Inflationary slow-roll formalism and perturbations in the Randall-Sundrum type II braneworld

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We formalize the Hubble slow-roll formalism for inflationary dynamics in Randall-Sundrum type II braneworld cosmologies, defining Hubble slow-roll parameters which can be used along with the Hamilton-Jacobi formalism. Focusing on the high-energy limit, we use these to calculate the exact power spectrum for power-law inflation, and then perturb around this solution to derive the higher-order expression for the density perturbations (sometimes called the Stewart-Lyth correction) of slow-roll braneworld models. Finally we apply our result to specific examples of potentials to calculate the correction to the amplitude of the power spectrum, and compare it with the standard cosmology. We find that the amplitude is not changed significantly by the higher-order correction.

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I. INTRODUCTION

New ideas in fundamental physics have suggested the possibility that our observable Universe lies on a ''brane'' within a higher-dimensional bulk space-time, and this idea may have serious ramifications for early Universe cosmology [1]. The most studied scenario, viewed as a toy model for more elaborate proposals, is the Randall-Sundrum type II (RS-II) model where there is a single brane living in an anti–de Sitter bulk [2]. In that case, the Friedmann equation is modified at high energy, potentially with significant consequences for our understanding of inflation and the perturbations it generates.

The most powerful tool for studying inflationary dynamics is the slow-roll approximation [3], which is expected to be extremely accurate for models capable of matching current observations such as those of the Wilkinson Microwave Anisotropy Probe (WMAP) [4]. This enables considerable analytical progress to be made, and predictions for the perturbations generated by inflation are most conveniently expressed in terms of slow-roll parameters which measure the accuracy of the approximation [5]. Such an approach was used by Maartens et al. [6] to describe the outcome of inflation in the RS-II braneworld scenario.

In that paper, the version of the slow-roll approximation used was based on derivatives of the potential driving inflation. In this paper we formulate the approximation in terms of derivatives of the Hubble parameter, which in the standard cosmology is an important tool for developing high-accuracy predictions for density perturbations. We take advantage of our formalism to carry out similar calculations for the high-energy limit of RS-II inflation models, deriving an exact result for power-law inflation and calculating the higher-order slow-roll correction to the perturbation amplitude (often called the Stewart-Lyth correction, as they carried out the original calculation for the standard cosmology [7]). We find that the correction is of a similar size to that in the standard cosmology (i.e. negligible in many inflationary models), and calculate it for some sample inflation models.

II. SLOW-ROLL FORMALISM

We follow the notation set down by Liddle and Taylor [8]. In the Randall-Sundrum type II model [2] the Friedmann equation receives an additional term quadratic in the density [9]. The Hubble parameter H is related to the energy density ρ by

$$H^2=\frac{8\pi}{3M_4^2}\rho\left(1+\frac{\rho}{2\lambda}\right), \tag{1}$$

where M_4 is the four-dimensional Planck mass and λ is the brane tension. We have set the four-dimensional cosmological constant to zero, and assumed that inflation rapidly makes any dark radiation term negligible. This reduces to the usual Friedmann equation for $\rho\ll\lambda$. We assume that the scalar field is confined to the brane, so it obeys the usual equation

$$\ddot{\phi}+3H\dot{\phi}=-V', \tag{2}$$

where prime indicates derivative with respect to ϕ, and dot a derivative with respect to time. Its energy density is $\rho=V+\dot{\phi}^2/2$.

For the standard cosmology, the Hubble slow-roll formalism was set down in detail in Ref. [10]. The first two parameters are defined as

$$\epsilon_{\rm H}\equiv 3\frac{\dot{\phi}^2/2}{V+\dot{\phi}^2/2}=\frac{M_4^2}{4\pi}\frac{H'^2}{H^2}, \tag{3}$$

$$\eta_{\rm H}\equiv -3\frac{\ddot{\phi}}{3H\dot{\phi}}=\frac{M_4^2}{4\pi}\frac{H''}{H}. \tag{4}$$

If these parameters are much less than one, they allow the neglect of the $\dot{\phi}$ term in the Friedmann equation, and the $\ddot{\phi}$ term in the scalar wave equation [3]. In addition, the condition for inflation, $\ddot{a}>0$, is conveniently expressed as $\epsilon_{\rm H}<1$.

We seek a generalization of those parameters suitable for use with the braneworld equations. We note that there is some arbitrariness to the definitions, at least in terms of the constant prefactor. However this can be removed by imposing the requirements that the parameters $\epsilon_{\rm H}$ and $\eta_{\rm H}$ continue to correspond to the conditions enabling neglect of terms in

the Friedmann and fluid equations respectively, that $\epsilon_{\rm H}<1$ remains the condition for inflation, and that in the slow-roll limit the density perturbation spectral index takes the same form as in the standard cosmology.

We will make use of an approach developed by Hawkins and Lidsey [11,12], who devised a formalism for braneworld inflation with many of the properties of the Hamilton-Jacobi approach used in the standard cosmology [13]. They define a quantity $y(\phi)$, which is to play a similar role to $H(\phi)$ in the standard cosmology, by

$$y^2=\frac{\rho/2\lambda}{1+\rho/2\lambda}. \tag{5}$$

The inverse relation is

$$\rho\equiv\frac{2\lambda y^2}{1-y^2}. \tag{6}$$

At low energies $\rho\ll\lambda$, $y^2\to 0$, while at high energies $\rho\gg\lambda$, $y^2\to 1$.

The Friedmann equation can be written as

$$H(y)=\left(\frac{16\pi\lambda}{3M_4^2}\right)^{1/2}\frac{y}{1-y^2}. \tag{7}$$

From the scalar wave equation and Eq. (6) one obtains

$$\dot\phi=-\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\frac{y'}{1-y^2}. \tag{8}$$

Taking the derivative with respect to the field in the Friedmann equation, one finds

$$H'=\left(\frac{16\pi\lambda}{3M_4^2}\right)^{1/2}\left[\frac{1+y^2}{(1-y^2)^2}\right]y', \tag{9}$$

and using Eq. (8)

$$H'=-\frac{4\pi}{M_4^2}\frac{(1+y^2)}{(1-y^2)}\dot\phi. \tag{10}$$

In Ref. [12], Hawkins and Lidsey define two parameters

$$\beta\equiv\frac{M_4^2}{4\pi}\frac{y'^2}{y^2},\qquad \gamma\equiv\frac{M_4^2}{4\pi}\frac{y''}{y^2}, \tag{11}$$

by analogy to the standard cosmology Hubble slow-roll parameters. These parameters prove useful in analyzing the exact dynamics of braneworld inflationary models. However, their smallness (as compared to unity) does not precisely correspond to the ability to neglect terms in the Friedmann and scalar wave equations, and this means they are not ideal for the purpose of analyzing perturbation generation. We therefore define new Hubble slow-roll parameters for the braneworld, which do have this property.

To do this, we define the parameters as ratios of terms in the Friedmann and wave equations, following Eqs. (3) and (4):

$$\epsilon_{\rm H}=C(y)\frac{\dot\phi^2/2}{V+\dot\phi^2/2},\qquad \eta_{\rm H}=-D(y)\frac{\ddot\phi}{3H\dot\phi}, \tag{12}$$

At this stage we have allowed the ''constant'' prefactors $C(y)$ and $D(y)$ to depend on y; we will next show how to fix them using the requirements that $\epsilon_{\rm H}<1$ corresponds to inflation, and that in the slow-roll limit the density perturbation spectral index takes its usual form. In order for these parameters to correspond to the ability to neglect terms, those prefactors should always be of order unity, and we will soon see that they are.

From the definition of $\epsilon_{\rm H}$, using Eqs. (7) and (10), one can find

$$\epsilon_{\rm H}=\frac{C(y)}{3}\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\frac{y}{(1+y^2)^2}\frac{H'^2}{H^3}. \tag{13}$$

The coefficient $C(y)$ can be determined by demanding that $\ddot a=0\Leftrightarrow\epsilon_{\rm H}=1$; taking the derivative of the Friedmann equation with respect to time gives

$$\frac{\ddot a}{a}=\frac{16\pi\lambda y^2}{M_4^2(1-y^2)^2}\left[\frac{-(1+y^2)}{C(y)}+\frac{1}{3}\right] \tag{14}$$

and so we require $C(y)\equiv 3(1+y^2)$. Its value ranges from 3 in the low-energy limit to 6 in the high-energy limit. Our definition for $\epsilon_{\rm H}$ is therefore

$$\epsilon_{\rm H}\equiv\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\frac{y}{(1+y^2)}\frac{H'^2}{H^3}. \tag{15}$$

In the low-energy limit, Eq. (15) becomes the usual expression Eq. (3).

For $\eta_{\rm H}$ we apply Eqs. (7) and (10) to obtain

$$\eta_{\rm H}=\frac{D(y)}{3}\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\left[\frac{y}{(1+y^2)}\frac{H''}{H^2}-\frac{4y^3}{(1+y^2)^3}\frac{H'^2}{H^3}\right]. \tag{16}$$

There are several ways one could aim to fix the constant $D(y)$ to establish a unique definition of $\eta_{\rm H}$. We choose to do so such that the slow-roll expression for the density perturbation spectral index takes its usual form, namely $n=1-4\epsilon_{\rm H}+2\eta_{\rm H}$, as was done for the potential slow-roll parameters in Ref. [6]. The slow-roll expression for the perturbation amplitude is [6]

$$\mathcal{P}_{\mathcal{R}}{}^{1/2}=\left.\frac{H^2}{2\pi\dot\phi}\right|_{k=aH}, \tag{17}$$

which in terms of those variables is exactly the usual result. The spectral index is defined as

$$n-1=\frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k}, \tag{18}$$

and using Eqs. (7) and (10) one obtains

$$n-1=-4\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\frac{y}{(1+y^2)}\frac{H'^2}{H^3}+2\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\left[\frac{y}{(1+y^2)}\frac{H''}{H^2}-\frac{4y^3}{(1+y^2)^3}\frac{H'^2}{H^3}\right]. \tag{19}$$

This agrees with the usual expression $n=1-4\epsilon_{\rm H}+2\eta_{\rm H}$ provided $D(y)$ is set to 3 for all regimes. That gives us our definition

$$\eta_{\rm H}\equiv\left(\frac{\lambda M_4^2}{3\pi}\right)^{1/2}\left[\frac{y}{(1+y^2)}\frac{H''}{H^2}-\frac{4y^3}{(1+y^2)^3}\frac{H'^2}{H^3}\right]. \tag{20}$$

This too reduces to the usual expression, Eq. (4), in the low-energy regime.

The two definitions Eqs. (15) and (20) define Hubble slow-roll parameters valid in all regimes of RS-II brane inflation, generalizing the usual ones while preserving many key results: they give the conditions for neglecting terms in the Friedmann and fluid equations, $\epsilon_{\rm H}<1$ corresponds to an inflationary expansion, and the slow-roll spectral index formula is always $n=1-4\epsilon_{\rm H}+2\eta_{\rm H}$.

III. EXACT AND HIGHER-ORDER PERTURBATIONS IN THE HIGH-ENERGY REGIME

In this section we exploit the formalism of the previous section to make accurate calculations of the density perturbations. Throughout this section we will restrict ourselves to the high-energy regime, obtained by taking $y\to 1$, where our slow-roll parameters can be written

$$\epsilon_{\rm H}=\frac{M_5^3}{4\pi}\frac{H'^2}{H^3},\quad \eta_{\rm H}+\epsilon_{\rm H}=\frac{M_5^3}{4\pi}\frac{H''}{H^2} \tag{21}$$

where $M_5\equiv(4\pi\lambda/3)^{1/6}M_4^{1/3}$ is the five-dimensional Planck mass. The high-energy versions of Eqs. (10) and (7) are

$$\dot\phi=-\frac{M_5^3}{4\pi}\frac{H'}{H} \tag{22}$$

and

$$H=\frac{4\pi}{3M_5^3}\rho. \tag{23}$$

All these expressions could have been obtained directly for the high-energy regime using the same criteria we set down for the general case in the previous section.

We will consider one case where the perturbations can be obtained exactly (namely power-law inflation, though in this case the corresponding potential is not exponential), and then carry out our main calculation which is to compute the correction to the density perturbation amplitude from next-order in slow roll. This type of calculation was first performed by Stewart and Lyth for the standard cosmology [7], and is often known as the Stewart-Lyth correction. We will compute its equivalent for the density perturbations in the high-energy regime of the RS-II model.

We will calculate the perturbations using a formalism due to Mukhanov [14]. He defined a new variable $u=a\delta\phi$ (where $\delta\phi$ is defined in the spatially-flat gauge; a gauge-invariant definition can be made which includes a contribution from the curvature perturbation), and demonstrated that in linear perturbation theory its Fourier modes obey the wave equation

$$\frac{d^2u_k}{d\tau^2}+\left(k^2-\frac{1}{z}\frac{d^2z}{d\tau^2}\right)u_k=0. \tag{24}$$

Here τ is the conformal time, and $z\equiv a\dot\phi/H$ encodes all the relevant information about how the background is evolving. It is by no means clear that the Mukhanov equation remains correct in the braneworld context; it encodes the backreaction from scalar metric perturbations on the scalar field evolution, assuming they take the 4D form. So far it has not proven possible to calculate the backreaction from five-dimensional gravity in order to assess whether it is different from the four-dimensional backreaction that the Mukhanov equation incorporates, other than in the large-scale limit where energy conservation alone is sufficient to ensure the perturbations remain constant [15]. Nevertheless, until such a calculation becomes possible it is a reasonable working hypothesis to use the Mukhanov equation,[1] and we have verified explicitly that the same equation does result using the modified Friedmann equation in the derivation as given in Ref. [16], as well as giving the correct result for the perturbation amplitude in the slow-roll limit. This however falls short of being a full five-dimensional calculation as would be required to fully verify the use of the equation, but as yet it is not known how to implement such a calculation.

Having adopted the Mukhanov equation, the first step is to write it in terms of the slow-roll parameters, with a lengthy calculation yielding

$$\frac{1}{z}\frac{d^2z}{d\tau^2}=2a^2H^2\left[1+\epsilon_{\rm H}-\frac{3}{2}\eta_{\rm H}-\frac{7}{2}\epsilon_{\rm H}\eta_{\rm H}+\frac{1}{2}\eta_{\rm H}^2+\frac{M_5^6}{32\pi^2}\frac{H'H'''}{H^4}\right], \tag{25}$$

which is an exact relation.

A. Exact mode equation solution

It is well known that the mode function can be solved exactly if the square-bracketed term in Eq. (25) is constant,

[1]Indeed, after the original version of this paper was submitted, Calcagni [17] analyzed the inflationary consistency equations for the braneworld to second order. To do so he made assumptions about how scalar and tensor perturbations might behave, and for the scalars his assumptions are equivalent to using the Mukhanov equation and give the same result as we obtain.

which in the standard cosmology corresponds to power-law inflation from an exponential potential [18], the calculation having first been performed by Lyth and Stewart [19]. To discover if there is an analogous result for the RS-II scenario, we set $\epsilon_{\rm H}=1/p$ where $p>1$ is a constant, and examine whether this makes the square bracket constant.

Taking the derivative of $\epsilon_{\rm H}$ with respect to the field, one can write

$$\epsilon_{\rm H}'=\frac{H'}{H}[2\eta_{\rm H}-\epsilon_{\rm H}]. \tag{26}$$

As we have demanded $\epsilon_{\rm H}$ is constant, this implies $\eta_{\rm H}=1/2p$. Similarly, differentiating $\eta_{\rm H}$ gives

$$\frac{M_5^6}{32\pi^2}\frac{H'H'''}{H^4}=\epsilon_{\rm H}\left[(\eta_{\rm H}+\epsilon_{\rm H})+\frac{H}{2H'}(\eta_{\rm H}'+\epsilon_{\rm H}')\right] \tag{27}$$

implying

$$\frac{M_5^6 H'H'''}{32\pi^2 H^4}=\frac{3}{2p^2}. \tag{28}$$

The square bracket of Eq. (25) therefore indeed is constant, so the equation can be solved exactly.

Before going on to do that, it is interesting to ask what potential gives this solution. Solving for the Hubble parameter from the definition of $\epsilon_{\rm H}$, and then substituting into the Hamilton-Jacobi equation in the high-energy limit, namely

$$H-\frac{M_5^3}{24\pi}\frac{H'^2}{H^2}=\frac{4\pi}{3M_5^3}V(\phi), \tag{29}$$

we find that the corresponding potential is

$$V(\phi)=\frac{1}{8}\frac{M_5^6(6p-1)}{\pi^2\phi^2}. \tag{30}$$

Instead of the exponential potential found in the standard cosmology, we have an inverse power-law potential. Nevertheless, the corresponding expansion law $a\propto t^p$ is power-law inflation as usual.

Following Refs. [7,19,20], the conformal time for constant $\epsilon_{\rm H}$ is given by

$$\tau=-\frac{1}{aH}\frac{1}{1-\epsilon_{\rm H}}, \tag{31}$$

and Eq. (25) with the values of the Hubble slow-roll parameters gives

$$\frac{1}{z}\frac{d^2z}{d\tau^2}=\frac{1}{4}\frac{8p^2+2p-1}{\tau^2(p-1)^2}. \tag{32}$$

This allows one to write the Mukhanov equation (24) as a Bessel-like one

$$\left[\frac{d^2}{d\tau^2}+k^2-\frac{(\nu^2-1/4)}{\tau^2}\right]u_k=0, \tag{33}$$

with $\nu=3p/2(p-1)$. The solution with the appropriate behavior at small scales can be written as

$$u_k(\tau)=\frac{\sqrt{\pi}}{2}e^{i(\nu+1/2)\pi/2}(-\tau)^{1/2}H_\nu^{(1)}(-k\tau), \tag{34}$$

where $H_\nu^{(1)}$ is the Hankel function of the first kind of order ν. The asymptotic form of this equation once the mode is outside of the horizon is obtained by taking the limit $k/aH\to 0$

$$u_k\to e^{i(\nu-1/2)\pi/2}2^{\nu-3/2}\frac{\Gamma(\nu)}{\Gamma(3/2)}\frac{1}{\sqrt{2k}}(-k\tau)^{-\nu+1/2}, \tag{35}$$

from which the corresponding form of the power spectrum using [20]

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k)=\sqrt{\frac{k^3}{2\pi^2}}\left|\frac{u}{z}\right| \tag{36}$$

yields

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k)=\frac{3^{\nu-1/2}\nu^{1/2-\nu}}{M_5^3}\frac{\Gamma(\nu)}{\Gamma(3/2)}\left.\frac{H^3}{|H'|}\right|_{k=aH}. \tag{37}$$

B. Higher-order perturbation calculation

We now perturb around the exact solution given above for small $\epsilon_{\rm H}$ and $\eta_{\rm H}$, following Refs. [7] and [20]. The expansion to lowest order of the conformal time gives

$$\tau=-\frac{1}{aH}(1+\epsilon_{\rm H}). \tag{38}$$

Applying this in Eq. (25) and truncating the expansion to first order, one arrives at another Bessel equation, now with ν given by

$$\nu\simeq\frac{3}{2}+2\epsilon_{\rm H}-\eta_{\rm H}. \tag{39}$$

Note that the final three terms of Eq. (25) do not affect the form of this expression. Then the solution Eq. (35) can be used with the new form for ν and the conformal time, expanding also the Gamma function and the other expressions to first order, to obtain the final result

$$\mathcal{P}_{\mathcal{R}}^{1/2}=[1-(2C+1)\epsilon_{\rm H}+C\eta_{\rm H}]\left.\frac{2H^3}{M_5^3|H'|}\right|_{k=aH} \tag{40}$$

where $C=-2+\ln 2+b\simeq-0.73$, with b the Euler-Mascheroni constant. The leading-order term, obtained by setting the square bracket to one, agrees with the result of Maartens *et al.* [6] in the high-energy limit.

From this expression we can also obtain the second-order formula for the spectral index of the perturbations in the high-energy regime, using the high-energy relation

$$\frac{d\phi}{d\ln k} = -\frac{M_5^3 H'}{4\pi H^2}(1+\epsilon_{\rm H}), \tag{41}$$

and keeping terms up to second order to obtain

$$n-1 = -4\epsilon_{\rm H} + 2\eta_{\rm H} + 2(8C+3)\epsilon_{\rm H}\eta_{\rm H} - 2(C+3)\epsilon_{\rm H}^2 - 4C\frac{M_5^6}{32\pi^2}\frac{H'H'''}{H^4}. \tag{42}$$

While formally the correction term in the square bracket of Eq. (40) looks exactly the same as in the standard cosmology [7,20], we should recall that the slow-roll parameters which appear in it are generalizations of those in the standard cosmology, and could have been defined in different ways. To get a feel for what this correction means, we need to evaluate it for some characteristic potentials, which we do in the next subsection.

C. Specific examples

To determine the typical size of the next-order correction, we study the monomial potentials $V \propto \phi^\alpha$ for $\alpha = 2$, 4 and 6, assuming inflation takes place well within the high-energy regime. For comparison, we also calculate the magnitude of the correction for the standard cosmology.

To calculate the size of the correction term, we can use the slow-roll approximation for $\epsilon_{\rm H}$ and $\eta_{\rm H}$, since any corrections to that will be of higher order. The simplest approach is to rewrite $\epsilon_{\rm H}$ and $\eta_{\rm H}$ in terms of the potential and its derivatives. We will make use of the equations

$$H \simeq \frac{4\pi}{3M_5^3}V, \tag{43}$$

$$3H\dot{\phi} \simeq -V', \tag{44}$$

$$\dot{\phi} = -\frac{M_5^3}{4\pi}\frac{H'}{H}. \tag{45}$$

The first and second of these use the slow-roll approximation, and the first and third use the high-energy approximation. These enable us to obtain the relations

$$\epsilon_{\rm H} \simeq \epsilon_{\rm V}, \quad \eta_{\rm H} \simeq \eta_{\rm V} - \epsilon_{\rm V}, \tag{46}$$

where the potential slow-roll parameters are

$$\epsilon_{\rm V} = \frac{3M_5^6}{16\pi^2}\frac{V'^2}{V^3}, \quad \eta_{\rm V} = \frac{3M_5^6}{16\pi^2}\frac{V''}{V^2}, \tag{47}$$

being the high-energy limit of the parameters as defined by Maartens et al. [6]. Using them, we can write Eq. (40) as

$$\mathcal{P}_{\mathcal{R}}^{1/2} = [1-(3C+1)\epsilon_{\rm V} + C\eta_{\rm V}]\frac{2H^3}{M_5^3|H'|}\bigg|_{k=aH}, \tag{48}$$

where we aim to calculate the square-bracketed term.

We assume that observable scales crossed the Hubble radius 50 *e*-foldings before the end of inflation, and need to compute the slow-roll parameters at that time. We take the potential as

$$V(\phi) = m\phi^\alpha, \tag{49}$$

where m is a constant. Putting this in Eq. (47) and setting $\epsilon_{\rm V} = 1$, which corresponds to the end of inflation, we can obtain the value of $\phi_{\rm end}$. We use this in the expression for the number of *e*-foldings in the high-energy limit [6]

$$N \simeq -\frac{16\pi^2}{3M_5^6}\int_{\phi_N}^{\phi_{\rm end}}\frac{V^2}{V'}d\phi. \tag{50}$$

Taking $N = 50$, this gives the value of ϕ_{50} and substituting in the equations for $\epsilon_{\rm V}$ and $\eta_{\rm V}$ one gets

$$\epsilon_{\rm V,50} = \frac{\alpha}{100+51\alpha}, \tag{51}$$

$$\eta_{\rm V,50} = \frac{\alpha-1}{100+51\alpha}. \tag{52}$$

In the case of standard cosmology, the same calculation is carried out using the corresponding expressions for the slow-roll parameters and the power spectrum given by Refs. [7,20]

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k) = [1-(2C+1)\epsilon_{\rm H} + C\eta_{\rm H}]\frac{2H^2}{M_4^2|H'|}\bigg|_{k=aH} \tag{53}$$

with $\epsilon_{\rm H}$ and $\eta_{\rm H}$, defined as in Eqs. (3) and (4), being the Hubble slow-roll parameters in the standard cosmology. To rewrite them in terms of the potential, we use the equations [20]

$$H^2 \simeq \frac{8\pi}{3M_4^2}V, \tag{54}$$

$$3H\dot{\phi} \simeq -V', \tag{55}$$

$$\dot{\phi} = -\frac{M_4^2}{4\pi}H', \tag{56}$$

which are the standard cosmology equivalents of Eqs. (43)–(45). This leads to the same relations as in the previous case

$$\epsilon_{\rm H} \simeq \epsilon_{\rm V}, \quad \eta_{\rm H} \simeq \eta_{\rm V} - \epsilon_{\rm V}, \tag{57}$$

with

$$\epsilon_{\rm V} = \frac{M_4^2}{16\pi}\frac{V'^2}{V^2}, \quad \eta_{\rm V} = \frac{M_4^2}{8\pi}\frac{V''}{V}. \tag{58}$$

TABLE I. Fractional correction to the power spectrum.

	Standard cosmology			High-energy braneworld		
	$\epsilon_{\mathrm{V},50}$	$\eta_{\mathrm{V},50}$	correction	$\epsilon_{\mathrm{V},50}$	$\eta_{\mathrm{V},50}$	correction
$\alpha=2$	$\frac{1}{101}$	$\frac{1}{101}$	1.0046	$\frac{1}{101}$	$\frac{1}{202}$	1.0082
$\alpha=4$	$\frac{1}{51}$	$\frac{3}{102}$	1.0019	$\frac{1}{76}$	$\frac{3}{304}$	1.0085
$\alpha=6$	$\frac{3}{103}$	$\frac{5}{103}$	0.9992	$\frac{3}{203}$	$\frac{5}{406}$	1.0086

Then the power spectrum can be written as

$$\mathcal{P}_{\mathcal{R}}^{1/2}(k)=\left[1-(3C+1)\epsilon_{\mathrm{V}}+C\eta_{\mathrm{V}}\right]\frac{2H^2}{M_4^2|H'|}\Bigg|_{k=aH} \tag{59}$$

where now $N\simeq-(8\pi/M_4^2)\int_{\phi_N}^{\phi_{\mathrm{end}}}(V/V')d\phi$.

The results for both regimes are shown in Table I. They show that the magnitude of the correction is similar in both cases, though it differs in detail. Nevertheless, these results confirm that the amplitude of the power spectrum is not changed significantly with respect to the slow-roll result by the higher-order correction.

D. Correction from the high-energy approximation

We end by mentioning that, especially when it is small, the higher-order slow-roll correction may well be subdominant to corrections coming from the high-energy approximation not being exact. The slow-roll approximation allows us to determine the size of such corrections; using Eq. (17) which is valid in any regime, we find the slow-roll perturbation amplitude to be

$$\mathcal{P}_{\mathcal{R}}^{1/2}\simeq\left[1+\frac{1}{2}\left(\frac{\lambda}{V}\right)^2\right]\frac{2H^3}{M_5^3|H'|}\Bigg|_{k=aH}, \tag{60}$$

where we have expanded the result in terms of the parameter λ/V which is small in the high-energy regime. Because the term linear in λ/V happens to cancel, the high-energy approximation is a better one than might have been expected, and we only need $V\gtrsim10\lambda$ to bring the correction from that approximation within one percent. More generally, comparison of Eqs. (40) and (60) allows a test of when the higher-order slow-roll correction dominates the correction to the high-energy approximation.

If one were very ambitious, one could also attempt to generalize our higher-order slow-roll calculation to be valid in any regime, but given the limited ability of observations to probe or distinguish amongst such small corrections, such a calculation does not seem worthwhile. The same remark applies to an attempt to calculate the higher-order gravitational wave spectrum correction; as gravitational waves are known to be subdominant such corrections are even less relevant, and also much harder to calculate due to the gravitational waves' ability to penetrate the bulk dimension.

IV. CONCLUSIONS

We have devised a Hubble slow-roll formalism for inflation in the RS-II braneworld cosmology, extending work by Hawkins and Lidsey [11,12] to define parameters which share the nice properties of those used in the standard cosmology, which are recovered in that limit. As an application, we have computed the density perturbation spectrum in the high-energy limit, both exactly for power-law inflation and to higher-order for general slow-roll inflation models. To do so we have used the Mukhanov equation; while no one has yet been able to show that this equation is still valid in the braneworld context, it is the best working hypothesis currently available. We have also quantified how well the high-energy approximation must hold in order for the higher-order slow-roll correction to be the dominant one.

It is interesting to note that, having defined the slow-roll parameters ϵ_{H} and η_{H} so as to give the usual spectral index formula for slow-roll perturbations, it turns out that the next-order correction is of the same form as in the standard cosmology. We are not aware of a physical reason which leads to this result. Nevertheless, for a given choice of potential one expects that observable perturbations are generated at a different location on that potential depending on the braneworld regime, and so predictions for both the spectral indices (see e.g. Ref. [21]) and for the higher-order corrections will be different. We have examined the magnitude of the correction for some simple potentials, and we conclude that there is no reason to believe that the higher-order correction might be more important in the high-energy regime than in the standard cosmology. As recent observations including WMAP have restricted viable inflation models to regions close to the slow-roll limit, such corrections are expected to be small.

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[1] For reviews, see F. Quevedo, Class. Quantum Grav. **19**, 5721 (2002); P. Brax and C. van de Bruck, *ibid.* **20**, R201 (2003).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[3] P.J. Steinhardt and M.S. Turner, Phys. Rev. D **29**, 2162 (1984); E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).
[4] C.L. Bennett *et al.*, Astrophys. J., Suppl. Ser. **148**, 1 (2003); D.N. Spergel *et al.*, *ibid.* **148**, 175 (2003); H.V. Peiris *et al.*, *ibid.* **148**, 213 (2003).
[5] A.R. Liddle and D.H. Lyth, Phys. Lett. B **291**, 391 (1992).
[6] R. Maartens, D. Wands, B.A. Bassett, and I.P.C. Heard, Phys. Rev. D **62**, 041301 (2000).
[7] E.D. Stewart and D.H. Lyth, Phys. Lett. B **302**, 171 (1993).
[8] A.R. Liddle and A.N. Taylor, Phys. Rev. D **65**, 041301 (2002).

[9] C. Csáki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999); J.M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999); P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000); T. Shiromizu, K.I. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).
[10] A.R. Liddle, P. Parsons, and J.D. Barrow, Phys. Rev. D **50**, 7222 (1994).
[11] R.M. Hawkins and J.E. Lidsey, Phys. Rev. D **63**, 041301 (2001).
[12] R.M. Hawkins and J.E. Lidsey, Phys. Rev. D **68**, 083505 (2003).
[13] D.S. Salopek and J.R. Bond, Phys. Rev. D **42**, 3936 (1990).
[14] V.F. Mukhanov, Pis'ma Zh. Éksp. Teor. Fiz. **41**, 402 (1985) [JETP Lett. **41**, 493 (1985)]; V.F. Mukhanov, Phys. Lett. B **218**, 17 (1989); see also M. Sasaki, Prog. Theor. Phys. **76**, 1036 (1986).
[15] D. Wands, K.A. Malik, D.H. Lyth, and A.R. Liddle, Phys. Rev. D **62**, 043527 (2000).
[16] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, England, 2000).
[17] G. Calcagni, J. Cosmol. Astropart. Phys. **0311**, 009 (2003).
[18] F. Lucchin and S. Matarrese, Phys. Rev. D **32**, 1316 (1985).
[19] D.H. Lyth and E.D. Stewart, Phys. Lett. B **274**, 168 (1992).
[20] J.E. Lidsey, A.R. Liddle, E.W. Kolb, E.J. Copeland, T. Barreiro, and M. Abney, Rev. Mod. Phys. **69**, 373 (1997).
[21] A.R. Liddle and A.J. Smith, Phys. Rev. D **68**, 061301(R) (2003).