Asteroseismological bound on $\dot{G}/G$ from pulsating white dwarfs

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We study the secular variation of the period of nonradial pulsations of white dwarfs when a temporal dependence of the gravitational constant G is assumed. We consider models that reproduce accurately the main characteristics of the best studied DA (hydrogen-rich atmosphere) white dwarf G117-B15A. For this object it has been possible to measure the secular variation of the main observed period of 215.2 s ($\dot{P}=2.3\pm1.4\times10^{-15}\ \mathrm{s}^{-1}$) with unprecedented accuracy. Comparing our models with observations we obtain that for $\dot{G}<0$ the allowed values are in the range $-2.5\times10^{-10}\ \mathrm{yr}^{-1}\lesssim\dot{G}/G\lesssim0$, whereas, for $\dot{G}>0$, the allowed values are $0\lesssim\dot{G}/G\lesssim4.0\times10^{-11}\ \mathrm{yr}^{-1}$ at the 2σ confidence level, which are comparable to other bounds established by independent methods. We also show that in order to improve this bound significantly an improvement in the precision of the observed change in $\dot{P}$ which is too large to be reached in the foreseeable future is required.

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I. INTRODUCTION

One of the most important challenges of modern physics is the quantization of the gravitational force. The ongoing attempts to create such theories have reopened the subject of varying fundamental constants. In this regard it is worth noticing that the constancy of the fundamental constants—and of the gravitational constant in particular—has been questioned for a long time [1] and that early attempts to unify gravity with electromagnetism [2,3] predicted such kinds of variations. Although modern theories, such as string theory and M theory, do not necessarily require a variation of the fundamental constants, they provide a natural and self-consistent framework for such variations (see, for instance, [4,5] for comprehensive descriptions of the theoretical background). Moreover, this issue has recently become a subject of intensive experimental and theoretical studies—see [6] for an excellent review. In particular, it is worth mentioning at this point that although there have been several attempts to measure the rate of variation of the fundamental constants, many of them have yielded just upper bounds on the absolute value of the rate of change—which by themselves are very interesting results.

Very recently, the rate of variation of the fine structure constant has been measured using high resolution spectroscopy of quasistellar object (QSO) absorption systems. To be specific a nonzero detection of the variation of the fine structure constant, $\Delta\alpha/\alpha\sim-10^{-5}$ at $z\sim1.5$, has been reported [5]. Although this result is still controversial, it is clear that it opens new and interesting possibilities since in several theoretical scenarios the time variations of α and G turn out to be closely related [7]. In this paper we present a new method to determine upper bounds to the rate of change of the gravitational constant G. In doing so we will use the pulsational properties of white dwarf stars.

White dwarfs have already been used for such a purpose [8]. In particular, the luminosity function of white dwarfs, which provides a measure of the total cooling ages of old white dwarfs, was used. The method used here uses, instead, a direct measure of the cooling rate. White dwarfs represent the final state of the evolution of objects with masses smaller than $\sim10M_\odot$. Consequently, most of the stars will end up their lives as a white dwarf. The general properties of white dwarfs were firmly established long ago. The most relevant one is the fact that their structure is largely supported against gravitational collapse by the pressure of degenerate electrons. It is, however, interesting to note that the outer layers are mildly degenerate. In the simplest picture, most of the energy release of white dwarfs results from the residual gravothermal energy, while nuclear energy release usually represents a minor contribution and it is non-negligible only for the hottest white dwarfs. Moreover, the outermost nondegenerate layers effectively control the energy leaks and, hence, are crucial in our understanding of their rate of cooling.

II. WHITE DWARF ASTEROSEISMOLOGY

Sometime ago it was discovered that some white dwarfs are, in fact, variable stars and, more specifically, nonradial pulsators. This has provided us with the interesting possibility of using seismological techniques to investigate the internal constitution of white dwarfs. As a matter of fact, the so-called *asteroseismology* of white dwarfs has proved to be a powerful tool in searching for the internal properties of these objects which would be otherwise unaccessible for us. Moreover, *all* white dwarfs should undergo nonradial pulsations as they cool down across the relevant range of effective temperatures. For a detailed description of the theory of nonradial pulsations we refer the reader to Ref. [9]. For our purposes it is enough to note here that the properties of the nonradial oscillations of white dwarfs are basically determined by the so-called Brunt-Väisälä frequency N, which is defined as

$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{d\ln p}{dr} - \frac{d\ln \rho}{dr}\right), \tag{1}$$

where g is the gravitational acceleration, Γ_1 is the first adiabatic exponent, p is the pressure, and ρ is the density. As the white dwarf cools down, the material tends to increase its degree of degeneracy, lowering the value of N which, in turn, makes the periods of oscillation longer.

For the problem we are dealing with here it is important to realize that if G has a secular variation, it will have a direct impact on the structure of white dwarfs. For instance, if G increases, the star will tend to shrink, the reverse being also true. In other words, a variation of G will have a direct impact on the degree of degeneracy of the white dwarf interior and on the local acceleration of gravity. This, in turn, will affect the value of N and, consequently, the period of oscillation and, more importantly, its temporal derivative $\dot{P}$. While the periods of oscillation of white dwarfs have a secular variation in the standard case of constant G, a nonzero $\dot{G}$ will produce a supplementary variation of P that can be used to set limits on the range of possible values of $\dot{G}$ that do not contradict the available observations.

For our purposes, hydrogen-rich (DA) variable white dwarfs are suitable objects because for most of them neutrino cooling is negligible, making the modeling of these objects quite robust and reliable. Among this kind of white dwarfs, G117-B15A is an ideal target because it has been continously monitored for more than 20 years. Its observed periods are 215.2, 271, and 304.4 s (together with harmonics and linear combinations of the quoted periods) which are interpreted [10,11] to correspond to dipolar g modes ($\ell=1$) with radial orders (number of nodes of the radial part of the eigenfunction) $k=2$, 3, and 4, respectively. The time base line for this particular star is long enough to detect the secular change of period of the 215.2 s mode due to its evolution $\dot{P}=(2.3\pm1.4)\times10^{-15}\ \mathrm{s}^{-1}$, making this object the most stable optical clock ever found [12]. The observational properties of G117-B15A are consistent with a model with a carbon-oxygen-rich core, surrounded by an almost pure helium layer embracing about 1% of the stellar mass. Finally, the outermost hydrogen layer has a mass of $M_{\mathrm{H}}/M_{*}\approx10^{-4}$, where M_{*} is the stellar mass which has been found [11] to be $0.525M_{\odot}$, while its effective temperature is $T_{\mathrm{eff}}=11\,800$ K.

III. MODELS AND RESULTS

We have followed the evolution of a model white dwarf adopting a varying G together with the standard system of differential equations for g-mode pulsation corresponding to linear, adiabatic nonradial oscillations (see [9] for details).

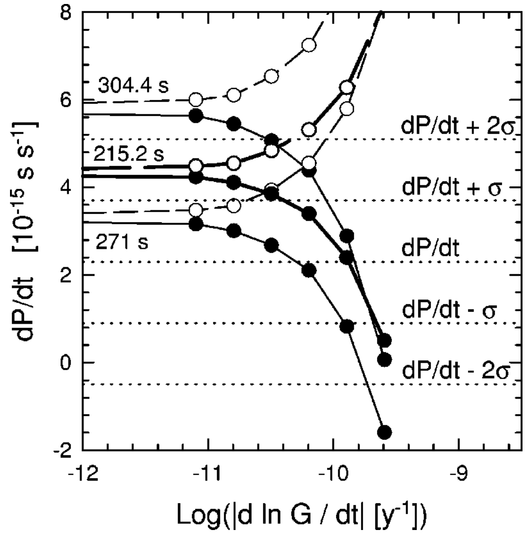

FIG. 1. The derivatives of the periods of oscillation for our set of white dwarf models as a function of $|d\ln G/dt|$. For the cases of $\dot{G}>0$ ($\dot{G}<0$) we used dashed (solid) lines. In order to emphasize the case of the main period (215.2 s) we used thick lines. Dots represent the values for which models have been computed. The observed secular variation of the main period of G117-B15A together with one and two standard deviations is depicted with dotted lines. For further details, see text.

The initial model of the evolutionary sequences for G117-B15A had the chemical composition discussed in [11] which was derived assuming a constant G. For the sake of simplicity, the variation of G has been considered to be linear:

$$G(t) = G_0\left[1 + \frac{\dot{G}}{G_0}(t - t_0)\right], \tag{2}$$

where G_0 is the present value of G. We have considered several values for $\dot{G}/G$ ranging from $\dot{G}/G=\pm2.0\times10^{-9}\ \mathrm{yr}^{-1}$ to $\pm8.0\times10^{-12}\ \mathrm{yr}^{-1}$, besides the case of constant G. For each of these values of $\dot{G}$ we have computed the corresponding evolutionary sequence which reproduces the present value of G and then we have computed the periods of the modes corresponding to $\ell=1$, $k=2,3,4$ when the model white dwarf reaches the effective temperature of G117-B15A. In doing so we have used our evolutionary-pulsational code—see [11] for a thorough description—which incorporates an up-to-date and detailed physical description of the constitutive physics of white dwarfs.

The main results of the present work are summarized in Fig. 1. For completeness the three periods present in G117-B15A have been included in the figure in spite of the fact that $\dot{P}$ is presently available only for the 215.2 s mode. In the case of constant G, the periods of the three modes undergo a secular increase due to the increase of the internal degeneracy of the white dwarf. Thus, if $\dot{G}>0$, degeneracy will increase faster, making the rate of increase of the periods larger. The opposite occurs when $\dot{G}<0$. Moreover, for large values of $\dot{G}$ it may occur that the secular rate of change of the period reverses its sign. Also, as should be expected, for low values of $\dot{G}$, the values of the derivatives of the periods tend asymptotically to those corresponding to the case of constant G.

In our analysis, the next step should be to impose bounds on $\dot{G}$ by asking for a value of $\dot{P}$ for the 215.2 s mode that does not differ by more than 2σ from the observed value. Here we face a fundamental difficulty in our analysis. While the results presented in Fig. 1 clearly indicate that the pulsational properties of G117-B15A are fairly sensitive to $\dot{G}$, at present the observed value and the computed ones differ by more than 1σ, even for the case of constant G. While a deep analysis of the reasons for this discrepancy is beyond the scope of the present work, it is convenient to discuss here several possibilities.

There are reasons to suspect that the observational determination of $\dot{P}$ for the 251 s mode may not be as reliable as previously thought. In particular, in [13] a very accurate observational analysis of the stability of the periods of another pulsating DA white dwarf (ZZ Ceti itself) was performed. For this particular white dwarf the observations also span for a considerably large time base line (30 years). The authors used several methods to derive $\dot{P}$ from the time series and found that, even if the nominal error bars were comfortably smaller than the value of $\dot{P}$, there was a considerable spread in the value of $\dot{P}$ itself, depending on the adopted method. Additionally, in [13] it is also discussed that as various seasons of observations are added, the nominal error bars decrease but the value of $\dot{P}$ fluctuates. The same behavior has been observed for G117-B15A (see the discussion of this issue in [14]). Also, as discussed in [13], in order to reduce the uncertainties by a sizable factor (say ~ 10), 95 years of continuous observations would be needed. Moreover, on the basis of new time-resolved optical spectrophotometric observations of G117-B15A, it has been recently found [15] that, indeed, the 215 s period corresponds to a dipolar mode, but at the same time, it has been found as well that a detailed analysis of the 272 s mode reveals that there is a very puzzling increase in its fractional amplitude that cannot be reproduced by the current models. It is important to recall at this point that in order to measure $\dot{P}$ it is required that the dominant modes of the star do not interact strongly with each other.

From the theoretical point of view it worth noting that in performing the study here presented, we have assumed a standard chemical composition for the white dwarf model (see Ref. [11] for details). This is the consequence of assuming a standard rate for the nuclear reaction $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ together with standard stellar evolution calculations—see, for instance, [16]. However, this nuclear reaction rate is not particularly well determined. It is well known (see, e.g., [14]) that changes in the internal composition of white dwarfs modify the rate of cooling and, consequently, the rate of variation of the pulsation periods. For example, if the fraction of oxygen in the core is larger than that considered here then, for a fixed mass, the star would have a smaller heat capacity. Thus, for a given luminosity value, the cooling would be accelerated. This would directly imply a moderate vertical displacement of the curves in Fig. 1 towards larger values of $\dot{P}$. It is important to mention here that, precisely, pulsating white dwarfs have been used to obtain information about the astrophysically important, but experimentally uncertain, $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction rate—see [17] and references therein. Certainly the problem of the effect of the uncertainty of the reaction rate $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ on the bound to the variation of the gravitational constant may be a interesting issue. However, in face of the large uncertainties on the variation of the period of the 215.2 s mode, we shall not explore this effect in detail here.

All in all and given both the theoretical and observational constraints and the results of Fig. 1, our determination of $\dot{P}$ is accurate enough for our purposes. Consequently, we will consider only bounds at the 2σ confidence level, and if we adhere to the 2σ criterion, which can be considered as safe, we conclude that for $\dot{G}<0$ the allowed values are in the range $-2.5\times 10^{-10}\ \mathrm{yr}^{-1} \lesssim \dot{G}/G \lesssim 0$, whereas for $\dot{G}>0$, the allowed values are $0 \lesssim \dot{G}/G \lesssim 4.0\times 10^{-11}\ \mathrm{yr}^{-1}$.

IV. DISCUSSION AND CONCLUSION

It has been recently claimed [18] that G117-B15A can provide the best upper bound on the rate of variation of G. More specifically, an upper bound of $|\dot{G}/G| \lesssim 3.0\times 10^{-13}\ \mathrm{yr}^{-1}$ was obtained. This upper bound was obtained using a simplified theoretical model for the cooling of white dwarfs [8]. Our detailed numerical results are in contradiction with those of [18]. The reason is quite simple. The secular rate of change of the period is given by

$$\frac{\dot{P}}{P} = -a\frac{\dot{T}}{T} + b\frac{\dot{R}}{R}, \tag{3}$$

where T is the temperature of the isothermal core, R is the radius of the white dwarf, and a and b are constants of order unity which depend on the chemical composition, thicknesses of the H and He buffers, equation of state, and other ingredients involved in the modeling of white dwarfs. In [18] it was proved, using the simplified model of [8], that the first term of Eq. (3) can be directly related to $\dot{G}/G$, whereas the second term was neglected. This can be done when G remains constant, as was proved in [14]. However, if G varies, it is obvious that R changes accordingly. It turns out that the second term in Eq. (3) must also be taken into account because the pulsational properties of G117-B15A are predominantly determined by the thicknesses of the H and He buffers (see [14] for an extensive discussion on this issue), which are mildly degenerate. Hence, the upper bound of [18] turns out to be very optimistic.

The next question we would like to address here is how the upper bounds obtained in this work compare with other results. The observational limits on $\dot{G}/G$ come from quite different times, scales, and methods [19–21]. In particular, determinations based on celestial mechanics, such as laser ranging of the Moon [22] and radar ranging of Mars [23], provide strong constraints on the variation of G, but tests based on neutron star masses [24], globular cluster ages [25], binary pulsar timing [26], and the white dwarf luminosity function [8] have been used. Typical upper bounds give $|\dot{G}/G| \lesssim 10^{-11}$ yr^{-1} [20]. Cosmological nucleosynthesis also offers another limit on the amount of variation of G. Generally speaking, the bounds derived from primordial nucleosynthesis arise from the sensitivity of the abundances of light elements produced at high temperature to the expansion rate of the Universe at those temperatures, especially ^{4}He—see the recent analysis presented in [27] for a detailed discussion. Perhaps the most stringent upper limit to the time variation of G comes from helioseismology [28]. In the case of a varying G the products of hydrogen burning in the main sequence would be different in the solar interior. Since helioseismology probes with extreme precision the structure of the solar interior, the observed p-mode oscillation frequencies constrain $\dot{G}$. Taking advantage of the very high accuracy of the helioseismological observations, a limit far stronger than the one we have found using pulsating white dwarfs can be obtained, $\dot{G}/G \lesssim 1.6\times 10^{-12}$ yr^{-1}. Note, however, that using pulsating white dwarfs we measure directly a dynamical quantity instead of the total accumulated change of G, as helioseismology does.

Finally we would like to remark that from the results of Fig. 1 it is clear that if we want to improve the limits here deduced we would need smaller error bars for the observed value of $\dot{P}$ and, simultaneously, the theoretical models should agree better with the observational value of $\dot{P}$. Regarding the first of these issues it must be said that it is obvious that in order to obtain a dramatic improvement in the accuracy of the observational determination of $\dot{P}$, we would unavoidably need a much longer time base line. Regarding the theoretical modeling it is clear as well that the theoretical uncertainties could only be eventually reduced by either obtaining a statistically representative sample of pulsating white dwarfs with well-determined periods, distances, masses, and period derivatives or by determining the period derivatives of the rest of the periods present in the pulsational spectrum of G117-B15A. All this would allow us to reduce the uncertainties in the theoretical modeling. All these reasons force us to conclude that the limits for the rate of change of G deduced from white dwarf asteroseismology are not expected to be significantly improved in the near future.

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