Covariant structure of light-front wave functions and the behavior of hadronic form factors

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We study the analytic structure of light-front wave functions (LFWFs) and its consequences for hadron form factors using an explicitly Lorentz-invariant formulation of the front form. The normal to the light front is specified by a general null vector ω^μ. The LFWFs with definite total angular momentum are eigenstates of a *kinematic* angular momentum operator and satisfy all Lorentz symmetries. They are analytic functions of the invariant mass squared of the constituents $M_0^2=(\Sigma k^\mu)^2$ and the light-cone momentum fractions $x_i = k_i\cdot\omega/p\cdot\omega$ multiplied by invariants constructed from the spin matrices, polarization vectors, and ω^μ. These properties are illustrated using known nonperturbative eigensolutions of the Wick-Cutkosky model. We analyze the LFWFs introduced by Chung and Coester to describe static and low momentum properties of the nucleons. They correspond to the spin locking of a quark with the spin of its parent nucleon, together with a positive-energy projection constraint. These extra constraints lead to an anomalous dependence of form factors on Q rather than Q^2. In contrast, the dependence of LFWFs on M_0^2 implies that hadron form factors are analytic functions of Q^2 in agreement with dispersion theory and perturbative QCD. We show that a model incorporating the leading-twist perturbative QCD prediction is consistent with recent data for the ratio of proton Pauli and Dirac form factors.

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I. INTRODUCTION

Light-front wave functions (LFWFs) are the interpolating functions connecting hadrons to their fundamental quark and gluon degrees of freedom in QCD. Many hadronic observables can be computed directly from these amplitudes. For example, matrix elements of local operators such as spacelike proton form factors, transition form factors such as $B \to \ell\bar{\nu}\pi$, and generalized parton distributions can be computed from the overlap integrals of the LFWFs. The determination of the hadron LFWFs from phenomenological constraints and from QCD itself is thus a central goal of hadron and nuclear physics. In principle, one can solve the hadronic LFWFs directly from fundamental theory using nonperturbative methods, such as discretized light-front quantization, the transverse lattice, lattice gauge theory moments, or Bethe-Salpeter or Dyson-Schwinger techniques. Reviews of nonperturbative light-front methods may be found in Refs. [1–3].

One of the central issues in the analysis of fundamental hadron structure is the presence of nonzero orbital angular momentum in the bound-state wave functions. The evidence for a ''spin crisis'' in the Ellis-Jaffe sum rule signals a significant orbital contribution in the proton wave function [4,5]. The Pauli form factor of nucleons is computed from the overlap of LFWFs differing by one unit of orbital angular momentum $\Delta L_z=\pm 1$. Thus the fact that the anomalous magnetic moment of the proton is not zero is an immediate signal for the presence of nonzero orbital angular momentum in the proton's LFWFs [6]. It should be noted that orbital angular momentum is treated explicitly in light-front quantization; it includes the orbital contributions induced by relativistic effects, such as the spin-orbit effects normally associated with the conventional Dirac spinors.

In this paper we shall show how orbital angular momentum is represented by light-front Fock state wave functions. A key tool will be the explicitly Lorentz-invariant formulation of the front form (see Ref. [1] for a review and references to original papers). The wave functions are defined at the light-front plane $\omega\cdot x=\sigma$, for which the orientation is determined by the null four-vector ω. Although LFWFs depend on the choice of the light-front quantization direction ω, all observables such as matrix elements of local current operators, form factors, and cross sections are light-front invariants—they must be ω independent. When the ω independence is violated in approximate calculations, one can

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still find the ω-independent form factors by separating them from the current operator matrix elements and omitting the nonphysical, ω-dependent contribution [7]. We shall show that the analytic form of LFWFs with nonzero orbital angular momentum is then constrained to a specific set of simple prefactors multiplying the scalar zero orbital angular momentum solutions. Knowing the general form of the LFWFs can be important for determining the hadron eigenstates from QCD using variational or other methods.

We begin by noting that eigensolutions of the Bethe-Salpeter equation have specific angular momentum as specified by the Pauli-Lubanski vector. The corresponding LFWF for an n-particle Fock state evaluated at equal light-front time $\sigma=\omega\cdot x$ can be obtained by integrating the Bethe-Salpeter solutions over the corresponding relative light-front energies. The resulting LFWFs $\psi_n^I(x_i,k_{\perp i})$ are functions of the light-cone momentum fractions x_i and the invariant mass squared of the constituents

$$M_0^2=\left(\sum_{i=1}^{n} k_i^{\mu}\right)^2=\sum_{i=1}^{n}\left[\frac{k_\perp^2+m^2}{x}\right]_i$$

and the light-cone momentum fractions $x_i=k\cdot\omega/p\cdot\omega$, each multiplying spin-vector and polarization tensor invariants that can involve ω^μ. The resulting LFWFs for bound states are eigenstates of a kinematic angular momentum operator. Thus LFWFs satisfy all Lorentz symmetries of the front form [8], including boost invariance, and they are proper eigenstates of angular momentum.

There is now heightened interest in the analytic form of the nucleon form factors. Recent measurements of the proton form factors at Jefferson Laboratory [9,10], using the polarization transfer method, show a surprising result—the ratio. $G_E(Q^2)/G_M(Q^2)$ falls faster in momentum transfer $Q^2=-q^2=-t$ than that found using the traditional Rosenbluth separation method. A possible source for this disparity is the QED radiative corrections, since these are more likely to affect the Rosenbluth method [11–13]. For example, the interference of one-photon and two-photon exchange amplitudes and the interference between proton and electron bremsstrahlung are present in the measured electron-proton cross section and can complicate the analysis of the energy and angular dependence required for the Rosenbluth separation.

If one translates the new polarization transfer results for G_E and G_M to the Pauli and Dirac form factors, the data appear to suggest the asymptotic behavior $QF_1(Q^2)/F_2(Q^2)\sim$const. In a recent paper, Miller and Frank [14] have shown that the three-quark model for the proton LFWF constructed by Chung and Coester (CC) [15] and extended by Schlumpf [16] leads to $QF_1(Q^2)/F_2(Q^2)\sim$const in the range of the Jefferson Laboratory experiment, thus providing an apparent explanation of the Jefferson Laboratory data.

In dispersion theory form factors are analytic functions of q^2, with a cut structure reflecting physical thresholds at timelike q^2. This is also apparent from the analytic structure of Feynman amplitudes in perturbation theory. A functional dependence in $Q=\sqrt{-q^2}$ does arise when there is a physical threshold at $q^2=0$, as in the case of gravitational [17,18] (or axial current) form factors due to the two-photon intermediate state; however, this would not be expected to occur for the vector current in QCD.

Chung and Coester [15] introduced their ansatz for the form of baryon LFWFs in order to describe the static and low momentum transfer properties of the nucleons. The LFWFs in the CC model have the effect of spin-locking a quark with the spin of its parent nucleon, together with a positive-energy projection constraint. As we shall show, if one extends these forms to large transverse momentum, the extra constraints lead to an anomalous linear dependence of LFWFs in the invariant mass of the constituents and an anomalous dependence of form factors on Q rather than Q^2. As we discuss in the Conclusions, the lack of analyticity in Q^2 is related to the breakdown of the crossing properties incorporated in field theory. The CC constraint may provide a reasonable model for computing static properties of hadrons, but it is not applicable to large momentum transfer observables.

We shall show that form factors computed from the overlap of LFWFs are analytic functions of q^2 due to their analytic dependence on the off-shell light-front energy and the general form of prefactors associated with nonzero orbital angular momentum. In particular, the general form of the LFWFs for baryons in QCD leads to a ratio of form factors $F_2(Q^2)/F_1(Q^2)$, which behaves asymptotically as an inverse power of Q^2 modulo logarithms, in agreement with the perturbative QCD (PQCD) analysis of Belitsky, Ji, and Yuan [19]. We also shall show that the form factor ratios obtained from the nonperturbative solutions to the Wick-Cutkosky model [20,21] have a similar behavior.

It should be noted that the analytic form predicted by perturbative QCD is compatible with the form factor ratio determined by the polarization transfer measurements. The detailed analysis of baryon form factors at large Q^2 based on perturbative QCD predicts the asymptotic behavior $Q^2F_2(Q^2)/F_1(Q^2)\sim\log^{2+8/(9\beta)}(Q^2/\Lambda^2)$, where $\beta=11-2n_f/3$ [19]. This asymptotic logarithmic form can be generalized to include the correct $Q^2=0$ limit and the cut at the two-pion threshold in the timelike region. Such a parametrization is

$$F_2/F_1=\kappa_p\frac{1+(Q^2/C_1)^2\log^{b+2}(1+Q^2/4m_\pi^2)}{1+(Q^2/C_2)^3\log^{b}(1+Q^2/4m_\pi^2)}, \qquad (1)$$

where for simplicity we have ignored the small factor $8/9\beta$, as do Belitsky *et al.* For the large-Q^2 region of the available data, this already reduces to the asymptotic form

$$F_2/F_1=\kappa_p\frac{C_2^3}{C_1^2}\frac{\log^2(Q^2/4m_\pi^2)}{Q^2}. \qquad (2)$$

Therefore, the values of C_1, C_2, and b are not tightly constrained, except for the combination C_2^3/C_1^2. A fit to the Jefferson Laboratory data yields $C_1=0.79$ GeV^2, $C_2=0.38$ GeV^2, and $b=5.1$. Thus, as shown in Fig. 1, one can

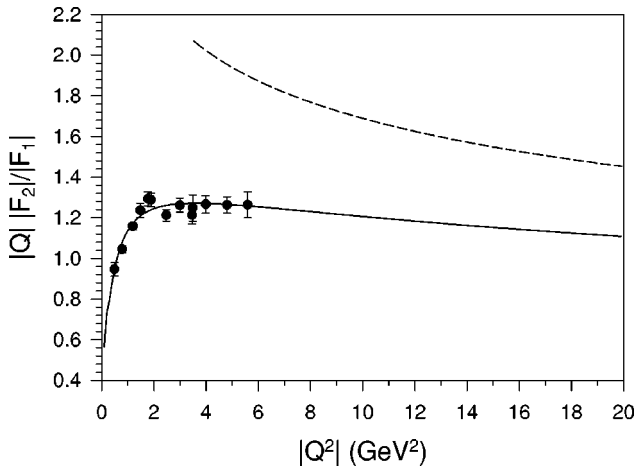

FIG. 1. Perturbative QCD motivated fit to the Jefferson Laboratory polarization transfer data [9,10]. The parametrization is given in Eq. (1). The dashed line shows the predicted form [22] for timelike $q^2=-Q^2$.

fit the form factor ratio over the entire measured range with an analytic form compatible with the predicted perturbative QCD asymptotic behavior.

The nominal $1/Q^2$ power-law fall-off of $F_2(Q^2)/F_1(Q^2)$ in perturbative QCD is a consequence of the underlying chiral structure of the vector interactions in QCD. The factorized structure of hard QCD amplitudes predicts hadron helicity conservation [23] at the leading twist and thus the relative suppression of the Pauli form factor since it is a helicity-flip amplitude. These nominal power-law forms are also properties of dimensional counting rules [24–26] for hard scattering amplitudes in QCD. The power-law suppression of $F_2(Q^2)/F_1(Q^2)$ is not generally true for Yukawa theories with scalar gluons or in quark-diquark models of the nucleon based on scalar diquarks since the effective interactions violate chirality conservation.

Iachello, Jackson, and Lande [27–29] have introduced a model for the nucleon form factors based on dimensional counting and perturbative QCD at high momentum plus the analytic structure due to vector meson intermediate states. The model gives an excellent phenomenological description of the individual form factors and the form factor ratio measured using polarization transfer.

Although the spacelike form factors of a stable hadron are real, the timelike form factors have a phase structure reflecting the final-state interactions of the outgoing hadrons. The analytic structure and phases of the form factors in the timelike regime are connected by dispersion theory to the spacelike regime. Each of the above models predicts a specific fall-off and phase structure of the form factors from $s\leftrightarrow t$ crossing to the timelike domain. As noted by Dubnickova, Dubnicka and Rekalo and by Rock [30], the phase of the form factor ratio G_E/G_M of spin-1/2 baryons in the timelike region can be determined from measurements of the polarization of one of the produced baryons in the exclusive process $e^-e^+\to B\bar{B}$, since the single-spin asymmetry normal to the scattering plane requires a nonzero phase difference between the G_E and G_M form factors. As demonstrated in Ref. [31], measurements of the proton polarization in $e^+e^-\to p\bar{p}$ will strongly discriminate between the analytic forms of the models that have been suggested to fit the proton G_E/G_M data in the spacelike region. The single-spin proton asymmetry is predicted to be large, of the order of several tens of percent.

The content of the remainder of the paper is as follows. The general construction of the wave functions appearing through the Fock decomposition of the state vector defined on the light front $\omega\cdot x=0$ is explained in Sec. II A. The angular momentum properties of LFWFs are discussed in Sec. II B. For simplicity, we present the LFWFs for bound states of scalar fields and for a spin-1/2 system in Yukawa theory, which can serve as a diquark-inspired model for a three-quark hadron. The consequences of the CC ansatz are explored in detail. Section III recasts this discussion in terms of the standard light-front Fock-state expansion and the associated LF spinors. This allows contact to be made with perturbation theory. The construction of form factors and their asymptotic behavior are described in Sec. IV. The LFWF for the valence Fock state is obtained by integrating the Bethe-Salpeter solutions over the relative light-front energies. The expectation that LFWFs are functions of M_0^2, rather than M_0, is demonstrated in Sec. V. The angular momentum properties of LFWFs are illustrated using the known nonperturbative eigensolutions of the Wick-Cutkosky model for nonzero angular momentum. Section VI presents the implications for the CC ansatz for a three-quark nucleon state, where we show that the anomalous asymptotic behavior of $QF_2/F_1\sim$const is a consequence of the extra constraints on the LFWFs imposed by the CC construction. Section VII contains our conclusions, and an appendix collects some useful definitions and intermediate results.

II. LIGHT-FRONT WAVE FUNCTIONS

A. General construction

The concept of a wave function of a hadron as a composite of relativistic quarks and gluons is naturally formulated in terms of the light-front Fock expansion at fixed light-front time, $\sigma=x\cdot\omega$ [1]. The null four-vector ω determines the orientation of the light-front plane; the freedom to choose ω provides an explicitly covariant formulation of light-front quantization. For a stationary state we can consider a fixed light-front time and set $\sigma=0$.

In this formulation, the eigensolution of a proton, projected on the eigenstates of the free Hamiltonian $H_{LC}^{QCD}(g=0)$, has the expansion

$$|p,\lambda\rangle=\sum_{n\geqslant 3}\int \psi^{\lambda}_{\lambda_1,\ldots,\lambda_n}(k_1,\ldots,k_n,p,\omega\tau)\times\delta^{(4)}\left(\sum_j^n k_j-p-\omega\tau\right)2(\omega\cdot p)d\tau\times\prod_{i=1}^n\frac{d^3k_i}{(2\pi)^3 2\varepsilon_{k_i}}a^\dagger_{\lambda_i}(\vec{k}_i)|0\rangle. \quad (3)$$

Here $a^\dagger$ is the usual creation operator and $\varepsilon_{k_i}=\sqrt{m_i^2+\vec{k}_i^2}$. All the four-momenta are on the corresponding mass shells: $k_j^2=m_j^2$, $p^2=M^2$, $(\omega\tau)^2=0$. The set of light-front Fock-state wave functions $\psi^{\lambda}_{\lambda_1,\ldots,\lambda_n}(k_1,\ldots,k_n,p,\omega\tau)$ represents the ensemble of quark and gluon states possible when the proton is intercepted at the light front. The λ_j label the light-front spin projections of the quarks and gluons along the light-front quantization direction. The scalar variable τ controls the off-shell continuation of the wave function. From the point of view of kinematics, the four-momentum $\omega\tau$ can be considered on equal ground with the particle four-momenta $k_1,\ldots,k_n,p$. Being expressed through them (by squaring the equality $\Sigma_j^n k_j=p+\omega\tau$), τ reads

$$\tau=\frac{M_0^2-M^2}{2\omega\cdot p},$$

where $M_0^2=(\Sigma_j^n k_j)^2$. The difference $M_0-M\sim\tau$ between the effective mass M_0 of constituents and their bound state mass M is just a measure of the off-energy-shell effect.

If the wave functions describe a system with spin composed of constituents with spin, they are represented through scalar functions multiplied by invariants constructed from the spin matrices, polarization vectors, and ω^μ. In general, the scalar functions depend on a set of scalar products of the four-momenta $k_1,\ldots,k_n,p,\omega\tau$ with each other. One should choose a set of independent scalar products. A convenient way to choose these variables is the following. We define

$$x_j=\frac{\omega\cdot k_j}{\omega\cdot p},\quad R_j=k_j-x_j p,$$

where $\Sigma_{j=1}^n x_j=1$ and $\Sigma_{j=1}^n R_j=\omega\tau$, and represent the spatial part of the four-vector R_j as $\vec{R}_j=\vec{R}_{\|j}+\vec{k}_{\perp j}$, where $\vec{R}_{\|j}$ is parallel to $\vec{\omega}$ and $\vec{k}_{\perp j}$ (with $\Sigma_j^n\vec{k}_{\perp j}=0$) is orthogonal to $\vec{\omega}$. Since $R_j\cdot\omega=R_{0j}\omega_0-\vec{R}_{\|j}\cdot\vec{\omega}=0$ by definition of R_j, it follows that $R_{0j}=|\vec{R}_{\|j}|$, and, hence, $\vec{k}_{\perp j}^2=-R_j^2$ and $\vec{k}_{\perp i}\cdot\vec{k}_{\perp j}=-R_i\cdot R_j$ are expressed though the squares and the scalar products of the four-vectors R_j. Hence, they are the Lorentz and rotation invariants. Therefore, the scalar functions should depend on x_j, $\vec{k}_{\perp j}^2$, and $\vec{k}_{\perp i}\cdot\vec{k}_{\perp j}$. In terms of these variables, the integral in Eq. (3) is transformed as

$$\int\ \ldots\delta^{(4)}\left(\sum_j^n k_j-p-\omega\tau\right)2(\omega\cdot p)d\tau\prod_{i=1}^n\frac{d^3k_i}{(2\pi)^3 2\varepsilon_{k_i}}$$

$$=\int\ \ldots 2\delta\left(\sum_j^n x_j-1\right)\delta^{(2)}\left(\sum_j^n\vec{k}_{\perp j}\right)\prod_{i=1}^n\frac{d^2\vec{k}_{\perp i}dx_i}{16\pi^3 x_i}.$$

In this way we find that ψ reduces to $\psi^{\lambda}_{\lambda_1,\ldots,\lambda_n}(x_j,\vec{k}_{\perp j})$. These wave functions are independent of the hadron's momentum $p^+=p\cdot\omega$ and $p_\perp$, reflecting the kinematical boost invariance of the front form. All observables must be invariant under variation of ω^μ; as we shall show, this generalized rotational invariance provides an elegant representation of angular momentum on the light front.

A scalar bound state of two spinless particles with $p^2=(k_a+k_b)^2=M^2$ can be described using the covariant Bethe-Salpeter function $\Phi(x_1,x_2,p)$, which in momentum space corresponds to $\Phi_{BS}(k_a,k_b)=\Phi(q,p)$, where $q=(k_a-k_b)/2$. For $J=0$ the Bethe-Salpeter function is a scalar function of the Feynman virtualities with $k_a^\mu+k_b^\mu=p^\mu$.

The explicitly covariant version of light-front dynamics [1] turns into the standard one at the particular value of $\omega=(1,0,0,-1)$. The conjugate direction is defined as ζ, where $\zeta\cdot\omega=2$ and $\zeta=(1,0,0,1)$ in the standard light-front frame. In general, the two-body scalar LFWF can only be a function of the corresponding off-shell light-front energy

$$M_0^2-M^2=\left(\sum_i^2 k_i\cdot\zeta-p\cdot\zeta\right)(\omega\cdot p)=2(\omega\cdot p)\tau \qquad (4)$$

and the momentum fractions $x_i=k_i\cdot\omega/p\cdot\omega$.

One can identify the corresponding two-parton LFWF by calculating the Fourier transform of $\Phi(x_1,x_2,p)$ with the arguments x_1, x_2 constrained to a fixed light-front time $\sigma=\omega\cdot x$, i.e., the transform of the function $\Phi(x_1,x_2,p)\delta(\omega\cdot x_1)\delta(\omega\cdot x_2)$. In momentum space it corresponds to integration over the minus component of the relative momentum k, or in the covariant form [1]

$$\psi=\frac{(\omega\cdot k_1)(\omega\cdot k_2)}{\pi(\omega\cdot p)}\int_{-\infty}^{+\infty}\Phi(k+\beta\omega,p)d\beta. \qquad (5)$$

The resulting LFWF describes a spin-zero bound state of on-shell partons with $k_a^2=m_a^2$, $k_b^2=m_b^2$.

The main dynamical dependence of a LFWF thus involves $M_0^2=(\Sigma k_i^\mu)^2$, the invariant mass squared of the partons. Since the equation of motion for the LFWF involves the off-shell LF energy $p\cdot\zeta$, the natural analytic dependence of φ is in the variable M_0^2, not M_0.

It is useful to examine the properties of the LFWF in the constituent rest frame [32,33]. We will distinguish the four-vector $\mathcal{P}^\mu=\Sigma k_i^\mu$ and the bound-state four-momentum p^μ. Since there is no conservation law for the minus components of momenta, we have $\zeta\cdot p\neq\zeta\cdot\mathcal{P}$, and the constituent rest frame ($\vec{\mathcal{P}}=\vec{0}$) and the rest frame ($\vec{p}=\vec{0}$) differ from each other. In the constituent rest frame where $\vec{\mathcal{P}}=\Sigma_i\vec{k}_i=\vec{0}$, we can identify

$$M_0=\mathcal{P}^0=\sum_i k_i^0=\sum_i\sqrt{\vec{k}_i^2+m_i^2}.$$

It is also convenient to make an identification of the front-form and the usual instant-form wave functions in this frame.

In general, LFWFs are eigenstates of the LF angular momentum operator [1,34]

$$\vec{J}=-i[\vec{k}\times\partial/\partial\vec{k}]-i[\vec{n}\times\partial/\partial\vec{n}]+\frac{1}{2}\vec{\sigma}, \qquad (6)$$

where $\vec{n}$ is the spatial component of ω in the constituent rest frame ($\vec{\mathcal{P}}=\vec{0}$). Although this form is written specifically in

the constituent rest frame, it can be generalized to an arbitrary frame by a Lorentz boost.

Normally the generators of angular rotations in the LF formalism contain interactions $J^{int}_{\mu\nu}$, as in the Pauli-Lubanski formulation; however, the LF angular momentum operator can also be represented in the kinematical form (6) without interactions. The key term is the generator of rotations of the LF plane $-i[\vec{n}\times\partial/\partial\vec{n}]$, which replaces the interaction term. In four-dimensional form the relation between them reads [34]

$$i\left(\omega_\mu\frac{\partial}{\omega_\nu}-\omega_\nu\frac{\partial}{\omega_\mu}\right)|p,\lambda\rangle=J^{int}_{\mu\nu}|p,\lambda\rangle. \tag{7}$$

The operator $i(\omega_\mu\partial/\omega_\nu-\omega_\nu\partial/\omega_\mu)$ (or its counterpart $-i[\vec{n}\times\partial/\partial\vec{n}]$ in the constituent rest frame) appears only in the explicitly covariant formulation where the dependence on $\vec{n}$ is present. Details can be found in Ref. [1].

We emphasize that the angular momentum operator in Eq. (6) has general validity in quantum field theory; in particular, we do not need to truncate the Fock space. The angular momentum operator (6) can be applied to each Fock component: each operator (for any given n) acts on the corresponding n-body Fock component in Eq. (3). For example, the application of the LF angular momentum operator (6) to the scalar function $\varphi(M_0^2,x)$ verifies that it describes a $J=0$ state.

The validity of the operator (6) can be also verified within the covariant Bethe-Salpeter formalism as follows. In Sec. V we will derive a two-body Fock component wave function [see Eq. (51) below] from the Bethe-Salpeter function in the Wick-Cutkosky model. The latter definitely corresponds to a specific angular momentum ($J=L=1$ in the particular case considered in Sec. V). One can check that this LFWF is an eigenfunction of the operators $\vec{J}^2$ and J_z, corresponding to (6) (omitting the Pauli matrix). This argument can be generalized to the case of arbitrary angular momentum.

B. The Yukawa model

The general form of the LFWF of a spin-1/2 system composed of spin-half and spin-zero constituents is given by

$$\psi(k_1,p)=\overline{u}(k_1)\left(\varphi_1+\frac{M\hat{\omega}}{\omega\cdot p}\varphi_2\right)u(p), \tag{8}$$

where $\omega=(\omega_0,\vec{\omega})$ is the four-vector determining the orientation of the light-front plane $\omega\cdot x=0$. Here $\hat{\omega}=\omega_\mu\gamma^\mu$, and $\overline{u}(k_1)$ is the conventional Dirac spinor of the spin-half constituent, and $u(p)$ is the spinor of the bound state.

The general wave function (8) in Yukawa theory is determined by two scalar components φ_1 and φ_2, each a function of M_0^2 and x. In contrast, the wave function corresponding to the CC ansatz used in Ref. [14] is equivalent[1] to the form

$$\psi=c_k c_p\overline{u}(k_1)\Lambda_+u(p)f_1, \tag{9}$$

where $\mathcal{P}=k_1+k_2$ is the sum of the constituent momenta, $M_0=\sqrt{\mathcal{P}^2}$, $c_k=1/\sqrt{m+\varepsilon_k}$, $c_p=1/\sqrt{M+\varepsilon_p}$, $\varepsilon_k=\sqrt{m^2+\vec{k}^2}$, and $\varepsilon_p=\sqrt{M^2+\vec{p}^2}$ (for simplicity we assume that constituents have equal masses $m_a=m_b=m$). The matrix Λ_+ in (9) is the projection operator

$$\Lambda_+=\frac{\hat{\mathcal{P}}+M_0}{2M_0}, \tag{10}$$

which in the constituent rest frame becomes

$$\Lambda_+=\begin{pmatrix}1&0\\0&0\end{pmatrix}. \tag{11}$$

Hence, in the constituent rest frame we obtain

$$c_k\overline{u}(k_1)\Lambda_+=\chi_1^\dagger(1,0),\quad c_p\Lambda_+u(p)=\begin{pmatrix}1\\0\end{pmatrix}\chi_N, \tag{12}$$

with χ_1 and χ_N being two-component spinors for constituent 1 and the nucleon, respectively. From (9) we reproduce the analog of the three-quark CC ansatz used in Ref. [14], which is

$$\psi^\sigma_{\sigma_1}=\chi^{\dagger\sigma_1}\chi^\sigma f_1=\delta^{\sigma_1\sigma}f_1. \tag{13}$$

This should be contrasted with the general form (in the constituent rest frame)

$$\psi^\sigma_{\sigma_1}=\chi^{\dagger\sigma_1}\left(f_1+\frac{i}{k}[\vec{n}\times\vec{k}]\cdot\vec{\sigma}f_2\right)\chi^\sigma, \tag{14}$$

where $\vec{k}=\vec{k}_1=-\vec{k}_2$ and $\vec{n}=\vec{\omega}/|\vec{\omega}|$ in this frame. The additional f_2 term represents a separate dynamical contribution with orbital angular momentum, to be contrasted with the purely kinematical contributions of orbital angular momentum from Melosh rotations. The form (14) can be found by substituting into Eq. (8) the explicit form of the Bjorken-Drell (BD) spinors, as given in the constituent frame by Eq. (A3) of the Appendix. The two functions f_1 and f_2 are then determined in terms of φ_1 and φ_2 as

$$\begin{aligned}f_1&=\frac{c_kc_p}{2M_0}(M_0+m)\{m(M_0+m)+M_0[xM+(1-x)M_0]\}\varphi_1\\&\quad+\frac{c_kc_p}{M_0}M(M_0+m)(xM_0+m)\varphi_2,\\f_2&=\frac{c_kc_p}{M_0}kM(M_0+M)\varphi_2-\frac{c_kc_p}{2M_0}k(M_0^2-M^2)\varphi_1,\end{aligned} \tag{15}$$

with $x=x_1=1/2-\vec{k}\cdot\vec{n}/M_0$. The inverse relations read

[1]Equivalence has been established [35] in the sense that equivalent results are obtained for form factors.

$$\varphi_1=\frac{c_k c_p}{2M_0}(M_0+M)f_1-\frac{c_k c_p}{2M_0 k}(M_0+M)(xM_0+m)f_2,$$

$$\varphi_2=\frac{c_k c_p}{4M_0 M}(M_0^2-M^2)f_1+\frac{c_k c_p}{4M_0 M k}(M_0+M)$$
$$\times\{m(M_0+m)+M_0[xM+(1-x)M_0]\}f_2. \quad (16)$$

Note that the wave functions φ_1 and φ_2 are Lorentz scalars and analytic functions of M_0^2 and x. The derived amplitudes f_1 and f_2 contain kinematic factors linear in M_0 that arise from the reduction of the covariant form to the Pauli spinor form in the constituent rest frame. The application of the LF angular momentum operator (6) to the LFWFs of the Yukawa theory in Eq. (14) verifies that these wave functions are in fact states with $J=1/2$, $J_z=\pm 1/2$.

In the nonrelativistic limit, where $M=2m$, $M_0=2m$, and $x=1/2$, we get

$$\varphi_1=\frac{1}{2\sqrt{2}m}f_1-\frac{1}{k\sqrt{2}}f_2, \quad \varphi_2=\frac{1}{k\sqrt{2}}f_2. \quad (17)$$

The component f_2 is of relativistic origin. In the nonrelativistic limit $c\to\infty$ (here c is the speed of light), the LF plane $t+z/c=0$ turns into $t=0$. Any $\vec{n}$ dependence in (14) should then disappear, and this happens if f_2 becomes negligible.

We see that in order to obtain the constrained CC form (13) from the general form (8), one should eliminate, according to (12), the second components of spinors and neglect the second component f_2 in (14). This is natural in a nonrelativistic approximation, when both the second components of spinors and the dependence on the light-front orientation indeed disappear. One can use this form of the wave function to estimate the influence of relativistic effects on the static nucleon properties [15]. However, there is no compelling reason to use the form (13) in the asymptotic relativistic domain.

The form (13), determined by the single component f_1, implies a relation between φ_1 and φ_2 in (8). Setting f_2 to zero in (16), we obtain

$$\varphi_1=\frac{c_k c_p}{2M_0}(M_0+M)f_1,$$

$$\varphi_2=\frac{c_k c_p}{4M_0 M}(M_0^2-M^2)f_1, \quad (18)$$

and, hence,

$$\varphi_2=\frac{M_0-M}{2M}\varphi_1. \quad (19)$$

Since M_0 is large for large momenta, we see that in the asymptotic regime the component φ_2 dominates: $\varphi_2\approx M_0\varphi_1/(2M)$.

Thus the ansatz (13) is equivalent to an assumption that the component φ_2 dominates in (8). We shall show below that this predominance of φ_2 over φ_1, generated by the wave function (13), results in the $QF_2/F_1\to$const asymptotic behavior.

III. LIGHT-FRONT SPINORS

The discussion of the preceding section can also be given in terms of a LF Fock-state expansion and the associated LF spinors. The LFWF of a hadron with spin projection $J_z=\pm\frac{1}{2}$ is represented by the function $\psi^{J_z}_{\lambda_1,\ldots,\lambda_n}(x_i,\vec{k}_{\perp i})$, where

$$k_i=(k_i^+,k_i^-,\vec{k}_{i\perp})=\left(x_i P^+,\frac{\vec{k}_{\perp i}^2+m_i^2}{x_i P^+},\vec{k}_{i\perp}\right) \quad (20)$$

specifies the momentum of each constituent and λ_i specifies its light-front helicity in the z direction. The light-front fractions $x_i=k_i^+/P^+$ are positive and satisfy $\Sigma_i x_i=1$. We note that $M_0^2=\Sigma_{i=1}^n[(k_{\perp i}^2+m_i^2)/x_i]=(\Sigma_i k_i)^2$ is Lorentz invariant, and the scalar part of the LFWF is a function of only x_i and M_0^2.

For a spin-1/2 state with two constituents in Yukawa theory, we write $\psi_\lambda^{J_z}(x,\vec{k}_\perp)\equiv\psi_\lambda^{J_z}(x_i,\vec{k}_{\perp i})$, where $\lambda=\lambda_1$ is the helicity of the fermion, $x=x_1$, and $\vec{k}_\perp=\vec{k}_{1\perp}$. (We use the subscript 1 for the fermion and 2 for the scalar constituent.) The two-particle Fock state with total momentum $(P^+,\vec{P}_\perp)$ and spin J_z is then given by

$$|P^+,\vec{P}_\perp=\vec{0}_\perp,J_z\rangle$$
$$=\int\frac{dx\,d^2k_\perp}{16\pi^3\sqrt{x(1-x)}}\sum_\lambda\psi_\lambda^{J_z}(x,\vec{k}_\perp)|xP^+,\vec{k}_\perp,\lambda\rangle. \quad (21)$$

The Fock-state ket on the right is defined by

$$|xP^+,\vec{k}_\perp,\lambda\rangle\equiv|k_1^+=xP^+,k_2^+=(1-x)P^+;$$
$$k_{1\perp}=\vec{k}_\perp,\vec{k}_{2\perp}=-\vec{k}_\perp;\lambda_1=\lambda\rangle \quad (22)$$

and normalized by

$$\langle k_i'^+,\vec{k}_{i\perp}',\lambda'|k_i^+,\vec{k}_{i\perp},\lambda\rangle=\prod_{i=1}^2 16\pi^3 k_i^+\delta(k_i'^+-k_i^+)$$
$$\times\delta(\vec{k}_{i\perp}'-\vec{k}_{i\perp})\delta_{\lambda',\lambda}. \quad (23)$$

The four functions $\psi^\uparrow_{\pm 1/2}$ and $\psi^\downarrow_{\pm 1/2}$ provide a representation of the LFWFs for $J_z=\uparrow,\downarrow$ [17]. The associated spinors are the LF spinors u^{LF} [36,2], which in the hadron rest frame $(\vec{p}=\vec{0})$ are given by

$$\overline{u}^{LF\sigma_1}(k_1)=\frac{1}{\sqrt{2(k_{10}+k_{1z})}}\chi^{\dagger\sigma_1}[m+k_{10}+\sigma_z(\vec{\sigma}\cdot\vec{k}_1)-(k_{10}-m)\sigma_z-\vec{\sigma}\cdot\vec{k}_1],$$

$$u^{LF\sigma}(p)=\frac{1}{\sqrt{2M}}\begin{pmatrix}2M\\0\end{pmatrix}\chi^{\sigma}. \tag{24}$$

The general form (8) has a simple form in the light-front spinor representation. A straightforward calculation gives[2]

$$\sqrt{1-x}\,\psi^{\uparrow}_{+1/2}(x,\vec{k}_\perp)=\left(M+\frac{m}{x}\right)\varphi_1+2M\varphi_2,$$

$$\sqrt{1-x}\,\psi^{\uparrow}_{-1/2}(x,\vec{k}_\perp)=-\frac{(+k_x+ik_y)}{x}\varphi_1,$$

$$\sqrt{1-x}\,\psi^{\downarrow}_{+1/2}(x,\vec{k}_\perp)=\frac{(+k_x-ik_y)}{x}\varphi_1,$$

$$\sqrt{1-x}\,\psi^{\downarrow}_{-1/2}(x,\vec{k}_\perp)=\left(M+\frac{m}{x}\right)\varphi_1+2M\varphi_2. \tag{25}$$

Note that in the perturbative Yukawa model[3] one obtains [17]

$$\varphi_1=\frac{g}{M^2-M_0^2},\quad \varphi_2=0. \tag{26}$$

In this way we reproduce the wave functions (44) and (46) of Ref. [17]. The general solution in (25), which does not require any assumptions, differs from the perturbative solution only by the contribution $2M\varphi_2$ in the components $\psi^{\uparrow}_{+1/2}$ and $\psi^{\downarrow}_{-1/2}$.

In terms of the LF spinor representation, the CC ansatz (13) implies particular forms for the components of (25). Substituting the expression (19) for φ_2 into Eqs. (25), we find

$$\sqrt{1-x}\,\psi^{\uparrow}_{+1/2}(x,\vec{k}_\perp)=\left(M_0+\frac{m}{x}\right)\varphi_1,$$

$$\sqrt{1-x}\,\psi^{\uparrow}_{-1/2}(x,\vec{k}_\perp)=-\frac{(+k_x+ik_y)}{x}\varphi_1,$$

$$\sqrt{1-x}\,\psi^{\downarrow}_{+1/2}(x,\vec{k}_\perp)=\frac{(+k_x-ik_y)}{x}\varphi_1,$$

$$\sqrt{1-x}\,\psi^{\downarrow}_{-1/2}(x,\vec{k}_\perp)=\left(M_0+\frac{m}{x}\right)\varphi_1. \tag{27}$$

We see that the CC ansatz in (13) is equivalent to the replacement $M\to M_0$ in the perturbative LF components given in Ref. [17]. The anomalous dependence on M_0 is the source of the discrepancy of the ansatz with the asymptotic behavior of the components with different angular momentum projections found in Ref. [37] from a perturbative model based on the iteration of the one-gluon exchange kernel. The perturbative QCD counting rules for the fall-off of hadronic LFWFs [37] and hard scattering exclusive amplitudes [24–26] at high transverse momentum can also be derived [38,39] without the use of perturbation theory using the conformal properties of the AdS conformal field theory (CFT) correspondence between gauge theory and string theory [40].

The component $\psi^{\uparrow}_{-1/2}$ corresponds to $L_z=1$, and the component $\psi^{\uparrow}_{+1/2}$ corresponds to $L_z=0$. The PQCD analysis based on the exchange of a gluon gives [37]

$$\psi(L_z=1)/\psi(L_z=0)=\psi^{\uparrow}_{-1/2}/\psi^{\uparrow}_{+1/2}\sim k_\perp\varphi_1/\varphi_2\sim 1/k_\perp, \tag{28}$$

which implies $Q^2F_2/F_1=\text{const}$ [37]. We confirm this in the next section. On the other hand, (19) or (27), corresponding to the CC ansatz, gives

$$\psi(L_z=1)/\psi(L_z=0)=\psi^{\uparrow}_{-1/2}/\psi^{\uparrow}_{+1/2}\sim k_\perp\varphi_1/\varphi_2\sim\text{const}. \tag{29}$$

In turn, the ratio (29) results in the asymptotic behavior $QF_2/F_1=\text{const}$, as we will see in the next section.

IV. FORM FACTORS AND THE M_0 DEPENDENCE OF LIGHT-FRONT WAVE FUNCTIONS

A. Dirac and Pauli form factors

In the case of a spin-$\frac{1}{2}$ composite system, the Dirac and Pauli form factors $F_1(q^2)$ and $F_2(q^2)$ are defined by

$$\langle P'|J^\mu(0)|P\rangle=\overline{u}(P')\left[F_1(q^2)\gamma^\mu+F_2(q^2)\frac{i}{2M}\sigma^{\mu\alpha}q_\alpha\right]u(P), \tag{30}$$

where $q^\mu=(P'-P)^\mu$, $u(P)$ is the bound-state spinor, and M is the mass of the composite system. In the light-front formalism it is convenient to identify the Dirac and Pauli form factors from the helicity-conserving and helicity-flip vector current matrix elements of the J^+ current component [6]

[2]The wave function (8) and the one given here are related as follows: $\psi(\text{in (8)})=\sqrt{x(1-x)}\,\psi(\text{here})$. In this expression, k_x and k_y are equivalent to k^1 and k^2, and J_z and L_z denote J^3 and L^3.

[3]In the case of perturbative models, a single-particle wave function $\psi^{J_z}_{\lambda_1}=\sqrt{Z}\,\delta^2(\vec{k}_\perp)\,\delta(1-x)\,\delta_{J_z\lambda_1}$ is present, where the normalization constant Z ensures unit probability. The perturbative Yukawa model wave functions can be formally differentiated with respect to the boson mass in order to simulate the fall-off of the wave function of a composite hadron and eliminate the single-particle Fock component.

$$\langle P+q,\uparrow|\frac{J^+(0)}{2P^+}|P,\uparrow\rangle=F_1(q^2), \tag{31}$$

$$\langle P+q,\uparrow|\frac{J^+(0)}{2P^+}|P,\downarrow\rangle=-(q^1-\mathrm{i}q^2)\frac{F_2(q^2)}{2M}. \tag{32}$$

We use the standard light-front frame ($q^\pm=q^0\pm q^3$) where

$$q=(q^+,q^-,\vec{q}_\perp)=\left(0,\frac{-q^2}{P^+},\vec{q}_\perp\right),$$

$$P=(P^+,P^-,\vec{P}_\perp)=\left(P^+,\frac{M^2}{P^+},\vec{0}_\perp\right), \tag{33}$$

and $q^2=-2P\cdot q=-\vec{q}_\perp^{\,2}$ is the square of the four-momentum transferred by the photon.

Using Eqs. (31) and (21) we have

$$F_1(q^2)=\int\frac{\mathrm{d}^2\vec{k}_\perp\mathrm{d}x}{16\pi^3}[\psi^{\uparrow *}_{+1/2}(x,\vec{k}'_\perp)\psi^{\uparrow}_{+1/2}(x,\vec{k}_\perp)+\psi^{\downarrow *}_{-1/2}(x,\vec{k}'_\perp)\psi^{\uparrow}_{-1/2}(x,\vec{k}_\perp)], \tag{34}$$

where

$$\vec{k}'_\perp=\vec{k}_\perp+(1-x)\vec{q}_\perp. \tag{35}$$

From Eqs. (32) and (21), we have

$$F_2(q^2)=\frac{-2M}{(q_x-\mathrm{i}q_y)}\int\frac{\mathrm{d}^2\vec{k}_\perp\mathrm{d}x}{16\pi^3}[\psi^{\uparrow *}_{+1/2}(x,\vec{k}'_\perp)\psi^{\downarrow}_{+1/2}(x,\vec{k}_\perp)+\psi^{\uparrow *}_{-1/2}(x,\vec{k}'_\perp)\psi^{\downarrow}_{-1/2}(x,\vec{k}_\perp)]. \tag{36}$$

The individual wave functions are given by (25); substitution yields

$$F_1(q^2)=\frac{1}{16\pi^3}\int\frac{d^2k_\perp dx}{x^2(1-x)}\left\{\left[x(2mM+xM^2)+m^2+k_\perp^2-\frac{1}{4}(1-x)^2Q^2\right]\varphi'_1\varphi_1+2Mx(m+xM)\times(\varphi'_1\varphi_2+\varphi'_2\varphi_1)+4M^2x^2\varphi'_2\varphi_2\right\}, \tag{37}$$

$$F_2(q^2)=\frac{M}{8\pi^3}\int\frac{d^2k_\perp dx}{x^2(1-x)}\left\{(1-x)(m+xM)\varphi'_1\varphi_1-2Mx\frac{\vec{k}_\perp\cdot\vec{q}_\perp}{Q^2}(\varphi_1\varphi'_2-\varphi_2\varphi'_1)+Mx(1-x)(\varphi_1\varphi'_2+\varphi_2\varphi'_1)\right\}, \tag{38}$$

where the $\vec{k}_\perp$ variable has been shifted by $-\frac{1}{2}(1-x)\vec{q}_\perp$, so that

TABLE I. The dependence of the asymptotic form of the form factor ratio on the asymptotic behavior of the LF components $\psi(L_z=1)/\psi(L_z=0)=\psi^{\uparrow}_{-1/2}/\psi^{\uparrow}_{+1/2}$ in the Yukawa model.

$\frac{\psi(L_z=1)}{\psi(L_z=0)}$	$k_\perp^2$	$k_\perp$	const	$\frac{1}{k_\perp}$	$\frac{1}{k_\perp^2}$
$\frac{F_2}{F_1}$	$\frac{1}{Q^3}$	$\frac{1}{Q^2}$	$\frac{1}{Q}$	$\frac{1}{Q^2}$	$\frac{1}{Q^3}$

$$\varphi_{1,2}=\varphi_{1,2}\left(x,\vec{k}_\perp-\frac{1}{2}(1-x)\vec{q}_\perp\right),$$

$$\varphi'_{1,2}=\varphi_{1,2}\left(x,\vec{k}_\perp+\frac{1}{2}(1-x)\vec{q}_\perp\right). \tag{39}$$

The required parton invariant masses M_0 and M'_0 are

$$M_0^2=\frac{\left[\vec{k}_\perp-\frac{1}{2}(1-x)\vec{q}_\perp\right]^2+m^2}{x(1-x)},$$

$$M'^2_0=\frac{\left[\vec{k}_\perp+\frac{1}{2}(1-x)\vec{q}_\perp\right]^2+m^2}{x(1-x)}. \tag{40}$$

Being expressed in terms of the invariant masses, the form factor integrands contain squares of masses: M_0^2 and M'^2_0.

One can see that if φ_2 is smaller than φ_1 or of the order of φ_1 for large $\vec{k}_\perp^{\,2}$, the leading terms in F_1 are $k_\perp^2$ and $-\frac{1}{4}(1-x)^2Q^2$. An analytical calculation of the asymptotic behavior of the form factors with the power-law wave function $\varphi_1=N/(M_0^2+\beta^2)^n$ shows that these two leading contributions cancel each other. Thus the leading term becomes $\sim\log Q^2$ instead of $\sim Q^2$ and the ratio F_2/F_1 becomes $\sim 1/\log(Q^2/m^2)$. The same result is found for pseudoscalar coupling, i.e., with the wave function which is obtained by inserting in (8) the matrix γ_5. As explained in the Introduction, the cancellation of the leading term in the Dirac form factor in the scalar gluon models is related to the violation of chirality conservation.

We summarize in Table I how the asymptotic behavior of the form factor ratio depends on the asymptotic properties of the LFWFs in the Yukawa model. Since the scalar part of the LFWF is a function of M_0^2 and x_i, and $\psi(L_z=\pm1)$ contains the $k_x\pm ik_y$ prefactor, $\psi(L_z=1)/\psi(L_z=0)$ can only be an odd power of $k_\perp$. The third and fourth columns of Table I correspond to the cases of (29) and (28), respectively.

The cancellation of the leading power-law contribution is specific to the scalar and pseudoscalar diquark models. In the case of a spin-1/2 system comprising a spin-1/2 quark and a spin-1 diquark, the hadron wave function is determined in general by six independent components. To see the effect coming from the spin-1 diquark, consider a wave function in the one-component form

$$\psi_{\sigma_1,\lambda}(k_1,k_2,p)=e^{*(\lambda)}_\nu(k_2)\bar{u}_{\sigma_1}(k_1)\gamma^\nu u(p)\varphi_1. \tag{41}$$

Here $e_\nu^{(\lambda)}(k_2)$ is the spin-1 polarization vector. The sum over polarizations results in the propagator $(g_{\nu\nu'}-k_{2\nu}k_{2\nu'}/\mu^2)$ in the form factor calculation. In this model we find for the form factor ratio

$$\frac{F_2}{F_1}\sim\frac{\log(Q^2/m^2)}{Q^2},$$

which is close to the fit (2), differing only by a factor $\log(Q^2/m^2)$. The same perturbative behavior (up to a coefficient) also occurs in the asymptotic behavior of the electron form factor in QED. If a scalar or pseudoscalar coupling is also present, the vector contribution will dominate the asymptotic behavior. We emphasize that for all three couplings, both form factors decrease as an integral power of Q^2 [modulo powers of $\log(Q^2/m^2)$], not as a power of Q. As shown in Fig. 1, a fit to the form factor ratio based on powers of Q^2 and powers of $\log Q^2$ describes the experimental data well.

B. Consequences of the CC constraint

The CC ansatz $f_2=0$ is equivalent to a quark–scalar-diquark model, but with the additional condition (19). As we have seen this introduces anomalous terms in the LFWFs, which are linear in M_0. Substituting φ_2 from (19) into Eqs. (37), (38), we obtain

$$F_1(q^2)=\frac{1}{32\pi^3}\int\frac{d^2k_\perp dx}{x^2(1-x)}[x(1-x)(M_0^2+M'^2_0)+2x^2M_0M_0'+2xm(M_0+M_0')-(1-x)^2Q^2]\varphi_1'\varphi_1, \tag{42}$$

$$F_2(q^2)=\frac{M}{16\pi^3}\int\frac{d^2k_\perp dx}{x^2(1-x)}\left[2(1-x)m+x(1-x)(M_0+M_0')-\frac{x^2}{Q^2}(M_0-M_0')^2(M_0+M_0')\right]\varphi_1'\varphi_1. \tag{43}$$

The same result is of course obtained by direct calculation with the wave function (9).

Because of the terms with M_0^2, M'^2_0, and M_0M_0', the form factor F_1 contains the second power Q^2 relative to the term $2(1-x)m$ in F_2, which does not contain M_0 or M_0'. This occurs independently of whether the cancellation between $k_\perp^2$ and $-\frac{1}{4}(1-x)^2Q^2$ takes place or not. If it does not take place, we get an extra contribution to the Q^2 power. This only changes the coefficient of Q^2. Similarly, because of the term with (M_0+M_0'), F_2 contains the first power of Q. This extra factor of M_0 (and, hence, Q) results in the nominal asymptotic behavior

$$\frac{QF_2}{F_1}=\text{const.} \tag{44}$$

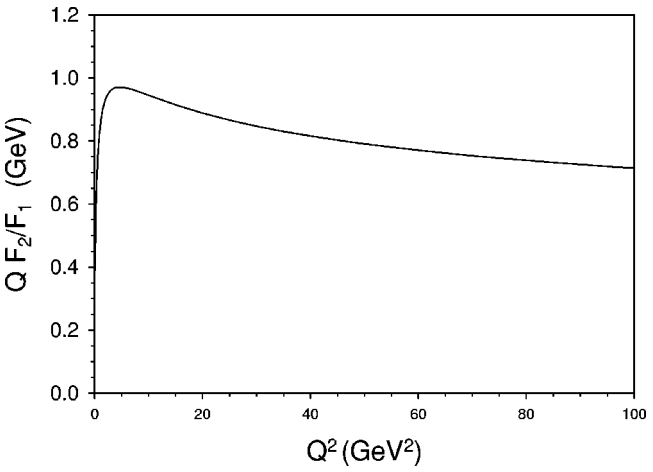

FIG. 2. Asymptotic behavior of the form factor ratio QF_2/F_1 as determined from Eqs. (37) and (38) of the text with $\varphi_1\propto[M_0^2+(0.6\ \text{GeV})^2]^{-3.5}$ and φ_2 fixed by Eq. (19). The constituent mass is $m=0.3$ GeV, and the bound-state mass is $M=0.94$ GeV. The choice of φ_2 corresponds to the Chung-Coester ansatz and nominally implies a constant asymptotic behavior for the ratio, but here is well fit by $Q^{-0.25}$.

We emphasize that this behavior follows from the CC ansatz (13) [or $f_2=0$ in (14)], which in turn, is equivalent to the relation (19). After the constraint is imposed, the asymptotic behavior (44) follows without further dynamical assumptions.

The coefficient functions φ_i are in general functions of M_0^2 since in the equations of motion only the quantity M_0^2 appears. In the next section this is shown explicitly in the case of the Wick-Cutkosky model. Explicit calculation of the form factors, with $\varphi_1=N/(M_0^2+\beta^2)^{3.5}$, as assumed in Ref. [14], and with φ_2 fixed by Eq. (19), qualitatively replicates the nonconstant, nonintegral asymptotic behavior found in Ref. [14]. Figure 2 shows the result for QF_2/F_1, which is approximately constant for a small range of intermediate Q^2 but is well fitted by $Q^{-0.25}$ for large Q^2. Similar results are obtained with an exponential wave function.

The arguments by Ralston *et al.* [41] and Kroll [42], in favor of QF_2/F_1 being asymptotically constant, reduce to a discussion of the following ratio of wave function matrix elements:

$$\frac{F_2}{F_1}=\frac{\langle\bar{\psi}_1\psi_0\rangle}{Q\langle b\bar{\psi}_0\psi_0\rangle}. \tag{45}$$

[See for example Eq. (5) of Ref. [41] and Eq. (11) of Ref. [42].] They arrive at this ratio by including an $L_z=1$ wave function in the contribution to F_2, so that there is an overlap with the $L_z=0$ wave function. However, the $L_z=1$ wave function $\psi^\uparrow_{-1/2}$ also contributes to F_1 in an overlap with itself, as given above in Eq. (34). This introduces an additional term of order Q^2 in the denominator and makes Q^2F_2/F_1 asymptotically constant.

V. THE WICK-CUTKOSKY MODEL AND M_0^2 DEPENDENCE

The Wick-Cutkosky model is based on the ladder approximation to the Bethe-Salpeter (BS) equation in $\phi^2\chi$ field theory. The LFWFs in this model for the $J=L=0$ and $J=L=1$ bound states have been computed in Ref. [32]. We shall show below that if the wave function is represented in four-dimensional form, the scalar components depend on M_0^2 as expected on general grounds. Indeed, for $J=1$ the BS function reads [20,43]

$$\Phi_{1\lambda}(q,p)=\tilde{q}Y^*_{1\lambda}\left(\frac{\tilde{\vec{q}}}{\tilde{q}}\right)\Phi_1(q,p), \tag{46}$$

$$\Phi_1(q,p)=-i\int_{-1}^{+1}\frac{g_1(z,M)dz}{(m^2-M^2/4-q^2-zp\cdot q-i\epsilon)^4},$$

where $g_1(z,M)$ is the spectral function satisfying a specific differential equation [20,43]. We will not need the explicit form of g_1. Here $q=(k_a-k_b)/2$, $\tilde{\vec{q}}=L^{-1}(\vec{v})\vec{q}$ has the sense of relative momentum in the rest frame given by $\vec{p}=\vec{0}$, and $L^{-1}(\vec{v})$ is the Lorentz boost with $\vec{v}=\vec{p}/\varepsilon_p$. In an arbitrary frame the spherical function in (46) can be replaced by

$$\tilde{q}Y^*_{1\lambda}\left(\frac{\tilde{\vec{q}}}{\tilde{q}}\right)\to-\sqrt{\frac{3}{4\pi}}e^{(\lambda)}_\mu(p)q^\mu, \tag{47}$$

where $e^{(\lambda)}_\mu(p)$ is the spin-1 polarization vector.

Using the relation (5) between the LFWF and the BS function and Eq. (47), we find the LFWF in the form

$$\psi_{1\lambda}=-\sqrt{\frac{3}{4\pi}}e^{(\lambda)}_\mu(p)\psi^\mu,\quad \psi^\mu=k^\mu\varphi_1+\frac{\omega^\mu}{\omega\cdot p}\varphi_2, \tag{48}$$

where

$$\varphi_1=\frac{2g_1(1-2x,M)}{3(M_0^2-M^2)^3x^2(1-x)^2},$$

$$\varphi_2=-\frac{g_1(1-2x,M)}{6(M_0^2-M^2)^2x^2(1-x)^2}-\frac{g_1'(1-2x,M)}{3(M_0^2-M^2)^2x(1-x)}. \tag{49}$$

The prime in $g_1'(z,M)$ means differentiation with respect to z. We see that the components φ_1 and φ_2 depend on M_0^2, as expected. A similar result holds for the $L=0$ state:

$$\psi=\frac{g(1-2x,M)}{2\sqrt{\pi}x(1-x)(M_0^2-M^2)^2}, \tag{50}$$

where $g(z,M)$ is the corresponding spectral function. It does not depend linearly on M_0.

The representation (48) for the Wick-Cutkosky wave function is analogous to the representation (8) for the Yukawa model. These results for the LFWFs of the bound states of the Wick-Cutkosky model can also be obtained by computing the instant-form wave function and boosting to infinite momentum, as in Weinberg's $P_z\to\infty$ method [44].

Note that the LFWF (48) in the constituent rest frame has the form

$$\psi_{1\lambda}(\vec{k},\vec{n})=f_1(\vec{k}^2,\vec{n}\cdot\vec{k})kY^*_{1\lambda}\left(\frac{\vec{k}}{k}\right)+f_2(\vec{k}^2,\vec{n}\cdot\vec{k})Y^*_{1\lambda}(\vec{n}), \tag{51}$$

where the scalar components $f_{1,2}$ can be expressed in terms of $\varphi_{1,2}$ from (48). This representation is analogous to the representation (14) for the Yukawa model. Since the BS function (46) describes a state with angular momentum $J=1$, the corresponding LFWF (51), which is derived from the BS function without any approximation, definitely has the same angular momentum. It is an eigenfunction of the angular momentum operator (6) (omitting the spin operator $\frac{1}{2}\vec{\sigma}$).

VI. CONSEQUENCES OF THE CC ANSATZ FOR THREE-BODY LIGHT-FRONT WAVE FUNCTIONS

The simplified three-quark LFWF of the nucleons introduced by Coester and Chung, and used in Ref. [14] for a model of the proton form factors, has the same spin structure as the quark-scalar-diquark model—two quarks form a spin-zero diquark, and the spin of the nucleon is determined by the spin of the third quark. The wave function is symmetrized relative to the permutations of the quarks according to $SU(6)$ flavor-spin symmetry. For example, in the constituent rest frame, the Pauli-spinor representation of the three-quark wave function corresponding to the state of zero spin and zero isospin of the quark pair 12 is

$$\psi(12,3)=\chi^{(S=0)}(1,2)\chi^\dagger_{\sigma_3}\delta^{\sigma_3\sigma}\cdot\xi^{(I=0)}(1,2)\xi^\dagger_{\tau_3}\delta^{\tau_3\tau},$$

where $\chi^{(S=0)}(1,2)=\chi^\dagger_{\sigma_1}i\sigma_y\chi^\dagger_{\sigma_2}$ is the spin-zero wave function of the two quarks and $\xi^{(I=0)}(1,2)=\xi^\dagger_{\tau_1}i\sigma_y\xi^\dagger_{\tau_2}$ is the isospin-zero wave function. The nucleon spin-isospin wave function is obtained by symmetrization:

$$\psi(1,2,3)=\psi(12,3)+\psi(23,1)+\psi(31,2).$$

Using the Fierz identities, we may transform it to the form

$$\psi=\frac{1}{\sqrt{72}}\psi_S[3+(\vec{\sigma}_{12}\cdot\vec{\sigma}_{3N})(\vec{\tau}_{12}\cdot\vec{\tau}_{3N})]. \tag{52}$$

This form is totally symmetric with respect to spin and isospin. We have introduced here the symmetric momentum-dependent part ψ_S. The factors in (52) should be understood as

$$1\equiv(\chi^\dagger_{\sigma_1}i\sigma_y\chi^\dagger_{\sigma_2})(\chi^\dagger_{\sigma_3}\chi_{\sigma_N}),$$

$$(\vec{\sigma}_{12}\cdot\vec{\sigma}_{3N})\equiv(\chi^\dagger_{\sigma_1}\vec{\sigma}\sigma_y\chi^\dagger_{\sigma_2})(\chi^\dagger_{\sigma_3}\vec{\sigma}\chi_{\sigma_N}),$$

and similarly for the isospin part. The wave function (52) is just the ansatz used in Ref. [14].

In the covariant Yukawa model, the general form of the two-parton LFWF (14) contains two components, in contrast to the one-component CC ansatz (13). The general form of the three-quark nucleon wave function contains sixteen components [35] in contrast to the one-component ansatz (52). Thus the CC ansatz (52) is even a stronger constraint than in the two-body Yukawa model.

As in the two-body case (9), the three-body wave function (52) can be recast in the four-dimensional form [35]

$$\psi = \frac{\psi_S}{\sqrt{72}} c_1 c_2 c_3 c_N \{3[\bar{u}(k_1)\Lambda_+ \gamma_5 U_c \bar{u}(k_2)] \times [\bar{u}(k_3)\Lambda_+ u_N(p)] - [\bar{u}(k_1)\Lambda_+ \gamma^\mu \Lambda_- U_c \bar{u}(k_2)] \times [\bar{u}(k_3)\Lambda_+ \gamma_\mu \gamma_5 \Lambda_+ u_N(p)](\vec{\tau}_{12}\cdot\vec{\tau}_{3N})\}, \quad (53)$$

where $\Lambda_- = (\hat{\mathcal{P}} - M_0)/2M_0$ and $U_c = \gamma_2\gamma_0$ is the charge conjugation matrix. The coefficients are $c_1 = 1/\sqrt{m+\varepsilon_{k_1}}$, etc. The projection operator Λ_+ is again given by Eq. (10); however, the two-body constituent momentum is replaced by the three-body momentum

$$\mathcal{P} = k_1 + k_2 + k_3, \quad M_0^2 = \mathcal{P}^2.$$

In the constituent rest frame, where $\vec{\mathcal{P}} = \vec{0}$, we have $\mathcal{P}_0 = M_0$, and the wave function (53) reduces to (52).

The second term in (53) does not contribute to the proton form factor [35,14]. The first term is factorized. We suppose that the photon interacts with the third quark. The first factor $[\bar{u}(k_1)\Lambda_+ \gamma_5 U_c \bar{u}(k_2)]$ is the same for F_1 and F_2, and, therefore, it does not change their ratio. Since only the second factor $[\bar{u}(k_3)\Lambda_+ u_N(p)]$ gives different contributions to F_1 and F_2, we rewrite the wave function in a factorized form

$$\psi \propto \psi_S \bar{u}(k_3)\Lambda_+ u_N(p). \quad (54)$$

This coincides with the CC-constrained Yukawa wave function (9).

The form factor calculation using the three-quark wave function determined by (52) and (54) differs from the two-body calculation in Sec. IV only in the use of three-body kinematics. The result is

$$F_1(q^2) = \frac{1}{2}\int [x_3(1-x_3)(M_0^2 + M'^2_0) + 2x_3^2 M_0 M'_0 + 2x_3 m(M_0 + M'_0) - (1-x_3)^2 Q^2] G(1,2,3,Q^2) D, \quad (55)$$

$$F_2(q^2) = M\int \left[2(1-x_3)m + x_3(1-x_3)(M_0 + M'_0) - \frac{x_3^2}{Q^2}(M_0 - M'_0)^2 (M_0 + M'_0)\right] G(1,2,3,Q^2) D. \quad (56)$$

Here D is the three-body phase volume, and $G(1,2,3,Q^2)$ is a function of the variables of all three quarks and of Q^2. We do not need the explicit form of G. M_0 is the effective three-body mass, which depends on $\vec{R}_{3\perp} - \frac{1}{2}(1-x_3)\vec{q}_\perp$, whereas M'_0 depends on $\vec{R}_{3\perp} + \frac{1}{2}(1-x_3)\vec{q}_\perp$. Because of factorization of the wave function (54), these formulas (found by direct calculation) coincide, after evident changes of notation, with the corresponding expressions (42) and (43) for the two-body form factors for the CC ansatz.

At $Q^2 \equiv \vec{q}_\perp^{\,2} \to \infty$, both M'_0 and M_0 tend to Q. For the leading term we find

$$F_1 \propto \ldots Q^2,$$

$$F_2 \propto \ldots Q, \quad (57)$$

which reproduce the ratio $QF_1/F_2 = \text{const}$. As before, this asymptotic behavior is a consequence of the specific constraints on the LFWFs.

On the other hand, we have mentioned in Sec. IV that scalar and pseudoscalar couplings result in the asymptotic ratio $F_2/F_1 \sim 1/\log(Q^2/m^2)$ and vector coupling in $F_2/F_1 \sim \log(Q^2/m^2)/Q^2$. These couplings are not really related to a hypothesis of predominance of diquarks, but simply represent different spin structures of the nucleon wave function. The total number of these structures is sixteen [35]. Some of these other structures, in addition to the vector one, may also contribute to the asymptotic behavior $F_2/F_1 \sim \log(Q^2/m^2)/Q^2$.

VII. CONCLUSIONS

The explicitly Lorentz-invariant formulation of the front form provides a general method for determining the general structure of light-front wave functions. We have also used the fact that the angular momentum of a bound state can be defined covariantly within the Bethe-Salpeter formalism: The LFWFs for an n-particle Fock state evaluated at equal light-front time $\sigma = \omega\cdot x$ can be obtained by integrating the covariant Bethe-Salpeter functions over the corresponding relative light-front energies. The resulting LFWFs are eigenstates of the kinematic angular momentum operator (6). The result is that the LFWFs are functions of the invariant mass squared of the constituents $M_0^2 = (\Sigma k^\mu)^2$ and the light-cone momentum fractions $x_i = k_i\cdot\omega/p\cdot\omega$, each multiplying spin-vector and polarization invariants involving ω^μ, where $\omega = (\omega_0, \vec{\omega})$ is the four-vector determining the orientation of the light-front plane $\omega\cdot x = 0$.

We have presented the structure of LFWFs for two- and three-particle bound states using the explicitly Lorentz-invariant formulation of the front form [1]. As examples we have given the explicit form of the LFWFs for spin-0 and spin-1 eigenstates of the nonperturbative eigensolutions of the Wick-Cutkosky model, as well as examples of spin-1/2 states constructed using perturbation theory. For example, the LFWF of a spin-1/2 system composed of spin-half and spin-zero constituents has the general form

$$\psi(k_1,p)=\bar{u}(k_1)\left(\varphi_1+\frac{M\hat{\omega}}{\omega\cdot p}\varphi_2\right)u(p),$$

where the φ_i are functions of the square of the invariant mass and the light-front momentum fractions. The orbital angular momentum prefactor in the constituent rest frame is proportional to $\vec{\omega}\times\vec{k}\cdot\vec{S}$.

An important test of the LF computations is light-front invariance—although the LFWFs depend on the choice of the light-front quantization direction, all observables, such as matrix elements of local current operators, form factors, and cross sections, must be independent of ω^μ. We have computed the large momentum transfer behavior of the ratio of Pauli and Dirac form factors of the nucleon using the exact relation for spacelike current matrix elements in terms of LFWFs. The dependence of the invariant mass squared implies that hadron form factors computed from the overlap integrals of LFWFs are analytic functions of Q^2 in agreement with dispersion theory, the PQCD analysis of Belitsky, Ji, and Yuan [19], and conformal arguments [39], as well as with the form factor ratios obtained using the nonperturbative solutions to the Wick-Cutkosky model. We have also shown that a fit to the Pauli to Dirac form factor ratio incorporating the predicted perturbative QCD $1/Q^2$ and $\log Q^2$ asymptotic dependence describes the recent Jefferson Laboratory polarization transfer data well. In contrast, we have shown that the LFWFs introduced by Chung and Coester to parametrize the static and low-momentum properties of the nucleons correspond to the spin-locking of a quark with the spin of its parent nucleon, together with a positive-energy projection constraint. These extra constraints lead to an anomalous linear dependence of the LFWFs on the invariant mass of the constituents and an anomalous dependence of form factors on Q rather than Q^2.

The CC construction of relativistic light-front wave functions was introduced to represent the properties of the nucleons in the low-momentum transfer domain. However, there are a number of difficulties with extending the CC form to the high-momentum transfer domain:

(1) If one applies the CC ansatz to a bound state of a spin-half quark and a scalar (the Yukawa model), the quark is constrained to have the same spin projection as the bound state nucleon, $S_q^z=S_p^z$, when one uses the conventional BD spinor representation. Thus the only orbital angular momentum allowed by the CC constraint is the kinematical angular momentum arising from the lower components of the BD spinor. The spin-locked CC constraint does not allow for the full degrees of freedom of a relativistic system.

(2) If one compares the CC ansatz to wave functions generated in perturbation theory, the net result is to replace the bound-state mass M in the numerator of the LFWFs by the invariant mass of the constituents

$$M_0=\sqrt{\sum_{i=1}^{n}\frac{k_{\perp i}^2+m_i^2}{x_i}}.$$

This replacement leads to the anomalous growth of the CC LFWF at large transverse momentum. For example, if one applies the CC ansatz to the QCD quark splitting function, the ultraviolet $\log Q^2$ behavior of $q(x,Q^2)$ will derive from two sources: the standard behavior in $k_\perp$ arising from perturbative QCD plus the presence of a term $M_0^2\propto k_\perp^2$ due to the CC ansatz. The presence of the latter source would destroy Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution in Q^2.

(3) The LFWFs of the $J=0$ and $J=1$ bound states can be obtained explicitly in the Wick-Cutkosky model. The form factor ratios of the spin-1 system obtained in this nonperturbative analysis is given by a quadratic Q^2 dependence and by $\log Q^2$ [21]. The application of the CC ansatz leads to terms in the wave function that are linear in M_0 rather than the quadratic dependence of the explicit solutions.

(4) In the case of the simple Yukawa theory, the effective interaction in the CC model has the form $H_I^{\rm eff}=g\bar{\psi}_q\Lambda\psi\phi$ where Λ is a nonlocal positive-energy projection operator. The presence of the projection operator Λ conflicts with the usual relations obtained from crossing and particle-antiparticle symmetry. For example, consider the electroproduction amplitude $\gamma^*p\to q+qq$, where, for simplicity, the qq diquark can be taken as the Yukawa scalar ϕ. The Born amplitude with quark exchange in the t channel has the form

$$M^\mu_{\gamma^*p\to q\phi}(p\cdot q,t,q^2)\propto\frac{e_q\bar{u}\gamma^\mu u}{t-m_q^2}\psi(x,k_\perp),$$

where

$$t-m_q^2=x(M^2-M_0^2),$$

and $x=x_{Bj}=-q^2/(2p\cdot q)$. The CC ansatz introduces a linear term in M_0 in the electroproduction amplitude. If we now use $s\to t$ crossing to obtain the process $\gamma^*\bar{q}\to\bar{p}\phi$, the presence of a linear term in M_0 in the LFWF gives a contribution to the amplitude $M_{\gamma^*\bar{q}\to\bar{p}\phi}(s,t,q^2)$ which is proportional to $\sqrt{s}$ at fixed momentum transfer t. This anomalous Regge behavior corresponds to fermion exchange in the t channel, which, however, is not present in this amplitude.

Again, we emphasize that these difficulties concern the extrapolation of the CC ansatz to the asymptotic region. These problems do not appear in the original work [15] where the CC ansatz was only applied to static nucleon properties and to form factors at relatively small momentum transfer.

Light-front wave functions are the fundamental amplitudes that relate hadrons to their fundamental quark and gluon degrees of freedom. We have shown how one can exhibit the general analytic structure of light-front wave functions, including states with nonzero orbital angular momentum. A key element of this analysis is the use of the explicitly Lorentz-invariant formulation of the front form where the normal to the light-front is specified by a general null vector ω^μ. The resulting LFWFs are functions $\psi_n(x_i,k_{\perp i})$ of the invariant mass squared of the constituents $M_0^2=(\Sigma k^\mu)^2$ and the light-cone momentum fractions $x_i=k_i\cdot\omega/p\cdot\omega$, which multiply invariant prefactors constructed from the spin matrices, polarization vectors, and ω^μ in the case of nonzero orbital angular momentum. The LFWFs corresponding to

definite total angular momentum are eigenstates of a kinematic angular momentum operator and satisfy all Lorentz symmetries of the front form, including boost invariance. We have illustrated these properties using known nonperturbative eigensolutions of the Wick-Cutkosky model for nonzero angular momentum. The dependence of LFWFs on the invariant mass squared implies that current matrix elements and hadron form factors are analytic functions of Q^2 in agreement with dispersion theory and perturbative QCD. We have also shown that a model incorporating this analytic property and leading-twist perturbative QCD constraints is consistent with recent data for the ratio of proton Pauli and Dirac form factors determined by the polarization transfer method.

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APPENDIX

We use the convention $a^{\pm}=a^0\pm a^3$, which gives $a\cdot b=\frac{1}{2}(a^+b^-+a^-b^+)-\vec{a}_\perp\cdot\vec{b}_\perp$, and use the γ matrices in the Dirac representation

$$\gamma^0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \vec{\gamma}=\begin{pmatrix}0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix}. \tag{A1}$$

A caret indicates the inner product of a four-vector with the γ matrices, so that $\hat{k}=k_\mu\gamma^\mu$.

The solutions of $\overline{u}(k_1)\hat{k}_1=m\overline{u}(k_1)$ and $\hat{p}u(p)=Mu(p)$, in terms of the BD spinors, are given by

$$\overline{u}^{\sigma_1}(k_1)=u^{\dagger\sigma_1}(k_1)\gamma^0=\sqrt{\varepsilon_{k_1}+m}\,\chi^{\dagger\sigma_1}\left(1,-\frac{\vec{\sigma}\cdot\vec{k_1}}{\varepsilon_{k_1}+m}\right),$$

$$u^\sigma(p)=\sqrt{\varepsilon_p+M}\begin{pmatrix}1\\ \dfrac{\vec{\sigma}\cdot\vec{p}}{\varepsilon_p+M}\end{pmatrix}\chi^\sigma, \tag{A2}$$

where χ^σ is a two-component spinor, $\varepsilon_k=\sqrt{\vec{k}^2+m^2}$, and $\varepsilon_p=\sqrt{\vec{p}^2+M^2}$. In the constituent rest frame, where $\vec{p}+\vec{\omega}\tau=\vec{k}_1+\vec{k}_2=\vec{0}$, we introduce the variables $\vec{k}$, $\vec{n}$, $\vec{k}_1\equiv\vec{k}$, and $\vec{\omega}=\vec{n}\omega_0$. We also find

$$\vec{p}=-\vec{\omega}\tau=-\vec{n}\frac{M_0^2-M^2}{2M_0},\quad \varepsilon_p=\frac{M_0^2+M^2}{2M_0}.$$

The value of τ here was obtained by squaring the equality $p+\omega\tau=k_1+k_2$, which gives $\tau=(M_0^2-M^2)/(2\omega\cdot p)$, and by using $\omega\cdot p=\omega\cdot(k_1+k_2)=\omega_0M_0$. In this way, we find the BD spinors in the constituent rest frame to be written as

$$\overline{u}^{\sigma_1}(k_1)=\sqrt{\varepsilon_k+m}\,\chi^{\dagger\sigma_1}\left(1,-\frac{\vec{\sigma}\cdot\vec{k}}{\varepsilon_k+m}\right),$$

$$u^\sigma(p)=\frac{1}{\sqrt{2M_0}}\begin{pmatrix}M_0+M\\ -(M_0-M)\vec{\sigma}\cdot\vec{n}\end{pmatrix}\chi^\sigma. \tag{A3}$$

The factor $M\hat{\omega}/(\omega\cdot p)$ in (8) is transformed as

$$\frac{M\hat{\omega}}{\omega\cdot p}=\frac{M}{M_0}(\gamma_0-\vec{n}\cdot\vec{\gamma})=\frac{M}{M_0}\begin{pmatrix}1 & -\vec{n}\cdot\vec{\sigma}\\ \vec{n}\cdot\vec{\sigma} & -1\end{pmatrix}.$$

We also introduce

$$x=\frac{\omega\cdot k_1}{\omega\cdot p}=\frac{1}{2}\left(1-\frac{\vec{n}\cdot\vec{k}}{\varepsilon_k}\right)=\frac{1}{2}-\frac{\vec{n}\cdot\vec{k}}{M_0}.$$

Substituting these expressions into Eq. (8), we reproduce the wave function (14) with the components f_1, f_2 given by Eqs. (15).

The light-front spinor is defined as [36]

$$u^{LF\sigma}(k)=\frac{1}{\sqrt{2(k^0+k^3)}}\begin{pmatrix}k^0+m+\vec{\sigma}\cdot\vec{k}\sigma^3\\ (k^0-m)\sigma^3+\vec{\sigma}\cdot\vec{k}\end{pmatrix}\chi^\sigma, \tag{A4}$$

which gives

$$\overline{u}^{LF\sigma_1}(k_1)=\frac{1}{\sqrt{2(k_1^0+k_1^3)}}\chi^{\dagger\sigma_1}(k_1^0+m+\sigma^3(\vec{\sigma}\cdot\vec{k}_1),$$
$$-(k_1^0-m)\sigma^3-\vec{\sigma}\cdot\vec{k}_1). \tag{A5}$$

The BD and LF spinors are connected by the following unitary relations (at $\vec{n}||z$):

$$\begin{pmatrix}u^{+1/2}(p)\\ u^{-1/2}(p)\end{pmatrix}=\frac{1}{\sqrt{2p^+(p^0+M)}}\begin{pmatrix}(p^++M) & -p^R\\ p^L & (p^++M)\end{pmatrix}\times\begin{pmatrix}u^{LF+1/2}(p)\\ u^{LF-1/2}(p)\end{pmatrix}, \tag{A6}$$

$$\begin{pmatrix}u^{LF+1/2}(p)\\ u^{LF-1/2}(p)\end{pmatrix}=\frac{1}{\sqrt{2p^+(p^0+M)}}\begin{pmatrix}(p^++M) & p^R\\ -p^L & (p^++M)\end{pmatrix}\times\begin{pmatrix}u^{+1/2}(p)\\ u^{-1/2}(p)\end{pmatrix}. \tag{A7}$$

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