Cosmological constraints on Chaplygin gas dark energy from galaxy cluster x-ray and supernova data

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The recent observational evidence for the present accelerated stage of the Universe has stimulated renewed interest in alternative cosmologies. In general, such models contain an unknown negative-pressure dark component that explains the supernova results and reconciles the inflationary flatness prediction ($\Omega_T = 1$) and the cosmic microwave background measurements with the dynamical estimates of the quantity of matter in the Universe ($\Omega_m \approx 0.3 \pm 0.1$). In this paper we study some observational consequences of a dark energy candidate, the so-called generalized Chaplygin gas, which is characterized by an equation of state $p_C = -A/\rho_C^{\alpha}$, where A and α are positive constants. We investigate the prospects for constraining the equation of state of this dark energy component by combining Chandra observations of the x-ray luminosity of galaxy clusters, independent measurements of the baryonic matter density, the latest measurements of the Hubble parameter as given by the HST Key Project, and data of the Supernova Cosmology Project. We show that very stringent constraints on the model parameters can be obtained from this combination of observational data.

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I. INTRODUCTION

One of the most important goals of current cosmological studies is to unveil the nature of the so-called dark energy or quintessence, the exotic negative-pressure component responsible for the accelerating expansion of our Universe. Over recent years, a number of candidates for this dark energy have been proposed in the literature [1], with the vacuum energy density (or cosmological constant) and a dynamical scalar field [2,3] apparently constituting the most plausible explanations. From the observational viewpoint, these two classes of models are currently considered our best description of the observed Universe, whereas from the theoretical viewpoint they usually face fine-tuning problems, notably, the so-called cosmological constant problem [4] as well as the cosmic coincidence problem, i.e., the question of explaining why the vacuum energy or the scalar field dominates the Universe only very recently. The latter problem happens even for tracker versions of scalar field models in which the evolution of the dark energy density is fairly independent of initial conditions [2,5].

Among the many dark energy candidates, a recent and very interesting proposal has been suggested by Kamenshchik *et al.* [6] and developed by Bilić *et al.* [7] and Bento *et al.* [8]. It refers to the so-called Chaplygin gas (C), an exotic fluid whose equation of state is given by

$$p_C = -A/\rho_C^{\alpha},\tag{1}$$

with $\alpha = 1$ and A a positive constant. Actually, the above equation for $\alpha \neq 1$ generalizes the original Chaplygin equa-

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tion of state proposed in Ref. [8], whereas for $\alpha = 0$ the model behaves like scenarios with cold dark matter plus a cosmological constant (Λ CDM).

In the context of the Friedman-Robertson-Walker (FRW) cosmologies, when one inserts Eq. (1) into the energy conservation law $(u_{\mu}T^{\mu\nu}_{;\nu}=0)$, the following expression for the energy density is immediately obtained:

$$\rho_C = \left[A + B \left(\frac{R_o}{R} \right)^{3(1+\alpha)} \right]^{1/(1+\alpha)}, \qquad (2)$$

or, equivalently,

$$\rho_{C} = \rho_{C_{o}} \left[A_{s} + (1 - A_{s}) \left(\frac{R_{o}}{R} \right)^{3(1+\alpha)} \right]^{1/(1+\alpha)}, \quad (3)$$

where ρ_{C_o} is the current energy density (from now on a subscript *o* means present day quantities, and *C* denotes either the Chaplygin gas or its generalized version). The function R(t) is the cosmic scale factor, $B = \rho_{C_o}^{1+\alpha} - A$ is a constant, and $A_s = A/\rho_{C_o}^{1+\alpha}$ is a quantity related to the present day Chaplygin adiabatic sound speed $(v_s^2 = \alpha A/\rho_{C_o}^{1+\alpha})$. As can be seen from the above equations, the *C* gas interpolates between nonrelativistic matter $[\rho_C(R \rightarrow 0) \approx \sqrt{B}/R^3]$ and negative-pressure dark component regimes $[\rho_C(R \rightarrow \infty) \approx \sqrt{A}]$. This particular behavior of the Chaplygin gas inspired some authors to propose a unified scheme for the cosmological "dark sector" [7–9], an interesting idea which has also been considered in many different contexts [10] (see, however, [11]).

On the theoretical front, a connection between the Chaplygin equation of state and string theory had long been identified by Bordemann and Hoppe [12] and Hoppe [13] (see also [14] for a detailed review). As explained in such refer-

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ences, a Chaplygin-gas-type equation of state is associated with a parametric description of the invariant Nambu-Goto *d*-brane action in a d+2 spacetime. In the light-cone parameterization, such an action is reduced to the action of a Newtonian fluid which obeys Eq. (1) with $\alpha = 1$, with the *C* gas corresponding effectively to a *d*-brane gas in a (d+2)-dimensional spacetime.

Another interesting connection is related to recent attempts at describing the dark energy component through the original Chaplygin gas or its generalized version. Such a possibility has provoked growing interest in exploring the observational consequences of this fluid in the cosmological context. For example, Fabris et al. [15] analyzed some consequences of such scenario using type Ia supernovae data (SNe Ia). Their results indicate that a Universe completely dominated by the Chaplygin gas is favored when compared with Λ CDM models. Recently, Avelino *et al.* [16] used a larger sample of SNe Ia and the shape of the matter power spectrum to show that such data restrict the model to a behavior that closely matches that of a Λ CDM model, while Bento *et al.* [17,18] showed that the location of the cosmic microwave background (CMB) peaks imposes tight constraints on the free parameters of the model. More recently, Dev, Alcaniz, and Jain [19] and Alcaniz, Jain, and Dev [9] investigated the constraints on the C gas equation of state from strong lensing statistics and high-z age estimates, respectively, while Silva and Bertolami [20] studied the use of future SNAP data together with the result of searches for strong gravitational lenses in future large quasar surveys to constrain C gas models. Makler et al. [21] also showed that such models are consistent with current SNe Ia data for a broad range of parameters. The trajectories of statefinder parameters [22] in this class of scenarios were studied in Ref. [23] while constraints involving cosmic microwave background data have also been extensively discussed by many authors [17,18,24,25].

In this work, we study the possibility of constraining the generalized Chaplygin equation of state from x-ray luminosity of galaxy clusters. With a basis in measurements of the mean baryonic mass fraction in clusters as a function of redshift, we consider the method originally proposed by Sasaki [26] and Pen [27], and further modified by Allen *et al.* [28,29] who analyzed the x-ray observations in some relaxed lensing clusters observed with Chandra in the redshift interval 0.1 < z < 0.5 (see also [30]). By inferring the corresponding gas mass fraction, Allen and collaborators placed observational limits on the total matter density parameter Ω_m , as well as on the density parameter Ω_{Λ} , associated with the vacuum energy density. More recently, a similar analysis has also been applied to conventional quintessence models with an equation of state $p_x = \omega \rho_x$ by Lima *et al.* [31].

The paper is organized as follows. In Sec. II we present the field equations and distance formulas necessary to our analysis. In Sec. III the corresponding limits on C gas models from x-ray luminosity of galaxy clusters are derived. We also examine the limits from a statistical combination between x-ray data and recent SNe Ia observations. Finally, in Sec. IV, we finish the paper by summarizing the main results and comparing our constraints with others derived from independent analyses.

II. THE CHAPLYGIN GAS MODEL

The FRW equation for spatially flat, homogeneous, and isotropic scenarios driven by nonrelativistic matter and a separately conserved C gas component reads

$$\left(\frac{\dot{R}}{R}\right)^{2} = H_{o}^{2} \left\{ \Omega_{m} \left(\frac{R_{o}}{R}\right)^{3} + (1 - \Omega_{m}) \left[A_{s} + (1 - A_{s})\right] \times \left(\frac{R_{o}}{R}\right)^{3(\alpha+1)} \right]^{1/(\alpha+1)} \right\}, \qquad (4)$$

where an overdot denotes the time derivative, $H_o = 100 \ h \ \mathrm{km \ s^{-1} \ Mpc^{-1}}$ is the present value of the Hubble parameter, Ω_m is the matter density parameter, and the dependence of the *C* gas energy density on the scale factor comes from Eq. (3).

The comoving distance $r_1(z)$ to a light source located at $r=r_1$ and $t=t_1$ and observed at r=0 and $t=t_o$ is given by

$$r_1(z) = \frac{1}{R_o H_o} \int_{x'}^1 \frac{dx}{x^2 \mathcal{F}(x, \Omega_m, A_s, \alpha)},$$
(5)

where $x' = R(t)/R_o = (1+z)^{-1}$ is a convenient integration variable and the dimensionless function $\mathcal{F}(x, \Omega_m, A_s, \alpha)$ is given by

$$\mathcal{F} = \left[\Omega_m x^{-3} + (1 - \Omega_m) \left(A_s + \frac{(1 - A_s)}{x^{3(\alpha + 1)}}\right)^{1/(\alpha + 1)}\right]^{1/2}.$$
 (6)

Now, in order to derive the constraints from x-ray gas mass fraction on the *C* gas, let us consider the concept of angular diameter distance $D_A(z)$. Such a quantity is defined as the ratio of the source diameter to its angular diameter, i.e.,

$$D_A = \frac{\ell}{\theta} = R(t_1)r_1 = (1+z)^{-1}R_o r_1(z), \tag{7}$$

which provides, when combined with Eq. (5),

$$D_A^C = \frac{H_o^{-1}}{(1+z)} \int_{x'}^1 \frac{dx}{x^2 \mathcal{F}(x,\Omega_m,A_s,\alpha)}.$$
 (8)

As one may check, for $A_s = 0$ and $\alpha = 1$ the above expressions reduce to the standard cold dark matter model (SCDM). In this case, the angular diameter distance can be written as

$$D_A^{\text{SCDM}} = \frac{2H_o^{-1}}{(1+z)^{3/2}} [(1+z)^{1/2} - 1].$$
(9)



FIG. 1. The model function $f_{\text{gas}}^{\text{mod}}$ as a function of the redshift for selected values of A_s and α and fixed values of $\Omega_m = 0.3$, $\Omega_b h^2 = 0.0205$, and h = 0.72.

III. LIMITS FROM X-RAY GAS MASS FRACTION

Following Allen *et al.* [28,29] and Lima *et al.* [31], we consider the Chandra data consisting of six clusters distributed over the redshift interval 0.1 < z < 0.5. The data are constituted of regular, relatively relaxed systems for which independent confirmation of the matter density parameter results is available from gravitational lensing studies. The x-ray gas mass fraction (f_{gas}) values were determined for a canonical radius r_{2500} , which is defined as the radius within which the mean mass density is 2500 times the critical density of the Universe at the redshift of the cluster. In order to generate the data set the SCDM model with $H_o = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ was used as the default cosmology (see [28] for details).

By assuming that the baryonic mass fraction in galaxy clusters provides a fair sample of the distribution of baryons at large scale (see, for instance, [32]) and that $f_{gas} \propto D_A^{3/2}$ [26], the model function is defined as [28]

$$f_{\rm gas}^{\rm mod}(z_i) = \frac{b\Omega_b}{(1+0.19h^{3/2})\Omega_m} \left[2h\frac{D_A^{\rm SCDM}(z_i)}{D_A^C(z_i)}\right]^{1.5}, \quad (10)$$

where the bias factor $b \approx 0.93$ [34] is a parameter motivated by gas dynamical simulations that takes into account the fact that the baryon fraction in clusters is slightly depressed with respect to the Universe as a whole [33]. The term $(2h)^{3/2}$ represents the change in the Hubble parameter between the default cosmology and quintessence scenarios and the ratio $D_A^{\text{SCDM}}(z_i)/D_A^C(z_i)$ accounts for deviations in the geometry of the universe from the default cosmology (SCDM model). In Fig. 1 we show the behavior of f_{gas}^{mod} as a function of the redshift for some selected values of A_s and α having the values of Ω_h and h fixed. For the sake of comparison, the current favored cosmological model, namely, a flat scenario with 70% of the critical energy density dominated by a cosmological constant (Λ CDM) is also shown. In order to have bidimensional plots we fix the value of Ω_m as suggested by dynamical estimates [35], i.e., 0.3 in Fig. 1, as well as in all statistical analyses involving the generalized Chaplygin gas. However, in the case of a conventional C gas ($\alpha = 1$), the



FIG. 2. (a) The $\Delta \chi^2$ contours for the α - A_s plane according to the x-ray data discussed in the text. The contours correspond to 68%, 95%, and 99% confidence levels. The value of the matter density parameter has been fixed at $\Omega_m = 0.3$. (b) Ω_m - A_s plane for the original *C* gas model (α =1). Note that the x-ray data tightly constrain the matter density parameter.

density parameter Ω_m has also been considered a free parameter to be adjusted by the data.

The cosmological parameters A_s and α are determined by using a χ^2 minimization with the priors $\Omega_b h^2 = 0.0205 \pm 0.0018$ [36] and $h = 0.72 \pm 0.08$ [37] for the range of A_s and α spanning the interval [0,1] in steps of 0.02,

$$\chi^{2} = \sum_{i=1}^{6} \frac{[f_{\text{gas}}^{\text{mod}}(z_{i}, \Omega_{m}, A_{s}, \alpha) - f_{\text{gas}, i}]^{2}}{\sigma_{f_{\text{gas}}, i}^{2}} + \left[\frac{\Omega_{b}h^{2} - 0.0205}{0.0018}\right]^{2} + \left[\frac{h - 0.72}{0.08}\right]^{2}, \quad (11)$$

where $\sigma_{f_{\text{gas}},i}$ are the symmetric root-mean-square errors for the SCDM data. The 68.3% and 95.4% confidence levels are defined by the conventional two-parameters χ^2 levels 2.30 and 6.17, respectively.

In Fig. 2(a) we show contours of constant likelihood (68%, 95%, and 99%) in the parameter space α - A_s for the x-ray data discussed earlier. From the above equation we find that the best fit model occurs for $A_s = 1$ which, according to Eq. (4), is independent of the index α and equivalent to a Λ CDM universe. This model corresponds to an accelerating scenario with the deceleration parameter $q_o = -0.55$.¹ From this figure, we also see that both A_s and α are quite insensitive to these data and that, at 95.4% C.L., one can limit the parameter A_s to be greater than 0.52. Figure 2b shows the plane Ω_m - A_s for the conventional *C* gas (α =1). As one should expect from different analyses [28,31], the matter density parameter is very well constrained by this data set while the

¹Note that for $A_s = 1$, Eq. (4) does not depend on the parameter α . Therefore, the smoothness of the curves at these points is a consequence of the step used for the parameters in the code.



FIG. 3. (a) The likelihood contours in the α - A_s plane for the joint x-ray + SNe Ia analysis described in the text. The contours correspond to 68%, 95%, and 99% confidence levels. (b) The Ω_m - A_s plane for the joint x-ray + SNe Ia analysis. The best fit values are located at A_s =0.98 and Ω_m =0.3. At 95.4% we find $A_s \ge 0.84$ and 0.273 $\le \Omega_m \le 0.329$.

parameter A_s remains quite insensitive to it. The best fit occurs for models lying in the interval $A_s = [0,1]$ and $\Omega_m = 0.3$. At 95.4% C.L., we find $0.268 \le \Omega_m \le 0.379$. For an x-ray analysis where the *C* gas plays the role of a unified model for dark matter and dark energy, see [38].

Joint analysis with SNe Ia

By combining the x-ray and SNe Ia data sets, more stringent constraints on the cosmological parameters Ω_m and A_s are obtained. As was shown elsewhere, the parameter α is highly insensitive to the SNe Ia data. To perform this analysis, we follow the conventional magnitude-redshift test (see, for example, [39]) and use the SNe Ia data set that corresponds to the primary fit C of Perlmutter *et al.* [40] together with the highest supernova observed so far, i.e., the 1997ff at z=1.755 and effective magnitude $m^{\text{eff}}=26.02\pm0.34$ [41] and two newly discovered SNe Ia, namely, SN 2002dc at z =0.475 and $m^{\text{eff}}=22.73\pm0.23$ and SN 2002dd at z=0.95and $m^{\text{eff}} = 24.68 \pm 0.2$ [42]. Figures 3(a), 3(b), and 4 show the results of our analysis. In Fig. 3(a) we display contours of the combined likelihood analysis for the parametric space A_s - α . In comparison with Fig. 2(a) we see that the available parameter space is reasonably modified with the value of A_s constrained to be greater than 0.73 at 95.4% C.L. and the entire interval of $\alpha = [0,1]$ allowed. The best fit model occurs for values of $A_s = 0.98$ and $\alpha = 0.93$ with $\chi^2_{\rm min} = 61.38$ and $\nu = 61$ degrees of freedom ($\chi^2_{\rm min}/\nu \simeq 1.0$). The most restrictive limits from this joint analysis are obtained for the original version of C gas ($\alpha = 1$). In this case, the plane Ω_m -A, [Fig. 3(b)] is tightly constrained with the best fit values located at $A_s = 0.98$ and $\Omega_m = 0.3$ with $\chi^2_{\min}/\nu \simeq 1.0$. At 95.4% this analysis also provides $A_s \ge 0.84$ and $0.27 \le \Omega_m \le 0.329$. Note that the contours α - A_s [Fig. 3(a)] and Ω_m - A_s [Fig. 2(b)] are almost orthogonal, thereby explaining the shape of the Ω_m - A_s plane appearing in Fig. 3b. We also observe that by extending the α -A_s plane to the interval [0,2] the new best



FIG. 4. The extended $A_s - \alpha$ plane for the joint x-ray + SNe Ia analysis. Although completely modified in comparison with the previous ones, the best fit values for this extended analysis ($A_s = 1.02$ and $\alpha = 0.45$) are still in agreement with the causality imposed by the adiabatic sound speed ($A_s \le 1/\alpha$). The dashed hyperbola corresponds to the limit condition $v_s^2 = 1$.

fit values ($A_s = 1.02$ and $\alpha = 0.45$), although completely modified in comparison with the previous ones, are still in agreement with the causality ($A_s \le 1/\alpha$) imposed by the fact that the adiabatic sound speed $v_s^2 = dp/d\rho$ in the medium must be lesser than or equal to the light velocity [see Eq. (1)]. Some basic results of the above analysis are displayed in Fig. 4.

IV. DISCUSSION AND CONCLUSIONS

Alternative cosmologies with a quintessence component (dark energy) may provide an explanation for the present accelerated stage of the Universe as suggested by the SNe Ia results. In this work we have focused our attention on a possible dark energy candidate, the so-called Chaplygin gas. The equation of state of this dark energy component has been constrained by combining Chandra observations of the x-ray luminosity of galaxy clusters and independent measurements of the Hubble parameter and of the baryonic matter density as well as from a statistical combination between x-ray data and recent SNe Ia observations. We have shown that stringent constraints on the free parameters of the model, namely, A_s , α , and Ω_m , can be obtained from this combination of observational data.

It is also interesting to compare the results derived here with another independent analyses. For example, using only SNe Ia data, Fabris *et al.* [15] found $A_s = 0.93^{+0.07}_{-0.20}$ for the original *C* gas model ($\alpha = 1$) with the matter density parameter constrained by the interval $0 \le \Omega_m \le 0.35$. The same analysis for $\Omega_m = \Omega_b = 0.04$ (in which the *C* gas plays the role of both dark matter and dark energy) provides A_s $= 0.87^{+0.13}_{-0.18}$. These values agree at some level with the ones obtained from statistics of gravitational lensing (SGL), i.e., $A_s \ge 0.72$ [19] and age estimates of high-*z* galaxies, A_s $\ge 0.85 - A_s \ge 0.99$ for the interval $\Omega_m = 0.2 - 0.4$ with lower values of A_s corresponding to lower Ω_m [9]. The original Chaplygin gas model, however, seems to be incompatible with the localization of the acoustic peak of CMB as given by WMAP [43] and BOOMERANG [44] data. For the case of a generalized component, the same analysis shows that for intermediary values of the spectral tilt n_s the *C* gas model is favored by this data set if $\alpha \approx 0.2$ [18]. A similar analysis for BOOMERANG and Arqueops [45] data implies $0.57 \leq A_s$ ≤ 0.91 for $\alpha \leq 1$ [8] whereas an investigation involving WMAP and SNe Ia data sets restricts α to be $0 \leq \alpha \leq 0.2$ [25].

It should be stressed that our results are in line with the above quoted independent studies. In particular, even considering that the parameter A_s is quite insensitive to the x-ray data alone, the matter density parameter is very well constrained. This result is also in agreement with the limits derived by Allen *et al.* [29] for Λ CDM models. In addition, as

shown in Fig. 3(b), by combining the x-ray and SNe Ia data sets, more stringent constraints on the parameters Ω_m and A_s are readily obtained. From the above analyses we also note that the α parameter is more strongly restricted if causality requirements ($v_s^2 \le 1$) are imposed (see Fig. 4). However, it seems that an even better method to place limits on such a parameter is through the physics of the perturbations, i.e., CMB and LSS data (see, e.g., [23,24]).

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