α^I [$I=1, \dots, (n-2)$] orthogonal to λ_α^\pm , the symplectic supertwistor Hamiltonian mechanics can be discussed in terms of the original phase space variables. Moreover, under Dirac brackets, supertwistors become self-conjugate variables and the symplectic structure of the phase space simplifies considerably. A natural application of the Hamiltonian approach developed here is the Becchi-Rouet-Stora-Tyutin (BRST) quantization of the $\Sigma^{((n(n+1)/2|n)}$ superstring model, which might provide a “higher spin” counterpart of the usual string field theory.

We have also presented a generalization of our $\Sigma^{((n(n+1)/2|n)}$ supersymmetric string model for supersymmetric p -branes in $\Sigma^{((n(n+1)/2|n)}$. They correspond to BPS states preserving all but $\tilde{n}(p)$ [see below Eq. (219)] supersymmetries, composites of $\tilde{n}(p)$ BPS preons [$\tilde{n}(2)=2, \tilde{n}(3)=4, \tilde{n}(5)=8$]. In particular, the $\Sigma^{(528|32)}$ supersymmetric membrane ($p=2$) also corresponds to a $\frac{30}{32}$ BPS state.

BPS states preserving 30 out of 32 supersymmetries have not been found yet among the solitonic solutions of the “usual” $D=11$ and $D=10$ dimensional supergravities, and the existence of such solutions is being discussed at present [32,33]. If found, it would be interesting to study a possible relation of the additional tensorial bosonic coordinate functions in our theory with such hypothetical solitonic solutions. In particular, an interesting question is to see how the Wess-Zumino term of the superbrane in the usual superspace is reproduced from a purely kinetic-like term in the action (18).

If, in contrast, these solutions do not exist, this could indicate, because of the special role of BPS preons in the algebraic classification of the M-theory BPS states [21], the necessity of a wider geometric framework for a description of M theory. In this case the proposed $\Sigma^{((n(n+1)/2|n)}$ supersymmetric string model could provide a part of such an extended framework, unifying M theory and higher spin theory ideas.

Note added

Two papers [68,69] have recently appeared. Reference [68] considers a spontaneous breaking of the $OSp(1|32)$ symmetry of the tensionless $\Sigma^{((n(n+1)/2|n)}$ supersymmetric p -brane models [44,34], and proposes an open tensionless $\Sigma^{((n(n+1)/2|n)}$ supersymmetric string action with an additional boundary term (or topological term; cf. [38]). These topological terms can be treated as describing superparticles attached to the endpoints of a tensionless $\Sigma^{((n(n+1)/2|n)}$ string (similar to quarks attached at the ends of a bosonic string or D0-branes at the ends of an open superstring; cf. [70,71]).

Reference [69] develops a formalism which looks promising for studying the relation of the BPS preon conjecture and the present approach with solitonic solutions of the standard $D=11$ supergravity. The authors of [69] deal with bosonic Killing spinors $\epsilon^{\alpha I}$, but some of their observations may be applied to the bosonic spinors λ_α (λ_α^r) characterizing the BPS preon(s). The Killing spinors will be orthogonal to λ_α (λ_α^r) and thus might be identified with the auxiliary $u^{\alpha I}$ variables of this paper (see [67]).

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APPENDIX A: BREAKING OF THE GENERALIZED SUPERCONFORMAL GROUP $OSp(2n|1)$ DOWN TO THE GENERALIZATION $\Sigma^{((n(n+1)/2|n)} \otimes Sp(n)$ OF THE SUPER-POINCARÉ GROUP

The $(2n+1) \times (2n+1)$ supermatrices $\mathcal{G}_\Sigma^{\Pi} \in OSp(2n|1)$ preserve the graded-antisymmetric matrix $\Omega_{\Sigma\Pi} = -(-1)^{\deg(\Sigma)\deg(\Pi)} \Omega_{\Pi\Sigma}$, “orthosymplectic metric,”

$$\mathcal{G}_\Sigma^{\Sigma'} \Omega_{\Sigma'\Pi'} \mathcal{G}_\Pi^{\Pi'} (-1)^{\deg(\Pi)[\deg(\Pi')+1]} = \Omega_{\Sigma\Pi}, \quad (\text{A1})$$

the canonical form of which is given by Eq. (125). The grading is defined by

$$(-1)^{\deg(\Sigma)} = \begin{cases} 1 & \text{for } \Sigma = 1, \dots, 2n, \\ -1 & \text{for } \Sigma = 2n+1, \end{cases}$$

and coincides with $\deg(\pm\Sigma)$ for $Y^{\pm\Sigma}$ [see below Eq. (71)]. The fundamental representation of $OSp(2n|1)$ acts on supertwistors

$$Y^\Sigma = (\mu^\alpha, \lambda_\alpha, \eta), \quad (\text{A2})$$

with even $\mu^\alpha, \lambda_\alpha$ and odd η . Near the unity,

$$\mathcal{G}_\Sigma^\Pi \sim \delta_\Sigma^\Pi + \Xi_\Sigma^\Pi, \quad (\text{A3})$$

where Ξ_Σ^Π is an element of the $osp(2n|1)$ superalgebra. It has the form

$$\Xi_\Sigma^\Pi = \begin{Bmatrix} G_{\alpha\beta} & K_{\alpha\beta} & \zeta_\alpha \\ A^{\alpha\beta} & -G_{\beta\alpha} & \epsilon^\alpha \\ i\epsilon^\beta & -i\zeta_\beta & 0 \end{Bmatrix}, \quad (\text{A4})$$

where the even $n \times n$ matrix $G_{\alpha\beta}$ is arbitrary and the even $n \times n$ $K_{\alpha\beta} = K_{\beta\alpha}$ and $A^{\alpha\beta} = A^{\beta\alpha}$ matrices are symmetric. They define a $gl(n)$ and two $sp(n)$ subalgebras of $osp(2n|1)$,

$$G_{\alpha\beta} \in gl(n), \quad A^{\alpha\beta} \in sp(n), \quad K_{\alpha\beta} \in sp(n). \quad (\text{A5})$$

Exploiting the analogy with the matrix representation of the standard 4-dimensional conformal algebra $su(2,2|N)$ and the 4-dimensional super-Poincaré algebra, one can look at the $gl(n)$ boxes G as a generalization of the $spin(1, D-1)$ and dilatation algebras ($L_{\alpha\beta} + \delta_{\alpha\beta} D$), at the elements $A^{\alpha\beta} \in sp(n)$ as a generalization of the translation one, and at $K_{\alpha\beta} \in sp(n)$ as a generalization of the special conformal transformations. Equation (A4) also contains two fermionic parameters, ϵ^α and ζ_α , which can be identified as those of the of ‘‘usual’’ and special conformal supersymmetries. A specific check is provided by the $n=2$ case, where $SL(2, \mathbb{R}) = Spin(1,2)$, the symmetric spin tensor provides an equivalent representation for a $SO(1,2)$ vector, and the superconformal group is $OSp(2|1)$.

If we now demand in addition that the degenerate matrix $C_{\Sigma\Pi}$ [Eq. (126)] is preserved,

$$\mathcal{G}_\Sigma^{\Sigma'} C_{\Sigma'\Pi'} \mathcal{G}_{\Pi'}^{\Pi} (-1)^{\deg(\Pi)[\deg(\Pi')+1]} = C_{\Sigma\Pi}, \quad (\text{A6})$$

we see that this is satisfied by the $osp(2n|1)$ elements of the form

$$\Xi_\Sigma^\Pi = \begin{Bmatrix} S_{\alpha\beta} & 0 & 0 \\ A^{\alpha\beta} & -S_{\beta\alpha} & \epsilon^\alpha \\ i\epsilon^\beta & 0 & 0 \end{Bmatrix} \equiv \Xi_\Sigma^\Pi(S, A, \epsilon), \quad (\text{A7})$$

where $S_{\alpha\beta} \in sp(n)$,

$$S^{\alpha\beta} \equiv C^{\alpha\gamma} S_{\gamma\beta} = S^{\beta\alpha}, \quad (\text{A8})$$

i.e. by those of Eq. (A4) with $K_{\alpha\beta} = 0$, $\zeta_\alpha = 0$ and $G_{\alpha\beta} = S_{\alpha\beta} \in sp(n)$. Thus the condition (A6) not only reduces $GL(n)$ symmetry down to $Sp(n)$, but also breaks the generalized special conformal transformations and the superconformal supersymmetry.

The right action of $\mathcal{G}_\Sigma^\Pi(S, A, \epsilon)$ [Eqs. (A3), (A7)] on the supertwistor (A2), $\delta Y^\Sigma = Y^\Pi \Xi_\Pi^\Sigma$, defines the generalized super-Poincaré transformation of the supertwistor components,

$$\delta \mu^\alpha = \mu^\beta S_{\beta\alpha} + \lambda_\beta A^{\beta\alpha} + i\epsilon^\alpha \eta,$$

$$\delta \lambda_\alpha = -S_{\alpha\beta} \lambda_\beta, \quad \delta \eta = \epsilon^\alpha \lambda_\alpha. \quad (\text{A9})$$

These can be reproduced from the following transformations of the $\Sigma^{((n(n+1)/2|n)}$ coordinates:

$$\delta X^{\alpha\beta} = A^{\alpha\beta} + i\theta^{(\alpha} \epsilon^{\beta)} + 2X^{(\alpha} \gamma S_{\gamma}^{\beta)},$$

$$\delta \theta^\alpha = \epsilon^\alpha + \theta^\beta A_{\beta\alpha} \quad (\text{A10})$$

using the generalization [22] of the Penrose correspondence relation [23,51] given in Eq. (119),

$$\mu^\alpha = X^{\alpha\beta} \lambda_\beta - \frac{i}{2} \theta^\alpha \theta^\beta \lambda_\beta, \quad \eta = \theta^\alpha \lambda_\alpha. \quad (\text{A11})$$

The transformations (A10) of the $\Sigma^{((n(n+1)/2|n)}$ variables are a straightforward generalization of the super-Poincaré transformations of the standard superspace coordinates. This justifies calling the resulting supergroup $\Sigma^{((n(n+1)/2|n)} \mathcal{D} Sp(n)$ a generalization of the super-Poincaré group.

Going back to $osp(2n|1)$, let us note that the generalized special superconformal transformations ($K_{\alpha\beta}, \zeta_\alpha$) act on the supertwistor components by

$$\delta \mu^\alpha = 0, \quad \delta \lambda_\alpha = \mu^\beta K_{\beta\alpha} - i\eta \zeta_\alpha, \quad \delta \eta = \mu^\beta \zeta_\beta. \quad (\text{A12})$$

Using Eq. (A11) one may find from (A12) the generalized special superconformal transformations of the $\Sigma^{((n(n+1)/2|n)}$ coordinates

$$\delta X^{\alpha\beta} = i\theta^{(\alpha} X^{\beta)\gamma} \zeta_\gamma - (X K X)^{\alpha\beta},$$

$$\delta \theta^\alpha = X^{\alpha\beta} \zeta_\beta - \frac{i}{2} (\theta \zeta) \theta^\alpha - (\theta K X)^\alpha. \quad (\text{A13})$$

Note that Eq. (A10) follows as well from a nonlinear realization of the generalized super-Poincaré group $\Sigma^{((n(n+1)/2|n)} \mathcal{D} Sp(n)$ on the $\Sigma^{((n(n+1)/2|n)}$ coset, i.e. from the left action of $\mathcal{G}_\Sigma^\Pi(S, A, \epsilon) \sim \delta_\Sigma^\Pi + \Sigma_\Sigma^\Pi(S, A, \epsilon)$ (A7) on $\mathcal{K}_\Sigma^\Pi(X, \theta) \sim \delta_\Sigma^\Pi + K_\Sigma^\Pi(X, \theta)$ with

$$K_\Sigma^\Pi(X, \theta) = \begin{Bmatrix} 0 & 0 & 0 \\ X^{\alpha\beta} & 0 & \theta^\alpha \\ i\theta^\beta & 0 & 0 \end{Bmatrix}. \quad (\text{A14})$$

Indeed, the infinitesimal form of

$$\mathcal{G}_\Sigma^\Pi(S, A, \epsilon) \mathcal{K}_\Sigma^\Pi(X, \theta) = \mathcal{K}_\Sigma^\Pi(X', \theta') \mathcal{G}_\Sigma^\Pi(A, 0, 0) \quad (\text{A15})$$

reads

$$K(\delta X, \delta \theta) = \Xi(0, A, \epsilon) + \Xi(0, A, \epsilon) K(X, \theta) + [\Xi(S, 0, 0), K(X, \theta)] \quad (\text{A16})$$

and reproduces the generalized super-Poincaré transformations (A10).

APPENDIX B: SOME TECHNICAL DETAILS

General solution of Eqs. (85)–(94) for the Lagrange multipliers
[Eqs. (96)–(102)]

$$\begin{aligned}
L^{\alpha\beta} = & b_{IJ} u^{\alpha I} u^{\beta J} + \frac{e_{\tau}^{++}}{e_{\sigma}^{++}} \{ e_{\sigma}^{++} \lambda^{-\alpha} \lambda^{-\beta} + 2[\lambda_{\gamma}^{-} \Pi_{\sigma}^{\gamma(\alpha} \lambda^{\beta)}] \\
& - (\lambda^{-} \Pi_{\sigma} \lambda^{+}) \lambda^{-(\alpha} \lambda^{\beta)} + (\lambda^{-} \Pi_{\sigma} \lambda^{-}) \lambda^{+(\alpha} \lambda^{\beta)} \} \\
& + \frac{e_{\tau}^{--}}{e_{\sigma}^{--}} \{ e_{\sigma}^{--} \lambda^{+\alpha} \lambda^{+\beta} - 2[\lambda_{\gamma}^{+} \Pi_{\sigma}^{\gamma(\alpha} \lambda^{\beta)}] \\
& - (\lambda^{+} \Pi_{\sigma} \lambda^{+}) \lambda^{-(\alpha} \lambda^{\beta)} + (\lambda^{+} \Pi_{\sigma} \lambda^{-}) \lambda^{+(\alpha} \lambda^{\beta)} \},
\end{aligned} \tag{B1}$$

$$\xi^{\alpha} = \kappa_I u^{\alpha I} + \frac{e_{\tau}^{++}}{e_{\sigma}^{++}} (\partial_{\sigma} \theta \lambda^{-}) \lambda^{+\alpha} - \frac{e_{\tau}^{--}}{e_{\sigma}^{--}} (\partial_{\sigma} \theta \lambda^{+}) \lambda^{-\alpha}, \tag{B2}$$

$$\begin{aligned}
l_{\alpha}^{+} = & \omega^{(0)} \lambda_{\alpha}^{+} + \frac{e_{\tau}^{--}}{e_{\sigma}^{--}} (\partial_{\sigma} \lambda_{\alpha}^{+} - \Omega_{\sigma}^{(0)} \lambda_{\alpha}^{+}) + \frac{e_{\tau}^{--}}{2e_{\sigma}^{--} e_{\sigma}^{--}} \\
& \times [-e_{\sigma}^{--} \Omega_{\sigma}^{++} - e_{\sigma}^{++} \Omega_{\sigma}^{--} + i \partial_{\sigma} \theta \lambda^{+} \partial_{\sigma} \theta \lambda^{-} \\
& - \Pi_{\sigma}^{\alpha\beta} (\partial_{\sigma} \lambda_{\alpha}^{+} \lambda_{\beta}^{-} - \lambda_{\alpha}^{+} \partial_{\sigma} \lambda_{\beta}^{-})] \lambda_{\alpha}^{-} \\
& + \frac{e_{\tau}^{++}}{2e_{\sigma}^{++} e_{\sigma}^{--}} [e_{\sigma}^{--} \Omega_{\sigma}^{++} + e_{\sigma}^{++} \Omega_{\sigma}^{--} \\
& + i \partial_{\sigma} \theta \lambda^{+} \partial_{\sigma} \theta \lambda^{-} + \Pi_{\sigma}^{\alpha\beta} (\partial_{\sigma} \lambda_{\alpha}^{+} \lambda_{\beta}^{-} - \lambda_{\alpha}^{+} \partial_{\sigma} \lambda_{\beta}^{-})] \lambda_{\alpha}^{-},
\end{aligned} \tag{B3}$$

$$\begin{aligned}
l_{\alpha}^{-} = & -\omega^{(0)} \lambda_{\alpha}^{-} + \frac{e_{\tau}^{++}}{e_{\sigma}^{++}} (\partial_{\sigma} \lambda_{\alpha}^{-} + \Omega_{\sigma}^{(0)} \lambda_{\alpha}^{-}) + \frac{e_{\tau}^{--}}{2e_{\sigma}^{++} e_{\sigma}^{--}} \\
& \times [-e_{\sigma}^{--} \Omega_{\sigma}^{++} - e_{\sigma}^{++} \Omega_{\sigma}^{--} + i \partial_{\sigma} \theta \lambda^{+} \partial_{\sigma} \theta \lambda^{-} \\
& - \Pi_{\sigma}^{\alpha\beta} (\partial_{\sigma} \lambda_{\alpha}^{+} \lambda_{\beta}^{-} - \lambda_{\alpha}^{+} \partial_{\sigma} \lambda_{\beta}^{-})] \lambda_{\alpha}^{+} \\
& + \frac{e_{\tau}^{++}}{2e_{\sigma}^{++} e_{\sigma}^{--}} [e_{\sigma}^{--} \Omega_{\sigma}^{++} + e_{\sigma}^{++} \Omega_{\sigma}^{--} \\
& + i \partial_{\sigma} \theta \lambda^{+} \partial_{\sigma} \theta \lambda^{-} + \Pi_{\sigma}^{\alpha\beta} (\partial_{\sigma} \lambda_{\alpha}^{+} \lambda_{\beta}^{-} - \lambda_{\alpha}^{+} \partial_{\sigma} \lambda_{\beta}^{-})] \lambda_{\alpha}^{+},
\end{aligned} \tag{B4}$$

$$L^{\pm\pm} = \partial_{\sigma} e_{\tau}^{\pm\pm} + 2e_{\tau}^{\pm\pm} \Omega_{\sigma}^{(0)} \pm 2e_{\sigma}^{\pm\pm} \omega^{(0)}, \tag{B5}$$

$$L^{(n)} = -4 \det(e_m^a) \equiv -2(e_{\tau}^{--} e_{\sigma}^{++} - e_{\tau}^{++} e_{\sigma}^{--}), \tag{B6}$$

$$l^{(0)} = 0, \tag{B7}$$

where, $\Omega_{\sigma}^{\pm\pm}$ and $\Omega_{\sigma}^{(0)}$ are defined in Eqs. (111), (112), namely

$$\Omega_{\sigma}^{++} := \partial_{\sigma} \lambda^{+} C \lambda^{+}, \quad \Omega_{\sigma}^{--} := \partial_{\sigma} \lambda^{-} C \lambda^{-} \tag{B8}$$

$$\Omega_{\sigma}^{(0)} := \frac{1}{2} (\partial_{\sigma} \lambda^{+} C \lambda^{-} - \lambda^{+} C \partial_{\sigma} \lambda^{-}). \tag{B9}$$

Dirac brackets of the second class constraints (189)–(192)

$$\begin{aligned}
[\Phi^{(0)}(\sigma), \mathcal{U}(\sigma')]_D = & -\frac{1}{2} \left(\frac{\partial_{\sigma} Y^{+\Sigma}(\sigma) \Omega_{\Sigma\Pi} Y^{+\Pi}(\sigma)}{e_{\sigma}^{++}(\sigma)} \right. \\
& \left. + \frac{\partial_{\sigma} Y^{-\Sigma}(\sigma) \Omega_{\Sigma\Pi} Y^{-\Pi}(\sigma)}{e_{\sigma}^{--}(\sigma)} \right) \delta(\sigma - \sigma') \\
= & -\frac{1}{2} \left(\frac{\Phi_{++}(\sigma)}{e_{\sigma}^{++}(\sigma)} + \frac{\Phi_{--}(\sigma)}{e_{\sigma}^{--}(\sigma)} + 2 \right) \\
& \times \delta(\sigma - \sigma') \approx -\delta(\sigma - \sigma'), \tag{B10}
\end{aligned}$$

$$\begin{aligned}
[\Phi^{(0)}(\sigma), \mathcal{N}(\sigma')]_D = & -\frac{1}{2} \left(\frac{\partial_{\sigma} Y^{+\Sigma}(\sigma) C_{\Sigma\Pi} Y^{+\Pi}(\sigma)}{e_{\sigma}^{++}(\sigma)} \right. \\
& \left. + \frac{\partial_{\sigma} Y^{-\Sigma}(\sigma) C_{\Sigma\Pi} Y^{-\Pi}(\sigma)}{e_{\sigma}^{--}(\sigma)} \right) \delta(\sigma - \sigma') \\
= & -\frac{1}{2} \left(\frac{A_{\sigma}^{++}(\sigma)}{e_{\sigma}^{++}(\sigma)} + \frac{A_{\sigma}^{--}(\sigma)}{e_{\sigma}^{--}(\sigma)} \right) \delta(\sigma - \sigma'), \tag{B11}
\end{aligned}$$

$$\begin{aligned}
[K^{(0)}(\sigma), \mathcal{U}(\sigma')]_D = & Y^{+\Sigma}(\sigma) \Omega_{\Sigma\Pi} Y^{-\Pi}(\sigma) \delta(\sigma - \sigma') \\
= & \mathcal{U} \delta(\sigma - \sigma') \approx 0, \tag{B12}
\end{aligned}$$

$$\begin{aligned}
[K^{(0)}(\sigma), \mathcal{N}(\sigma')]_D = & Y^{+\Sigma}(\sigma) C_{\Sigma\Pi} Y^{-\Pi}(\sigma) \delta(\sigma - \sigma') \\
= & (\mathcal{N} + 1) \delta(\sigma - \sigma') \approx \delta(\sigma - \sigma'), \tag{B13}
\end{aligned}$$

$$\begin{aligned}
[K^{(0)}(\sigma), \Phi^{(0)}(\sigma')]_D = & \frac{1}{2} [\partial_{\sigma} Y^{+\Sigma}(\sigma) \Omega_{\Sigma\Pi} Y^{-\Pi}(\sigma) \\
& - Y^{+\Sigma}(\sigma) \Omega_{\Sigma\Pi} \partial_{\sigma} Y^{-\Pi}(\sigma)] \\
& \times \delta(\sigma - \sigma') \\
= & \frac{1}{2} \Phi^{(0)} \delta(\sigma - \sigma') \approx 0. \tag{B14}
\end{aligned}$$

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