Five-dimensional rotating black hole in a uniform magnetic field: The gyromagnetic ratio

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In four-dimensional general relativity, the fact that a Killing vector in a vacuum spacetime serves as a vector potential for a test Maxwell field provides one with an elegant way of describing the behavior of electromagnetic fields near a rotating Kerr black hole immersed in a uniform magnetic field. We use a similar approach to examine the case of a five-dimensional rotating black hole placed in a uniform magnetic field of configuration with biazimuthal symmetry that is aligned with the angular momenta of the Myers-Perry spacetime. Assuming that the black hole may also possess a small electric charge we construct the five-vector potential of the electromagnetic field in the Myers-Perry metric using its three commuting Killing vector fields. We show that, like its four-dimensional counterparts, the five-dimensional Myers-Perry black hole rotating in a uniform magnetic field produces an inductive potential difference between the event horizon and an infinitely distant surface. This potential difference is determined by a superposition of two independent Coulomb fields consistent with the two angular momenta of the black hole and two nonvanishing components of the magnetic field. We also show that a weakly charged rotating black hole in five dimensions possesses two independent magnetic dipole moments specified in terms of its electric charge, mass, and angular momentum parameters. We prove that a five-dimensional weakly charged Myers-Perry black hole must have the value of the gyromagnetic ratio $g=3$.

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I. INTRODUCTION

Black holes originally predicted in four-dimensional general relativity have subsequently become an inseparable part of all higher dimensional gravity theories. The fundamental features of black holes in four dimensions, such as the equilibrium and uniqueness properties, and quantum properties following from Hawking's effect of evaporation of microscopic black holes, have revealed an intimate connection between spacetime geometry, quantum field theory, and thermodynamics [1–4] (see Refs. [5–7] for comprehensive reviews). Certainly, these properties of black holes might have played a crucial role in the analysis of dynamics of higher dimensional gravity theories, as well as in the compactification process. For instance, to test the novel predictions of superstring theory which, it is believed, provides a consistent quantum theory of gravity in higher dimensions [8], microscopic black holes may serve as good theoretical laboratories. Therefore over current years higher dimensional black holes have been widely considered as very interesting objects to be investigated in detail. Many interesting black hole solutions in various higher dimensional theories can be found in [9].

On the other hand, the interest in higher dimensional black holes gained new impetus after the advent of brane-world gravity theories [10–13] (see also Refs. [14,15]). The brane-world theories are built on the basic idea that the geometry of our physical Universe is indeed a "three-brane"—a (3+1)-dimensional hypersurface embedded in fundamental higher dimensional space. The size of the extra spatial dimensions may be much larger than the conventional Planckian length ($\sim 10^{-33}$ cm). Thus, in contrast to the original Kaluza-Klein scenario, in the brane-world models the extra dimensions are supposed to manifest themselves as physical ones. One of the dramatic consequences of these models is that the fundamental scale of quantum gravity might become as low as the weak interaction scale (of the order of TeV). This, in turn, raises the problem of TeV-size black holes, and the exciting signature of such mini black holes is that they can be directly probed in cosmic ray experiments or at future high energy colliders [16,17]. It has also been argued that one can describe these black holes by the classical solutions of higher dimensional vacuum Einstein equations provided that the radius of the event horizon is much smaller than the size scale of the extra dimensions. In light of all this, it is obvious that further knowledge of the special properties of black hole solutions in higher dimensional vacuum gravity is of great importance.

The first black hole solution to the higher dimensional Einstein equations is the static and hyperspherically symmetric Schwarzschild-Tangherlini solution [18], which has been found a long time ago within the Kaluza-Klein program of extension of four dimensional general relativity. As in four dimensions, one could expect the generalization of the static hyperspherical black hole solution to include rotational dynamics. In 1986 Myers and Perry discovered the exact solution of Einstein's equations describing such rotating black holes [19]. It is important that the Myers-Perry solution is supposed to be the most relevant to describe the "laboratory" black holes in high energy experiments [17]. In this context some essential features of the Myers-Perry solution in five dimensions, such as the existence of a Killing tensor and the separability of variables in the Hamilton-Jacobi equations of motion, as well as quantum radiation from a five-dimensional black hole, were explored in [20,21].

However, the Myers-Perry solution is not unique in five dimensions, unlike its four-dimensional counterpart, the Kerr solution. Emparan and Reall [22] found a rotating black ring solution in five dimensions with the horizon topology of $S^2\times S^1$, which could have the same mass and spin as the Myers-Perry solution. As for the static case, the authors of [23,24] have proved that in this case, remarkably, the uniqueness property survives. Another essential problem is the stability of higher dimensional rotating black hole solutions. Recently, in [25] it has been argued that the Myers-Perry solution becomes unstable for the case of an arbitrarily large rotation parameter for a fixed mass. It is clear that, in the general case, the full analytic theory of perturbations of higher dimensional rotating black holes is needed to resolve the stabilitiy problem. In the static case the stability of higher dimensional black holes was proved in [26].

In this paper we shall study further properties of a five-dimensional rotating black hole in the presence of an external magnetic field, which is supposed to be uniform at infinity. We shall consider the configuration of the magnetic field aligned with the angular momenta of the black hole. In other words, the magnetic field shares the biazimuthal symmetry of the Myers-Perry spacetime and has only two nonvanishing components. In order to construct the corresponding solution of the Maxwell field equations in the background of the Myers-Perry metric, we shall appeal to the well known fact [27] that in a vacuum spacetime one can obtain the solution for the Maxwell test field by using only isometries of that spacetime. Earlier, this fact was used in four-dimensional general relativity to construct the solution for the electromagnetic field around a Kerr black hole immersed in a uniform magnetic field [28]. In this analysis the temporal and axial Killing vectors of the Kerr spacetime were used as a vector potential for the Maxwell test field. In particular, it has been found that the so-called Wald effect occurs; the rotation of a black hole in an asymptotically uniform magnetic field causes an inductive potential difference between the event horizon and infinity, whereby the black hole may acquire an inductive electric charge.

In four dimensions the rotation group is $SO(3)$ and there always exists a rotation axis consistent with only one independent Casimir invariant. However, in five dimensions the rotation group is $SO(4)$, which possesses two independent Casimir invariants. These two Casimir invariants, in turn, are associated with two independent rotations of the system. In other words, a rotating black hole in five dimensions may have two distinct planes of rotation specified by appropriate azimuthal coordinates, rather than an axis of rotation. In accordance with this, the stationary and asymptotically flat Myers-Perry metric admits three commuting Killing vector fields, which reflect the time-translation invariance and biazimuthal symmetry of this metric in five dimensions. We shall use these Killing vectors to construct the five-vector potential for a test electromagnetic field, when the Myers-Perry black hole is placed in an asymptotically uniform magnetic field of a configuration with biazimuthal symmetry. We shall show that a rotation in two distinct planes of a five-dimensional black hole immersed in a uniform magnetic field produces an inductive electric field which is determined by the superposition of two independent Coulomb parts consistent with the two angular momentum parameters and two nonvanishing components of the magnetic field.

We shall also examine the case when a five-dimensional black hole may have an electric charge small enough that the spacetime is still well described by the Myers-Perry solution. In this case the rotation of the black hole must produce a dipole magnetic field. We shall establish that the black hole, in fact, possesses two independent magnetic dipole moments determined only by its charge, mass, and angular momentum.

The paper is organized as follows. In Sec. II we recall some of the properties of the Myers-Perry metric for a five-dimensional black hole. We describe the Killing isometries of the metric, its mass parameter, and specific angular momenta. We also give the precise definition of the angular velocities for stationary observers in the Myers-Perry metric and define locally nonrotating orthonormal frames (LNRFs) and their dual basis forms. In Sec. III we begin with a brief description of a uniform magnetic field in five dimensions and construct the five-vector potential for the Maxwell test field using the Killing isometries of the Myers-Perry spacetime. Here we also calculate the magnetic flux crossing a portion of the black hole horizon. The dominant orthonormal components of electric and magnetic fields in the asymptotic rest frame of a weakly charged Myers-Perry black hole, as well as its magnetic dipole moments, are calculated in Sec. IV. Finally, in Sec. V we prove that a five-dimensional weakly charged Myers-Perry black hole must possess the value of the gyromagnetic ratio $g=3$.

II. FIVE-DIMENSIONAL ROTATING BLACK HOLE

A. Myers-Perry metric

The metric of a rotating black hole in five dimensions follows from the general asymptotically flat solutions to $(N+1)$-dimensional vacuum gravity found by Myers and Perry [19]. In Boyer-Lindquist type coordinates it takes the simplest form given by

$$\begin{aligned} ds^2 = & -dt^2+\Sigma\left(\frac{r^2}{\Delta}dr^2+d\theta^2\right)+(r^2+a^2)\sin^2\theta\,d\phi^2 \\ & +(r^2+b^2)\cos^2\theta\,d\psi^2 \\ & +\frac{m}{\Sigma}(dt-a\sin^2\theta\,d\phi-b\cos^2\theta\,d\psi)^2, \end{aligned} \tag{1}$$

where

$$\Sigma=r^2+a^2\sin^2\theta+b^2\cos^2\theta,$$

$$\Delta=(r^2+a^2)(r^2+b^2)-mr^2, \tag{2}$$

and m is a parameter related to the physical mass of the black hole, while the parameters a and b are associated with its two independent angular momenta. For the metric determinant we have

$$\sqrt{-g}=r\Sigma \sin\theta\cos\theta. \tag{3}$$

The components of the inverse metric have the following forms:

$$g^{tt}=-\left(1+\frac{m}{\Sigma}+\frac{mr^2}{\Delta\Sigma}\right), \quad g^{rr}=\frac{\Delta}{r^2\Sigma},$$

$$g^{\theta\theta}=\frac{1}{\Sigma}, \quad g^{\phi\psi}=-\frac{mab}{\Delta\Sigma},$$

$$g^{t\phi}=-\frac{ma(r^2+b^2)}{\Delta\Sigma},$$

$$g^{\phi\phi}=\frac{1}{\Sigma}\left[\frac{1}{\sin^2\theta}+\frac{(r^2+b^2)(b^2-a^2)-mb^2}{\Delta}\right],$$

$$g^{t\psi}=-\frac{mb(r^2+a^2)}{\Delta\Sigma},$$

$$g^{\psi\psi}=\frac{1}{\Sigma}\left[\frac{1}{\cos^2\theta}+\frac{(r^2+a^2)(a^2-b^2)-ma^2}{\Delta}\right]. \tag{4}$$

The event horizon of the black hole is a null surface determined by the equation $g^{rr}=0$, which implies that

$$\Delta=(r^2+a^2)(r^2+b^2)-mr^2=0. \tag{5}$$

The largest root of this equation gives the radius of the black hole's outer event horizon. We have

$$r_h^2=\frac{1}{2}\left(m-a^2-b^2+\sqrt{(m-a^2-b^2)^2-4a^2b^2}\right). \tag{6}$$

Notice that the horizon exists if and only if

$$a^2+b^2+2|ab|\leqslant m, \tag{7}$$

so that the condition $m=a^2+b^2+2|ab|$ or, equivalently, $r_h^2=|ab|$ defines the extremal horizon of a five-dimensional black hole.

In the absence of the black hole $(m=0)$, the metric (1) reduces to the flat one written in oblate bipolar coordinates. The latter can be readily cast in the Minkowski form using the transformation of coordinates [20]

$$x=\sqrt{r^2+a^2}\sin\theta\cos\tilde{\phi}, \quad y=\sqrt{r^2+a^2}\sin\theta\sin\tilde{\phi},$$

$$z=\sqrt{r^2+b^2}\cos\theta\cos\tilde{\psi}, \quad w=\sqrt{r^2+b^2}\cos\theta\sin\tilde{\psi}, \tag{8}$$

where

$$\tilde{\phi}=\phi+\tan^{-1}(a/r), \quad \tilde{\psi}=\psi+\tan^{-1}(b/r).$$

On the other hand, for $a=b=0$ we have the Schwarzschild-Tangherlini static solution in spherical bipolar coordinates. It is clear that, in general, the metric (1) admits two orthogonal two-planes of rotation, the x-y plane, $z=w=0$ and the z-w plane, $x=y=0$, which are specified by the azimuthal angles ϕ and ψ, respectively. These angles both range from 0 to 2π, while θ is the angle between the two orthogonal two-planes varies in the interval $[0,\pi/2]$. As a consequence, the metric reveals the following obvious invariance properties: under the simultaneous inversion of the time coordinate $t\to -t$ and the angles $\phi\to-\phi$, $\psi\to-\psi$, and under the transformation

$$a\leftrightarrow b, \quad \phi\leftrightarrow\psi, \quad \theta\leftrightarrow\frac{\pi}{2}-\theta. \tag{9}$$

The biazimuthal symmetry properties of the five-dimensional black hole metric (1) along with its stationarity imply the existence of the three commuting Killing vectors

$$\xi_{(t)}=\partial/\partial t, \quad \xi_{(\phi)}=\partial/\partial\phi, \quad \xi_{(\psi)}=\partial/\partial\psi. \tag{10}$$

The various scalar products of these Killing vectors are expressed through the corresponding metric components as follows:

$$\xi_{(t)}\cdot\xi_{(t)}=g_{tt}=-1+\frac{m}{\Sigma},$$

$$\xi_{(\phi)}\cdot\xi_{(\phi)}=g_{\phi\phi}=\left(r^2+a^2+\frac{ma^2}{\Sigma}\sin^2\theta\right)\sin^2\theta,$$

$$\xi_{(t)}\cdot\xi_{(\phi)}=g_{t\phi}=-\frac{ma}{\Sigma}\sin^2\theta,$$

$$\xi_{(\psi)}\cdot\xi_{(\psi)}=g_{\psi\psi}\left(r^2+b^2+\frac{mb^2}{\Sigma}\cos^2\theta\right)\cos^2\theta,$$

$$\xi_{(t)}\cdot\xi_{(\psi)}=g_{t\psi}=-\frac{mb}{\Sigma}\cos^2\theta,$$

$$\xi_{(\phi)}\cdot\xi_{(\psi)}=g_{\phi\psi}=\frac{mab}{\Sigma}\sin^2\theta\cos^2\theta. \tag{11}$$

The Killing vectors (10) can be used to give a physical interpretation of the parameters m, a, and b involved in the metric (1); namely, following the analysis given in [29], one can obtain coordinate-independent definitions for these parameters. We have the integrals

$$m=\frac{1}{4\pi^2}\oint \xi_{(t)}^{\mu;\nu}d^3\Sigma_{\mu\nu} \tag{12}$$

and

$$j_{(a)}=am=-\frac{1}{4\pi^2}\oint \xi^{\mu;\nu}_{(\phi)}d^3\Sigma_{\mu\nu},$$

$$j_{(b)}=bm=-\frac{1}{4\pi^2}\oint \xi^{\mu;\nu}_{(\psi)}d^3\Sigma_{\mu\nu}, \tag{13}$$

where the integrals are taken over the three-sphere at spatial infinity,

$$d^3\Sigma_{\mu\nu}=\frac{1}{3!}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta\gamma}\,dx^\alpha\wedge dx^\beta\wedge dx^\gamma, \tag{14}$$

the semicolon denotes covariant differentiation, and we have introduced the two specific angular momentum parameters $j_{(a)}$ and $j_{(b)}$ associated with rotations in the ϕ and ψ directions, respectively. We note that with these definitions the relation between the specific angular momentum and the mass parameter looks exactly like the corresponding relation $(J=aM)$ of four-dimensional Kerr metric.

To show that the definitions given in Eqs. (12) and (13) do in fact correctly describe the mass and angular momenta parameters we can calculate the integrands in the asymptotic region $r\to\infty$. The dominant terms in the asymptotic expansion have the form

$$\xi^{t;r}_{(t)}=\frac{m}{r^3}+\mathcal{O}\left(\frac{1}{r^5}\right),$$

$$\xi^{t;r}_{(\phi)}=-\frac{2am\sin^2\theta}{r^3}+\mathcal{O}\left(\frac{1}{r^5}\right),$$

$$\xi^{t;r}_{(\psi)}=-\frac{2bm\cos^2\theta}{r^3}+\mathcal{O}\left(\frac{1}{r^5}\right). \tag{15}$$

The substitution of these expressions into the formulas (12) and (13) verifies them. On the other hand, the relation of the above parameters to the total mass M and the total angular momenta $J_{(a)}$ and $J_{(b)}$ of the black hole can be established using the formulas given in [19]. We obtain that

$$m=\frac{8}{3\pi}M,\quad j_{(a)}=\frac{4}{\pi}J_{(a)},\quad j_{(b)}=\frac{4}{\pi}J_{(b)}. \tag{16}$$

These relations confirm the interpretation of the parameters m, a, and b as being related to the physical mass and angular momenta of the metric (1).

B. Locally nonrotating observers

To examine further properties of the five-dimensional Myers-Perry black hole, as well as physical processes near such a black hole, it is useful to introduce a family of locally nonrotating observers. In the four-dimensional case a locally nonrotating observer has a vector of velocity orthogonal to the $t=\text{const}$ surface in the Kerr geometry and its angular momentum vanishes [30,31]. We define a locally nonrotating observer in the Myers-Perry metric in a similar manner. Let us write

$$u^\mu=u^\mu(r,\theta)=\alpha(\xi^\mu_{(t)}+\Omega_{(a)}\xi^\mu_{(\phi)}+\Omega_{(b)}\xi^\mu_{(\psi)}), \tag{17}$$

where u^μ is a unit vector of the five-velocity of a locally nonrotating observer, and α is a normalization constant determined by the condition $u^2=-1$. The orthogonality to the $t=\text{const}$ surface implies $u^r=u^\theta=0$ and

$$g_{t\phi}u^t+g_{\phi\phi}u^\phi+g_{\phi\psi}u^\psi=0,$$

$$g_{t\psi}u^t+g_{\psi\psi}u^\psi+g_{\psi\phi}u^\phi=0. \tag{18}$$

The simultaneous solution of these equations determines $u^\mu(r,\theta)$. Thus we obtain

$$\Omega_{(a)}=\frac{u^\phi}{u^t}=\frac{g_{t\psi}g_{\phi\psi}-g_{t\phi}g_{\psi\psi}}{g_{\phi\phi}g_{\psi\psi}-g^2_{\phi\psi}}=\frac{am(r^2+b^2)}{\Delta\Sigma+m(r^2+a^2)(r^2+b^2)},$$

$$\Omega_{(b)}=\frac{u^\psi}{u^t}=\frac{g_{t\phi}g_{\phi\psi}-g_{t\psi}g_{\phi\phi}}{g_{\phi\phi}g_{\psi\psi}-g^2_{\phi\psi}}=\frac{bm(r^2+a^2)}{\Delta\Sigma+m(r^2+a^2)(r^2+b^2)}. \tag{19}$$

In the case that either $b=0$ or $a=0$, instead of the above expressions we have either

$$\Omega_{(a)}=-\frac{g_{t\phi}}{g_{\phi\phi}}\quad\text{or}\quad\Omega_{(b)}=-\frac{g_{t\psi}}{g_{\psi\psi}}, \tag{20}$$

which are closely reminiscent of the corresponding relation for the Kerr black hole in four dimensions. At large distances from the black hole, the relations (19) can be written in the form

$$\Omega_{(a)}=\frac{j_{(a)}}{r^4}+\mathcal{O}\left(\frac{1}{r^6}\right),\quad \Omega_{(b)}=\frac{j_{(b)}}{r^4}+\mathcal{O}\left(\frac{1}{r^6}\right), \tag{21}$$

which reveals the remarkable property of the Myers-Perry black hole, namely, *the dragging of inertial frames* in both ϕ and ψ two-planes of rotation. Clearly, the effect of "bidragging" disappears at spatial infinity. However, toward the event horizon of the black hole it increases, tending to its constant value on the event horizon $(\Delta=0)$. From Eqs. (19) we obtain

$$\Omega_{(a)h}=\frac{a}{r_+^2+a^2},\quad \Omega_{(b)h}=\frac{b}{r_+^2+b^2}. \tag{22}$$

These quantities can be interpreted as angular velocities of the black hole [19]. In order to show this, one needs to know the isometry properties of the horizon geometry. Following the Hawking approach in four-dimensional geometry of the Kerr black hole [3], we suppose that the isometry of the event horizon of a five-dimensional black hole is described by a Killing vector, which must be a linear combination of the three Killing vectors given in Eq. (10). Thus, we can take it in the form

$$\chi=\xi_{(t)}+\Omega_{(a)h}\xi_{(\phi)}+\Omega_{(b)h}\xi_{(\psi)}. \tag{23}$$

One can easily verify that this vector becomes null at the surface $\Delta=0$, i.e., it is tangent to the null surface of the horizon. This means that stationary observers moving in the ϕ and ψ two-planes of rotation become corotating along with the horizon with the local angular velocities (22). In other words, the event horizon of the five-dimensional Myers-Perry black hole is, in fact, the Killing horizon determined by the vector χ.

From what we said above about the locally nonrotating observers, it also follows that with any such observer one can associate an orthonormal frame. The form of the metric given in (1) enables us to choose the appropriate basis one-forms of these frames. In particular, for a locally nonrotating observer moving in the ϕ two-plane we can choose an orthonormal frame (LNRF) with the following basis one-forms:

$$\begin{aligned}
\omega^{\hat{t}}&=\left|g_{tt}-\frac{g_{t\psi}^2}{g_{\psi\psi}}-\Omega_{(a)}^2\frac{g_{\phi\phi}g_{\psi\psi}-g_{\phi\psi}^2}{g_{\psi\psi}}\right|^{1/2}dt,\\
\omega^{\hat{\phi}}&=\left(\frac{g_{\phi\phi}g_{\psi\psi}-g_{\phi\psi}^2}{g_{\psi\psi}}\right)^{1/2}(d\phi-\Omega_{(a)}\,dt),\\
\omega^{\hat{\psi}}&=(g_{\psi\psi})^{1/2}\left(d\psi+\frac{g_{t\psi}}{g_{\psi\psi}}dt+\frac{g_{\phi\psi}}{g_{\psi\psi}}d\phi\right),\\
\omega^{\hat{r}}&=(g_{rr})^{1/2}\,dr,\\
\omega^{\hat{\theta}}&=(g_{\theta\theta})^{1/2}\,d\theta,
\end{aligned} \tag{24}$$

while the dual basis is given by

$$\begin{aligned}
e_{\hat{t}}&=\left|g_{tt}-\frac{g_{t\psi}^2}{g_{\psi\psi}}-\Omega_{(a)}^2\frac{g_{\phi\phi}g_{\psi\psi}-g_{\phi\psi}^2}{g_{\psi\psi}}\right|^{-1/2}\\
&\quad\times\left(\frac{\partial}{\partial t}+\Omega_{(a)}\frac{\partial}{\partial\phi}-\frac{g_{t\psi}+\Omega_{(a)}g_{\phi\psi}}{g_{\psi\psi}}\frac{\partial}{\partial\psi}\right),\\
e_{\hat{\phi}}&=\left(\frac{g_{\phi\phi}g_{\psi\psi}-g_{\phi\psi}^2}{g_{\psi\psi}}\right)^{-1/2}\left(\frac{\partial}{\partial\phi}-\frac{g_{\phi\psi}}{g_{\psi\psi}}\frac{\partial}{\partial\psi}\right),\\
e_{\hat{\psi}}&=\frac{1}{(g_{\psi\psi})^{1/2}}\frac{\partial}{\partial\psi},\\
e_{\hat{r}}&=\frac{1}{(g_{rr})^{1/2}}\frac{\partial}{\partial r},\\
e_{\hat{\theta}}&=\frac{1}{(g_{\theta\theta})^{1/2}}\frac{\partial}{\partial\theta}.
\end{aligned} \tag{25}$$

We note that the corresponding basis one-forms and their duals associated with an orthonormal frame of a locally nonrotating observer moving in the ψ direction are obtained from the expressions given above simply by the transformation $\phi\leftrightarrow\psi$.

III. UNIFORM MAGNETIC FIELD IN THE BACKGROUND OF A FIVE-DIMENSIONAL BLACK HOLE

A. Uniform magnetic field in a five-dimensional flat spacetime

In four-dimensional gravity the behavior of electromagnetic fields in the background of a rotating black hole described by the Kerr metric has been investigated by many authors [28,32–34] (see also Ref. [35] for a review). In particular, Wald has proposed the most elegant way of describing the behavior of electromagnetic fields near a rotating black hole which is placed in an originally uniform magnetic field aligned with the axis of symmetry of the black hole [28]. The Wald approach stems from the well known fact [27] that a Killing vector in a vacuum spacetime serves as a vector potential for a Maxwell test field in that spacetime's background. Therefore one can construct a solution for the Maxwell test field in the background of a vacuum spacetime simply by using the isometries of this spacetime.

We shall apply this approach to examine the behavior of a Maxwell test field around a five-dimensional black hole described by the metric (1) when the black hole is immersed in an asymptotically uniform magnetic field. We recall that, in general, the electromagnetic field in five dimensions can be described by an electric one-form field and magnetic two-form field. In other words, four of the total ten independent components of the electromagnetic field tensor $F_{\mu\nu}$ describe the electric field, while the remaining six components correspond to the magnetic field. We shall consider the case of a magnetic field configuration which is stationary and uniform at infinity and possesses biazimuthal symmetry as well. Then it is clear that the corresponding electromagnetic field tensor must share all isometries of the black hole's spacetime. On these grounds, it is also clear that there must exist only two nonvanishing components of the field tensor that describe a uniform magnetic field in our case. That is, we have

$$B=F_{xy}, \quad H=F_{zw}, \tag{26}$$

where we have used the notations B and H for the magnetic field strengths associated with the x-y and z-w two-planes, respectively. We note that these quantities are reminiscent of the two independent angular momenta given in Eq. (13), which are, in turn, the nonvanishing components of the underlying angular momentum two-form. In this respect, the magnetic field components in our model are aligned with the corresponding angular momenta.

In the following we shall need the expression for the components of the electromagnetic two-form field F written down in bipolar coordinates. By making use of the transformations (8) we obtain, instead of Eq. (26), an expression of the form

$$F = Br \sin\theta(\sin\theta\, dr\wedge d\phi + r\cos\theta\, d\theta\wedge d\phi) + Hr\cos\theta(\cos\theta\, dr\wedge d\psi - r\sin\theta\, d\theta\wedge d\psi), \quad (27)$$

which describes a uniform magnetic field of a configuration with biazimuthal symmetry in a flat five-dimensional spacetime.

B. Five-vector potential

It is remarkable that using only the isometries of the Myers-Perry spacetime described by the temporal Killing vector $\xi_{(t)}$ and two azimuthal Killing vectors $\xi_{(\phi)}$ and $\xi_{(\psi)}$ one can construct the five-vector potential for the Maxwell test field in this spacetime. Indeed, the homogeneous Maxwell equations, in the Lorentz gauge

$$A^{\mu}_{;\mu} = 0, \quad (28)$$

have the form

$$A^{\mu;\nu}{}_{;\nu} - R^{\mu}{}_{\nu} A^{\nu} = 0. \quad (29)$$

On the other hand, any Killing vector ξ satisfies the equation

$$\xi^{\mu;\nu}{}_{;\nu} + R^{\mu}{}_{\nu}\, \xi^{\nu} = 0. \quad (30)$$

Comparing the two equations (29) and (30) one sees that they are the same in a vacuum spacetime $(R^{\mu}{}_{\nu}=0)$. Thus, one can use a Killing vector as a vector potential for a test Maxwell field [27]. Following this fact, we shall seek for a five-vector potential of the form

$$A^{\mu} = \alpha \xi^{\mu}_{(t)} + \beta \xi^{\mu}_{(\phi)} + \gamma \xi^{\mu}_{(\psi)}, \quad (31)$$

where α, β, and γ are arbitrary parameters. Let us emphasize that in the general case this vector potential will also describe a test electric field so that the black hole is charged. We assume that this electric charge is small enough, $Q \ll M$, that the spacetime can still be adequately described by the unperturbed metric (1).

To determine the unknown parameters in Eq. (31) first we calculate the corresponding electromagnetic two-form field F in the metric (1). We obtain

$$F = \frac{2mr}{\Sigma^2}\mathcal{A}\, dt\wedge dr + \frac{m\sin 2\theta}{\Sigma^2}[\alpha(b^2-a^2) + \beta a(r^2+a^2) - \gamma b(r^2+b^2)]dt\wedge d\theta + 2r\sin^2\theta\left(\beta + \frac{am}{\Sigma^2}\mathcal{A}\right)dr\wedge d\phi$$
$$+ 2r\cos^2\theta\left(\gamma + \frac{bm}{\Sigma^2}\mathcal{A}\right)dr\wedge d\psi + \sin 2\theta\left[\beta(r^2+a^2) + \frac{am}{\Sigma^2}[\mathcal{B} - \alpha(r^2+a^2) - \gamma b(r^2+b^2)]\right]d\theta\wedge d\phi$$
$$- \sin 2\theta\left[\gamma(r^2+b^2) + \frac{bm}{\Sigma^2}[\mathcal{B} - \alpha(r^2+b^2) - \beta a(r^2+a^2)]\right]d\theta\wedge d\psi, \quad (32)$$

where for the sake of brevity we have used the notation

$$\mathcal{A} = \alpha - \beta a\sin^2\theta - \gamma b\cos^2\theta \quad (33)$$

and

$$\mathcal{B} = \beta a(r^2+a^2+\Sigma)\sin^2\theta + \gamma b(r^2+b^2+\Sigma)\cos^2\theta. \quad (34)$$

In the asymptotic region $r\to\infty$, this expression takes the form

$$F = 2\beta r\sin\theta(\sin\theta\, dr\wedge d\phi + r\cos\theta\, d\theta\wedge d\phi) + 2\gamma r\cos\theta(\cos\theta\, dr\wedge d\psi - r\sin\theta\, d\theta\wedge d\psi). \quad (35)$$

From a comparison of Eqs. (27) and (35), it follows that $\beta = B/2$, $\gamma = H/2$. The remaining parameter α can be determined from examining the integrals (12) and (13) along with the integral for the electric charge of the black hole

$$Q = \frac{1}{4\pi^2}\oint F^{\mu\nu} d^3\Sigma_{\mu\nu}. \quad (36)$$

Having done this, we obtain

$$\alpha = -\frac{Q}{2m} + \frac{aB}{2} + \frac{bH}{2}. \quad (37)$$

Finally, the five-vector potential for the electromagnetic field around a rotating and weakly charged black hole in a uniform magnetic field can be written in the form

$$A^{\mu} = -\frac{Q}{2m}\ \xi^{\mu}_{(t)} + \frac{B}{2}(\xi^{\mu}_{(\phi)} + a\xi^{\mu}_{(t)}) + \frac{H}{2}(\xi^{\mu}_{(\psi)} + b\xi^{\mu}_{(t)}). \quad (38)$$

From this expression, it follows that the five-vector potential in the background of the Myers-Perry spacetime consists of a superposition of the Coulomb-type part and the asymptotically uniform magnetic field. It is important to note that the Coulomb-type part, which is generated by the temporal Killing vector, does not vanish even when the electric charge of the black hole is zero $(Q=0)$. There are two independent contributions to the Coulomb field of the black hole. The physical reason underlying this phenomenon is that a rotation of a five-dimensional black hole in a uniform magnetic field produces an inductive electric field associated with the two independent ϕ and ψ two-planes of rotation of the black hole as well as two independent components of the electromagnetic field tensor. It is seen that in our model the black hole acts like a "dynamo" that causes an electrostatic potential difference between the event horizon of the black hole and an infinitely distant surface. Following the approach in four-dimensional case [36], we shall define the electrostatic potential of the event horizon with respect to the Killing vector (23) as follows:

$$\Phi_h = A\cdot\chi = A_0 + \Omega_{(a)h}A_\phi + \Omega_{(b)h}A_\psi. \tag{39}$$

Then, for the electrostatic potential difference between the event horizon and an infinitely distant surface, we find

$$\Delta\Phi = \Phi_h - \Phi_\infty = \frac{Q - amB - bmH}{2m}. \tag{40}$$

This potential difference is exactly of the same form as if it were produced by the electric charge

$$\tilde{Q} = Q - amB - bmH. \tag{41}$$

It is obvious that this charge $\tilde{Q}$ will be quickly neutralized (the potential difference vanishes) due to a selective accretion process of charged particles, provided that the black hole is surrounded by an ionized medium.[1] As a result of this the black hole will acquire the physical electric charge

$$Q = amB + bmH = j_{(a)}B + j_{(b)}H. \tag{42}$$

We note that in the degenerate case when $a=b$ and $B=H$ the above expression goes over into the form

$$Q = 2amB, \tag{43}$$

which is reminiscent of its counterpart for a Kerr black hole [28].

Accordingly, in terms of the total mass and angular momenta of the black hole Eq. (42) can be written as

$$Q = \frac{8}{3\pi}(aMB + bmH) = \frac{4}{\pi}(J_{(a)}B + J_{(b)}H). \tag{44}$$

Thus, a five-dimensional black hole rotating in a uniform magnetic field of a configuration with biazimuthal symmetry will be charged up to the value given by Eq. (44).

To conclude this section we shall calculate the magnetic flux crossing a portion Σ of the black hole event horizon. This flux is governed by the line integral on the horizon

$$\mathcal{F} = \int_{\partial\Sigma} A, \tag{45}$$

where the potential one-form A is determined through Eq. (38) with $Q=0$ and $\partial\Sigma$ is the boundary of Σ. For our purpose, it is convenient to rewrite the potential in terms of the Killing vector χ defined in Eq. (23). We obtain

$$A = \frac{1}{2}(aB + bH)\chi + \tilde{A},$$

$$\tilde{A} = \frac{B}{2}(\xi_{(\phi)} - a\Omega_{(a)h}\xi_{(\phi)} - a\Omega_{(b)h}\xi_{(\psi)}) + \frac{H}{2}(\xi_{(\psi)} - b\Omega_{(b)h}\xi_{(\psi)} - b\Omega_{(a)h}\xi_{(\phi)}), \tag{46}$$

where we have used the same notation for Killing one-form fields. The first term in this expression is proportional to χ and therefore its contribution to the flux $\mathcal{F}$ at the horizon vanishes. For the rest, taking into account Eq. (22) we obtain

$$\tilde{A} = \frac{1}{2}\left(\frac{Br_h^2 - Hab}{r_h^2 + a^2}\xi_{(\phi)} + \frac{Hr_h^2 - Bab}{r_h^2 + b^2}\xi_{(\psi)}\right). \tag{47}$$

This expression, and hence the flux (45), vanishes precisely at the extremal horizon of the black hole ($r_h^2 = ab$), provided that $B=H$. Thus, when the magnetic field strengths associated with the $x-y$ and $z-w$ two-planes of rotation are equal in magnitude, the magnetic flux is expelled from a five-dimensional black hole as the extremality in its rotation is approached. In this case, a portion of the black hole horizon, like its four-dimensional counterpart, acts as the surface of a perfectly diamagnetic object [37].

IV. MAGNETIC DIPOLE MOMENTS

We now turn to the consideration of a five-dimensional weakly charged rotating black hole. It is clear that the rotation of such a black hole must produce a dipole type magnetic field around itself. Since the charged black hole is characterized by two independent rotation parameters; accordingly, one may expect that it will acquire two independent magnetic dipole moments as well. We shall determine the value of these magnetic moments.

We begin with the expression for the electromagnetic two-form field

$$F = \frac{Q}{\Sigma^2}[r\,dr\wedge dt + (b^2 - a^2)\sin\theta\cos\theta\,d\theta\wedge dt] - \frac{Qa\sin\theta}{\Sigma^2}[r\sin\theta\,dr\wedge d\phi - (r^2+a^2)\cos\theta\,d\theta\wedge d\phi] - \frac{Qb\cos\theta}{\Sigma^2}[r\cos\theta\,dr\wedge d\psi + (r^2+b^2)\sin\theta\,d\theta\wedge d\psi], \tag{48}$$

which is obtained from Eq. (32) with $B=0$, $H=0$. The associated potential one-form can be written as

[1] In the brane-world scenario, when charged particles can live only on the brane, this conclusion also remains true. In the presence of a plasma of charged particles located on the brane, there will be a selective accretion process reducing the charge $\tilde{Q}$.

$$A=-\frac{Q}{2\Sigma}(dt-a\sin^2\theta\,d\phi-b\cos^2\theta\,d\psi). \quad (49)$$

In obtaining this expression we have gauged the potential (38) according to the transformation

$$A=\hat{A}-\frac{Q}{2m}dt \quad (50)$$

to provide its vanishing behavior at infinity. We shall also need the contravariant components of the electromagnetic field tensor, which are given by

$$F^{tr}=\frac{Q(r^2+a^2)(r^2+b^2)}{r\Sigma^3}, \quad F^{t\theta}=\frac{Q(b^2-a^2)\sin 2\theta}{2\Sigma^3},$$

$$F^{r\phi}=-\frac{Qa(r^2+b^2)}{r\Sigma^3}, \quad F^{r\psi}=-\frac{Qb(r^2+a^2)}{r\Sigma^3},$$

$$F^{\theta\phi}=\frac{Qa\cot\theta}{\Sigma^3}, \quad F^{\theta\psi}=-\frac{Qb\tan\theta}{\Sigma^3}. \quad (51)$$

Next, we shall define the electric field, as well as the dipole magnetic field, in the asymptotic rest frame of the black hole. We start with the electric one-form field $\hat{E}$ which, in the spacetime of dimensions D, can be defined as follows:

$$\hat{E}=-i_{\xi_{(t)}}F=(-1)^{D\star}(\xi_{(t)}\wedge{}^\star F), \quad (52)$$

where $\xi_{(t)}=\xi_{(t)\mu}\,dx^\mu$ is the timelike Killing one-form field and the $\star$ operator denotes the Hodge dual. Substituting Eq. (48) in Eq. (52) we obtain the following expression for the electric one-form field in the metric (1):

$$\hat{E}=\frac{Q}{\Sigma^2}[r\,dr+(b^2-a^2)\sin\theta\cos\theta\,d\theta]. \quad (53)$$

The orthonormal components of the electric field in the asymptotic rest frame of the black hole are obtained by projecting (53) on the basis (25). We have

$$E_{\hat{r}}=F_{\hat{r}\hat{t}}=\frac{Q}{r^3}+\mathcal{O}\left(\frac{1}{r^5}\right),$$

$$E_{\hat{\theta}}=F_{\hat{\theta}\hat{t}}=\mathcal{O}\left(\frac{1}{r^5}\right),$$

$$E_{\hat{\phi}}=F_{\hat{\phi}\hat{t}}=0. \quad (54)$$

We note that the dominant component of the electric field is purely radial and the associated Gaussian flux of this radial field gives the correct value for the electric charge of the black hole.

The dipole magnetic field of the black hole is described by the magnetic two-form defined as

$$\hat{B}=-i_{\xi_{(t)}}{}^\star F={}^\star(\xi_{(t)}\wedge F). \quad (55)$$

This can also be rewritten in the alternative form

$$\hat{B}=\frac{1}{4}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta\gamma}\xi^{\mu}_{(t)}F^{\nu\alpha}\,dx^\beta\wedge dx^\gamma. \quad (56)$$

Substituting into this expression the contravariant components of the electromagnetic field tensor (51), we obtain

$$\hat{B}=\frac{Qb\sin\theta}{\Sigma^2}[r\sin\theta\,dr\wedge d\phi-(r^2+a^2)\cos\theta\,d\theta\wedge d\phi]$$
$$+\frac{Qa\cos\theta}{\Sigma^2}\cos\theta[r\cos\theta\,dr\wedge d\psi$$
$$+(r^2+b^2)\sin\theta\,d\theta\wedge d\psi], \quad (57)$$

which in the asymptotic rest frame of the black hole has the following orthonormal components:

$$B_{\hat{r}\hat{\psi}}=F_{\hat{\theta}\hat{\phi}}=\frac{F_{\theta\phi}}{r^2\sin\theta}=\frac{Qa}{r^4}\cos\theta+\mathcal{O}\left(\frac{1}{r^6}\right),$$

$$B_{\hat{\theta}\hat{\psi}}=F_{\hat{\phi}\hat{r}}=\frac{F_{\phi r}}{r\sin\theta}=\frac{Qa}{r^4}\sin\theta+\mathcal{O}\left(\frac{1}{r^6}\right),$$

$$B_{\hat{\phi}\hat{r}}=F_{\hat{\theta}\hat{\psi}}=\frac{F_{\theta\psi}}{r^2\cos\theta}=-\frac{Qb}{r^4}\sin\theta+\mathcal{O}\left(\frac{1}{r^6}\right),$$

$$B_{\hat{\theta}\hat{\phi}}=F_{\hat{r}\hat{\psi}}=\frac{F_{r\psi}}{r\cos\theta}=-\frac{Qb}{r^4}\cos\theta+\mathcal{O}\left(\frac{1}{r^6}\right), \quad (58)$$

with all others vanishing. The above expressions describe the dipole magnetic field created by a five-dimensional weakly charged rotating black hole. We see that far from the black hole the dominating behavior of the magnetic field is determined only by the two independent quantities

$$\mu_{(a)}=Qa, \quad \mu_{(b)}=Qb, \quad (59)$$

which can be thought of as the magnetic dipole moments of the black hole. We conclude that a weakly charged rotating black hole in five dimensions possesses two independent magnetic moments specified only in terms of the electric charge of the black hole and its two rotation parameters.

V. GYROMAGNETIC RATIO

It is now natural to address the gyromagnetic ratio of the five-dimensional weakly charged rotating black hole we discussed above. We recall that one of the remarkable facts about a charged rotating black hole of four-dimensional general relativity is that it can be assigned a gyromagnetic ratio $g=2$ just like the electron in Dirac theory [38]. The parameter g is defined as a constant of proportionality in the equation

$$\mu = g\frac{QJ}{2M}, \tag{60}$$

where M is the mass, J is the angular momentum, and Q is the electric charge of the four-dimensional black hole.

Turning now to the case of a weakly charged black hole in five dimensions and comparing Eqs. (13) and (59), we see that the coupling of rotation parameters of the black hole to its mass parameter to give the specific angular momenta looks exactly the same as their coupling to the electric charge to define the magnetic dipole moments. Thus, we may write the analogue of Eq. (60) in five dimensions as follows:

$$\mu_{(i)} = \frac{Qj_{(i)}}{m} = 3\frac{QJ_{(i)}}{2M}, \tag{61}$$

where we have used the relations (16) and the subscript index i refers to either the parameter a or b. From a comparison of this equation with the classical relation (60), it becomes apparent that a five-dimensional weakly charged rotating black hole can be assigned a gyromagnetic ratio $g = 3$.

Next, following the basic arguments of [28], we shall prove the value $g=3$. For this purpose, we shall define the twist [6] of a timelike Killing one-form field $\xi_{(t)}$, which in five dimensions is the two-form field given by

$$\Omega = \frac{1}{2}\ {}^{\star}(\hat{\xi}_{(t)} \wedge d\hat{\xi}_{(t)}). \tag{62}$$

Physically, this quantity measures the failure of the timelike Killing one-form field to be hypersurface orthogonal. Evaluating this quantity in the metric (1), we obtain

$$\Omega = -\frac{bmr}{\Sigma^2}\sin^2\theta\, dr \wedge d\phi + \frac{bm(r^2+a^2)}{\Sigma^2}\sin\theta\cos\theta\, d\theta \wedge d\phi - \frac{amr}{\Sigma^2}\cos^2\theta\, dr \wedge d\psi - \frac{am(r^2+b^2)}{\Sigma^2}\sin\theta\cos\theta\, d\theta \wedge d\psi, \tag{63}$$

which implies the existence of the twist potential one-form

$$\omega = \frac{m}{2\Sigma}(b\sin^2\theta\, d\phi + a\cos^2\theta\, d\psi). \tag{64}$$

The components of this quantity in the asymptotic rest frame of the black hole show that the failure of the timelike Killing vector to be hypersurface orthogonal is completely determined by the specific angular momenta $j_{(a)} = am$ and $j_{(b)} = bm$ of the black hole.

On the other hand, the magnetic two-form field (57) implies the magnetic potential one-form determined through the equation

$$B = -d\varphi, \tag{65}$$

where

$$\varphi = \frac{Q}{2\Sigma}(b\sin^2\theta\, d\phi + a\cos^2\theta\, d\psi). \tag{66}$$

From this expression it follows that in the asymptotic rest frame of the black hole the magnetic potential one-form determines the two magnetic dipole moments, just as the twist one-form (64) determines the two specific angular momenta of the black hole. From Eqs. (64) and (66) we read off the relation

$$\varphi = \frac{Q}{m}\omega, \tag{67}$$

which, obviously, can also be rewritten in the form of (61). This proves that a five-dimensional Myers-Perry black hole endowed with a small enough electric charge must have a gyromagnetic ratio of value $g=3$. The same value of gyromagnetic ratio has been found for a supersymmetric rotating black hole [39] described by the Breckenridge-Myers-Peet-Vafa (BMPV) five-dimensional solution [40].

VI. CONCLUSIONS

We have discussed the special properties of a five-dimensional rotating black hole described by the Myers-Perry metric in the presence of an originally uniform magnetic field. The configuration of the magnetic field is supposed to have biazimuthal symmetry, just like the black hole spacetime itself. In this case the magnetic field has only two nonvanishing components aligned with the two angular momenta of the black hole. We have also allowed the black hole to have an electric charge small enough that the spacetime can still be described by the Myers-Perry solution.

We have constructed the five-vector potential describing the test Maxwell field in the Myers-Perry spacetime using the Killing isometries of this spacetime. The intriguing feature of this model is the appearance of nontrivial gravitomagnetic phenomena; a rotation of a five-dimensional black hole in a uniform magnetic field of given configuration produces an inductive electrostatic potential difference between the event horizon and an infinitely distant surface. This potential difference comes from the superposition of two independent Coulomb fields arising due to rotations in two distinct two-planes and two nonvanishing components of the magnetic field. Of course, in the case of an ionized medium surrounding the black hole, the potential difference will be quickly neutralized by a selective accretion process, thereby providing a mechanism for charging up the black hole.

We have also described a dipole magnetic field around a weakly charged rotating black hole in five dimensions and, as expected, it turned out that the black hole possesses two independent magnetic dipole moments determined only by its electric charge, mass, and angular momentum parameters. In many aspects the gravitomagnetic phenomena described are qualitatively closely reminiscent of their counterparts for a four-dimensional Kerr black hole immersed in a uniform magnetic field. However, there also exist some essential differences. In particular, we have shown that the gyromagnetic ratio for a five-dimensional weakly charged Myers-Perry black hole is $g=3$.

In four-dimensional gravity there exist stable circular orbits in the equatorial plane of a Kerr black hole. Furthermore, the presence of a uniform magnetic field around the Kerr black hole has its greatest effect in enlarging the region of stability of the circular orbits toward the horizon [41]. However, there are no stable circular orbits around a five-dimensional rotating black hole, at least in the equatorial planes [20]. Therefore, it would be interesting to use the results of this paper to study the effect of an external magnetic field on the stability of circular motion around a five-dimensional Myers-Perry metric.

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