Cosmological perturbations from inhomogeneous reheating, freeze-out, and mass domination

Gia Dvali and Andrei Gruzinov
Center for Cosmology and Particle Physics, Department of Physics, New York University, New York 10003, USA

Matias Zaldarriaga
Center for Cosmology and Particle Physics, Department of Physics, New York University, New York 10003, USA;
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA;
and Department of Astronomy, Harvard University, Cambridge, Massachusetts 02138, USA
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We generalize a recently proposed mechanism for the origin of primordial metric perturbations in inflationary models. Quantum fluctuations of light scalar fields during inflation gives rise to superhorizon fluctuations of masses and reaction rates of various particles. Reheating, freeze-out, and matter-domination processes become inhomogeneous and generate superhorizon metric perturbations. We also calculate the degree of non-Gaussianity f_{nl} for this new model of cosmological perturbations. The precise value of f_{nl} depends on the specific models, but $|f_{nl}|\sim$ few is a natural lower bound for our mechanisms. This is much larger than the currently assumed theoretical value $f_{nl}\sim$ tilt$\lesssim 0.05$, and is thought to be observable. In a particularly attractive model of inhomogeneous mass domination, the non-Gaussianity of perturbations generated by our mechanism is simply $f_{nl}=5$, irrespective of the detailed structure of the underlying field theory.

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I. INTRODUCTION

During inflation the energy density is dominated by the potential energy of a slowly rolling scalar field, the inflaton. At the end of inflation this energy density has to be converted into normal particles, reheating the Universe and starting the standard phase of the hot big bang. In [1] we suggested that if the inflaton decay rate Γ varied in space, density perturbations would be generated during reheating independently of those generated by the standard inflationary mechanism. If two different regions of the Universe had different Γ's then effectively the inflaton would decay into radiation first in one region and then in the other. During the time one region is filled with radiation while the other one is not, the Universe expands at a different rate in each region, resulting in density perturbations when reheating is finished. The decay rate of the inflaton is determined by the expectation values of some scalar fields. If those scalar fields were light during inflation they fluctuated, leading to density perturbations through the proposed mechanism. Here we extend and generalize this model (Sec. II). Generation of superhorizon curvature perturbations in preheating models has been discussed in the past [2], but the preheating models seem to be more involved and thus less conclusive than our simple analysis.

We also calculate the non-Gaussianity of the metric perturbations generated by our mechanism. The standard inflationary model predicts unobservably small non-Gaussianities, while our models generically predict non-Gaussianities of potentially observable magnitude (Sec. III).

II. MASS-DOMINATION MECHANISM

A. The mechanism

We first discuss a generalization of our original mechanism [1]. Assume that density perturbations created during inflation, as well as during reheating, are negligible. So right after reheating the Universe is filled with radiation of uniform energy density and temperature T_R. Assume that the mass and the decay rate (Γ) of some of the created particles (call them ψ) is set by the vacuum expectation value (VEV) of a scalar field ϕ,

$$M=\lambda\phi, \tag{1}$$

where λ is a coupling constant. We shall assume that the mass of ϕ during inflation is smaller than the Hubble parameter. Thus, inflationary fluctuations of ϕ on superhorizon scales get imprinted into the mass of ψ quanta. As long as the temperature is much larger than M, the perturbations in mass do not contribute into the energy density. However, once the temperature drops below M, the mass fluctuations become important. For the mass fluctuations to be imprinted into the density perturbations, it is essential that ψ dominates (or at least becomes a significant component of) the energy density of the Universe during some period. That is, the decay rate of ψ must satisfy

$$\Gamma<H_D, \tag{2}$$

where H_D the Hubble parameter at the moment when ψ starts dominating the energy density. For simplicity, we shall assume that the annihilation rate of ψ is much smaller than M^2/M_{Pl}. Then ψ start dominating as soon as they become non-relativistic ($T\sim M$, assuming that ψ were thermalized at some early time). The corresponding Hubble parameter is $H_D\sim M^2/M_{Pl}$.

If Eq. (2) is satisfied, the perturbations in M and Γ get translated into the density perturbations. Due to the variation of M and Γ, the interval of ψ domination in different regions of the Universe will be different leading to density perturbations at the end of the process.

We shall now derive the magnitude of the resulting density perturbations. We compare final radiation energies (after ψ decay) in different superhorizon regions, for the same value of the scale factor a. It is simplest to compare the energy density in any given region to the one in radiation, which scales as

$$\rho_{rad} \propto a^{-4}, \tag{3}$$

and is the same in all the regions of interest. The ψ-energy density in a region with mass M and decay rate Γ scales as radiation for all the values of a, except the interval of its domination, that is, between the domination and the decay. Domination starts at $a = a_{domination}$ when

$$\rho = \rho_{domination} = M^4 \tag{4}$$

and ends at $a = a_{decay}$, when

$$\rho = \rho_{decay} = \Gamma^2 M_{Pl}^2. \tag{5}$$

Outside this interval $a_{decay} > a > a_{domination}$, the energy density in all the domains scales as radiation and is independent of either M or Γ. However, during the domination interval energy of ψ scales as matter

$$\rho \propto a^{-3} \tag{6}$$

and becomes different in different domains. Thus, we have

$$\left(\frac{a_{decay}}{a_{domination}}\right)^3 = \frac{\rho_{domination}}{\rho_{decay}} = M^4 \Gamma^{-2} M_{Pl}^{-2}. \tag{7}$$

The final energy density stored in ψ, right before the decay, is

$$\rho \propto \frac{a_{decay}}{a_{domination}} \rho_{rad} = M^{4/3} \Gamma^{-2/3} M_{Pl}^{-2/3} \rho_{rad}. \tag{8}$$

The resulting density perturbations are given by

$$\frac{\delta\rho}{\rho} = \frac{4}{3}\frac{\delta M}{M} - \frac{2}{3}\frac{\delta\Gamma}{\Gamma}. \tag{9}$$

Depending on the underlying model, one can consider several possibilities. For instance, for $M = \text{const}$, this recovers our previous result [1], where now we are interpreting Γ as the decay rate of the inflaton rather than ψ. The results carry over from one scenario to the other because during reheating, when the oscillating inflaton dominates the energy density we also have $\rho \propto a^{-3}$. Another interesting case for the realistic model building is $\Gamma \propto M$, for which we get

$$\frac{\delta\rho}{\rho} = \frac{2}{3}\frac{\delta M}{M}. \tag{10}$$

B. Constraints

The necessary requirement for our mechanism is that an effective mass of ϕ must remain smaller than the Hubble parameter during inflation. However, this relation may or may not be violated during the ψ-domination period. If it is violated, then ϕ will start oscillations during the ψ domination epoch. The calculation of the resulting density perturbations for oscillating ϕ is more involved and will not be discussed here.

The simplest situation is when the effective mass of ϕ remains smaller than the Hubble parameter all the way until the ψ particles decay. This requirement strongly constraints the initial value of ϕ, provided the ϕ dependence of the ψ mass is significant. Below we shall estimate the constraint for the case of maximal dependence, that is when the entire mass of ψ comes from ϕ according to Eq. (1). Generalization for the case of milder dependence is obvious.

In many cases this puts a non-trivial constraint on the initial value of ϕ, as we shall now demonstrate. The crucial point is that at high temperature the non-zero density of ψ particles generates a large thermal mass for ϕ. For instance, every scalar degree of freedom that is in thermal equilibrium at temperature T, and is coupled to ϕ, generates the following contribution to the ϕ mass:

$$m_\phi^2 \sim \lambda^2 T^2. \tag{11}$$

This contribution is present irrespective of ϕ itself being in thermal equilibrium. This mass may exceed the value of the Hubble parameter during the ψ domination, unless the following condition is met:

$$\lambda T_D < H_D, \tag{12}$$

where $T_D = \lambda \phi$ is the domination temperature. This implies that ϕ must satisfy the following condition:

$$1 < \frac{\phi}{M_{Pl}}. \tag{13}$$

Notice that Eq. (13) guarantees that the condition $m_\phi < H$ will be satisfied both before and throughout the domination. This is easy to understand. Above the domination temperatures the thermal mass scales as temperature, and quickly becomes negligible relative to the Hubble parameter, which scales as T^2. During the domination period ψ particles are out of thermal equilibrium and their energy scales as nonrelativistic matter, according to Eq. (6). So does the mass of ϕ, $m_\phi^2 \propto \rho_\psi$. Thus, m_ϕ and H scale in the same way, and relation $m_\phi < H$ is maintained throughout the domination.

For typical ϕ dependence of M and Γ, $\phi \sim M_{Pl}$ implies that the amount of density perturbations created by the standard inflationary scenario will not be subdominant. So, our mechanism will be the primary source of perturbations only in inflationary scenarios that have no fluctuating inflaton field, e.g., such as recently proposed "self-terminated inflation" [3]. Even in this case the level of the gravitational wave background should be significant.

We should again stress that the above constraint is absent in the models in which M is independent of ϕ. In these class of models our source of density perturbations can dominate over the standard inflationary mechanism by many orders of magnitude.

C. Models

We shall now consider some practical implementations of our mechanism in realistic models. As is obvious from the above, we are interested in theories in which masses of dominating particles $M(\phi)$ and/or their decay rate(s) $\Gamma(\phi)$ are functions of the fluctuating flat-direction field ϕ. Since, in general the ϕ dependence of functions $M(\phi),\Gamma(\phi)$ can be very different, it is hard to talk about more than an order of magnitude predictions. However, there is a sub-class of theories in which all the mass scales (at least during the epoch of interest) are set by ϕ alone. In such a case $M(\phi)\propto\phi,\Gamma(\phi)\propto\phi$, and density perturbations via our mechanism acquire an especially simple form, practically independent of either the coupling constants or the field content. Interestingly, the standard model, as well as its minimal supersymmetric extension, in which ϕ is identified with a flat direction field, fall within this category.

Before proceeding we have to stress that there can be corrections to the masses and decay rates from other existing scales in the theory. For instance, non-perturbative gravity effects can contribute into the decay rates of the standard model particles. If such corrections are there in the first place, their relative value will depend on many factors, such as the existence of discrete gauge symmetries, masses of decaying particles, etc. It is not hard within a concrete model to make these corrections sub-dominant. This is beyond the scope of the present work, in which we shall deal only with the simplest minimal case.

We shall now apply our mechanism to the standard model, and to its minimal supersymmetric (MSSM) extension. We assume that (1) during inflation at least some of the MSSM flat direction fields (call them ϕ) have masses much less than H, and (2) after the inflaton decay, the Universe is left with a thermal gas of SM particles (and their superpartners) of uniform energy density. Under these assumptions we will demonstrate that the amount of the density perturbations generated through the cooling process, *irrespective* of the coupling constants or the nature of flat direction, is given by

$$\frac{\delta\rho}{\rho}=\frac{2}{3}\frac{\delta\phi}{\phi}, \tag{14}$$

provided some of the particles dominate the Universe for a short period.

To show this, let us first ignore superpartners and only consider standard model particles. Let ϕ be a flat direction corresponding to the standard model Higgs field. That is, we shall assume that the value of the Higgs boson mass during inflation satisfies our requirements, and that the Universe is reheated by producing some of the standard model particles. When the Universe cools down, the heavy particles decouple and decay into the lighter ones. The heavy states that have small annihilation and decay rates can dominate the Universe. Such can only be fermions (quarks and leptons), since the gauge bosons decay through the order one gauge couplings and decay before domination.

For a given species to dominate it must have frozen out but still have not had enough time to decay. Thus both its annihilation and decay rates must be less than H. This situation can always be achieved for some species for the appropriate initial value of ϕ. Assume for definiteness that $\phi\sim M_P$, and, of course, we shall assume that $T_R\ll M_P$, but is big enough that some of the unstable fermions are produced. We shall now discuss under what conditions some of the species can undergo a brief interval of domination. It is useful to consider electrically charged fermions and the neutral ones separately.

Charged fermions can annihilate into photons so their freeze-out abundance will be much smaller than M^3 unless their mass is of order $M\sim\alpha_{EM}^2 M_{Pl}$. Their mass, however, has to be smaller than the reheating temperature at the end of inflation for them to be in thermal equilibrium to start with, so $M<T_{rh}\lesssim\sqrt{H_{inf}M_{PL}}\lesssim 10^{-2}M_{PL}$. This case is only marginally possible so we will concentrate on uncharged fermions, i.e., neutrinos.

The story with neutrinos is very different, as we shall now discuss. In order to understand how neutrinos can dominate in the very early Universe, we shall specify the origin of their mass. For definiteness, we shall assume the standard "seesaw" mechanism [4], in which the neutrino masses are generated by mixing to the heavy gauge-singlet fermions (right-handed neutrinos). The relevant terms in the Lagrangian are

$$\lambda_\nu\phi\overline{\nu_L}\nu_R+M_R\nu_R\nu_R+\ \dots, \tag{15}$$

where M_R is the Majorana mass of the gauge-singlet fermion, λ is an Yukawa coupling constant, and generation indices are suppressed.

In today's Universe

$$\lambda_\nu\phi\ll M_R, \tag{16}$$

and heavy neutrinos can be integrated out. As a result of this integration the light neutrinos acquire small Majorana masses given by

$$M_\nu\sim\frac{(\lambda_\nu\phi)^2}{M_R}. \tag{17}$$

However, in the epoch of our interest $\phi\sim M_{Pl}$, and the condition (16) can be violated. Depending whether this is the case, the light neutrino will either continue to be a Majorana particle with mass (17), or will effectively become a Dirac particle of mass

$$M_\nu\sim\lambda_\nu\phi. \tag{18}$$

The neutrino annihilation rate into the fermions happens through the exchange of heavy Z,W bosons, with masses $\sim M_{Pl}$, and the annihilation into the Higgs particles is suppressed by λ_ν^4/M_R^2. So it is safe to assume that their abundance froze out early on.

The decay rate of neutrinos requires some attention. In the early Universe some of the neutrinos that are stable today could have been unstable. For instance, since the electron and quark masses scale linearly with ϕ as opposed to neutrino masses that scale quadratically, the electron neutrino ν_e can become heavier and decay into the electron and the quark–anti-quark pair

$$\nu_e \to e^- + u + \overline{d}. \tag{19}$$

In general, the standard model fermions decay through the exchange of W bosons and the decay rate is

$$\Gamma = f M^5/\phi^4, \tag{20}$$

where f is a small constant that depends on mixing angles, and is different for different fermions. The crucial point is that because M is set by ϕ, Γ is linear in ϕ (for neutrino this requires $\lambda_\nu \phi > M_R$)

$$\Gamma = f\lambda^5 \phi. \tag{21}$$

As a result of this, irrespective of which fermion happens to dominate, the imprint of density perturbation is *universally* given by Eq. (14). It is easy to check that for $\phi \sim M_{Pl} \sim 10^4 H$, the domination condition is always satisfied by at least some SM fermions. For large values of ϕ and H, our mechanism can operate several times and each time imprint the density perturbations.

Interestingly, going to MSSM does not modify this general result, irrespective of which flat direction develops large VEV during inflation. This can be understood in the following way. Let ϕ be some MSSM flat direction. ϕ breaks gauge symmetry and gives masses proportional to ϕ to some gauge fields and fermions (and their superpartners). Since, for the needed magnitude of perturbations the flat direction VEV must be much greater than H, the supersymmetry breaking effects can be ignored. So we can neglect the mass splittings between the superpartners. Thus, irrespective of the particular flat direction, the ϕ dependence of Γ will be the same. This is obvious since the masses of particles are set by ϕ, and heavy particles decay into the light ones through the exchange of the massive gauge bosons.[1] This leads us to a conclusion that the density perturbations generated via our mechanism is independent of the detailed structure of the couplings as well as the nature of dominating flat directions, and is always given by Eq. (14).

III. NON-GAUSSIANITIES

The WMAP satellite has provided bounds for the degree of non-Gaussianity of the primordial cosmological perturbations. Assuming that the super-horizon gravitational potential fluctuations (during the matter-dominated era) are of the form

$$\Phi = g + f_{nl} g^2, \tag{22}$$

where g is Gaussian, the WMAP results are $-58 < f_{nl} < 134$ at 95% confidence limit [5]. The Sloan Digital Sky Survey should provide similar accuracy [6].

The standard one-field slow-roll inflation predicts the degree of non-Gaussianity which corresponds to $f_{nl} \sim$ tilt of the perturbation spectrum [7]. The power spectrum measured by WMAP is consistent with scale invariant; however, when WMAP data are combined with other probes of large scale structure small tilts may be preferred [8]. It is fair to say that the largest tilts still allowed by the data are of order $|n-1| \lesssim 0.05$. Thus the current theoretical expectation is that primordial non-Gaussianity is unobservably small. We will show that in our model non-Gaussianities are much larger.

We will consider non-Gaussianities assuming that the fluctuations of the light field ϕ, which is responsible for fluctuations of the coupling constants and/or masses, are purely Gaussian. The case of non-Gaussianities of ϕ will be discussed elsewhere.

To talk about non-Gaussianities, one needs to define a gauge-invariant quantity to quadratic order. We will follow [7,10,11], and use a gauge invariant variable ζ that remains constant outside the horizon (in the absense of entropy perturbations). It is defined as follows: e^ζ is proportional to the local scale factor measured on uniform local Hubble parameter hypersurfaces. To linear order, ζ is proportional to the gravitational potential Φ. During matter domination $\zeta = -(5/3)\Phi$, during radiation domination $\zeta = -(3/2)\Phi$ [12].

In our original scenario [1], the energy density fluctuations on hypersurfaces of a constant scale factor a are

$$\rho \propto \Gamma^{-2/3}. \tag{23}$$

Since $\rho \propto a^{-4}$, this gives

$$\zeta = -(1/6)\log \Gamma. \tag{24}$$

The inflaton reheating rate $\Gamma = \lambda^2 m_\phi$, where m_ϕ is the inflaton mass at its minimum, and λ is the coupling constant of the inflaton decay. We have to consider two cases. First assume that fluctuations of λ are negligible, and $m_\phi \propto \phi$ dominates the fluctuations of Γ. Then, up to second order,

$$\zeta = -\frac{1}{6}\left(\delta_\phi - \frac{1}{2}\delta_\phi^2\right), \tag{25}$$

where $\delta_\phi \equiv (\phi/\langle\phi\rangle - 1)$ is assumed to be Gaussian. The definition of f_{nl} used by [9] corresponds to [7]

$$\zeta = g - (3/5) f_{nl} g^2, \tag{26}$$

and gives

$$f_{nl} = -5 \tag{27}$$

for this scenario. If, on the other hand, $\lambda \propto \phi$ dominates the fluctuations of the reheating rate, one obtains a smaller non-Gaussianity,

$$f_{nl} = -5/2. \tag{28}$$

This shows the general trend: less efficient mechanisms of translation of the ϕ fluctuation into the metric fluctuation give greater non-Gaussianities (assuming the inefficient mechanism is still the dominant one). Our assumptions that

[1]The massless exchange cannot lead to a particle decay, due to the fact that the massless gauge bosons couple through unbroken generators, which commute with the Hamiltonian.

$m \propto \phi$ and $\lambda \propto \phi$ can also be generalized to include non-linear terms, leading to different values of f_{nl}.

The new mass-domination scenario described in Sec. II is of particular interest because of its high universality, based on ϕ being the only mass scale of the theory. From $\rho \propto \phi^{2/3}$, we get a unique answer independent of the Yukawa couplings:

$$f_{nl} = 5. \tag{29}$$

IV. CONCLUSIONS

We proposed a new scenario for the generation of metric perturbations after inflation, whereby superhorizon fluctuations of the light fields generated during inflation are translated into metric fluctuations during reheating, mass domination, or freeze-out.

We calculated non-Gaussianities for this model. Our result is $f_{nl} = 5$ for the mass-domination mechanism. Other scenarios give different values, and non-Gaussianities of the light fields ϕ also lead to partial compensation or enhancement of the intrinsic non-Gaussianities of our mechanism, but an observable value $|f_{nl}| \sim$ few is a natural lower bound.

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[1] G. Dvali, A. Gruzinov, and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004)
[2] F. Finelli and R. Brandeberger, Phys. Rev. D **62**, 083502 (2000) and references therein.
[3] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and G. Gabadadze, hep-th/0209227; N. Arkani-Hamed, S. Dimopoulos, G. Dvali, G. Gabadadze, and A. Linde (in preparation).
[4] M. Gell-Mann *et al.*, in *Supergravity*, Stony Brook, NY, 1997; T. Yanagida, in *Proceedings of the workshop on unified theory and baryon number in the Universe*, KEK, 1979; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[5] E. Komatsu *et al.*, Astrophys. J., Suppl. Ser. **148**, 119 (2003).
[6] Roman Scoccimarro (private communication).
[7] J. Maldacena, J. High Energy Phys. **05**, 013 (2003).
[8] D.N. Spergel *et al.*, Astrophys. J., Suppl. Ser. **148**, 175 (2003).
[9] E. Komatsu and D.N. Spergel, ''The cosmic microwave background bispectrum as a test of the physics of inflation and probe of the astrophysics of the low-redshift universe,'' astro-ph/0012197.
[10] J.M. Bardeen, P.J. Steinhardt, and M.S. Turner, Phys. Rev. D **28**, 679 (1983).
[11] D.S. Salopek and J.R. Bond, Phys. Rev. D **42**, 3936 (1990).
[12] V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger, Phys. Rep. **215**, 203 (1992).