27-plet baryons from chiral soliton models

Bin Wu
Department of Physics, Peking University, Beijing 100871, China

Bo-Qiang Ma*
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
and Department of Physics, Peking University, Beijing 100871, China

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We use the perturbation method to calculate the masses and widths for 27-plet baryons with spin $\frac{3}{2}$ from chiral soliton models. According to the masses and quantum numbers, we find all the candidates for nonexotic members of the 27-plet. The calculation of the widths shows that these candidates manifest an approximate symmetry of the 27 representation of the SU(3) group, and the quantum numbers of $\Xi(1950)$ seem to be $I(J^P)=\frac{1}{2}(\frac{3}{2}^+)$. Up to leading order of the strange quark mass, we find that the exotic members have widths much larger than those of the antidecuplet members.

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The possible observation [1–5] of an exotic baryon with a narrow width and a positive strangeness number $S=+1$, the $\Theta^+(1540)$, has led to an explosion of interest this year. In chiral soliton models, the antidecuplet baryon multiplet [6,7] is the lightest one next to the octet and decuplet, and Θ^+ is the lightest member of the antidecuplet. In quark language, Θ^+ is of the minimal five-quark configuration $|uudd\overline{s}\rangle$ [8,9]; thus, Θ^+ has the exotic strangeness number $S=+1$ and, if it exists, must be the lightest exotic baryon of pentaquark states. Predictions about the mass and width of Θ^+ from chiral soliton models [10–13] have played an important role in the searches of Θ^+. Especially, in Ref. [12], Diakonov, Petrov, and Polyakov predicted that Θ^+ has a mass around 1.53 GeV and a width $\Gamma_{\Theta^+}<15$ MeV, which is surprisingly close to experimental observations. The experimental results seemed to support chiral soliton models. Following this success, some authors [14,15] studied baryons in the 27-plet and 35-plet, which are the next baryon multiplets to the antidecuplet from chiral soliton models. However, a recent report [16] on the existence of a narrow $\Xi^-\pi^-$ baryon resonance with a mass of 1.862 ± 0.003 GeV and width below the detector resolution of about 0.018 GeV, if confirmed and identified as a member of the antidecuplet, seems to imply that identifying the nucleon resonance $N(1710)$ with a member of the antidecuplet needs revision [17,18].

The purpose of the present Brief Report is to give a clear picture of all the 27-plet baryons from chiral soliton models and check the validity of this picture by symmetry. Though there are criticisms of the validity of chiral soliton models to study pentaquark states [19], we find that we can identify candidates for all nonexotic members in the 27-plet with spin 3/2, consistent with the experimental results in [16]. We also make predictions about the masses and widths for all exotic members in the 27-plet.

The action of Skyrme model is of the form [20,21]

$$I=\frac{f_\pi^2}{4}\int d^4x\,\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)$$
$$+\frac{1}{32e^2}\int d^4x\,\mathrm{Tr}[\partial_\mu UU^\dagger,\partial_\nu UU^\dagger]^2+N_c\Gamma, \tag{1}$$

and the SU(3) chiral field is expressed as

$$U(x)=\exp\left[i\frac{\lambda_b\phi_b(x)}{f_\pi}\right]=A(t)U_1(\mathbf{x})A(t)^{-1},\quad A\in\mathrm{SU}(3), \tag{2}$$

where $f_\pi\approx 93$ MeV is the observed pion decay constant, the dimensionless parameter e is introduced to stabilize the solitons by Skyrme, Γ is the Wess-Zumino term, λ_b are the eight Gell-Mann SU(3) matrices, $\phi_b(x)$ are the eight pseudoscalar meson fields, and $U_1(\mathbf{x})$ is a solitonic solution (with unit baryonic charge) of the equation of motion [22]

$$U_1(\mathbf{x})=\begin{pmatrix} \exp[i(\hat{\mathbf{r}}\cdot\tau)F(r)] & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0 & 0\end{matrix} & 1\end{pmatrix}, \tag{3}$$

where $F(r)$ is the spherical-symmetric profile of the soliton, $\boldsymbol{\tau}$ are the three Pauli matrices, and $\hat{\mathbf{r}}$ is the unit vector in space. The eigenvalues of the collective Hamiltonian are [22]

$$E_J^{(p,q)}=M_{cl}+\frac{1}{6I_2}\left[p^2+q^2+pq+3(p+q)-\frac{1}{4}(N_cB)^2\right]$$
$$+\left(\frac{1}{2I_1}-\frac{1}{2I_2}\right)J(J+1), \tag{4}$$

where (p,q) denotes an irreducible representation of the SU(3) group, M_{cl}, I_1, and I_2 are given by the three-dimensional space coordinate integrals of even functions of $F(r)$ and e, treated model independently and fixed by experimental data, M_{cl} is the classical soliton mass, and I_1 and I_2 are moments of inertia. From the energy eigenvalues above,

*Corresponding author. Electronic address: mabq@phy.pku.edu.cn

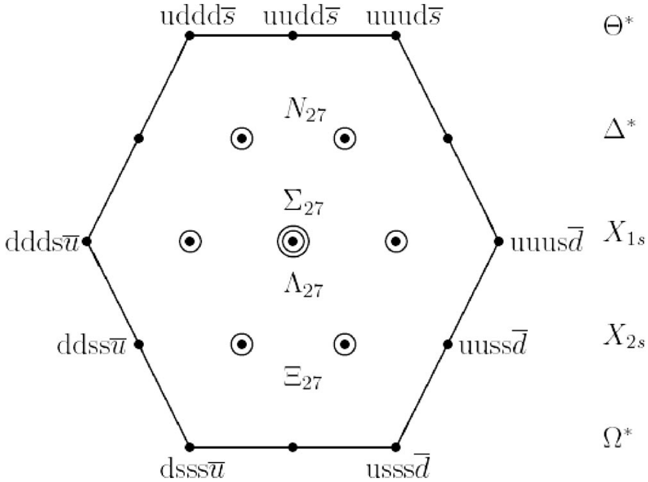

FIG. 1. The quark content of the {27} multiplet baryons.

it can be seen that the {27} multiplet with spin 3/2 is next to the antidecuplet, whose quark content is suggested in Fig. 1.

Using the wave function $\Psi^{(\mu)}_{\nu\nu'}$ of baryon B in the collective coordinates,

$$\Psi^{(\mu)}_{\nu\nu'}(A)=\sqrt{\dim(\mu)}D^{(\mu)}_{\nu\nu'}(A), \tag{5}$$

where (μ) denotes an irreducible representation of the SU(3) group, ν and ν' denote (Y,I,I_3) and $(1,J,-J_3)$ quantum numbers collectively, Y is the hypercharge of B, I and I_3 are the isospin and its third component of B, respectively, J_3 is the third component of spin J, and $D^{(\mu)}_{\nu\nu'}(A)$ are representation matrices, we can deal with the symmetry breaking Hamiltonian [23] perturbatively [24]:

$$H'=\alpha D^{(8)}_{88}+\beta Y+\frac{\gamma}{\sqrt{3}}\sum_{i=1}^{3}D^{(8)}_{8i}J^i, \tag{6}$$

where the coefficients $\alpha,\ \beta,\ \gamma$ are proportional to the strange quark mass and are model dependent. In this Brief Report they are treated model independently and fixed by experiments. $D^{(8)}_{ma}(A)=\frac{1}{2}\,\mathrm{Tr}(A^{\dagger}\lambda^{m}A\lambda^{a})$ is the adjoint representation of the SU(3) group.

We can use the relations between the masses of the octet and decuplet baryons; then, we need two additional equations to fix the parameters. Up to know, we have two methods to fix all the parameters in Eqs. (4) and (6) model independently: (I) take Θ^+ as the member of the antidecuplet, and use relations about the mass (1.54 GeV) and width (<25 MeV) of Θ^+; (II) take both Θ^+ and the candidate for $\Xi_{3/2}$ [16] as members of the antidecuplet and only use the mass relations of the antidecuplet baryons [17,18]. For method I, we have the results

$$\alpha=-766\ \text{MeV},\quad \beta=22\ \text{MeV},\quad \gamma=254\ \text{MeV},$$

$$1/I_1=154\ \text{MeV},\quad 1/I_2=376\ \text{MeV},$$

and for method II we have the results

$$\alpha=-663\ \text{MeV},\quad \beta=-12\ \text{MeV},\quad \gamma=185\ \text{MeV},$$

$$1/I_1=154\ \text{MeV},\quad 1/I_2=399\ \text{MeV}.$$

The predicted mass of $\Xi_{3/2}$ from method I is 1.81, which is compatible with the experimental observation [16] and can be further adjusted to meet the data within uncertainties.

We find that the masses of the 27-plet calculated by the two methods are nearly equal, shown in Table I. These results are close to those calculated by Walliser and Kopeliovich [14], with difference in the mass of Δ_{27}. This shows the validity of the use of the perturbation method in chiral soliton models. In Table I, we also list the candidates for the 27-plet baryons. We can find all the candidates for the nonexotic members by considering their masses and $I(J^P)$ in the baryon listing [25]. To verify this identification, we calculate the widths of the 27-plet baryons.

TABLE I. The masses (GeV) of baryons in the {27} multiplet.

	$\langle B\lvert H'\rvert B\rangle$	Method I	Method II	Candidate	$I(J^{PC})$	PDG
Δ^*	$\frac{13}{112}\alpha+\beta-\frac{65}{224}\gamma$	1.62	1.64	$\Delta(1600)$	$\frac{3}{2}(\frac{3}{2}^+)$	1.55–1.70
N_{27}	$\frac{1}{28}\alpha+\beta-\frac{5}{56}\gamma$	1.73	1.73	$N(1720)$	$\frac{1}{2}(\frac{3}{2}^+)$	1.65–1.75
Σ_{27}	$-\frac{1}{56}\alpha+\frac{5}{112}\gamma$	1.79	1.80	$\Sigma(1840)$	$1(\frac{3}{2}^+)$	1.72–1.93
Ξ_{27}	$-\frac{17}{112}\alpha-\beta+\frac{85}{224}\gamma$	1.95	1.96	$\Xi(1950)$	$\frac{1}{2}(\frac{3}{2}^+)(?^?)$	1.95 ± 0.015
Λ_{27}	$-\frac{1}{14}\alpha+\frac{5}{28}\gamma$	1.86	1.86	$\Lambda(1890)$	$0(\frac{3}{2}^+)$	1.85–1.91
Θ^*	$\frac{\alpha}{7}+2\beta-\frac{5}{14}\gamma$	1.61	1.60	?	$1(\frac{3}{2}^+)?(?^?)$	?
X_{1s}	$\frac{5}{56}\alpha-\frac{25}{112}\gamma$	1.64	1.68	?	$2(\frac{3}{2}^+)?(?^?)$	?
X_{2s}	$-\frac{1}{14}\alpha-\beta+\frac{5}{28}\gamma$	1.84	1.87	?	$\frac{3}{2}(\frac{3}{2}^+)?(?^?)$	?
Ω^*	$-\frac{13}{56}\alpha-2\beta-\frac{65}{112}\gamma$	2.06	2.07	?	$1(\frac{3}{2}^+)?(?^?)$	?

TABLE II. The widths (MeV) of baryons in the 27-plet.

	PDG estimation	Modes	Branching ratios	Γ_i from data	Width$\leqslant$calculation
$\Delta(1600)$	250–450	$N\pi$	10%–25%	25 to 113	130
$N(1720)$	100–200	$N\pi$	10%–20%	10 to 40	19
		$N\eta$	(4.0 ± 1.0)%	3 to 10	66
		ΛK	1%–15%	1 to 30	18
		ΣK			0.39
$\Sigma(1840)$	65–120	$N\overline{K}$	0.37 ± 0.13	11 to 60	50
		$\Lambda\pi$			0
$\Lambda(1890)$	60–200	$N\overline{K}$	20%–35%	12 to 70	46
		$\Sigma\pi$	3%–10%	2 to 30	5
$\Xi(1950)$	60 ± 20	$\Lambda\overline{K}$	seen		90
		$\Sigma\overline{K}$	possibly seen		6.5
		$\Xi\pi$	seen		8.3
Θ^*	?	KN	?	?	79
X_{1s}	?	$\Sigma\pi$	?	?	96
X_{2s}	?	$\Xi\pi$	?	?	58
		$\Sigma\overline{K}$	?	?	36
Ω^*	?	$\Xi\overline{K}$	?	?	107

The decay of a 27-plet baryon B to an octet baryon B' and a pseudoscalar meson m is controlled by a pseudoscalar Yucawa coupling [12,26]:

$$\hat{g}_A \propto G_0 D^{(8)}_{m3} - G_1 d_{3ab} D^{(8)}_{ma} J_b - \frac{G_2}{\sqrt{3}} D^{(8)}_{m8} J_3, \tag{7}$$

where d_{iab} is the SU(3) symmetric tensor, $a,b=4,5,6,7$, and J_a are the generators of the infinitesimal $SU_R(3)$ rotations. G_1, G_2 are dimensionless constants, and $1/N_c$ suppressed relative to G_0. Here G_2 is neglected; then, G_0 and G_1 can be fixed by experiments. Up to leading order of the strange quark mass, we have

$$\begin{aligned}\Gamma(B\to B'm) = {} & \frac{G_s^2}{4\pi}\frac{|\mathbf{p}|}{m_B}[(m_{B'}^2+\mathbf{p}^2)^{1/2}-m_{B'}] \\ & \times\left\{\frac{\dim(\mu')}{\dim(\mu)}\left|\sum_\gamma \begin{pmatrix} 8 & \mu' \\ Y_m I_m & Y_\rho I_\rho \end{pmatrix}\middle| \begin{matrix}\mu_\gamma \\ Y_\nu I_\nu\end{matrix}\right) \right. \\ & \left.\times \begin{pmatrix} 8 & \mu' \\ 01 & 1J_\rho \end{pmatrix}\middle|\begin{matrix}\mu_\gamma \\ 1J_\nu\end{matrix}\right)\Bigg|^2\Bigg\},\end{aligned} \tag{8}$$

where we postulate B with $(Y,I,I_3;J^P,-J_3) = (Y_\nu,I_\nu,I_{\nu3};J_\nu^+,-J_{\nu3})$, B' with $(Y,I,I_3;J^P,-J_3) = (Y_\rho,I_\rho,I_{\rho3};J_\rho^+,-J_{\rho3})$, and m with $(Y,I,I_3;J^P,-J_3) = (Y_m,I_m,I_{m3};0^-,0)$, and $G_s^2=3.84(G_0-\frac{1}{2}G_1)^2$. If we postulate the width of Θ^+ $\Gamma_{\Theta^+}<25$ MeV, we can calculate the upper bounds of widths for all 27-plet baryons, listed in Table II. We can see that the candidates for nonexotic baryons manifest the approximate symmetry of the 27 representation of the SU(3) group. In the results above, we only consider the flavor SU(3) as an exact symmetry. If we take into account the effects of flavor asymmetry, the width of Θ^* will fall by about 30% [27].

In summary, we use the perturbation method to deal with the 27-plet baryons with spin 3/2 from chiral soliton models. Calculations of the widths of the candidates for the nonexotic members manifest an approximate symmetry of the 27 representation of the SU(3) group. Thus, it seems that chiral soliton models are able to give us a clear picture of the 27-plet with spin 3/2, as well as the antidecuplet [17,18], beyond their validity of describing the octet and decuplet baryons. We also predict the masses and widths of the exotic members in 27-plet. The exotic members seem to be more difficult to be found experimentally for their larger widths compared with those of the antidecuplet members. If this picture is right, the nonexotic member $\Xi(1950)$ should be with $J^P=\frac{3}{2}^+$.

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