

Practical problems to compute the ghost propagator in $SU(2)$ lattice gauge theory

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In $SU(2)$ lattice pure gauge theory we study numerically the dependence of the ghost propagator $G(p)$ on the choice of Gribov copies in the Lorentz (or Landau) gauge. We find that the effect of Gribov copies is essential in the scaling window region; however, it tends to decrease with increasing β . On the other hand, we find that at larger β values very strong fluctuations appear which can make problematic the calculation of the ghost propagator.

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I. INTRODUCTION

The nonperturbative study of the ghost propagator is of great interest for the understanding of the mechanism of confinement. The Kugo-Ojima confinement criterion [1,2] is formulated in terms of the ghost propagator $G^{ab}(p)$ at $p \rightarrow 0$ and expresses the absence of colored states from the spectrum of physical states. Moreover, confinement of gluons has been related more directly to the suppression of the gluon propagator in the limit $p \rightarrow 0$ [3]. In both cases, the propagator in question is defined in the Landau (or Lorentz) gauge.

In a series of papers Zwanziger [4] has suggested that this behavior might result from the restriction of the fields in the transversal plane $\Gamma = \{A: \partial_\mu A_\mu = 0\}$ to the Gribov region $\Omega = \{A: \partial_\mu A_\mu = 0, M \geq 0\}$, where the Faddeev-Popov operator M is non-negative.

From studies of the coupled Dyson-Schwinger equations for gluons and ghosts [5,6] it is well known that the infrared behavior of gluon and ghost propagators is closely related [7]: the gluon propagator $D_{\mu\nu}^{ab}(p) = (\delta_{\mu\nu} - p_\mu p_\nu / p^2) Z_{gl}(p^2) / p^2$ is damped in the infrared as $Z_{gl}(p^2) \propto (p^2)^{2\kappa}$, while the ghost propagator $G^{ab}(p) = \delta^{ab} G(p) = \delta^{ab} Z_{gh}(p^2) / p^2$ is more singular than the free propagator $Z_{gh}(p^2) \propto (p^2)^{-\kappa}$. In a particular truncation scheme $\kappa = 0.595$ has been determined [8,9].

There are only relatively few previous lattice studies of the ghost propagator [10–13], in contrast with numerous investigations of the gluon propagator [14–16]. As for the latter, it is not yet clear from the lattice [17] whether

$Z_{gl}(p^2) / p^2 \rightarrow 0$ or $\neq 0$ with $p^2 \rightarrow 0$.¹ The lattice volumes might still be insufficient to decide this question. The singular behavior of $Z_{gh}(p^2)$ is seen to become stronger with increasing volume [11]. This supports the expectation [19] that the sample of physically important gauge field configurations $A \in \Gamma$, which constitutes the Euclidean functional integral, in the thermodynamical limit $V \rightarrow \infty$ is concentrating toward the edge of the Gribov region, the first Gribov horizon $\partial\Omega$ where the lowest nonvanishing eigenvalue of the Faddeev-Popov operator is approaching zero. This statement is the content of Zwanziger's horizon condition [19,20], which can be related to the Kugo-Ojima criterion.

All this is complicated by the nonuniqueness, first pointed out by Gribov [3], of the intersection with Γ of the gauge orbit A^g of any gauge field A , even if restricted to the Gribov region Ω . Practically, the Landau gauge is implemented by maximizing (with respect to gauge transformations g) a certain gauge functional. Usually, such a problem leads to more than a single maximum, which are gauge copies (Gribov copies) of each other, and hence to a nonunique definition of gauge dependent observables. Thus, in a lattice investigation one has to determine which observables are really subject to the so-called Gribov problem which reflects the dependence of an observable on the restriction (if possible) to the copy corresponding to the absolute maximum of the gauge functional. More precisely, one has to study whether this dependence disappears when one is approaching the continuum and/or infinite volume limit. Otherwise this would indicate the persistence of a real Gribov problem to which Gribov has drawn the attention. On the lattice, the structure of the Gribov region has been investigated more closely in this respect only by Cucchieri [21] some years ago.

Here we are mainly dealing with the infrared behavior of the calculated ghost propagator. In the result of a study for

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$SU(2)$ gluodynamics [11], Cucchieri came to the conclusion that the ghost propagator depends on the selection of the highest among more and more maxima of the gauge functional while the gluon propagator does not so depend. This study was restricted on one hand to the strong coupling region ($\beta=0.0, 0.8, 1.6$) where these observations apply, and $\beta=2.7$ where no gauge copy dependence was seen at all. These β values are outside the physically interesting scaling region. In a more recent paper [13], it has been reported that the gauge copy dependence of the ghost propagator in the more interesting scaling region (at $\beta=2.15, 2.2, 2.3$, and 2.4 for lattices $16^3 \times 32$) has been found to be within the statistical errors, on a level which is called the Gribov noise.

In the present paper we reanalyze the scaling region at $\beta=2.2, 2.3, 2.4, 2.5$, and 2.6 for lattices 8^4 and 16^4 by comparing two ensembles of gauge-fixed field configurations. One ensemble (“fc”) consists of an arbitrary maximum (usually the first to be found), and the other consists of the best (relative) maximum (“bc”) among N_{copy} local maxima of the gauge functional. We find that the difference of the ensemble averages of the ghost propagator for the lowest nonvanishing lattice momentum between the two ensembles does not vanish, except for the highest β value. Hence the Gribov problem remains a serious obstacle for a unique definition of the $SU(2)$ ghost propagator in the scaling region. More serious is an unexpected observation in the higher- β region. We find intermittent behavior of the ghost propagator estimator for the lowest nonvanishing momentum, signaled by anomalously large, isolated fluctuations of the ghost propagator $G(p_{min})$ (see below) within the time history of uncorrelated configurations. We stress already here that this behavior is *not* a Gribov copy problem since the anomalous peaks of $G(p_{min})$ are observed both for the first and the best Gribov copies, entering the ensembles “fc” and “bc”, respectively. We have tested whether this is correlated with various infrared observables. For the time being, two hypothetical causes must be excluded as viable explanations of the phenomenon.

In Sec. II we recall the definition of the gluon field A_μ , the definition of the Lorentz (or Landau) gauge, the structure of the Faddeev-Popov operator, and the definition of the ghost propagator. Details of the simulations, the gauge fixing, and the observation of Gribov copies are reported in Sec. III. In Sec. IV we discuss the results for the ghost propagator. We conclude in Sec. V.

II. FADDEEV-POPOV OPERATOR AND GHOST PROPAGATOR

A. Definition of the gluon field and Faddeev-Popov operator

For the Monte Carlo generation of ensembles of non-gauge-fixed gauge field configurations we use the standard Wilson action [22], which for the case of an $SU(N)$ gauge group is written

$$S = \beta \sum_x \sum_{\mu > \nu} \left[1 - \frac{1}{N} \text{Re Tr}(U_{x\mu} U_{x+\mu;\nu} U_{x+\nu;\mu}^\dagger U_{x\nu}^\dagger) \right],$$

$$\beta = 2N/g_0^2. \quad (1)$$

Here g_0 is a bare coupling constant and $U_{x\mu} \in SU(N)$ are the link variables. The field variables $U_{x\mu}$ transform as follows under gauge transformations g_x :

$$U_{x\mu} \xrightarrow{g} U_{x\mu}^g = g_x^\dagger U_{x\mu} g_{x+\mu}, \quad g_x \in SU(N). \quad (2)$$

For $SU(2)$ gauge links $U_{x\mu}$, a standard definition [23] of the lattice gauge field (vector potential) $\mathcal{A}_{x+\hat{\mu}/2;\mu}$ is

$$\mathcal{A}_{x+\hat{\mu}/2;\mu} = \frac{1}{2i} (U_{x\mu} - U_{x\mu}^\dagger). \quad (3)$$

Therefore, for $SU(2)$, the link can be written

$$U_{x\mu} = b_{x\mu}^0 \hat{1} + i \vec{b}_{x\mu} \vec{\sigma} = b_{x\mu}^0 \hat{1} + i \mathcal{A}_{x+\hat{\mu}/2;\mu},$$

$$b_{x\mu}^0 = \frac{1}{2} \text{Tr} U_{x\mu}. \quad (4)$$

In lattice gauge theory the usual choice of the Landau gauge condition is [23]

$$(\partial \mathcal{A})_x = \sum_{\mu=1}^4 (\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}) = 0, \quad (5)$$

which is equivalent to finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V_4} \sum_{x\mu} \frac{1}{N} \text{Re Tr} U_{x\mu}^g \quad (6)$$

with respect to gauge transformations g_x . After replacing $U \Rightarrow U^g$ at the extremum the gauge condition (5) is satisfied. In what follows this gauge is referred to as the Landau gauge.

The lattice expression of the Faddeev-Popov operator M^{ab} corresponding to $M^{ab} = -\partial_\mu D_\mu^{ab}$ in the continuum theory (where D_μ^{ab} is the covariant derivative in the adjoint representation) is given by

$$M_{xy}^{ab} = \sum_\mu \{ (\bar{S}_{x\mu}^{ab} + \bar{S}_{x-\hat{\mu};\mu}^{ab}) \delta_{x;y} - (\bar{S}_{x\mu}^{ab} - \bar{A}_{x\mu}^{ab}) \delta_{y;x+\hat{\mu}} - (\bar{S}_{x-\hat{\mu};\mu}^{ab} + \bar{A}_{x-\hat{\mu};\mu}^{ab}) \delta_{y;x-\hat{\mu}} \}, \quad (7)$$

where

$$\bar{S}_{x\mu}^{ab} = \delta^{ab} \frac{1}{2} \text{Tr} U_{x\mu}, \quad \bar{A}_{x\mu}^{ab} = -\frac{1}{2} \epsilon^{abc} A_{x+\hat{\mu}/2;\mu}^c. \quad (8)$$

From the form (8) it follows that a trivial zero eigenvalue is always present, such that at the Gribov horizon $\partial\Gamma$ the first nontrivial zero eigenvalue appears. Conversely, it is easy to see that for constant field configurations, with $b_{x\mu}^0 = \bar{b}_\mu^0$ and $b_{x\mu}^a = \bar{b}_\mu^a$ independent of x , there exist eigenmodes of M with a vanishing eigenvalue. Thus, if the Landau gauge is properly implemented, $M[U]$ is a symmetric and semipositive definite matrix.

B. Ghost propagator

The ghost propagator $G^{ab}(x,y)$ is defined as [10,19]

$$G^{ab}(x,y) = \delta^{ab} G(x-y) \equiv \langle (M^{-1})_{xy}^{ab}[U] \rangle, \quad (9)$$

where $M[U]$ is the Faddeev-Popov operator. Note that the ghost propagator becomes translationally invariant (i.e., dependent only on $x-y$) and diagonal in color space only as a result of averaging over the ensemble of gauge-fixed representatives (first or best gauge-fixed copies) of the original Monte Carlo gauge configurations. The ghost propagator in momentum space can be written as

$$G(p) = \frac{1}{3V} \sum_{x,y} e^{-2\pi i p \cdot (x-y)} \langle (M^{-1})_{xy}^{aa}[U] \rangle, \quad (10)$$

where $V=L^4$ is the lattice volume, and the coefficient $1/3V$ is taken for a full normalization, including the indicated color average over $a=1, \dots, 3$.

We mentioned above that $M[U]$ is a symmetric and semi-positive definite matrix. In particular, this matrix is positive definite in the subspace orthogonal to constant vectors. The latter are zero modes of $M[U]$. Therefore, it can be inverted by using a conjugate-gradient method, provided that both the source $\psi^a(y)$ and the initial guess of the solution are orthogonal to zero modes. As the source we adopted the one proposed by Cucchieri [11]:

$$\psi^a(y) = \delta^{ac} e^{2\pi i p \cdot y}, \quad p \neq (0,0,0,0), \quad (11)$$

for which the condition $\sum_y \psi^a(y) = 0$ is automatically imposed. Choosing the source in this way allows one to save computer time since, instead of the summation over x and y in Eq. (10), only the scalar product of $M^{-1}\psi$ with the source ψ itself has to be evaluated. In general, the gauge-fixed configurations can be used in a more efficient way when the inversion of M is done on sources for $c=1, \dots, 3$ such that the (adjoint) color averaging, formally required in Eq. (10), will be *explicitly* performed.

III. SIMULATION DETAILS

The numerical simulations have been done for $SU(2)$ pure gauge theory using the standard Wilson action, for lattice volumes L^4 with $L=8$ and $L=16$. At a given lattice size L for each β value we have generated N_{conf} independent mother configurations, for which the Landau gauge was fixed $N_{copy}=20$ times, each time starting from a random gauge transformation of the mother configuration, obtaining in this way N_{copy} Landau-gauge-fixed copies.

Two consecutive configurations (considered as independent) were separated by 100 and 200 sweeps for lattice sizes 8^4 and 16^4 , respectively. Each sweep consisted of one local heat bath update followed by four or eight microcanonical updates [24] for 8^4 or 16^4 lattices. In all our runs we have measured the integrated autocorrelation time for the plaquette, for the Polyakov loop, and for the ghost propagator (separately for each momentum p). In all cases, the relation $\tau_{int} \sim 0.5$ was observed, showing that the consecutive

TABLE I. The table shows in the third column the number $N_{multiple\ copies}^{conf}$ of configurations for which nonequivalent Gribov copies have actually been obtained, out of a total number N^{conf} of configurations that underwent gauge fixing; and in the fourth column the total number $N_{nonequiv}^{copies}$ of nonequivalent Gribov copies out of a total number $N_{total}^{copies} = N^{conf} \times N_{copy}$ of gauge copies under investigation. The last column presents the number of times out of N^{conf} that the first copy was not identical to the best (relative maximum) copy.

Size	β	$N_{multiple\ copies}^{conf}/N^{conf}$	$N_{nonequiv}^{copies}/N_{total}^{copies}$	$N_{fc \neq bc}$
8^4	1.6	500/500	8263/10000	446
	2.0	490/500	4431/10000	354
	2.1	468/500	3460/10000	311
	2.2	426/500	2180/10000	235
	2.3	301/500	1364/10000	150
	2.4	184/500	877/10000	92
16^4	2.0	25/25	500/500	25
	2.1	25/25	500/500	25
	2.2	25/25	500/500	23
	2.3	25/25	494/500	25
	2.4	25/25	337/500	23
	2.5	24/25	169/500	14

configurations are effectively independent.

The actual measurements of the ghost propagator were done for the “first”, i.e., in fact an arbitrary gauge copy, and for the “best” one among the N_{copy} copies. If the first copy turned out to be the best, the ghost propagator was measured only once, and the result simultaneously entered the two different gauge-fixed ensemble averages. In the following the two ensembles are labeled “fc” and “bc”, referring to the first or the best gauge copy, respectively. In Table I we give, for each set of simulation parameters (L, β), the number of times the first copy produced turned *not* out to be the best, i.e., did not correspond to the relative maximum of $F_U(g_i)$ among the N_{copy} copies.

As the gauge fixing procedure we used standard Los Alamos type overrelaxation with $\omega=1.7$. The iterations were stopped when the following transversality condition was satisfied:

$$\max_{x,a} \left| \sum_{\mu=1}^4 (A_{x+\hat{\mu}/2;\mu}^a - A_{x-\hat{\mu}/2;\mu}^a) \right| < \epsilon_{lor}. \quad (12)$$

We used the parameters $\epsilon_{lor} = 10^{-10}$ or 10^{-9} for lattice size 8^4 or 16^4 , respectively. In our test runs it was found that further decreasing ϵ_{lor} does not affect the results for the ghost propagator. Also it was checked that these values of ϵ_{lor} are sufficient for identifying, according only to the values of $F_U(g_i)$, Gribov copies which are actually global gauge transformations of each other, and conversely for distinguishing this from the case of actually inequivalent lattice Gribov copies.

In Table I, for each set of simulation parameters (L, β), we present also the number of configurations for which Gribov copies have been found and the total number of different Gribov copies.

TABLE II. The ghost propagator $G(p)$ from Eq. (10) as a function of $k_4=1,2,3,4$. We have set the momentum $p=(0,0,0,k_4/L)$, where $L=8,16$ is the lattice size. The averages over the gauge configurations in Eq. (10) were taken in two different ways: “fc” means the average taking only the gauge-fixed copy generated first for each configuration; “bc” means the average over only the best (relative maximum) copy among 20 different gauge-fixed copies that we have generated.

8 ⁴ lattice						
β	n_{meas}	Copy	$k_4=1$	$k_4=2$	$k_4=3$	$k_4=4$
1.6	500	bc	6.58(4)	1.327(5)	0.628(2)	0.501(1)
1.6	500	fc	7.02(6)	1.363(5)	0.638(1)	0.508(1)
2.0	500	bc	5.15(3)	1.013(2)	0.491(1)	0.3970(4)
2.0	500	fc	5.46(9)	1.028(3)	0.495(1)	0.3995(6)
2.1	500	bc	4.62(3)	0.920(2)	0.4545(6)	0.3701(4)
2.1	500	fc	4.89(7)	0.935(3)	0.4573(8)	0.3719(5)
2.2	500	bc	4.06(3)	0.823(2)	0.4189(4)	0.3444(3)
2.2	500	fc	4.26(4)	0.833(2)	0.4203(5)	0.3450(3)
2.3	500	bc	3.60(4)	0.744(1)	0.3903(3)	0.3238(2)
2.3	500	fc	3.65(4)	0.747(2)	0.3909(4)	0.3241(2)
2.4	500	bc	3.38(5)	0.691(1)	0.3710(4)	0.3098(2)
2.4	500	fc	3.47(7)	0.692(2)	0.3712(4)	0.3099(2)
16 ⁴ lattice						
β	n_{meas}	Copy	$k_4=1$	$k_4=2$	$k_4=3$	$k_4=4$
2.2	296	bc	20.1(1)	3.87(1)	1.494(2)	0.8078(6)
2.2	296	fc	21.3(1)	3.97(1)	1.509(2)	0.8115(6)
2.3	270	bc	17.3(1)	3.29(1)	1.303(1)	0.7248(4)
2.3	270	fc	18.0(1)	3.33(1)	1.310(2)	0.7268(5)
2.4	370	bc	14.8(1)	2.83(1)	1.165(1)	0.6673(3)
2.4	370	fc	15.6(1)	2.87(1)	1.171(1)	0.6690(3)
2.5	294	bc	13.7(2)	2.56(1)	1.088(1)	0.6353(3)
2.5	294	fc	13.9(2)	2.58(1)	1.090(1)	0.6358(3)
2.6	229	bc	13.6(4)	2.41(1)	1.043(2)	0.6161(5)
2.6	229	fc	13.8(4)	2.41(1)	1.044(2)	0.6164(5)

The momenta p for the propagator $G(p)$ were taken with all spatial components put equal to zero: $p=(0,0,0,k_4/L)$, where k_4 was restricted to $k_4=1,2,3,4$.

IV. DISCUSSION OF THE RESULTS

From Table I one can learn that at the lattice size 8⁴ the fraction of Monte Carlo configurations which are represented by more than one gauge-fixed configuration (among 20 attempts to find copies) drastically begins to decrease at $\beta=2.3$. Parallel to this the multiplicity of *actually different* copies among the 20 also drops down. The decrease of the number of available *basins of attraction* for the gauge-fixing process is a finite-volume effect. For the bigger lattice size (16⁴) one sees that the fraction of Monte Carlo configurations with more than one gauge-fixed configuration practically does not depend on β . However, the multiplicity of

nonequivalent copies among the 20 obtained copies starts to decrease from $\beta=2.3$.

From Table II one can see for separate small momenta how the average value of the ghost propagator differs between the two ways to deal with the Gribov copy problem: to ignore it ($N_{copy}=1$) or to inspect $N_{copy}=20$ copies. For all momenta, the ensemble consisting of the first copies (“fc”) turns out to give slightly larger values than the ensemble including always the best copy (“bc”). For the lowest non-vanishing momentum this is shown in Fig. 1 for the lattice 8⁴. It is clear that for $\beta \in [1.5, 2.4]$ the difference of $G(p_{min})$ between the two ways of averaging is well outside the statistical error.

In Fig. 2, for the bigger lattice 16⁴, the ghost propagator values for the two lowest momenta are compared with respect to the dependence on Gribov copies for $\beta \in [2.2, 2.6]$. Whereas for the lowest momentum the results resemble

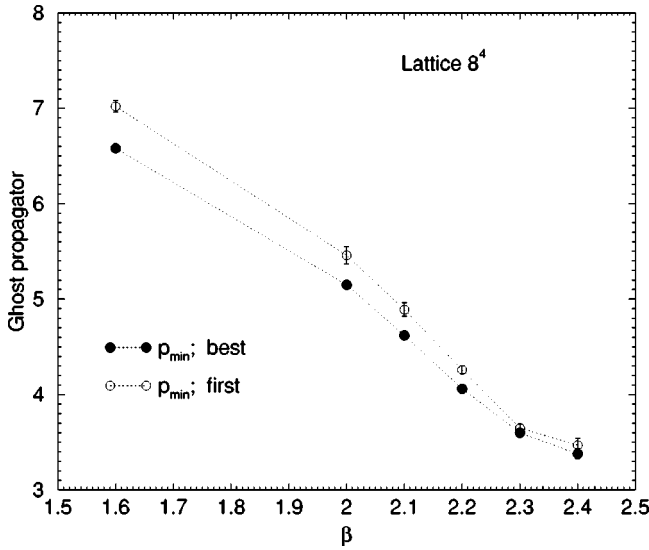


FIG. 1. The β dependence and Gribov copy sensitivity of the ghost propagator $G(p_{min})$ at minimal momentum p_{min} on the 8^4 lattice. Filled symbols correspond to the “bc” ensemble, open symbols to the “fc” ensemble (see the text).

those of the smaller lattice, for the second lowest momentum they are practically indistinguishable at the given scale. For increasing β the difference becomes of the order of the statistical error (Gribov noise). At $\beta=2.6$ the ghost propagator data even for the *lowest* momentum fall together within error bars. This indicates that the Gribov problem has disappeared for the ghost propagator there.

Instead, at $\beta=2.6$ a new problem arises which can already be recognized in Fig. 2, where we also demonstrate how, at $\beta=2.6$, the average for the ghost propagator at the lowest momentum would be influenced by the removal of “exceptional configurations.” These are signaled as spikes in the Monte Carlo time histories of the corresponding observable shown in Fig. 3 for $\beta=2.6$. Precursors of this phenomenon are visible there at lower β too, but for $\beta=2.6$ the effect becomes notable. We notice that these spikes occur in the first as well as in the best gauge-fixed copy. Therefore, the existence of these “exceptional configurations” is definitely *not* a result of gauge fixing.

In order to explore what the essence of these “exceptional configurations” is, we have looked for correlations with certain “toron” excitations on one hand and with different Polyakov loops on the other.

In the first case we followed the procedure applied by Kovacs [25] for extracting the toron content of Monte Carlo gauge field configurations.² We evaluated for all four direc-

²In an attempt to reconstruct hadronic correlators from model configurations derived from lattice Monte Carlo configurations, he found it necessary to augment the instanton content of the latter—as extracted via smoothing—by an appropriate “toron” field extracted as we explain in the text. Indeed, this mixture turned out essential to reproduce mesonic correlators in his “instanton plus toron” model of the vacuum.

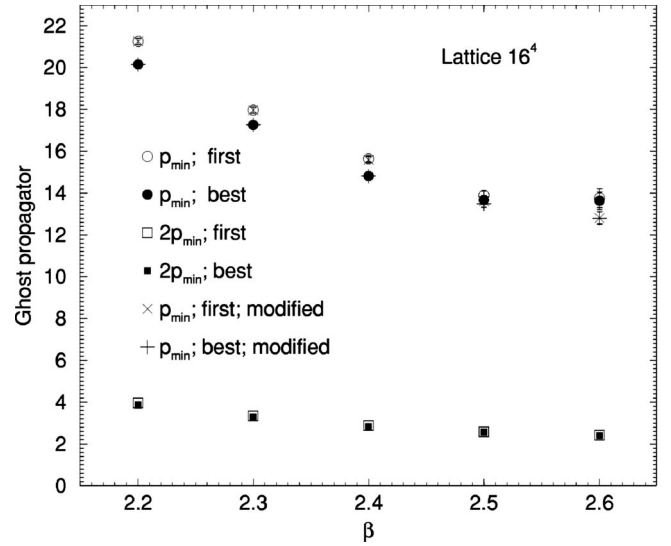


FIG. 2. The β dependence and Gribov copy sensitivity of the ghost propagator $G(p)$ at minimal momentum p_{min} and next-to-minimum momentum $2p_{min}$ on the 16^4 lattice. Filled symbols correspond to the “bc” ensemble, open symbols to the “fc” ensemble. At $\beta=2.6$ the minimal momentum ghost propagator is sensitive to the inclusion or exclusion of “exceptional configurations” (see text).

tions μ on the lattice the corresponding holonomies over a μ slice fixed at $x_\mu=1$:

$$\mathcal{P}_\mu(x) = \prod_{s=0}^{L-1} U_{x+s\hat{\mu};\mu}. \quad (13)$$

We averaged this quantity over the μ slice,

$$\bar{\mathcal{P}}_\mu = \frac{1}{L^3} \sum_{x; x_\mu=1} \mathcal{P}_\mu(x). \quad (14)$$

These *gauge dependent* quantities were normalized to $SU(2)$ in the usual way,

$$\bar{\mathcal{P}}_\mu \Rightarrow \bar{\mathcal{P}}_\mu / \sqrt{\det \bar{\mathcal{P}}_\mu}. \quad (15)$$

Then the anticipated homogeneous toron field is given by links $\bar{U}_{x\mu}$ independent of x , which are required to reproduce $\bar{\mathcal{P}}_\mu$ as follows:

$$(\bar{U}_{x\mu})^L = \bar{\mathcal{P}}_\mu. \quad (16)$$

The corresponding toron gluon field can be extracted as

$$\mathcal{A}_{x+\hat{\mu}/2;\mu}^{toron} = \frac{1}{2i} (\bar{U}_{x\mu} - \bar{U}_{x\mu}^\dagger). \quad (17)$$

We plotted the time history of the lowest-momentum ghost propagator together with the toron observable

$$T_\mu = \text{Tr}(\mathcal{A}_{x+\hat{\mu}/2;\mu}^{toron})^2, \quad (18)$$

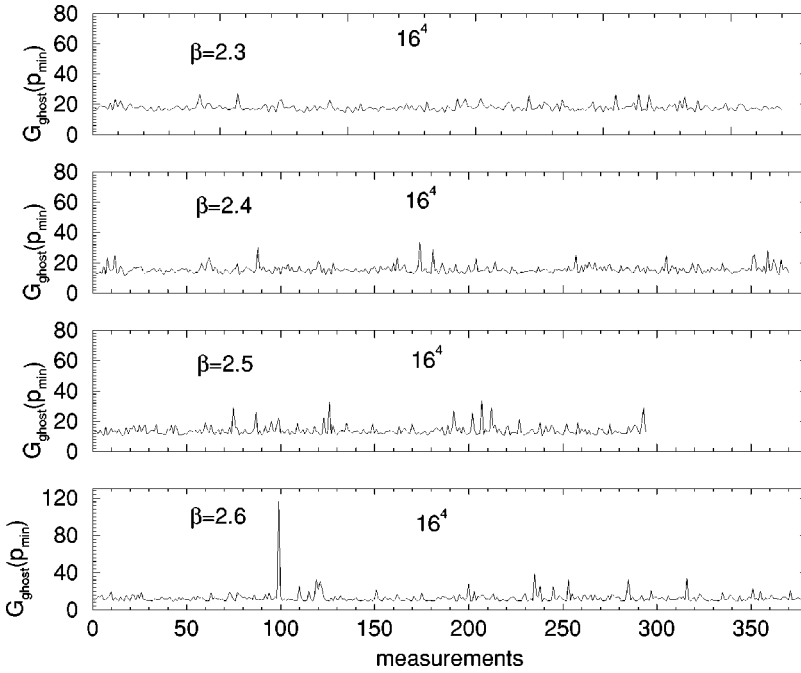


FIG. 3. Monte Carlo time histories of the ghost propagator $G(p_{min})$ for various β on the 16^4 lattice. The frequency of the occurrence of “exceptional configurations” increases with higher β .

defined separately for the four Euclidean directions. We noticed that the previously mentioned spikes (“exceptional configurations”) occur independent of the spikes of this toron observable in each of the Euclidean directions. We demonstrate this in Fig. 4 which shows the Monte Carlo history of the lowest-momentum ghost propagator (upper panel) together with the histories of the toron fields T_μ for $\mu=4$ (middle) and $\mu=1$ (lower panel).

We also checked the Monte Carlo sample for eventual correlations with the average Polyakov loop

$$L_\mu = \frac{1}{2} \text{Tr} \bar{P}_\mu. \quad (19)$$

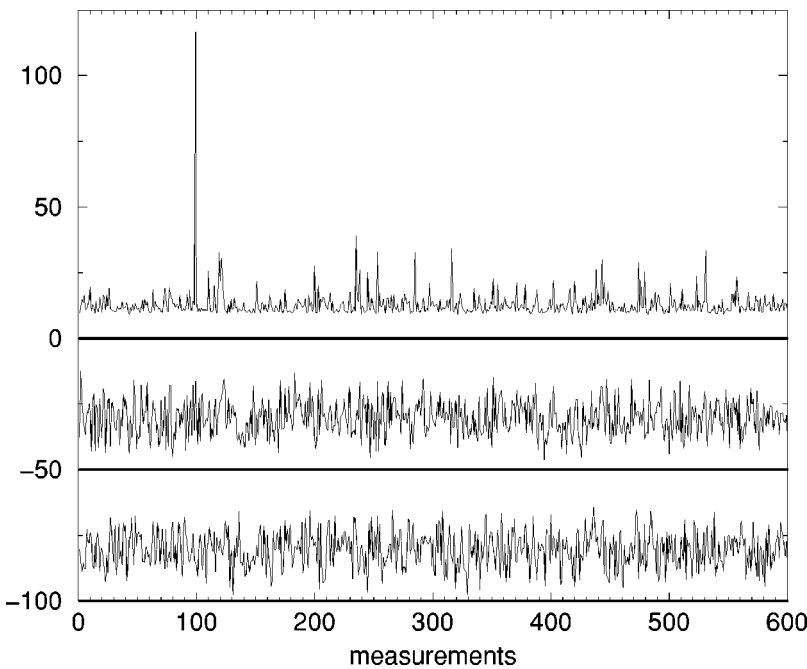


FIG. 4. Monte Carlo time histories of the ghost propagator $G(p_{min})$ (above) for $\beta=2.6$ on the 16^4 lattice, compared with the histories of the toron fields T_4 (middle) and T_1 (below). The fluctuations of the latter have been arbitrarily rescaled and shifted for better visual inspection.

Similarly, we illustrate in Fig. 5 that there are no correlations between the spikes of the lowest-momentum ghost propagator with extremal fluctuations of the average Polyakov loop in any of the four directions. Shown in Fig. 5 are, in addition to the history of the lowest-momentum ghost propagator (upper panel), the histories of the average Polyakov lines L_μ for $\mu=4$ (middle) and $\mu=1$ (lower panel).

V. CONCLUSIONS

In this work we studied numerically the dependence of the ghost propagator $G(p)$ in pure gauge $SU(2)$ theory on the choice of Gribov copies in the Lorentz (or Landau) gauge

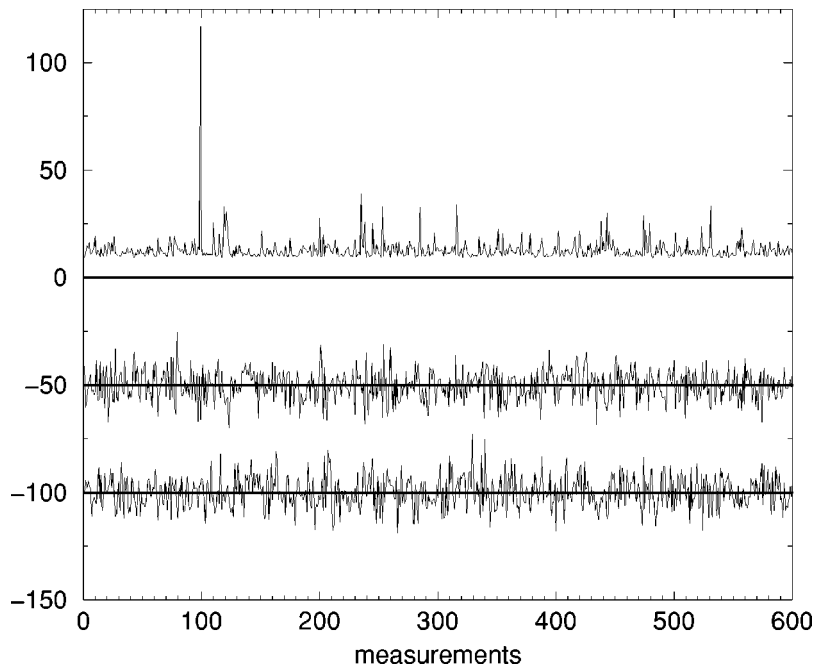


FIG. 5. Monte Carlo time histories of the ghost propagator $G(p_{min})$ (above) for $\beta=2.6$ on the 16^4 lattice, compared with the histories of the average Polyakov lines L_4 (middle) and L_1 (below). The fluctuations of the latter have been arbitrarily rescaled and shifted for better visual inspection.

with special focus on the physically interesting scaling region. All simulations were performed on the 8^4 and 16^4 lattices.

We found that the effect of Gribov copies is essential in the scaling window region. Therefore, the Gribov problem remains a serious obstacle for a unique definition of the $SU(2)$ ghost propagator in the scaling region. However, it tends to decrease with increasing β values.

Another—and more serious—problem is presented by the unexpected observation, in the higher- β region, of anomalously large, isolated fluctuations of the ghost propagator $G(p_{min})$ within the time history of uncorrelated configurations. These strong fluctuations make problematic the calculation of the ghost propagator.

We believe that this problem deserves a more thorough study, in particular of how to interpret the relevant configurations. If there is nothing physically wrong with them, much more statistics is necessary to get a reliable result.

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