$\pi\pi$ invariant mass spectrum in $V' \to V\pi\pi$ and the $f_0(600)$ pole

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We consider the phenomenological description of the two pion invariant mass spectrum in the $V' \to V\pi\pi$ decays. We study the parametrization of the amplitude involving both S and D wave contributions. From a fit to the two pion decays of the $\Upsilon(nS)$ and $\Psi(nS)$ we determine the $f_0(600)$ mass and width to be $m_{f_0}=528 \pm 32$ MeV and $\Gamma_{f_0}=413 \pm 45$ MeV. The mass and width values we report correspond, respectively, to the real and imaginary part of the S matrix pole.

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I. INTRODUCTION

The experimental identification of low mass scalar resonances is a long-standing puzzle whose origin can be traced back to some of the following characteristics: a large decay width, possible mixing with multiquark or glueballs, overlap of resonances, and the opening of channels, which is responsible for the appearance of cusp effects [1]. In particular the $f_0(600)$ has a long history; it has been included in some issues of the Particle Data Group Book but it has also been excluded for long periods by the Particle Data Group. Recently, experimental evidence from different corners of particle physics has accumulated confirming the existence of the $f_0(600)$ resonance. There have been attempts to interpret the low-lying scalars as multiquark states [2,3] or $K\bar{K}$ bound states [4,5]. Also, models exist based on chiral symmetry [6–8] or else unitarized models where the scalar nonet arises [9,10]. However, the understanding of actual processes involving scalar mesons starting from first principles has not been achieved. Effective theories are not well suited to deal with these scalars. The use of sum rules is perhaps the best approach to the problem [11]; however, no predictions for all of the scalars have been advanced in that framework. Furthermore, the phenomenological description of broad resonance faces severe problems. The determination of the physical parameters—mass and width—is a nontrivial problem that requires the study of the nonresonant background dependence.

For small invariant mass of the pion pair, experimental data for $\pi\pi$ scattering is obtained from the K_{e4} ($K \to \pi\pi e^+ \nu_e$) [12,13]. Theoretical studies have been carried in this kinematical region using chiral symmetry and Roy equations—solid theoretical tools—establishing thus a firm result to be considered by other analysis [14]. Inclusion of the di-pion low invariant mass favor a light and very broad $f_0(600)$ ($m \approx 470 \pm 30$ MeV and $\Gamma/2 \approx 295 \pm 20$ MeV). Below 1 GeV, information on the $\pi\pi$ phase shift is extracted from πN scattering, $P\bar{P}$ annihilation at rest, and central production; these data allow the existence of a broad ($\Gamma \approx 500$ MeV) scalar meson resonance [15,16]. Decay of pseudoscalar charmed mesons are also a source of valuable data involving scalar mesons, although the $f_0(600)$ has only been reported by the E-791 Collaboration after the Dalitz plot analysis of $D \to \pi^+\pi^-\pi^+$ [17]. Another important source of information is the vector meson decays. The processes considered involve either the scalars themselves or pion pairs together with photons or vector mesons. Among these we can mention $(\rho,\omega,\Phi) \to P\bar{P}\gamma$ [18–22], $J/\Psi \to P\bar{P}V$, $\Psi' \to \Psi P\bar{P}$, and $\Upsilon(nS) \to \Upsilon(mS)P\bar{P}$ [23], where P stands for a pseudoscalar (π or K) and V for a vector meson (ρ,ω). Experimental evidence for the contribution of scalar resonances to some of these processes has been reported; for example [24], in the $J/\Psi \to \omega\pi^+\pi^-$ decay a bump is observed at low $\pi^+\pi^-$ mass which could be interpreted as the σ, a result that seems to be confirmed by the BES Collaboration [25]. Also a recent analysis of the data for $\Upsilon(nS) \to \Upsilon(mS)P\bar{P}$ concludes the contribution of the $f_0(600)$ with a large uncertainty in the width ($m=526^{+48}_{-37}$, $\Gamma=301^{+145}_{-100}$ MeV) [26]. It should be noticed that in the experimental data for $\psi(2S) \to \pi^+\pi^- J/\Psi$ reported by the BES Collaboration [27], no evidence for the $f_0(600)$ contribution is foreseen.

In this paper we concentrate on the decay of heavy quark vector meson resonances (Υ and Ψ), where a pair of pions is produced with invariant mass ranging below the 1-GeV region. The kind of processes we are interested in has been considered by a number of authors using techniques as diverse as pure chiral symmetry [28], nonrelativistic theory assuming the existence of an Adler zero [29], the color field multipole expansion in the nonrelativistic limit [30,31], effective Lagrangian based on chiral symmetry and the heavy quark expansion [32–34], and also a purely phenomenological description based on a Breit-Wigner parametrization [26]. The framework so developed is used then to describe the two pion invariant mass spectrum and, in some cases, also the angular distribution. The latter is important since existing experimental data could discriminate the models. Furthermore, as far as we know, the complete parametrization of the amplitude has not been discussed; in fact in the framework of effective field theories where an expansion in terms of the number of derivatives is performed, some confusion arose concerning the "S" and "D" wave contributions to the amplitude [34].

Our purpose is to perform an analysis as general as possible, including both S and D waves in the $(P\overline{P})$ system. We show that Lorentz covariance fixes the parametrization of the amplitude in terms of four form factors. Using the two gluon mechanism required by the OZI rule—which implies that the form factors depend only upon the $V'-V$ momentum transfer or equivalently on the $\pi\pi$ invariant mass—the spin 0 (di-pion S wave) and spin 2 (di-pion D wave) are unambiguously identified since no angular dependence is involved in the form factors. As far as the angular dependence is concerned, including two or four form factors makes no difference, and since the $\pi\pi$ invariant mass dependence will be parametrized, for simplicity we will restrict our analysis to two form factors, $a_0(m_{\pi\pi})$ and $a_2(m_{\pi\pi})$, which are associated to S and D waves, respectively. In order to parametrize a_0 and a_2 we rely on the pole approach, i.e., we assume that the $f_0(600)$ pole dominates the $\pi\pi$ invariant mass distribution of the processes under consideration. For this reason we propose that $a_0(m_{\pi\pi})$ amounts to a Breit-Wigner plus a soft background while for "D" wave channel, where no resonance is expected, $a_2(m_{\pi\pi})$ is parametrized in terms of a soft background. In this way we claim that crossed channel as well as higher scalar resonance contributions are taken into account since we know that the amplitude associated to these phenomena behave as a soft function in the 500–800-MeV range, where the $f_0(600)$ is expected to lie.

Our strategy is to perform a joint fit to data from different processes, involving 165 points and 33 parameters. The set of data points we consider include the $\pi-\pi$ invariant mass distribution of the $\Upsilon(3S)\to\Upsilon(1S)+\pi\pi$ [23], $\Upsilon(3S)\to\Upsilon(2S)+\pi\pi$ [23], $\Upsilon(2S)\to\Upsilon(1S)+\pi\pi$ [23,35], $\psi(2S)\to J/\psi+\pi\pi$ [27] decays. The following characteristics are worth mentioning: (i) we are considering flavor conserving processes, and all of them are expected to proceed through two gluons. This point will be relevant when discussing the parametrization of the form factors. (ii) The smallness of the phase space available for the processes under consideration ($2m_\pi\leqslant\sqrt{s}\leqslant 0.9$ GeV). Note that the expected central value and the large width of the $f_0(600)$ would imply non-negligible resonance effects in these processes; and (iii) the invariant mass distribution for $s\to s_{th}$, where s_{th} stands for the threshold value of the di-pion invariant mass- shows a peculiar behavior to be contrasted with the typical ($s-2m_\pi^2$) expected in processes involving soft pions. Compare, for example, in Fig. 1 the threshold behavior of the $\sqrt{s}$ distribution for $\Upsilon(2S)\to\Upsilon(1S)+\pi\pi$ or $\psi(2S)\to J/\psi+\pi\pi$ with $\Upsilon(3S)\to\Upsilon(1S)+\pi\pi$.

In order to understand the nature of the problem that we face, we remark that the more recent data [27,35] have been analyzed in terms of the scale anomaly. Indeed, these processes can be fitted without difficulty using the scale anomaly, which brings the question if the full set of data we consider can be explained using the same formalism. Our results show that this is not the case and that inclusion of the $f_0(600)$ improves our understanding of the data. Thus, as far as we can see, any attempt to provide a successful description of the flavor conserving two pion decays of the Υ and ψ families should consider the full set of data, since the foreseen physical mechanisms (scale anomaly, scalar resonance exchange) could contribute in all cases under consideration.

II. PARAMETRIZATION OF THE AMPLITUDE

We are interested in the decay $V'(p',\eta')\to V(p,\eta)\pi(p_1)\pi(p_2)$ where the letters in parentheses stand for the four-momenta and polarization of the corresponding particles. We introduce $q\equiv p_1-p_2$ and $Q\equiv p_1+p_2$. $Q^2=s$, and $p'^2=m'^2$, $p^2=m^2$, $p_1^2=p_2^2=m_\pi^2$. In order to obtain the general parametrization it is convenient to consider the amplitude for the exchange of arbitrary spin-0 and spin-2 mesonlike objects (although we do not consider the actual exchange of any particle). Let us first consider the S wave contribution. The amplitude for the $V'\to V+$ scalar can be written as

$$\mathcal{M}_0=\eta'^{\mu}\eta^{\nu}t_{\mu\nu}.$$

Covariance allows us to write $t_{\mu\nu}$ in terms of $g_{\mu\nu}$ and the independent (p',p) four-momenta. Imposing $p'\cdot\eta'=0$, $p\cdot\eta=0$, and using $p'^{\mu}t_{\mu\nu}=0$, which follows from the fact that $V'(p')$ is produced through a virtual photon in an e^+e^- machine, we obtain

$$\mathcal{M}_0=a_0\left(\eta\cdot\eta'-\frac{(p'\cdot\eta)(p\cdot\eta')}{p\cdot p'}\right). \tag{1}$$

Already at this point we encounter differences with the parametrizations used in the literature [36], where only the $\eta\cdot\eta'$ term is considered. Although sizable effects are not produced by the extra term it is important to work with the proper Lorentz invariant amplitude. In particular, differences could become relevant when polarization measurements are involved [39].

For the D wave contribution we obtain (see the Appendix for details)

$$\begin{aligned}\mathcal{M}_2=\Bigg[&b\left(\eta'^{\mu}-\frac{p'^{\mu}}{p\cdot p'}(\eta'\cdot p)\right)\eta^{\nu}\\&+c\left(\eta'^{\mu}-p'^{\mu}\frac{(\eta'\cdot p)}{p\cdot p'}\right)(\eta\cdot p')p'^{\nu}\\&+\frac{a_2}{m'^2-m^2}\left((\eta\cdot\eta')-\frac{(\eta\cdot p')(\eta'\cdot p)}{p\cdot p'}\right)p'^{\mu}p'^{\nu}\Bigg]\Pi_{\mu\nu}.\end{aligned} \tag{2}$$

In order to obtain the decay rate, we carry out the integration over the p_1,p_2 Lorentz invariant phase space in the two pion center of mass reference frame, i.e., $\vec{Q}=0$, $q_0=0$. We obtain

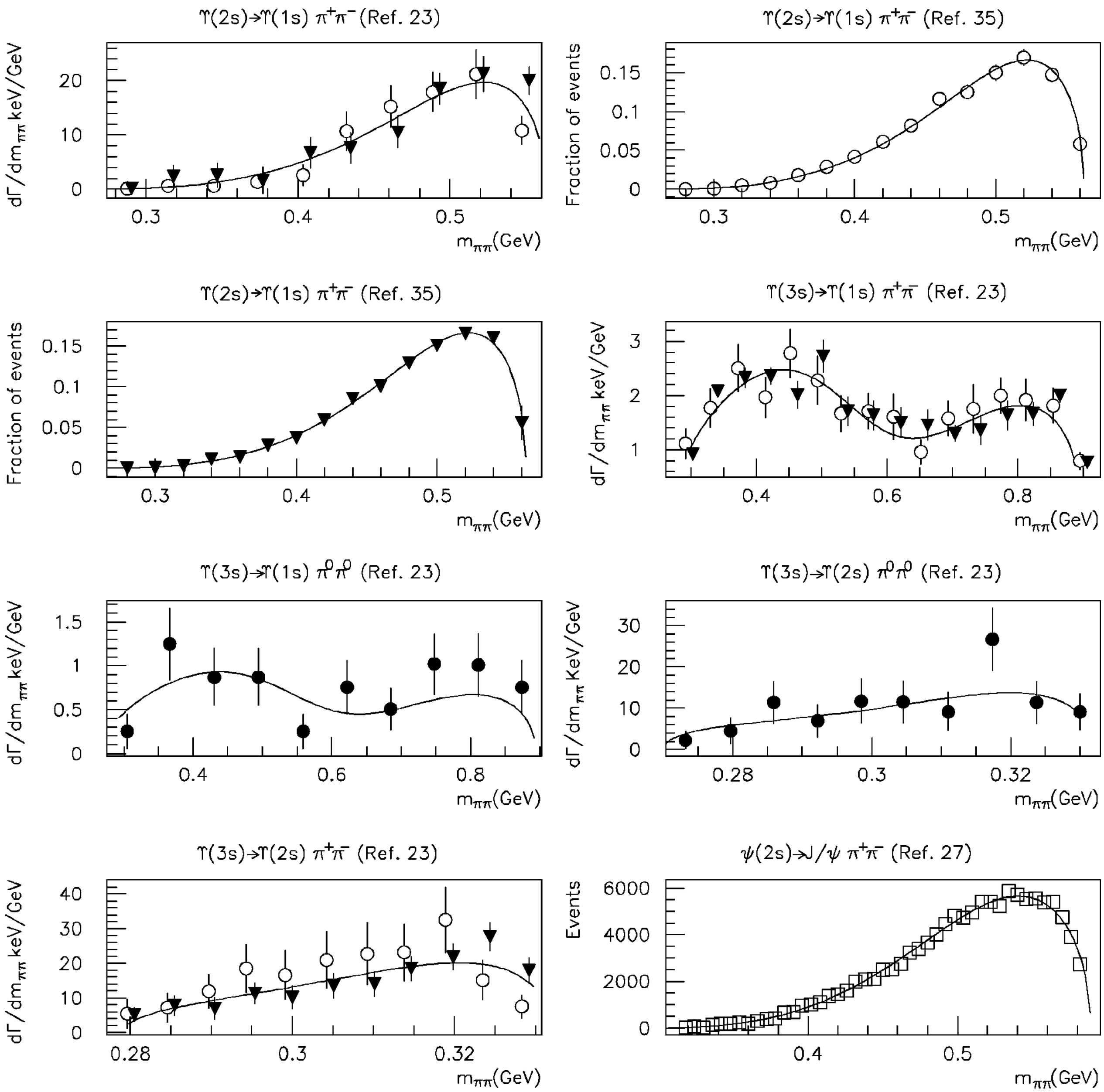

FIG. 1. Data points used in the analysis [23,27,35] and the resulting fit (solid line). In the horizontal axes we plot $m_{\pi\pi}=\sqrt{s}$ whereas the vertical refers to the differential decay rate $d\Gamma/d\sqrt{s}$, or number of events, as shown in the plots and discussed in the quoted references. For the same reaction, open circles and solid triangles refer to data obtained from exclusive and inclusive processes, respectively.

$$\Gamma_0 \equiv \frac{1}{2m'}\frac{1}{3}\int \sum_{pol} |\mathcal{M}_0|^2 \frac{d^3p}{(2\pi)^3 2p_0}\frac{d^3p_1}{(2\pi)^3 2p_{10}} \times \frac{d^3p_2}{(2\pi)^3 2p_{20}}(2\pi)^4\delta^4(Q-p_1-p_2)$$
$$= \frac{1}{48\pi m'}\int \sum_{pol} |\mathcal{M}_0|^2 \frac{d^3p}{(2\pi)^3 2p_0}\left(1-\frac{4m_\pi^2}{Q^2}\right)^{1/2}.$$

The S-D wave interference vanishes upon integration. Indeed, the decay rate is proportional to

$$\Gamma_{int} \sim \frac{1}{3m'}\int \sum_{pol} (R_e\mathcal{M}_0\Pi_{\mu\nu})\frac{d^3p_1}{(2\pi)^3 2p_{10}}\frac{d^3p_2}{(2\pi)^3 2p_{20}} \times (2\pi)^4\delta^4(Q-p_1-p_2)$$
$$= a g_{\mu\nu} + b Q_\mu Q_\nu .$$

The last equality follows from covariance. Using $Q^\mu\Pi_{\mu\nu} = g^{\mu\nu}\Pi_{\mu\nu}=0$, it follows that $a=b=0$. For the D wave we proceed along the same lines,

$$\Gamma_2 = \frac{1}{2m'}\frac{1}{3}\sum_{pol}\int \frac{d^3p}{(2\pi)^3(2p_0)}(A^{\mu\nu}A^{*\rho\sigma})\overline{\Pi}_{\mu\nu\rho\sigma},$$

where

$$\overline{\Pi}_{\mu\nu\rho\sigma}=\int\sum_{pol}(\Pi_{\mu\nu}\Pi_{\rho\sigma})\frac{d^3p_1}{(2\pi)^3(2p_{10})}\frac{d^3p_2}{(2\pi)^3(2p_{20})}\times(2\pi)^4\delta^4(Q-p_1-p_2)=\mathbf{x}\Pi_{\mu\nu\rho\sigma}.$$

$\mathbf{x}$ is determined using the reference frame where $\vec{Q}=0$, $q_0=0$. For example,

$$\overline{\Pi}_{3333}=\frac{2}{3}\mathbf{x}=\int\sum_{pol}(\Pi_{33}\Pi_{33})\frac{d^3p_1}{(2\pi)^3(2p_{10})}\frac{d^3p_2}{(2\pi)^3(2p_{20})}\times(2\pi)^4\delta^4(Q-p_1-p_2).$$

In this way we obtain

$$\mathbf{x}=\frac{Q^4}{60\pi}\left(1-\frac{4m_\pi^2}{Q^2}\right)^{5/2}.$$

Using the $q_0=0,\vec{Q}=0$ reference frame, it is easy to show that $\Pi_{\mu\nu}\sim Y_2(\theta,\phi)$, i.e., it is associated to the di-pion D wave. Furthermore, since the OZI rule allows us to conclude that a_2, b, and c can only depend upon $s\equiv(p_1+p_2)^2$ (recall that $s=Q^2$, stands for the $\pi\pi$ invariant mass), then we conclude that $\mathcal{M}_2$ describes the di-pion spin-2 wave. In the following we consider the particular case where only one Lorentz invariant amplitude is included in $A_{\mu\nu}$. To this end we set $b=c=0$ in Eq. (2) or Eq. (A3). This is the simplest way to consistently introduce D wave effects. Currently the experimental data are not precise enough to take into account precise D wave effects. Hence the choice $b=c=0$ is justified. Within this approximation we obtain for the di-pion invariant mass distribution

$$\frac{d\Gamma}{d\sqrt{s}}=\frac{(m'p)\sqrt{s}}{3(4m'\pi)^3}\sum_{pol}\left|(\eta\cdot\eta')-\frac{(\eta\cdot p')(\eta'\cdot p)}{p\cdot p'}\right|^2\times(SW+DW), \tag{3}$$

where p stands for the three-momentum ($p=|\vec{p}|$) and

$$SW=|a_0(s)|^2\left(1-\frac{4m_\pi^2}{s}\right)^{1/2}, \tag{4}$$

$$DW=\frac{\left|\frac{a_2(s)}{m'^2-m^2}\right|^2}{180}\left(1-\frac{4m_\pi^2}{s}\right)^{5/2}\times\{[s-(m'^2+m^2)]^2-4m'^2m^2\}^2, \tag{5}$$

with

$$2m'p=[(s-m'^2-m^2)^2-4m'^2m^2]^{1/2}, \tag{6}$$

$$\sum_{pol}\left|\eta\cdot\eta'-\frac{(\eta\cdot p')(\eta'\cdot p)}{p\cdot p'}\right|^2=2+\frac{4m^2m'^2}{(m^2+m'^2-s)^2}. \tag{7}$$

We have now a general expression describing the decay $V'\to V\pi\pi$, which involves two invariant amplitudes, associated to the S and D wave, respectively. Given that $\sqrt{s}\leqslant 0.9$ GeV, we assume that the S wave is composed of the $f_0(600)$ and a nonresonant background. This is an approximation since other resonances could contribute to the amplitude in the kinematical region considered. However, it is reasonable to expect that the contribution of the $f_0(980)$ ($\Gamma_{f_0(980)}\approx 100$ MeV) and higher states decaying in two pions behave softly in the neighborhood of the $f_0(600)$, even if the latter is a broad resonance. Crossed channel contributions are treated in a similar way, i.e., considered as soft functions of the di-pion invariant mass. Therefore we parametrize the form factor associated to the S wave in the following way:

$$a_0^{(i)}=\left(\frac{a_i m_0^2}{D(s)}+\frac{b_i}{1-\frac{c_i s}{m_0^2}}\right). \tag{8}$$

m_0 is introduced for dimensional reasons and is fixed to $m_0=0.5$ GeV. For $D(s)$ we used two different expressions:

$$D(s)=s-m^2+\Pi(s), \tag{9a}$$

$$D(s)=s-m_p^2+im_p\Gamma_p, \tag{9b}$$

where $\Pi(s)$ stands for the $f_0(600)$ self-energy which involves a loop of kaons and pions. Note that Eq. (9b) defines the mass m_p and width Γ_p of the resonance in terms of the real and imaginary part of the pole of the S matrix. If a pole exists, it should be independent of the process where it is observed. However, neither the residue nor the background have to be the same for different processes. For this reason we include the index i which is associated to the physical decay under consideration. In the kinematical region of interest the D channel is nonresonant, thus we can approximate it by a soft background:

$$a_2^{(i)}=\left(\frac{f_i}{1-\frac{g_i s}{m_0^2}}\right). \tag{10}$$

It should be clear that our approach is a phenomenological one, and that we have not attempted to explicitly incorporate the scale anomaly. In fact, it can be argued that such a contribution is included within the background. Before turning to the fit, a few words regarding the pole parametrization are in order. Different parametrizations can be used to fit the data, however, the parameters that have physical relevance are the mass and width of the resonance, which are identified with the real and imaginary part pole position. One can use, for example, an s dependent width, or include theoretical expression for the full self-energy, however, at the end the parameters resulting from the fit have to be used to obtain the pole (see, for example, Ref. [37]). One expects that the results thus obtained coincide, independently of the parametrization used (an example is worked in detail in Ref. [38]).

For completeness, we include the s distribution predicted by the scale anomaly; further details can be found in the original literature [30,31]. The decay rate for the decay $V' \to V\pi\pi$, with V' and V vector mesons, is given by

$$\frac{d\sigma}{d\sqrt{s}} = \frac{\sqrt{s-4m^2_\pi}}{4m^2_{V'}\pi^3 f^4_\pi}|\vec{q}|\left(\tilde{g}_1+\frac{\tilde{g}_2}{6}\right)^2\left\{\left[s-\tilde{c}_1(s+|\vec{q}|^2)\right.\right.$$
$$\left.\left.\times\left(1+\frac{2m^2_\pi}{s}\right)+\tilde{c}_2 m^2_\pi\right]^2+\frac{1}{5}\tilde{c}^2_1|\vec{q}|^4\left(1-\frac{4m^2_\pi}{s}\right)^2\right\}, \tag{11}$$

with

$$\tilde{c}_1 = -\frac{2\tilde{g}_2}{6\tilde{g}_1+\tilde{g}_2}, \quad \tilde{c}_2 = -\frac{4(3\tilde{g}_1+\tilde{g}_2-3\tilde{g}_3)}{6\tilde{g}_1+\tilde{g}_2}$$

and

$$|\vec{q}| = \frac{1}{2m_{V'}}\{[m^2_{V'}-(\sqrt{s}-m^2_V)][m^2_{V'}-(\sqrt{s}+m^2_V)]\}^{1/2}.$$

TABLE I. Parameters resulting from the fit. The normalization factors refer to: N_a data from Ref. [35], N_2 to the $\Upsilon(3S) \to \Upsilon(1S)+\pi^0\pi^0$, and N_3 to $\Upsilon(3S)\to\Upsilon(2S)+\pi^0\pi^0$. The parameters are defined by Eqs. (8) and (10). Processes are labeled 1: $\Upsilon(2S)\to\Upsilon(1S)\pi\pi$; 2: $\Upsilon(3S)\to\Upsilon(1S)\pi\pi$; 3: $\Upsilon(3S)\to\Upsilon(2S)\pi\pi$; and 4: $\Psi(2S)\to J/\Psi\pi\pi$.

N_a	8.4×10^{-3}
N_2	0.36
N_3	0.62
a_1	8.2×10^3
b_1	$5.3\times10^3+i5.1\times10^3$
c_1	-0.14
f_1	$-1.6\times10^3+i3.8\times10^3$
g_1	0.4
a_2	7.6×10^2
b_2	$-1.1\times10^3+i8.4\times10^2$
c_2	5.4×10^{-2}
f_2	$2.3+i2.8$
g_2	0.33
a_3	-1.2×10^5
b_3	$-7.8\times10^4-i1.0\times10^5$
c_3	-1.1
f_3	$-4.8\times10^3-i9.0\times10^5$
g_3	-5.0×10^4
a_4	4.8×10^4
b_4	$3.2\times10^4+i3.7\times10^4$
c_4	-0.48
f_4	$3.2\times10^4-i3.2\times10^4$
g_4	-4.9×10^{-2}

III. THE FIT

We consider the following decays: $\Upsilon(3S)\to\Upsilon(1S)+\pi\pi$ [23], $\Upsilon(3S)\to\Upsilon(2S)+\pi\pi$ [23], $\Upsilon(2S)\to\Upsilon(1S)+\pi\pi$ [23,35], $\psi(2S)\to J/\psi+\pi\pi$ [27], resulting in a total of 165 points. All the data points for the different decays but the last one have been obtained from plots since no listings of the data are available. It is important to mention that previous fits [26] of the $\psi(2S)\to J/\psi+\pi\pi$ included only a subset of the experimental data.

Following the analysis in Refs. [27,35] we first attempted a fit of the 165 data points in terms of the scale anomaly [see Eq. (11)] [30,31]. The parameters of the fit are $\tilde{g}_1,\tilde{g}_2,\tilde{g}_3$ (or equivalently $\tilde{c}_1,\tilde{c}_2$, and $\tilde{c}_3=6\tilde{g}_1+\tilde{g}_2$) for each process plus normalization factors since some data are reported as number of events, leading to a total of 15 parameters. The fit produces a $\chi^2_{d.o.f.}>2$. This is not an unexpected result, the amplitude associated with the scale anomaly is derived under the assumption that the pions are soft, and this is not the case for all the data under consideration. In fact, according to Ref. [26] and previous analysis [1], the $f_0(600)$ is expected to contribute to these processes. This is the motivation to try a fit in terms of the pole approach, as discussed in the previous section.

The fit using the pole approach is based on Eqs. (3)–(5), and (8)–(10). With the current data, the fit can be done with Eq. (9b): $D(s)=s-m^2_p+im_p\Gamma_p$. We need more precise data to use Eq. (9a) because $\Pi(s)$ is described in terms of $m,g_{f\pi\pi}$, and g_{fkk} with too many parameters. For this reason our analysis is restricted to $D(s)$ given by Eq. (9b) where the mass (m_p) and width (Γ_p) are obtained directly from the fit.

The fit involves 33 parameters. We consider four processes [$i=1$–4 in Eqs. (8) and (10)] and for each of these we require seven parameters (b and f which are complex, and a,c,g). Three normalization factors are also parameters of the fit—because some of the reported data refer to number of events, not to a differential decay rate—and finally the mass and width of the resonance. The fit leads to a pole located at $m_p=528\pm32$ MeV and $\Gamma_p=413\pm45$ MeV together with the parameters reported in Table I, and a $\chi^2_{d.o.f.}=1.12$. The result of the joint fit are shown as the solid lines in Fig. 1. We do not report the result of the fit for the S wave alone, however, we should mention that including D wave effects improves the total χ^2 but the $\chi^2_{d.o.f.}$ remains unchanged. Note in this respect that the fit parameters are obtained from the di-pion invariant mass spectrum, the angular distributions are not involved. Once the fit parameters are fixed, numerical integration over the di-pion invariant mass is performed and we obtain—up to normalization again—the following angular distributions (in GeV units) and the curves shown in Fig. 2:

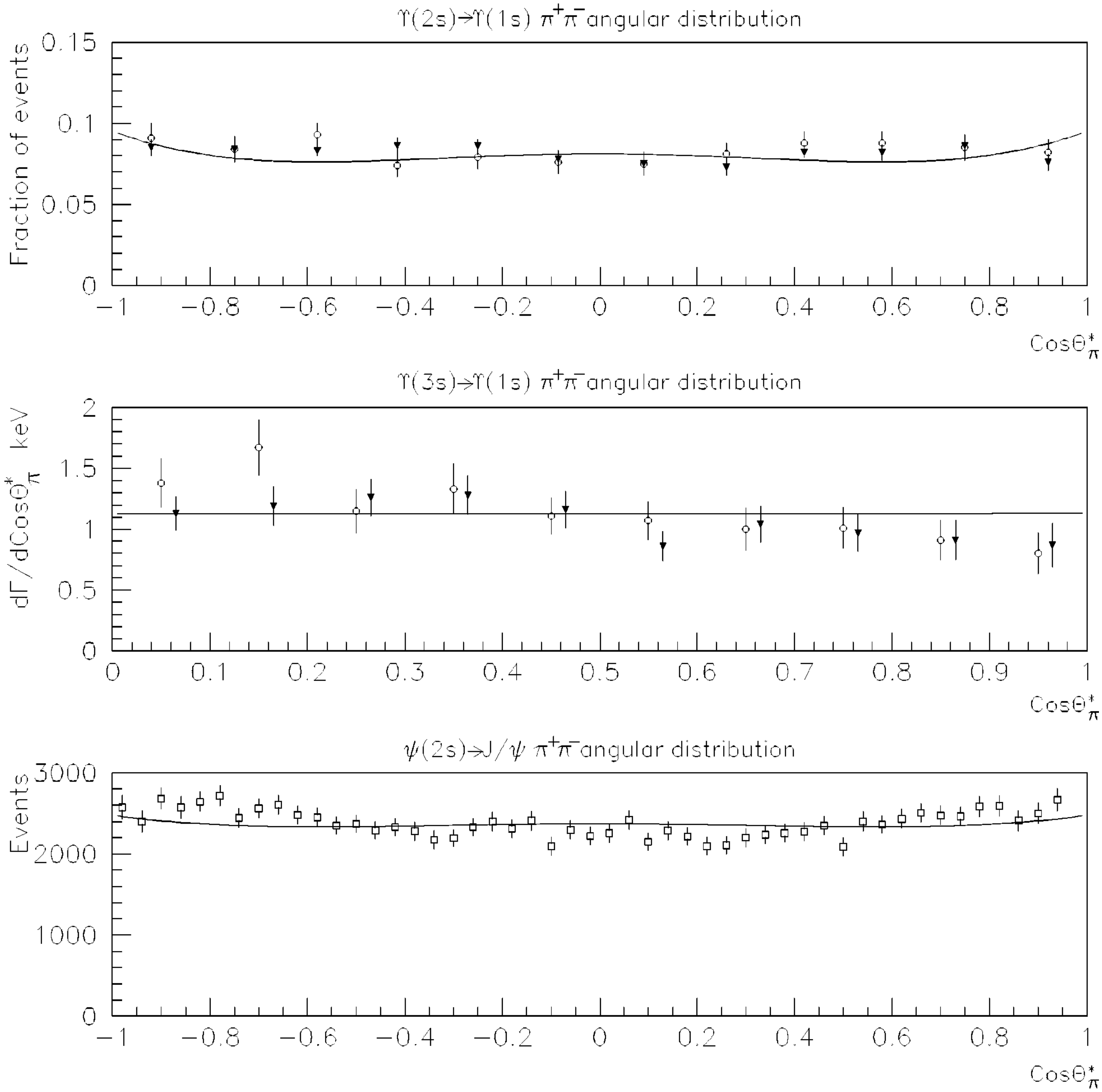

FIG. 2. Angular distribution as obtained from Eqs. (12)–(14) and compared to data from Refs. [23,27,35]. Open circles and solid triangles refer to data obtained from exclusive and inclusive processes, respectively.

$\Upsilon(2S)\to\Upsilon(1S)\pi\pi$ [35]:

$$\frac{d\Gamma}{d(\cos\theta)}=0.07[0.0674(3\cos^2\theta-1)^2+1.09], \tag{12}$$

$\Upsilon(3S)\to\Upsilon(1S)\pi\pi$ [23]:

$$\frac{d\Gamma}{d(\cos\theta)}=2.2[0.0009(3\cos^2\theta-1)^2+0.511], \tag{13}$$

$\Psi(2S)\to J/\Psi\pi\pi$ [27]:

$$\frac{d\Gamma}{d(\cos\theta)}=6.5[5.49(3\cos^2\theta-1)^2+359]. \tag{14}$$

Besides the pole position and the quality measure through the $\chi^2_{d.o.f.}$, it is instructive to analyze the different contributions to the decay rate. These are shown in Fig. 3 where the dots corresponds to the pole contribution, the dashed line to the background, and resulting fit is represented by the continuous line. From these plots one can see that the pole and background contributions must interfere destructively in all cases, except for the $\Upsilon(3S)\to\Upsilon(1S)$ transition in which the

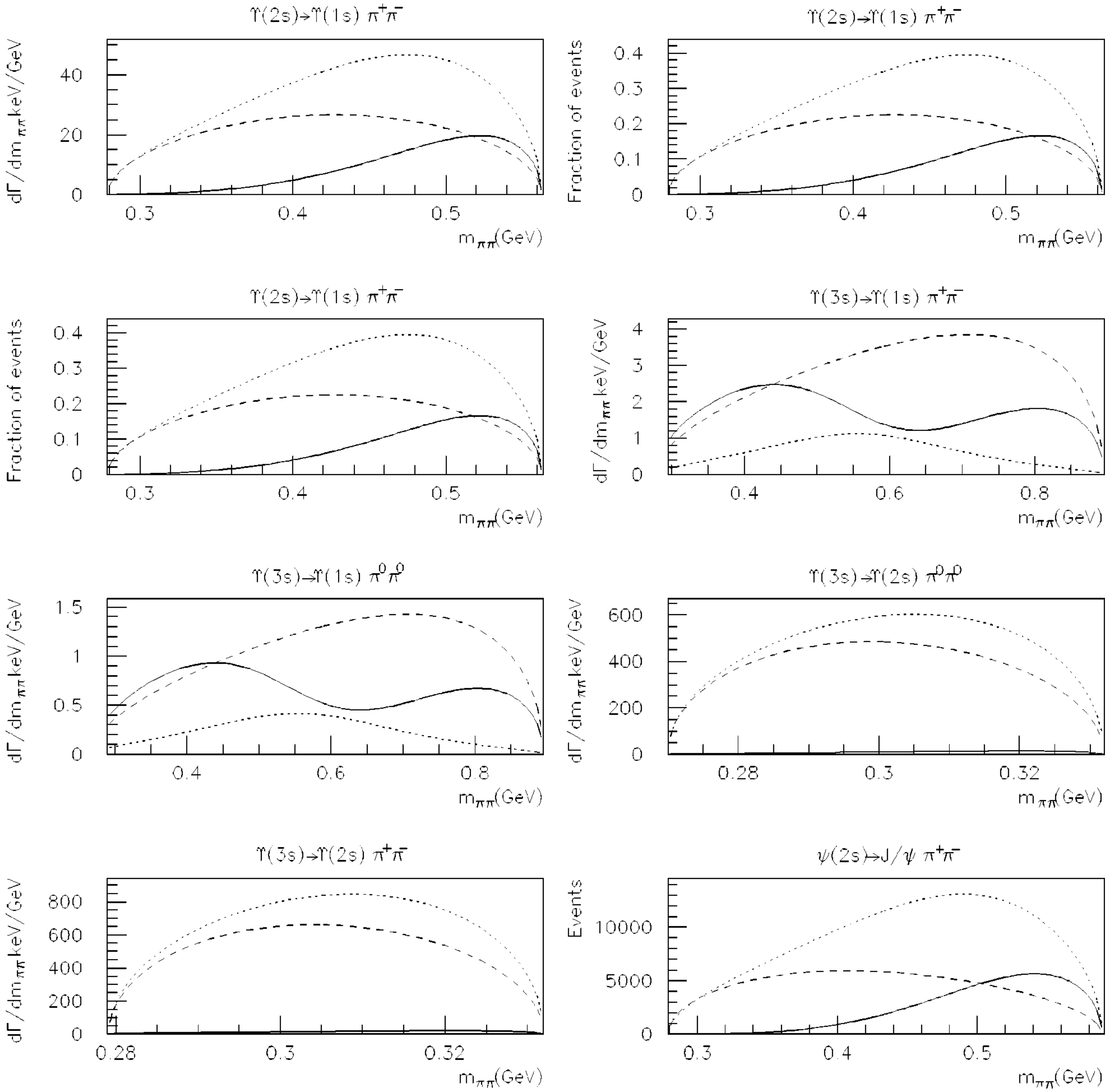

FIG. 3. The curve resulting from the fit to the data (solid line), and for comparison the pole (dotted line) and background (dashed line) contributions. Note that for the $\Upsilon(3S) \to \Upsilon(2S) + \pi\pi$ the fit curve is close to the axes, which implies a strong destructive interference among the pole and the background contributions.

kinematical region includes the pole position and both, constructive (below the pole mass) and destructive (above the pole) interferences, leading thus to the structure around 650 MeV. In Table II we quote the contributions to the χ^2 from the different processes. Of course anomaly effects can be incorporated in the background appearing in Eq. (8).

TABLE II. $\chi^2_{d.o.f.}$ for each separate process as obtained from fit to the di-pion invariant mass distribution.

Process	N_{data}	$\chi^2_{d.o.f.}$
$\Upsilon(2s) \to \Upsilon(1s)\pi\pi$	48	0.74
$\Upsilon(3s) \to \Upsilon(1s)\pi\pi$	40	1.3
$\Upsilon(3s) \to \Upsilon(2s)\pi\pi$	32	1.1
$\psi(2s) \to J/\psi\pi\pi$	45	1.5

IV. SUMMARY

In this paper we consider the phenomenological description of the following decays: $\Upsilon(3S) \to \Upsilon(1S) + \pi + \pi$, $\Upsilon(3S) \to \Upsilon(2S) + \pi + \pi$, $\Upsilon(2S) \to \Upsilon(1S) + \pi + \pi$, $\psi(2S) \to J/\psi + \pi + \pi$. As far as we can see, it is not possible to obtain a good quality fit in terms of the scale anomaly alone. Using general arguments, we derived an expression for the invariant amplitude describing flavor conserving processes of the type $V' \to V + \pi + \pi$, including both S and D waves, which involves two invariant amplitudes. We parametrized the S wave form factor with a pole plus a soft background and the D wave form factor by pure soft background. Fitting the data yields a pole in $m_p=528\pm32$ MeV and $\Gamma_p=413\pm45$ MeV with $\chi^2_{d.o.f.}=1.12$. We remark that a strong interference among the pole and the background is required to fit the data. Thus our analysis seems to indicate that physics in S wave pion-pion interaction below 800 MeV is governed by a subtle interplay between $f_0(600)$ meson contributions and a big background, difficult to understand in terms of conventional physics and which could be associated to the scale anomaly. In fact, one expects the dominant contribution to arise from the pole, while a background as important as the pole (as in the present case) can be taken as an indication of a nonperturbative phenomenon, like the scale anomaly.

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APPENDIX

In this appendix we present details of the derivation of the $\mathcal{M}_2$ amplitude. For the D wave contribution we write

$$\mathcal{M}_2=A^{\mu\nu}\Pi_{\mu\nu\rho\sigma}B^{\rho\sigma}. \tag{A1}$$

Using Lorentz covariance and imposing the conditions $p'\cdot\eta'=0$, $p\cdot\eta=0$ we find the general structure for $A_{\mu\nu}$, and through Eq. (A1) the di-pion D wave amplitude (note that $Q^\mu\Pi_{\mu\nu}=Q^\nu\Pi_{\mu\nu}=0$, therefore $p'^\mu\Pi_{\mu\nu}=p^\mu\Pi_{\mu\nu}$):

$$\mathcal{M}_2=\left[b\left(\eta'^\mu-\frac{p'^\mu}{p\cdot p'}(\eta'\cdot p)\right)\eta^\nu + c\left(\eta'^\mu-p'^\mu\frac{(\eta'\cdot p)}{p\cdot p'}\right)(\eta\cdot p')p'^\nu + \frac{a_2}{m'^2-m^2}\left((\eta\cdot\eta')-\frac{(\eta\cdot p')(\eta'\cdot p)}{p\cdot p'}\right)p'^\mu p'^\nu\right]\Pi_{\mu\nu}, \tag{A2}$$

where we introduced the m'^2-m^2 factor to work with a dimensionless a_2. $A^{\mu\nu}$ in Eq. (A1) describes the $V'(p')\to V(p)+D(Q)$, where D stands for a spin-2 mesonlike object and is given by

$$A^{\mu\nu}=b\left(\eta'^\mu-\frac{\eta'\cdot p}{p'\cdot p}p'^\mu\right)\eta^\nu+c\left(\eta'^\mu-p'^\mu\frac{(\eta'\cdot p)}{p\cdot p'}\right)\times(\eta\cdot p')p'^\nu+d\left(\eta\cdot\eta'-\frac{(\eta\cdot p')(\eta'\cdot p)}{p\cdot p'}\right)p'^\mu p'^\nu. \tag{A3}$$

In Eq. (A1) $B^{\rho\sigma}$ describes the $D\to\pi\pi$ amplitude and $\Pi_{\mu\nu\rho\sigma}$ is the spin two projector:

$$\Pi_{\mu\nu\rho\sigma}\equiv\sum_{\lambda=1}^{5}h_{\mu\nu}(\lambda)h_{\rho\sigma}(\lambda). \tag{A4}$$

The polarization tensor $h_{\mu\nu}$ has the following properties:

$$h_{\mu\nu}(\lambda)=h_{\nu\mu}(\lambda),\quad Q^\mu h_{\mu\nu}(\lambda)=g^{\mu\nu}h_{\mu\nu}(\lambda)=0. \tag{A5}$$

Using these relations together with the projector property of $\Pi_{\mu\nu\rho\sigma}$ one finds

$$\Pi_{\mu\nu\rho\sigma}\equiv\frac{1}{2}P_{\mu\rho}P_{\nu\sigma}+\frac{1}{2}P_{\mu\sigma}P_{\nu\rho}-\frac{1}{3}P_{\mu\nu}P_{\rho\sigma}, \tag{A6}$$

with

$$P_{\mu\nu}=g_{\mu\nu}-\frac{Q_\mu Q_\nu}{Q^2}. \tag{A7}$$

On the other hand, by Lorentz covariance $B_{\rho\sigma}\propto q_\rho q_\sigma$, and for convenience we introduced

$$\Pi_{\mu\nu}\equiv\Pi_{\mu\nu\rho\sigma}B^{\rho\sigma}=q_\mu q_\nu-\frac{1}{3}P_{\mu\nu}q^2. \tag{A8}$$

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