Two-hadron semi-inclusive production including subleading twist contributions

Alessandro Bacchetta*
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

Marco Radici†
Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, I-27100 Pavia, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy

(Received 29 December 2003; published 27 April 2004)

We extend the analysis of two-hadron fragmentation functions to the subleading twist, discussing also the issue of color gauge invariance. Our results can be used anywhere two unpolarized hadrons are semi-inclusively produced in the same fragmentation region, also at moderate values of the hard scale Q. Here, we consider the example of polarized deep-inelastic production of two hadrons and we give a complete list of cross sections and spin asymmetries up to the subleading twist. Among the results, we highlight the possibility of extracting the transversity distribution with longitudinally polarized targets and also the twist-3 distribution $e(x)$, which is related to the pion-nucleon σ term and to the strangeness content of the nucleon.

DOI: 10.1103/PhysRevD.69.074026 PACS number(s): 13.87.Fh, 11.80.Et, 13.60.Hb

I. INTRODUCTION

The study of the distribution of hadrons produced in the fragmentation of a quark offers the opportunity to understand the mechanism of hadronization as well as to extract information about the partonic structure of hadrons; both issues are a manifestation of confinement in QCD, a yet unexplained phenomenon. So far, parametrizations are available only for the distribution of the longitudinal momentum of only one of the final-state hadrons, the familiar unpolarized fragmentation function $D_1(z)$ [1,2]. Clearly, most of the complexity of the fragmentation process lies unexplored.

When the transverse momentum of one of the outgoing hadrons is measured, a new fragmentation function can be introduced relating the transverse polarization of the parent quark to the distribution of the produced hadron in the transverse direction [3]. This so-called Collins function acts as an analyzing power and it is perhaps the simplest observable that reveals the role of the quark's spin in the hadronization process. It also acts as a filter to measure the still unknown distribution of the transverse spin of quarks (transversity, for a review, see Ref. [4]) and the tensor charge of the hadron thereof [3]. However, the price to pay is the complete knowledge of the transverse dynamics of the detected leading hadron inside the jet. This creates problems both experimentally, as it is evident, and also theoretically, because the introduced dependence upon an intrinsic (nonperturbative) transverse momentum complicates the treatment of color gauge invariance [5,6] and evolution equations [7–10].

When two final-state hadrons are measured, in principle the number of variables doubles. For instance, it is possible to measure the relative transverse momentum of the pair, as well as its center-of-mass transverse momentum. Therefore, even after integrating upon the center-of-mass transverse momentum, a transverse vector is still available to establish a relation with the transverse polarization of the fragmenting quark [11–13].

Already from this intuitive discussion it is evident that two-hadron fragmentation functions can be important in studying spin effects in hadronization. They are perhaps more challenging to measure, in as much as they require the simultaneous detection of two hadrons inside the same jet. On the other side, the integration upon the center-of-mass transverse momentum removes the above-mentioned difficulty about the evolution equations, and it avoids, at least at the leading twist, the potential loss of universality implied by a correct treatment of color gauge invariance [5,6], as will also be discussed in Sec. III.

Another class of functions that deserves much attention is that of polarized fragmentation functions. In this case, the spin of the final-state hadron is measured and its relation with the hadronization dynamics can be investigated. However, in general the spin of a final-state hadron can be analyzed only through the decay into two or more hadronic byproducts. In this sense, polarized fragmentation functions can be thought of as specific examples of multihadron fragmentation functions. For instance, the polarization of a vector meson (e.g., ρ^0) is reflected in the angular distribution of its decay products (e.g., $\pi^+\pi^-$). As a consequence, spin-1 polarized fragmentation functions [14–16] correspond to the relative p-wave part of two-hadron fragmentation functions [17]. At present, however, the formalism of two-hadron fragmentation functions cannot comprise parity-violating decays, such as the extremely important case of the Λ baryon [14,18–22].

Two-hadron fragmentation functions were first introduced in Ref. [23], but with no quark polarization. Extension of the original functions to include polarization effects (usually known as interference fragmentation functions) were studied in Refs. [11,12,24,25]. The complete leading-twist analysis has been carried out in Ref. [13] and employed in semi-inclusive DIS [26,27] and electron-positron annihilation [28]. Positivity bounds and the expansion in the partial wave

*Electronic address: alessandro.bacchetta@physik.uni-regensburg.de
†Electronic address: radici@pv.infn.it

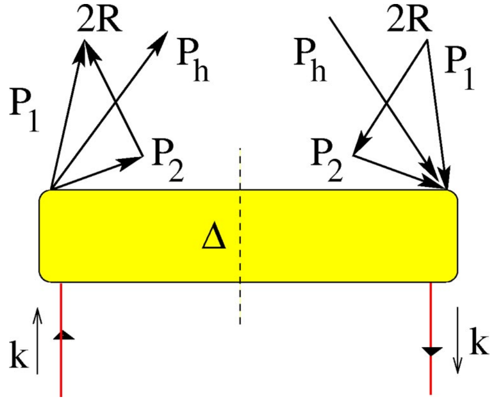

FIG. 1. Quark-quark correlation function Δ for the fragmentation of a quark with momentum k into a pair of hadrons with total momentum $P_h = P_1 + P_2$ and relative momentum $R = (P_1 - P_2)/2$.

of the two hadrons were presented in Ref. [17]. Very recently, a study of collinear fragmentation into two hadrons has been performed [29], demonstrating the factorization of two-hadron fragmentation functions at next-to-leading order in α_S and calculating their evolution, originally studied in Ref. [30]. In this article, we are going to extend the existing treatment to the subleading-twist level, but integrating upon the transverse momentum. The way we proceed is very similar to what was done in Ref. [31], for one-hadron production (see also Ref. [32]), and in Ref. [5], for the issue of color gauge invariance of the quark-quark correlator, even though we will only present results integrated upon the transverse momentum. The extension to the subleading twist is an important step not only from a formal point of view, but also because the measurement of two-hadron leptoproduction can be attempted in experiments at moderate Q^2, where subleading-twist contributions should not be neglected.

The paper is organized as follows. In Sec. II, we will briefly review the kinematics for the semi-inclusive production of two unpolarized hadrons inside the same current jet. In Sec. III, we present the complete twist analysis up to subleading order of the quark-quark and quark-gluon-quark fragmenting correlators, discussing also the issue of color gauge invariance and of partial-wave expansion. In Sec. IV, the explicit expression of the hadronic tensor for the semi-inclusive production of two unpolarized hadrons in deep-inelastic scattering (DIS) is shown, including leading- and subleading-twist contributions. In Sec. V, the corresponding cross sections and spin asymmetries are discussed for different polarization states of the beam and the target. Finally, in Sec. VI some conclusions are drawn.

II. KINEMATICS

The fragmentation process is schematically represented in Fig. 1, where a quark with mass m and momentum k fragments into two unpolarized hadrons with masses M_1, M_2, and momenta P_1, P_2. We introduce the vectors $P_h = P_1 + P_2$ and $R = (P_1 - P_2)/2$. Using two dimensionless lightlike vectors n_+ and n_- (satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$), we describe a four-vector a as $[a^-, a^+, \vec{a}_T]$ in terms of its light-cone components $a^{\pm} = a \cdot n_{\mp} = (a^0 \pm a^3)/\sqrt{2}$ and a bidimensional vector $\vec{a}_T$.

For later use, transverse projection operators can be defined as

$$g_T^{\mu\nu} = g^{\mu\nu} - n_+^{\{\mu} n_-^{\nu\}}, \qquad \epsilon_T^{\mu\nu} \equiv \epsilon^{\rho\sigma\mu\nu} n_{+\rho} n_{-\sigma}, \tag{1}$$

where the braces indicate symmetrization upon the included indices and $\epsilon^{0123} = 1$. In general, the fragmentation process is described in the frame where the hadronic final state has no transverse component, i.e., the frame where $\vec{P}_{hT} = \vec{0}$. We will see in the following that in actual measurements the natural choice is different and the required boost introduces effects that have to be consistently taken into account when extending the analysis to the subleading twist.

We also define the variables $z = P_h^- / k^-$, the light-cone fraction of fragmenting quark momentum carried by the hadron pair, and the variable $\zeta = 2R^- / P_h^-$, which describes how the total momentum of the pair is split into the two single hadrons. Therefore, the relevant momenta can be parametrized as

$$k^\mu = \left[\frac{P_h^-}{z}, \frac{z(k^2 + \vec{k}_T^{\,2})}{2P_h^-}, \vec{k}_T \right],$$

$$P_h^\mu = \left[P_h^-, \frac{M_h^2}{2P_h^-}, \vec{0} \right], \tag{2}$$

$$R^\mu = \left[\frac{\zeta}{2} P_h^-, \frac{(M_1^2 - M_2^2) - (\zeta/2) M_h^2}{2P_h^-}, \vec{R}_T \right],$$

where M_h is the pair invariant mass. Not all components of the four-vectors are independent. In particular, we note that

$$R^2 = \frac{M_1^2 + M_2^2}{2} - \frac{M_h^2}{4},$$

$$\vec{R}_T^2 = \frac{1}{2}\left[\frac{(1-\zeta)(1+\zeta)}{2} M_h^2 - (1-\zeta) M_1^2 - (1+\zeta) M_2^2 \right],$$

$$P_h \cdot R = \frac{M_1^2 - M_2^2}{2}, \tag{3}$$

$$P_h \cdot k = \frac{M_h^2}{2z} + z \frac{k^2 + \vec{k}_T^{\,2}}{2},$$

$$R \cdot k = \frac{(M_1^2 - M_2^2) - (\zeta/2) M_h^2}{2z} + z\zeta \frac{k^2 + \vec{k}_T^{\,2}}{4} - \vec{k}_T \cdot \vec{R}_T.$$

The positivity requirement $\vec{R}_T^2 \geqslant 0$ imposes the further constraint

$$M_h^2 \geqslant \frac{2}{1+\zeta} M_1^2 + \frac{2}{1-\zeta} M_2^2. \tag{4}$$

Note that to avoid the introduction of a new hard scale in the process, all invariants listed above have to be small compared to the hard scale Q of the process (where $Q^2=-q^2$, with q the momentum transfer).

III. THE FRAGMENTATION CORRELATOR UP TO THE SUBLEADING TWIST

The soft processes underlying the fragmentation are symbolically represented by the shaded blob in Fig. 1 and are described in terms of hadronic matrix elements of nonlocal quark operators as

$$\Delta(k,P_h,R)=\sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi}\langle 0|\psi(\xi)|P_h,R;X\rangle \times\langle X;P_h,R|\overline{\psi}(0)|0\rangle, \tag{5}$$

where ψ is the quark field operator. The above correlator is not color gauge invariant, as the quark fields are evaluated at two light-front-separated space-time points, 0 and ξ. To restore color gauge invariance, the so-called gauge link operator must be included,

$$U_{[0,\xi]}=\mathcal{P}\exp\left(-ig\int_0^{\xi} dw\cdot A(w)\right), \tag{6}$$

where A is the gluon field with coupling constant g, and $\mathcal{P}$ indicates a path-ordered exponential. It symbolically corresponds to attaching all possible soft gluon lines to the soft blob of Fig. 1 and resumming their contribution. As such, the corresponding diagrams will still be considered as tree-level contributions, since the coupling g can be reabsorbed in the definition of the correlator itself.

The quark line in the fragmentation correlator has to come from a hard process that determines a dominant lightlike direction. For the final state in a semi-inclusive DIS process, the hard scale Q selects the n_- direction as the dominant one with respect to the transverse and n_+ ones, which are suppressed as $O(M_h)$ and $O(1/Q)$, respectively. The integration upon the suppressed n_+ components of the momenta can be performed up to $O(1/Q)$, leading to

$$\Delta(z,\vec{k}_T,R)=\sum_X \int \frac{d\xi^+ d\vec{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi}\langle 0|\psi(\xi)|P_h,R;X\rangle \times\langle X;P_h,R|\overline{\psi}(0)|0\rangle|_{\xi^-=0}. \tag{7}$$

Similarly, in the gauge link it was usually assumed that the A^+ component of the gluon field is suppressed, and, by neglecting the $\vec{A}_T$ component and by imposing the choice $A^-=0$ (the so-called light-cone gauge), the gauge link was reduced to unity. Recently, the problem of the evaluation of such an operator and of the gauge-invariant description of the quark-quark correlator has been studied in Refs. [5,33,34]. We will address it following the analysis of Ref. [5] for the case of semi-inclusive DIS; the results can be easily generalized to the case of electron-positron annihilation. The proof in Ref. [5] relies on counting the powers in

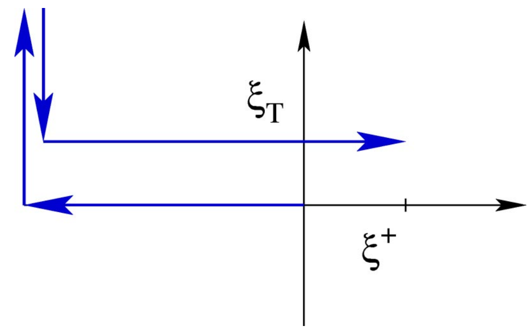

FIG. 2. Link structure for the leading-twist color gauge-invariant quark-quark correlator for the fragmentation of a quark into a pair of hadrons.

$1/Q$ of the product of the fragmenting quark propagator and of the various components of the gluon fields attached to the soft blob, retaining only the leading and subleading contributions; these arguments are independent of the hadronic final state, and they are valid also for two-hadron production, as long as the additional vector R does not introduce any new hard scale.

It turns out that only a combined analysis of leading and subleading contributions involving both A^- and $\vec{A}_T$ components of the gluon field leads to a color gauge-invariant expression for the fragmentation correlator. Following Ref. [5] (and generalizing its notation to the case of two-hadron production), a color gauge-invariant object is obtained at the leading twist by connecting the 0 and ξ points along the + direction running through $-\infty$ and through the transverse directions at $\xi^+=-\infty$ (see Fig. 2 for a schematic picture of the link path), namely

$$\Delta^{[-]}(z,\vec{k}_T,R) =\sum_X \int \frac{d\xi^+ d\vec{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi}\langle 0|U^+_{[-\infty,\xi]}U^T_{[\infty,\xi]}\psi(\xi)|P_h,R;X\rangle \times\langle X;P_h,R|\overline{\psi}(0)U^T_{[0,\infty]}U^+_{[0,-\infty]}|0\rangle|_{\xi^-=0}, \tag{8}$$

where

$$U^+_{[a,b]}=\mathcal{P}\exp\left(-ig\int_a^b dw^+A^-(w)\Big|_{\substack{w^-=b^-=a^-\\ \vec{w}_T=\vec{b}_T=\vec{a}_T}}\right),$$

$$U^T_{[a,b]}=\mathcal{P}\exp\left(-ig\int_a^b d\vec{w}_T\cdot\vec{A}_T(w)\Big|_{\substack{w^+=b^+=a^+\\ w^-=b^-=a^-}}\right). \tag{9}$$

Note that by reabsorbing the product of gauge links, U^TU^+, into a redefinition of the quark field ψ, the quark-quark correlator of Eq. (8) falls back into the expression of Eq. (7), but for the [−] superscript specifying the gauge link direction. Therefore, it still leads to a semipositive definite matrix in Dirac space [35] and the probabilistic interpretation of its leading-twist projections can be retained. The dependence on the direction of the gauge link is due to the contribution of

U^T, i.e., of the transverse component of the gluon field at $\xi^+=-\infty$, which plays a crucial role in T-odd effects since it introduces nontrivial phases in the scattering amplitude. The direction of the gauge link depends on the considered process, potentially posing a threat to the universality of the definition of the soft correlator. For example, when considering the e^+-e^- annihilation into one pair of hadrons in the same jet, the correlator of Eq. (8) will depend on a gauge link running through $\xi^+=+\infty$, therefore displaying the $[+]$ superscript. However, it has been explicitly shown that universality is preserved at the one-loop level [6]. Moreover, when integrating $\Delta^{[-]}(z,\vec{k}_T,R)$ of Eq. (8) upon $d\vec{k}_T$, the displacement of the quark fields is confined to the light-cone $+$ direction. Hence, the two gauge links U^T and U^+ will merge into a single operator connecting the points 0 and ξ along a straight line:

$$\Delta(z,R)=z^2\int d\vec{k}_T\Delta^{[-]}(z,\vec{k}_T,R)=z^2\int d\vec{k}_T\Delta^{[+]}(z,\vec{k}_T,R)$$

$$=z^2\sum\int_X\int\frac{d\xi^+}{2\pi}e^{\mathrm{i}k\cdot\xi}\langle 0|U^+_{[0,\xi]}\psi(\xi)|P_h,R;X\rangle \times\langle X;P_h,R|\overline{\psi}(0)|0\rangle|_{\xi^-=\vec{\xi}_T=\vec{0}}. \tag{10}$$

Therefore, in principle $\vec{k}_T$-integrated functions are insensitive to the link path and should represent universal functions. This is certainly true at the leading twist, but it is still matter of debate at the subleading level because of the presence of $\vec{k}_T$-weighted contributions [36], as we shall see below. Therefore, here in the following we will omit an explicit dependence on the gauge link path for transverse-momentum integrated quantities only when the issue is settled and commonly accepted.

At the subleading twist, both the combinations of the transverse components of the quark propagator with A^-, and of the n_+ projection of the quark propagator with $\vec{A}_T$, generate a color gauge-invariant operator involving the field strength tensor $G^{\mu\nu}$. After the $\vec{k}_T$ integration, this correlator reads

$$\Delta_A^{[-]\alpha}(z,R)=\int dz_1\frac{\mathrm{i}}{z_1+\mathrm{i}\epsilon}\Delta_G^{\alpha}(z,z_1,R)$$
$$=\int dz_1\frac{\mathrm{i}}{z_1+\mathrm{i}\epsilon}z^2\sum\int_X\int\frac{d\xi^+}{2\pi}\frac{d\eta^+}{2\pi}\times e^{\mathrm{i}k\cdot\xi}e^{\mathrm{i}k_1\cdot(\eta-\xi)}\times\langle 0|U^+_{[0,\xi]}\psi(\xi)U^+_{[\xi,\eta]}gG^{-\alpha}(\eta)|P_h,R;X\rangle\times\langle X;P_h,R|\overline{\psi}(0)|0\rangle|_{\xi^-=\eta^-=\vec{\xi}_T=\vec{\eta}_T=\vec{0}}, \tag{11}$$

where $z_1=P_h^-/k_1^-$. This correlator starts contributing at twist 3; it is considered a tree-level contribution, as already explained at the beginning of this section. Again, by absorbing the gauge link U^+ in a redefinition of the quark field ψ, the well known expression of the quark-gluon-quark correlator is recovered [37,38]. Introducing the covariant derivative $\mathrm{i}D^\mu(\xi)=\mathrm{i}\partial^\mu+gA^\mu(\xi)$ we can recover also the relation

$$\Delta_A^{[-]\alpha}(z,R)=\Delta_D^{\alpha}(z,R)-\Delta_\partial^{[-]\alpha}(z,R), \tag{12}$$

where

$$\Delta_D^{\alpha}(z,R)=z^2\sum\int_X\int\frac{d\xi^+}{2\pi}e^{\mathrm{i}k\cdot\xi}\langle 0|U^+_{[0,\xi]}\psi(\xi)\mathrm{i}D^\alpha(\xi)|P_h,R;X\rangle\times\langle X;P_h,R|\overline{\psi}(0)|0\rangle|_{\xi^-=\vec{\xi}_T=\vec{0}}, \tag{13}$$

$$\Delta_\partial^{[-]\alpha}(z,R)=z^2\int d\vec{k}_T k_T^\alpha\Delta^{[-]}(z,\vec{k}_T,R), \tag{14}$$

with the covariant derivative $D(\xi)$ acting on the left on the quark field $\psi(\xi)$. Note that, after integrating upon $\vec{k}_T$, the term Δ_D^α becomes insensitive to the gauge link path. It is possible to relate the quark-gluon-quark correlator to the quark-quark one using the equation of motion of QCD, $(\mathrm{i}\not{D}-m)\psi=0$. Therefore, $\Delta_A^{[-]\alpha}(z,R)$ does not introduce any new fragmentation functions, but it turns out that it plays an essential role in ensuring electromagnetic gauge invariance up to subleading twist. In the following, both the quark-quark and quark-gluon-quark correlators of Eqs. (8) and (11), respectively, will be analyzed in detail for the semi-inclusive two-hadron production, including the expansion in the partial waves of the pair.

A. The correlator Δ

The most general parametrization of $\Delta^{[\pm]}(k,P_h,R)$ in Eq. (5), compatible with Hermiticity and parity invariance, is given by

$$\Delta^{[\pm]}(k,P_h,R)=M_hC_1^{[\pm]}1+C_2^{[\pm]}\not{P}_h+C_3^{[\pm]}\not{R}+C_4^{[\pm]}\not{k}+\frac{C_5^{[\pm]}}{M_h}\sigma_{\mu\nu}P_h^\mu k^\nu+\frac{C_6^{[\pm]}}{M_h}\sigma_{\mu\nu}R^\mu k^\nu+\frac{C_7^{[\pm]}}{M_h}\sigma_{\mu\nu}P_h^\mu R^\nu+\frac{C_8^{[\pm]}}{M_h^2}\gamma_5\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_{h\nu}R_\rho k_\sigma, \tag{15}$$

where the coefficients $C_i^{[\pm]}$ are real scalar functions of all the possible independent invariants, namely $k^2,k\cdot P_h,k\cdot R,M_h^2,M_1^2,M_2^2$. Integrating Eq. (15) upon the suppressed

k^+ direction and, consequently, taking the lightlike separation $\xi^-=0$, we get for, e.g., the DIS process the following decomposition:

$$\Delta^{[-]}(z,\vec{k}_T,R)=\frac{1}{32\pi z}\int dk^+\Delta^{[-]}(k,P_h,R)|_{k^-=P_h^-/z}$$
$$=\frac{1}{16\pi}\Bigg\{D_1\not{n}_-+H_1^{\sphericalangle\,\prime}\frac{\mathrm{i}}{2M_h}[\not{R}_T,\not{n}_-]+H_1^{\perp}\frac{\mathrm{i}}{2M_h}[\not{k}_T,\not{n}_-]+G_1^{\perp}\gamma_5\frac{\epsilon_T^{\rho\sigma}R_{T\rho}k_{T\sigma}}{M_h^2}\not{n}_-\Bigg\}$$
$$+\frac{M_h}{16\pi P_h^-}\Bigg\{E+D^{\sphericalangle\,\prime}\frac{\not{R}_T}{M_h}+D^{\perp}\frac{\not{k}_T}{M_h}+H\frac{\mathrm{i}}{2}[\not{n}_-,\not{n}_+]+H^{\sphericalangle}\frac{\mathrm{i}}{2M_h^2}[\not{R}_T,\not{k}_T]+G^{\sphericalangle\,\prime}\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho R_{T\sigma}}{M_h}+G^{\perp}\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\Bigg\}. \tag{16}$$

The first group of terms inside the curly brackets represents the leading-twist contribution and includes the usual interference fragmentation functions (IFF) discussed elsewhere [13,17,27]. They can be obtained by projecting out of Eq. (16) the usual Dirac structures $\Gamma=\gamma^-,\gamma^-\gamma_5,\mathrm{i}\sigma^{i-}\gamma_5$, where i means a transverse component. The second group shows the $1/P_h^-\sim 1/Q$-suppressed fragmentation functions that arise from the Dirac structures $\Gamma=1,\gamma^i,\sigma^{-+},\sigma^{ij},\gamma^i\gamma_5$, respectively. Note that the structures $\Gamma=\mathrm{i}\gamma_5,\sigma^{i+}$ give no contribution at this level. The functions $H_1^{\sphericalangle\,\prime},H_1^{\perp},G_1^{\perp},H,H^{\sphericalangle},G^{\sphericalangle\,\prime},G^{\perp}$, are T-odd, while $H_1^{\sphericalangle\,\prime},H_1^{\perp},E,H,H^{\sphericalangle}$, are chiral-odd.

Because of the constraints imposed by kinematics and by the k^+ integration, the fragmentation functions in Eq. (16) actually depend on five variables, namely $z,\zeta,M_h^2,\vec{k}_T^{\,2},\vec{k}_T\cdot\vec{R}_T$, and they can generally be decomposed as

$$D_1(z,\zeta,M_h^2,\vec{k}_T^{\,2},\vec{k}_T\cdot\vec{R}_T)=D_1^e(z,\zeta,M_h^2,\vec{k}_T^{\,2},(\vec{k}_T\cdot\vec{R}_T)^2)+\frac{\vec{k}_T\cdot\vec{R}_T}{M_h^2}D_1^o(z,\zeta,M_h^2,\vec{k}_T^{\,2},(\vec{k}_T\cdot\vec{R}_T)^2), \tag{17}$$

and similarly for the other functions. Both D_1^e and D_1^o, are even functions of $\vec{k}_T$.

By integrating Eq. (16) upon the transverse momentum $\vec{k}_T$, we get

$$\Delta(z,R)\equiv z^2\int d\vec{k}_T\Delta^{[-]}(z,\vec{k}_T,R)$$
$$=\frac{1}{16\pi}\Bigg\{D_1\not{n}_-+H_1^{\sphericalangle}\frac{\mathrm{i}}{2M_h}[\not{R}_T,\not{n}_-]\Bigg\}+\frac{M_h}{16\pi P_h^-}\times\Bigg\{E+D^{\sphericalangle}\frac{\not{R}_T}{M_h}+H\frac{\mathrm{i}}{2}[\not{n}_-,\not{n}_+]+G^{\sphericalangle}\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho R_{T\sigma}}{M_h}\Bigg\}$$
$$\equiv\Delta_1(z,\zeta,M_h^2,\phi_R)+\Delta_2(z,\zeta,M_h^2,\phi_R), \tag{18}$$

where

$$H_1^{\sphericalangle}\equiv H_1^{\sphericalangle\,\prime e}+H_1^{\perp o(1)},$$
$$D^{\sphericalangle}\equiv D^{\sphericalangle\,\prime e}+D^{\perp o(1)}, \tag{19}$$
$$G^{\sphericalangle}\equiv G^{\sphericalangle\,\prime e}+G^{\perp o(1)},$$

and each term now depends on z,ζ,M_h^2. We define the moment of a fragmentation function as

$$H_1^{\perp(1)}(z,\zeta,M_h^2)=\int d\vec{k}_T\frac{\vec{k}_T^{\,2}}{2M_h^2}H_1^{\perp}(z,\zeta,M_h^2,\vec{k}_T^{\,2},\vec{k}_T\cdot\vec{R}_T), \tag{20}$$

and similarly for the other fragmentation functions. The resulting functions $H_1^{\sphericalangle},H,G^{\sphericalangle}$ are still T-odd, while $H_1^{\sphericalangle},E,H$ are chiral-odd. For later convenience, the leading-twist contribution is indicated by Δ_1 and the subleading-twist one by Δ_2, respectively.

B. The subleading-twist correlator $\Delta_A^{[-]\alpha}$

As already anticipated above, the color gauge-invariant correlator $\Delta_A^{[-]\alpha}$ of Eq. (11) is suppressed by one power of $1/Q$ with respect to the leading twist $\Delta^{[-]}$ of Eq. (8). Therefore, it must be consistently included when extending the analysis to the subleading twist. For the sake of simplicity, only the $\vec{k}_T$-integrated result will be shown. Since the gauge links can be absorbed in a redefinition of the quark fields, both Δ_D^{α} and $\Delta_\partial^{[-]\alpha}$ in Eqs. (13) and (14), respectively, can be worked out in a way similar to the one-hadron emission. By projecting out the usual Dirac structures $\Gamma=\gamma^-,\gamma^-\gamma_5,\mathrm{i}\sigma^{i-}\gamma_5$, the following decomposition results,

$$\Delta_A^{[-]\alpha}(z,R)=\Delta_D^{\alpha}(z,\zeta,M_h^2,\phi_R)-\Delta_\partial^{[-]\alpha}(z,\zeta,M_h^2,\phi_R)$$
$$=\frac{M_h}{16\pi z}\Bigg\{\tilde{D}^{\sphericalangle}\frac{R_T^\alpha}{M_h}\not{n}_-+\tilde{G}^{\sphericalangle}\frac{\epsilon_T^{\alpha\beta}R_{T\beta}}{M_h}\gamma_5\not{n}_--(\tilde{E}-\mathrm{i}\tilde{H})\frac{\gamma^\alpha\not{n}_-}{2}-\mathrm{i}H_1^{\sphericalangle o(1)}\frac{R_T^\alpha\not{R}_T}{M_h^2}\not{n}_-\Bigg\}, \tag{21}$$

where the functions with tildes denote

$$\tilde{D}^{\sphericalangle} \equiv D^{\sphericalangle} - z D_1^{o(1)},$$

$$\tilde{G}^{\sphericalangle} \equiv G^{\sphericalangle} - z G_1^{\perp(1)} - z \frac{m}{M_h} H_1^{\sphericalangle}, \tag{22}$$

$$\tilde{E} \equiv E - z \frac{m}{M_h} D_1,$$

$$\tilde{H} \equiv H + 2z H_1^{\perp(1)},$$

and are pure twist-3 fragmentation functions depending on z, ζ, M_h^2. They all vanish in the Wandzura-Wilczek approximation.

C. Partial-wave expansion

If the invariant mass M_h is not very large, the hadron pair can be assumed to be in a channel corresponding to a relative s or p wave. Consequently, two-hadron fragmentation functions can be decomposed in partial waves [17]. In the center-of-mass (c.m.) frame of the two hadrons, the emission occurs back-to-back and the key variable is the angle θ between the directions of the emission and of P_h. The kinematics described in Sec. II can be easily adjusted to the c.m. frame of the two hadrons; the most important modifications are

$$\vec{R}_T = \vec{R} \sin\theta,$$

$$|\vec{R}| = \frac{1}{2M_h}\sqrt{M_h^2 - 2(M_1^2 + M_2^2) + (M_1^2 - M_2^2)^2}, \tag{23}$$

$$\zeta = \frac{1}{M_h}\left(\sqrt{M_1^2 - |\vec{R}|^2} - \sqrt{M_2^2 - |\vec{R}|^2} - 2|\vec{R}|\cos\theta\right),$$

where the crucial remark is that ζ is at most a linear polynomial in $\cos\theta$ with coefficients that depend only on invariant masses. This suggests that the dependence upon ζ in the fragmentation functions should be conveniently replaced by an expansion in the Legendre polynomials in $\cos\theta$ and, consequently, the cross section kept differential in $d\cos\theta$. The Jacobian $d\zeta/d\cos\theta = 2|\vec{R}|/M_h$ can be absorbed in a redefinition of the fragmentation functions.

The partial-wave expansion of the leading-twist fragmentation functions has been given in Ref. [17], namely,[1]

$$D_1 \to D_{1,oo} + D_{1,ol}\cos\theta + D_{1,ll}\frac{1}{4}(3\cos^2\theta - 1),$$

$$H_1^{\sphericalangle} \to H_{1,ot}^{\sphericalangle} + H_{1,lt}^{\sphericalangle}\cos\theta. \tag{24}$$

Extending the analysis to the subleading-twist functions is straightforward:

$$H_1^{\sphericalangle o(1)} \to H_{1,ot}^{\sphericalangle o(1)} + H_{1,lt}^{\sphericalangle o(1)}\cos\theta, \tag{25}$$

$$\tilde{D}^{\sphericalangle} \to \tilde{D}_{ot}^{\sphericalangle} + \tilde{D}_{lt}^{\sphericalangle}\cos\theta, \tag{26}$$

$$\tilde{G}^{\sphericalangle} \to \tilde{G}_{ot}^{\sphericalangle} + \tilde{G}_{lt}^{\sphericalangle}\cos\theta, \tag{27}$$

$$\tilde{H} \to \tilde{H}_{oo} + \tilde{H}_{ol}\cos\theta + \tilde{H}_{ll}\frac{1}{4}(3\cos^2\theta - 1), \tag{28}$$

$$\tilde{E} \to \tilde{E}_{oo} + \tilde{E}_{ol}\cos\theta + \tilde{E}_{ll}\frac{1}{4}(3\cos^2\theta - 1). \tag{29}$$

IV. HADRONIC TENSOR FOR SEMI-INCLUSIVE LEPTOPRODUCTION

When the semi-inclusive production of two hadrons happens via a DIS process, an electron with momentum l scatters off a target nucleon with mass M, polarization S, and momentum P, via the exchange of a virtual photon with momentum transfer $q = l - l'$. Inside the target, it is assumed that the photon hits a quark with momentum p, changing it to a state with momentum $k = p + q$ before the fragmentation [see Fig. 3(a)]. We define the variable $x = p^+/P^+$, which represents the light-cone fraction of the target momentum carried by the initial quark. As already anticipated in Sec. II, it is customary to consider the frame where all the hadronic systems have no transverse components, i.e., where $\vec{P}_T = \vec{P}_{hT} = \vec{0}$, while the virtual photon has a nonvanishing component $\vec{q}_T$. A convenient parametrization for the momenta referred to the initial hadronic system is

$$P^\mu = \left[\frac{M^2}{2P^+}, P^+, \vec{0}\right]$$

$$p^\mu = \left[\frac{p^2 + \vec{p}_T^{\,2}}{2xP^+}, xP^+, \vec{p}_T\right]. \tag{30}$$

However, when calculating the hadronic tensor (and, consequently, the cross section) it is more convenient to consider the frame where the $\hat{z}$ axis is antiparallel to the direction of the virtual photon momentum (see Fig. 4). By denoting the momenta in this frame with the subscript $_\perp$, we have, therefore, $\vec{P}_\perp = \vec{q}_\perp = \vec{0}$ and $\vec{P}_{h\perp} \simeq -z\vec{q}_T$. The difference between the T and the $\perp$ frames is a boost that introduces corrections suppressed as $1/Q$; therefore, it can be neglected at the leading twist, but it must consistently be included when extending the analysis at the subleading twist. The boost amounts to the following modifications,

[1]At variance with Ref. [17], here we use lowercase indices for the polarization of the relative partial wave, in order to avoid confusion with the polarization state of the beam and/or the target in the expression of the cross section (see the following Sec. V).

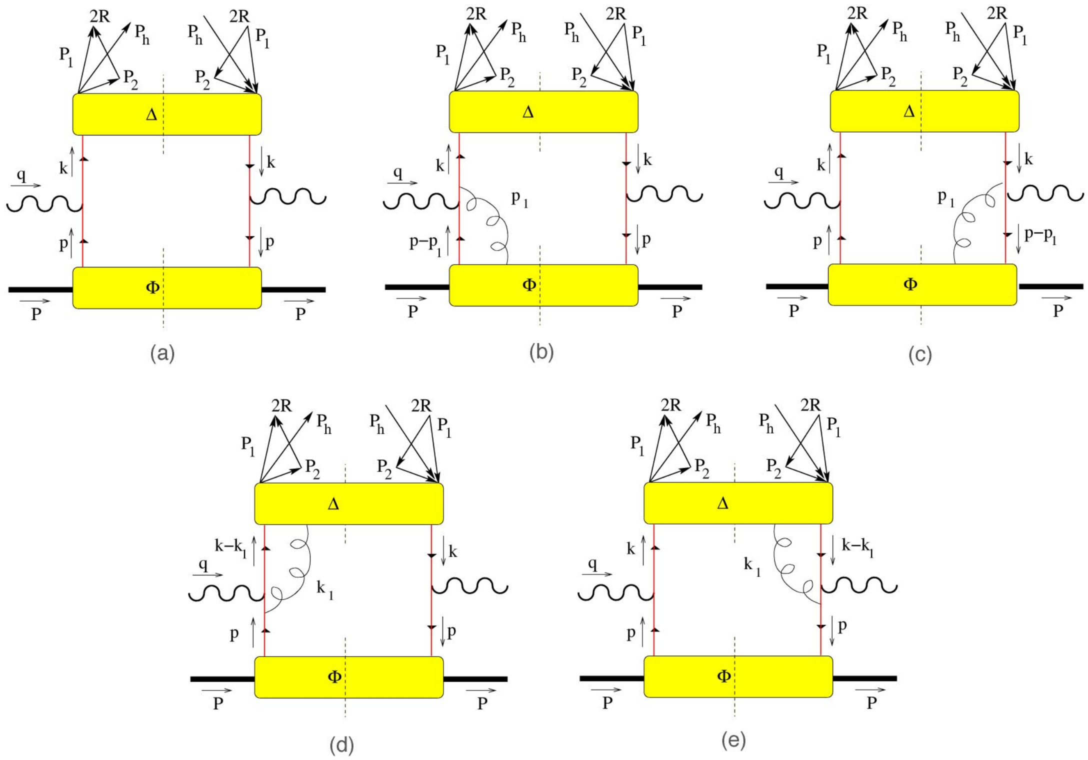

FIG. 3. Relevant diagrams at the leading and subleading twist for the semi-inclusive DIS of a lepton on a hadronic target with detection of two hadrons in the same current fragmentation region. The shaded blobs represent the contribution of all unsuppressed longitudinal gluons, while the gluon lines represent all possible contributions from transverse gluon fields (see text).

$$
\begin{aligned}
n_-^\mu &\sim n'^{\,\mu}_- - \frac{\sqrt{2}}{Q} q_T^\mu = n'^{\,\mu}_- + \frac{\sqrt{2}}{Q}(p_T - k_T)^\mu, \\
n_+^\mu &\sim n'^{\,\mu}_+ , \\
a_T^\mu &\sim g_\perp^{\mu\nu} a_{T\nu} - \frac{\sqrt{2}}{Q}\vec{a}_T\cdot\vec{q}_T n_+^\mu \\
&\equiv a_{T\perp}^\mu + \frac{\sqrt{2}}{Q}\vec{a}_T\cdot(\vec{p}_T - \vec{k}_T) n_+^\mu ,
\end{aligned}
\tag{31}
$$

where a_T^μ is a generic transverse four-vector, $n'_\pm$ are the lightlike vectors considered in the $\perp$ frame, and the analogue of the transverse projection operators of Eq. (1) are

$$
g_\perp^{\mu\nu} = g^{\mu\nu} - n_+^{\{\mu} n'^{\,\nu\}}_- , \qquad \epsilon_\perp^{\mu\nu} \equiv \epsilon^{\rho\sigma\mu\nu} n_{+\rho} n'_{-\sigma} . \tag{32}
$$

As an example of the difference between the T and the $\perp$ frames, in Fig. 5 we sketch the vectors R_T and $R_{T\perp}$. As expressed in Eq. (31), the difference between the two vectors is of order $1/Q$ (exagerated in the drawing). The difference between the angles ϕ_R and $\phi_{R\perp}$ and between $|\vec{R}_T|$ and $|\vec{R}_{T\perp}|$ is of order $1/Q^2$; therefore it can be neglected in our analysis.

The hadronic tensor, integrated upon the transverse cm momentum of the hadron pair, reads

$$
\begin{aligned}
2MW^{\mu\nu} = {} & 32z\,\mathrm{Tr}\left[z^2 \int d\vec{p}_T d\vec{k}_T \Phi^{[+]}(x,\vec{p}_T,S)\gamma^\mu \Delta^{[-]}(z,\vec{k}_T,R)\gamma^\nu\right] - 32z\,\mathrm{Tr}\left[\gamma_\alpha \frac{\gamma^-}{Q\sqrt{2}}\gamma^\nu \Phi_A^{[+]\alpha}(x,S)\gamma^\mu \Delta(z,R)\right] \\
& - 32z\,\mathrm{Tr}\left[\gamma^\mu \frac{\gamma^-}{Q\sqrt{2}}\gamma_\alpha \Delta(z,R)\gamma^\nu \gamma^0 \Phi_A^{[+]\alpha\dagger}(x,S)\gamma^0\right] - 32z\,\mathrm{Tr}\left[\gamma^\nu \frac{\gamma^+}{Q\sqrt{2}}\gamma_\alpha \Phi(x,S)\gamma^\mu \gamma^0 \Delta_A^{[-]\alpha\dagger}(z,R)\gamma^0\right] \\
& - 32z\,\mathrm{Tr}\left[\gamma_\alpha \frac{\gamma^+}{Q\sqrt{2}}\gamma^\mu \Delta_A^{[-]\alpha}(z,R)\gamma^\nu \Phi(x,S)\right].
\end{aligned}
\tag{33}
$$

Each contribution corresponds to a specific class of diagrams in Fig. 3. For sake of simplicity, the blobs in the diagrams represent all the connected lines related to unsuppressed longitudinal gluons, namely the lower blob includes all lines with A^+ gluons and the upper blob all lines with A^- gluons. Therefore, the diagram in Fig. 3(a) corresponds to the first term in Eq. (33) involving the leading-twist color gauge-invariant correlators Φ (which will be described in Sec. IV A) and Δ of Eq. (10). It is important to perform the integration upon the transverse momenta only after including the effect of the boost in Eq. (31), as it will turn out that the correlator Δ contains a $1/Q$-suppressed $\vec{p}_T$ dependence.

At subleading twist, the contribution of transverse gluons $\vec{A}_T$ is symbolically indicated by a line attached to the lower blob, corresponding to the color gauge-invariant quark-gluon-quark correlators $\Phi_A^{[+]\alpha}$ (which will be described in Sec. IV A), or to the upper blob, corresponding to $\Delta_A^{[-]\alpha}$ in Eq. (11). Therefore, the second and the third terms in Eq. (33) correspond to diagrams in Figs. 3(b) and 3(c), while the fourth and fifth ones to diagrams in Figs. 3(d) and 3(e), respectively.

The correlators Δ and $\Delta_A^{[-]\alpha}$ have already been discussed in Secs. III A and III B, respectively. In the following, the missing terms will be described in detail, leading to the final expression of $W^{\mu\nu}$ in terms of distribution and fragmentation functions.

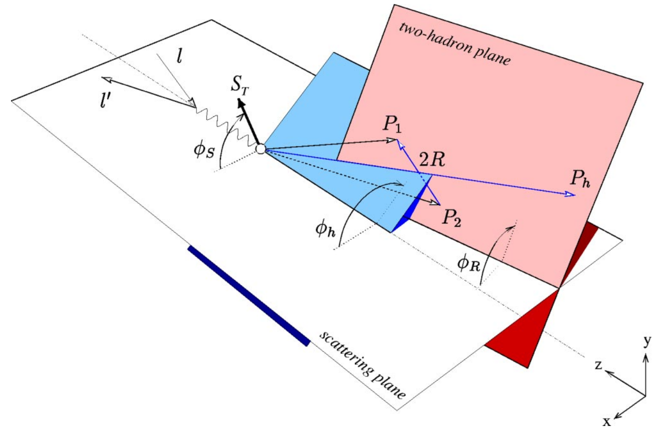

FIG. 4. Kinematics for the SIDIS of the lepton l on a (un)polarized target leading to two hadrons inside the same current jet.

A. The quark-quark correlators for the initial and the final states

The color gauge-invariant quark-quark correlator for the initial state, $\Phi(x,S)$, corresponding to the lower blob in Figs. 3(a), 3(d), and 3(e), reads [5]

$$\begin{aligned}\Phi(x,S) &= \int d\vec{p}_T \Phi^{[+]}(x,\vec{p}_T,S)\\ &= \frac{1}{2}\{f_1(x)\not{n}_+ + S_L g_1(x)\gamma_5\not{n}_+ + h_1(x)\gamma_5\not{S}_\perp\not{n}_+\}\\ &\quad + \frac{\sqrt{2}xM}{2Q}\{e(x) + g_T(x)\gamma_5\not{S}_\perp + S_L h_L(x)\gamma_5\not{n}_+\not{n}_-\}\\ &\quad + \frac{\sqrt{2}xM}{2Q}\{-\mathrm{i}S_L e_L(x)\gamma_5 - f_T(x)\epsilon_T^{\alpha\beta}\gamma_\alpha S_{\perp\beta}\\ &\quad + \mathrm{i}h(x)\not{n}_+\not{n}_-\}\\ &\equiv \Phi_1(x,S) + \Phi_2(x,S), \qquad (34)\end{aligned}$$

where $S_{L/\perp}$ are the longitudinal/transverse components of the target polarization, respectively. The first group (Φ_1) represents the contribution of the leading-twist distribution functions and it appears in the first, fourth, and fifth terms of Eq. (33), corresponding to the diagrams of Fig. 3(a), 3(d), and 3(e), respectively. The other terms (Φ_2) represent the contribution of the subleading-twist distribution functions, including also the $\vec{p}_T$-integrated T-odd functions $h(x)$, $f_T(x)$, and $e_L(x)$, which are vanishing if the gauge link is the only source of the T-odd behavior. The Φ_2 contributes only to the first term of Eq. (33), corresponding to the diagram of Fig. 3(a). Since the nonintegrated $\Phi^{[+]}(x,\vec{p}_T,S)$ involves scalar products of transverse vectors and commutators between transverse vectors and the lightlike vector n_+, it is easy to check that the boost transformations in Eq. (31) do not add other subleading-twist terms and leave $\Phi(x,S)$ unaltered. Moreover, because of the $\vec{p}_T$ integration the latter is insensitive to the direction of the link integration path.

At the subleading twist, the quark-gluon-quark correlator $\Phi_A^{[+]\alpha}$ also comes into play, appearing in the diagrams of Figs. 3(b) and 3(c). Similarly to the previous case, the redefinition of the quark fields including the gauge links and the integration upon $\vec{p}_T$ allows us to keep the same relation $\Phi_A^{[+]\alpha} = \Phi_D^\alpha - \Phi_\partial^{[+]\alpha}$ dictated by the QCD equations of motion as in the color-gauge-noninvariant case. The quark-gluon-quark correlator can be parametrized as [5]

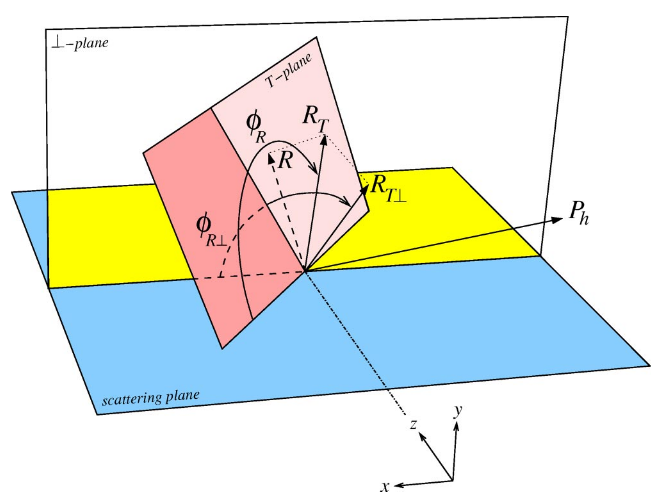

FIG. 5. Description of the angles ϕ_R and $\phi_{R\perp}$.

$$
\begin{aligned}
\Phi_A^{[+]\alpha}(x,S) &= \frac{M}{2}\Bigg\{\left(x g_T(x) - \frac{m}{M} h_1(x) - g_{1T}^{(1)}(x)\right) S_\perp^\alpha \gamma_5 \not n_+ \\
&\quad + S_L\left(x h_L(x) - \frac{m}{M} g_1(x) + 2 h_{1L}^{\perp(1)}(x) + \mathrm{i} x e_L(x)\right) \\
&\quad \times \tfrac{1}{2}\gamma_5 \gamma^\alpha \not n_+ \\
&\quad + \left(\frac{m}{M} f_1(x) - x e(x) + 2\mathrm{i} h_1^{\perp(1)}(x) + \mathrm{i} x h(x)\right)\tfrac{1}{2}\gamma^\alpha \not n_+ \\
&\quad - (f_{1T}^{\perp(1)}(x) + x f_T(x))\,\epsilon_T^{\alpha\beta} S_{\perp\beta} \not n_+ \Bigg\},
\end{aligned} \tag{35}
$$

where, as usual, we define the moments

$$
g_{1T}^{(1)}(x) = \int d\vec{p}_T \frac{\vec{p}_T^{\,2}}{2M^2} g_{1T}(x, \vec{p}_T^{\,2}), \tag{36}
$$

and similarly for the other distribution functions.

As for the fragmentation into two hadrons, the leading-twist correlator Δ_1 of Eq. (18) occurs in the diagrams of Figs. 3(a), 3(b), and 3(c), while the subleading-twist Δ_2 occurs together with Φ_1 only in the diagram of Fig. 3(a). However, the boost transformation (31) induces two additional contributions to the correlator of Eq. (16), which are suppressed as $1/Q$ and, therefore, must be consistently included in the analysis at the subleading twist before performing the integration upon $\vec{k}_T$. The final $\vec{k}_T$-integrated result reads

$$
\begin{aligned}
\tilde{\Delta}_2^{[-]}(z,R) &= \frac{\sqrt{2} M_h}{16\pi Q}\Bigg\{ - G_1^{\perp(1)} \gamma_5 \frac{\epsilon_T^{\mu\rho}\gamma_\mu R_{T\rho}}{M_h} - D_1^{o(1)} \frac{\not R_T}{M_h} \\
&\quad + \mathrm{i} H_1^{\sphericalangle\ o(1)} \frac{\vec{R}_T^{\,2}}{M_h^2} \frac{1}{2}[\not n'_-, \not n_+] + \mathrm{i} H_1^{\perp(1)} [\not n'_-, \not n_+] \Bigg\},
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
p_T^\alpha \tilde{\Delta}_{2\ \alpha}(z,R) &= \frac{\sqrt{2}}{16\pi Q} p_T^\alpha \Bigg\{ D_1 \gamma_\alpha + \mathrm{i} H_1^{\sphericalangle} \frac{1}{2M_h} [\not R_T, \gamma_\alpha] \\
&\quad + \mathrm{i} \frac{R_{T\alpha}}{M_h} H_1^{\sphericalangle} \frac{1}{2} [\not n'_-, \not n_+] \Bigg\}.
\end{aligned} \tag{38}
$$

Such contributions appear in the first term of Eq. (33), corresponding to the diagram of Fig. 3(a). The former couples to Φ_1, while $\tilde{\Delta}_{2\ \alpha}$, due to the presence of p_T^α, couples to

$$
\begin{aligned}
\Phi_\partial^{[+]\alpha}(x,S) &\equiv \int d\vec{p}_T p_T^\alpha \Phi^{[+]}(x, \vec{p}_T, S) \\
&= \frac{M}{2}\{ f_{1T}^{(1)}(x)\, \epsilon_T^{\alpha\beta} S_{T\beta} \not n_+ + g_{1T}^{(1)}(x) S_\perp^\alpha \gamma_5 \not n_+ \\
&\quad - S_L h_{1L}^{(1)}(x) \gamma_5 \gamma^\alpha \not n_+ - \mathrm{i} h_1^{\perp(1)} \gamma^\alpha \not n_+ \}.
\end{aligned} \tag{39}
$$

To complete the picture about the fragmentation at subleading twist, the quark-gluon-quark correlator $\Delta_A^{[-]\alpha}$ of Eq. (21) must be included in the fourth and fifth terms of Eq. (33), corresponding to diagrams 3(d) and 3(e).

B. The hadronic tensor

Putting together in a consistent way all the contributions discussed above up to the subleading twist, we get for the hadronic tensor the following expression:

$$
\begin{aligned}
2MW^{\mu\nu} &= \frac{16z}{4\pi}\Bigg\{ - g_\perp^{\mu\nu} f_1 D_1 + \mathrm{i}\epsilon_\perp^{\mu\nu} S_L g_1 D_1 - \frac{R_{T\perp}^{\{\mu}\epsilon_\perp^{\nu\}\rho} S_{\perp\rho} + S_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho} R_{T\perp\rho}}{2M_h} h_1 H_1^{\sphericalangle} + S_L \frac{2\hat{t}^{\ \{\mu}\epsilon_\perp^{\nu\}\rho} R_{T\perp\rho}}{Q}\left(\frac{M}{M_h} x h_L H_1^{\sphericalangle} + g_1 \frac{\tilde{G}^{\sphericalangle}}{z}\right) \\
&\quad + \frac{2M_h \hat{t}^{\ \{\mu}\epsilon_\perp^{\nu\}\rho} S_{\perp\rho}}{Q}\left[h_1\left(\frac{\tilde{H}}{z} + \frac{\vec{R}_T^{\,2}}{M_h^2} H_1^{\sphericalangle\ o(1)}\right) - \frac{M}{M_h} x f_T D_1\right] + \frac{2\hat{t}^{\ \{\mu} R_{T\perp}^{\nu\}}}{Q}\left(f_1 \frac{\tilde{D}^{\sphericalangle}}{z} + \frac{M}{M_h} x h H_1^{\sphericalangle}\right) \\
&\quad + \mathrm{i} S_L \frac{2\hat{t}^{\ [\mu}\epsilon_\perp^{\nu]\rho} R_{T\perp\rho}}{Q}\left(g_1 \frac{\tilde{D}^{\sphericalangle}}{z} - \frac{M}{M_h} x e_L H_1^{\sphericalangle}\right) - \mathrm{i}\frac{2\hat{t}^{\ [\mu} R_{T\perp}^{\nu]}}{Q}\left(\frac{M}{M_h} x e H_1^{\sphericalangle} + f_1 \frac{\tilde{G}^{\sphericalangle}}{z}\right) \\
&\quad + \mathrm{i}\frac{2M\hat{t}^{\ [\mu}\epsilon_\perp^{\nu]\rho} S_{\perp\rho}}{Q}\left(x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z}\right)\Bigg\}.
\end{aligned} \tag{40}
$$

The leading-twist contribution in the above formula involves color-gauge-invariant quantities that are independent of the properties of the gauge link; under the hypothesis of factorization, it represents a universal response. At the subleading twist, the issue is still under debate [5,6,36]. However, it is interesting to note that only $\vec{k}_T$-integrated fragmentation functions appear via $\Delta_\partial^{[-]\alpha}$ in Eq. (21), leading to the "tilde" functions of Eq. (22), that might depend on the considered process. No $\vec{p}_T$-integrated distribution functions appear via the corresponding $\Phi_{\partial}^{[+]\alpha}$, because these contributions in Eq. (35) are exactly cancelled by the ones generated by coupling $\tilde{\Delta}_{2\ \alpha}$ of Eq. (38) to $\Phi_{\partial}^{[+]\alpha}$ of Eq. (39) in the first, second, and third contributions of Eq. (33) [see also Eq. (64) of Ref. [5]]. The net result is that the functions $h(x)$, $f_T(x)$, and $e_L(x)$, are the only T-odd distributions in the hadronic tensor and, if not vanishing, they must be generated by a dynamical mechanism that has nothing to do with the sensitivity to the link path.

V. CROSS SECTION AND SPIN ASYMMETRIES

The cross section for SIDIS of polarized leptons off polarized hadronic targets with two unpolarized hadrons in the same current fragmentation region, reads

$$\frac{d^7\sigma}{d\zeta dM_h^2 d\phi_R dz dx dy d\phi_S}=\sum_a \frac{\alpha^2 y e_a^2}{32zQ^4} L_{\mu\nu} 2MW_a^{\mu\nu}, \tag{41}$$

where α is the fine structure constant, $y=(E-E')/E$ is the fraction of beam energy transferred to the hadronic system and is related to the lepton scattering angle in the target rest frame, ϕ_S is the azimuthal angle of the target polarization with respect to the scattering plane, ϕ_R is the azimuthal angle of the $\vec{R}_T$ vector with respect to the scattering plane, measured either around the P_h direction or around the $\hat{z}$ direction (see Fig. 5). The indicated sum runs over the quark and antiquark flavors a. The hadronic tensor $W_a^{\mu\nu}$ of Eq. (40) is contracted with the lepton tensor

$$L^{\mu\nu}=\frac{Q^2}{y^2}\Big[-2A(y)g_\perp^{\mu\nu}+4B(y)\hat{t}^\mu\hat{t}^\nu+4B(y)\big(\hat{x}^\mu\hat{x}^\nu+\tfrac{1}{2}g_\perp^{\mu\nu}\big)+V(y)\hat{t}^{\{\mu}\hat{x}^{\nu\}}+2\mathrm{i}\lambda C(y)\epsilon_\perp^{\mu\nu}-\mathrm{i}\lambda W(y)\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}\hat{x}_\rho\Big], \tag{42}$$

where λ is the lepton helicity, $\hat{x}$ the spatial unit vector, $\hat{t}^\mu=(n_+^\mu+n'^{\ \mu}_-)/\sqrt{2}$, and

$$\begin{aligned}
A(y)&=\left(1-y+\frac{y^2}{2}\right),\\
B(y)&=(1-y),\\
C(y)&=y\left(\frac{y}{2}-1\right),\\
V(y)&=2(2-y)\sqrt{1-y},\\
W(y)&=2y\sqrt{1-y}.
\end{aligned} \tag{43}$$

For convenience, in the following we will indicate the unpolarized or longitudinally polarized states of the beam with the labels O and L, respectively. Similarly, we will use the labels O,L, and T to indicate an unpolarized, longitudinally polarized, and transversely polarized target, respectively. We can then deduce the following list of cross sections:[2]

$$d^7\sigma_{OO}=\frac{\alpha^2}{2\pi Q^2 y}\sum_a e_a^2\left\{A(y)f_1(x)D_1(z,\zeta,M_h^2)-V(y)\cos\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{1}{z}f_1(x)\tilde{D}^{\sphericalangle}(z,\zeta,M_h^2)+\frac{M}{M_h}xh(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)\right]\right\}, \tag{44}$$

$$d^7\sigma_{OL}=\frac{\alpha^2}{2\pi Q^2 y}S_L\sum_a e_a^2V(y)\sin\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{M}{M_h}xh_L(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)+\frac{1}{z}g_1(x)\tilde{G}^{\sphericalangle}(z,\zeta,M_h^2)\right], \tag{45}$$

$$\begin{aligned}
d^7\sigma_{OT}=\frac{\alpha^2}{2\pi Q^2 y}|\vec{S}_\perp|\sum_a e_a^2\Bigg\{&B(y)\sin(\phi_R+\phi_S)\frac{|\vec{R}_T|}{M_h}h_1(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)\\
&+V(y)\sin\phi_S\frac{M_h}{Q}\left[h_1(x)\left(\frac{1}{z}\tilde{H}(z,\zeta,M_h^2)+\frac{|\vec{R}_T|^2}{M_h^2}H_1^{\sphericalangle\ o(1)}(z,\zeta,M_h^2)\right)-\frac{M}{M_h}xf_T(x)D_1(z,\zeta,M_h^2)\right]\Bigg\},
\end{aligned} \tag{46}$$

$$d^7\sigma_{LO}=\frac{\alpha^2}{2\pi Q^2 y}\lambda\sum_a e_a^2W(y)\sin\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{M}{M_h}xe(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)+\frac{1}{z}f_1(x)\tilde{G}^{\sphericalangle}(z,\zeta,M_h^2)\right], \tag{47}$$

[2]The distribution and fragmentation functions are understood to have a flavor index a.

$$d^7\sigma_{LL}=\frac{\alpha^2}{2\pi Q^2 y}\lambda S_L\sum_a e_a^2\left\{C(y)g_1(x)D_1(z,\zeta,M_h^2)-W(y)\cos\phi_R\frac{|\vec{R}_T|}{Q}\left[\frac{1}{z}g_1(x)\tilde{D}^{\sphericalangle}(z,\zeta,M_h^2)\right.\right.$$
$$\left.\left.-\frac{M}{M_h}xe_L(x)H_1^{\sphericalangle}(z,\zeta,M_h^2)\right]\right\}, \tag{48}$$

$$d^7\sigma_{LT}=\frac{\alpha^2}{2\pi Q^2 y}\lambda|\vec{S}_\perp|\sum_a e_a^2 W(y)\cos\phi_S\frac{M_h}{Q}\left[-\frac{M}{M_h}xg_T(x)D_1(z,\zeta,M_h^2)-\frac{1}{z}h_1(x)\tilde{E}(z,\zeta,M_h^2)\right]. \tag{49}$$

In the above formula, we stress again that we have included also the contributions of the $\vec{k}_T$-integrated T-odd distribution functions $h(x)$, $f_T(x)$, and $e_L(x)$, which are vanishing if the gauge link is the only source of a T-odd behavior. It would be interesting to experimentally check this feature.

Several useful spin asymmetries can also be built out of the previous formulas. In Eq. (46) for $d^7\sigma_{OT}$, the transversity h_1 can be isolated at leading twist through the fragmentation function $H_1^{\sphericalangle}$ in a $\sin(\phi_R+\phi_S)$ spin asymmetry. This asymmetry has been already discussed in leading-order analyses [17,27] and seems very promising with respect to the Collins asymmetry, since it does not need to keep memory of the $\vec{k}_T$ dependence but rather of the direction of $\vec{R}_T$.

While data from purely transversely polarized targets are not yet available, the HERMES collaboration has performed spin asymmetry measurements with targets longitudinally polarized along the lepton beam [39–41], hence with a polarization three-vector $\vec{S}=(S_x,0,S_z)$ in the lepton scattering plane ($\phi_S=0$) and with a transverse component S_x with respect to the direction of the momentum transfer along $\hat{z}$. Because of the kinematics setup, S_x is suppressed by $1/Q$ with respect to S_z [42]. In the present case of detection of two hadrons in the same jet, therefore, both the leading twist $d^7\sigma_{OT}$ and subleading twist $d^7\sigma_{OL}$ of Eqs. (46) and (45), respectively, should be consistently considered at the same time when looking for a $\sin\phi_R$ asymmetry. However, $d^7\sigma_{OT}$ is considerably simpler than the corresponding cross section for one-hadron SIDIS, because the information about the transversity is not contaminated by other contributions, as it happens with the Collins and Sivers effects. Moreover, in the Wandzura-Wilzcek approximation the fragmentation function $\tilde{G}^{\sphericalangle}$ vanishes inside $d^7\sigma_{OL}$; therefore, a $\sin\phi_R$ spin asymmetry for two-hadron SIDIS in the HERMES kinematics would approximately lead to the product of the fragmentation function $H_1^{\sphericalangle}$ times the transversity h_1 and the distribution h_L, which is anyway related to h_1 itself via a Wandzura-Wilzcek integral relation.

Again, if we neglect $\tilde{G}^{\sphericalangle}$, a $\sin\phi_R$ spin asymmetry with polarized beam and unpolarized target would give access to the chiral-odd distribution $e(x)$, always through the chiral-odd fragmentation function $H_1^{\sphericalangle}$, as is evident from inspection of $d^7\sigma_{LO}$ in Eq. (47). The function $e(x)$ has recently attracted a lot of interest [43], because it is directly related to the soft physics of chiral symmetry breaking [44]. Its first isoscalar Mellin moment gives the scalar form factor. Although this form factor (describing the elastic scattering off a spin-$\frac{1}{2}$ target via the exchange of a spin-0 particle) has not yet been measured, its value at $t=-Q^2=0$, the so-called σ term, can be deduced by low-energy theorems from the experimental pion-nucleon scattering in the timelike region at the so-called Chen-Dashen point $t=-Q^2=2m_\pi^2$, with m_π the pion mass [45–47]. Unexpectedly, the σ term turns out very big (50–70 MeV) [48,49] with respect to the average value of available lattice calculations [50], suggesting that approximately 20% of the nucleon mass M could be due to the strange quark content of the nucleon. Therefore, having experimental access to $e(x)$ is of great importance. This distribution could be extracted at subleading twist through the Collins function by a beam spin asymmetry in one-hadron SIDIS for longitudinally polarized beams and unpolarized targets [31,51], provided that the transverse momentum of the detected hadron is measured. This asymmetry contains another contribution that was neglected until recently [52,53]. Once again, the case of one-hadron SIDIS is complicated by the dependence upon the partonic transverse momentum. For the case of two-hadron SIDIS, it is possible to integrate upon the transverse total momentum of the pair and still build an azimuthal asymmetry using $\vec{R}_T$. In fact, Eq. (47) looks simpler than the corresponding one for the one-hadron case, and it could eventually represent the cleanest channel to look at in order to extract $e(x)$.

Finally, when expanding the fragmentation functions in partial waves and making the cross section differential in $\cos\theta$, the different dependence upon θ allows us to distinguish the contributions pertaining to pure s waves, pure p waves, and s-p interferences. For instance, by substituting Eqs. (24) and (23) into Eq. (46) it is possible to check that the asymmetry will be dominated by an s-p interference fragmentation function at $\theta=\pi/2$, and by a p-wave interference fragmentation function at $\theta=\pi/4$.

VI. CONCLUSIONS

Fragmentation functions are universal, process-independent objects [54] containing crucial information about the hadronization mechanism and, ultimately, about the confinement of partons inside hadrons. They appear in semi-inclusive processes such as, e.g., DIS or electron-positron annihilation, and they can act also as a sort of ''analyzing power'' for the polarization state of the fragmenting quark [14,15,18,32]. The typical example is the so-called Collins effect [3,24] relating the transverse polarization of the parent quark to the transverse-momentum-dependent Collins function, which describes a (nonperturbative) azimuthal asymmetry in the distribution of the detected leading hadron. Two-hadron fragmentation functions can also be defined, among which the so-called interference fragmentation functions [11–13] lead to interesting single-spin asymmetries even after integrating upon the transverse total momentum of the pair [27,28], thus avoiding the complications introduced by the intrinsic nonperturbative transverse-momentum dependence of the Collins function.

In this paper, we have extended the analysis of two-hadron fragmentation functions to the subleading-twist level, discussing also the issue of color gauge invariance but eventually integrating upon the transverse total momentum of the pair. Our results are theoretically interesting because the absence of an intrinsic nonperturbative dependence upon transverse momenta cancels, at leading twist, also any dependence upon the properties of the gauge link operator necessary to restore gauge invariance, allowing for a truly universal definition of these objects; a debate is still ongoing to check if this property holds true also at subleading twist [5,6,36]. The extension to the subleading twist is also experimentally important, because it can represent a non-negligible contribution when performing measurements at moderate Q^2.

We have analyzed both the quark-quark and the suppressed quark-gluon-quark correlator, relating the latter to the former by means of the QCD equations of motion. We have presented the full decomposition up to the subleading-twist level of these correlators in terms of fragmentation functions integrated upon the intrinsic transverse momentum. As previously stressed, these functions are universal certainly at twist 2 and maybe also at twist 3.

As an application of our results, we have calculated the hadronic tensor and the cross section for all possible combinations of polarization states of the beam-target system in the case of deep-inelastic semi-inclusive leptoproduction of two unpolarized hadrons, by integrating upon the two-hadron center-of-mass transverse momentum. Our results can be used to distinguish $1/Q$-suppressed contributions in experimental measurements, in order to extract more clearly leading-twist contributions, or in order to study interesting subleading-twist terms. An example of the former case is the possibility of extracting the transversity distribution in spin asymmetries also with longitudinally polarized targets (as they have been measured at HERMES [41] for the case of one-hadron production); an example of the latter is the possibility of extracting from beam-spin asymmetries (probably in the cleanest possible way [52,53]) the twist-3 chiral-odd distribution function $e(x)$ [51], related to the mechanism of the spontaneous breaking of the QCD chiral symmetry and, ultimately, to the strange-quark content of the nucleon [44].

As a last step, we have performed a partial-wave expansion of leading- and subleading-twist two-hadron fragmentation functions, in order to distinguish the interference coming from the s-s, p-p, and s-p channels in the relative partial wave of the hadron pairs. Each component carries information on different mechanisms, such as the polarization transfer to spin-1 resonances (for p-p interference) or T-odd effects from different kinds of final-state interactions. Therefore, extracting this information from data would allow for the exploration of different aspects of the physics of the fragmentation process.

ACKNOWLEDGMENTS

Several discussions with D. Boer, P. J. Mulders, and F. Pijlman are gratefully acknowledged. This work has been partially supported by the TMR network HPRN-CT-2000-00130 and by the BMBF. M.R. thanks the Institute for Nuclear Theory at the University of Washington (Seattle, USA) for its hospitality and the Department of Energy for partial support during the completion of this work.

[1] E. Christova, S. Kretzer, and E. Leader, Eur. Phys. J. C **22**, 269 (2001).
[2] B.A. Kniehl, G. Kramer, and B. Poetter, Nucl. Phys. **B582**, 514 (2000).
[3] J.C. Collins, Nucl. Phys. **B396**, 161 (1993).
[4] V. Barone, A. Drago, and P.G. Ratcliffe, Phys. Rep. **359**, 1 (2002).
[5] D. Boer, P.J. Mulders, and F. Pijlman, Nucl. Phys. **B667**, 201 (2003).
[6] A. Metz, Phys. Lett. B **549**, 139 (2002).
[7] A.A. Henneman, D. Boer, and P.J. Mulders, Nucl. Phys. **B620**, 331 (2002).
[8] R. Kundu and A. Metz, Phys. Rev. D **65**, 014009 (2002).
[9] D. Boer, Nucl. Phys. **B603**, 195 (2001).
[10] K. Goeke, A. Metz, P.V. Pobylitsa, and M.V. Polyakov, Phys. Lett. B **567**, 27 (2003).
[11] J.C. Collins and G.A. Ladinsky, hep-ph/9411444.
[12] R.L. Jaffe, X. Jin, and J. Tang, Phys. Rev. Lett. **80**, 1166 (1998).
[13] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, Phys. Rev. D **62**, 034008 (2000).
[14] X. Ji, Phys. Rev. D **49**, 114 (1994).
[15] A.V. Efremov and O.V. Teryaev, Sov. J. Nucl. Phys. **36**, 140 (1982).
[16] A. Bacchetta and P.J. Mulders, Phys. Rev. D **62**, 114004 (2000).
[17] A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003).
[18] X. Artru and M. Mekhfi, Z. Phys. C **45**, 669 (1990).
[19] R.L. Jaffe, Phys. Rev. D **54**, 6581 (1996).
[20] D. de Florian, M. Stratmann, and W. Vogelsang, Phys. Rev. D **57**, 5811 (1998).
[21] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, and F.

Murgia, Phys. Lett. B **509**, 246 (2001).
[22] M. Anselmino, D. Boer, U. D'Alesio, and F. Murgia, Phys. Rev. D **65**, 114014 (2004).
[23] K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. **78B**, 243 (1978).
[24] J.C. Collins, S.F. Heppelmann, and G.A. Ladinsky, Nucl. Phys. **B420**, 565 (1994).
[25] X. Artru and J.C. Collins, Z. Phys. C **69**, 277 (1996).
[26] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, Phys. Rev. D **62**, 034009 (2000).
[27] M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D **65**, 074031 (2002).
[28] D. Boer, R. Jakob, and M. Radici, Phys. Rev. D **67**, 094003 (2003).
[29] D. de Florian and L. Vanni, Phys. Lett. B **578**, 139 (2004).
[30] I. Vendramin, Nuovo Cimento A **66**, 339 (1981).
[31] P.J. Mulders and R.D. Tangerman, Nucl. Phys. **B461**, 197 (1996); **B484**, 538(E) (1996).
[32] R.L. Jaffe and X. Ji, Phys. Rev. Lett. **71**, 2547 (1993).
[33] X. Ji and F. Yuan, Phys. Lett. B **543**, 66 (2002).
[34] A.V. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. **B656**, 165 (2003).
[35] A. Bacchetta, M. Boglione, A. Henneman, and P.J. Mulders, Phys. Rev. Lett. **85**, 712 (2000).
[36] D. Boer, hep-ph/0312149.
[37] R.K. Ellis, W. Furmanski, and R. Petronzio, Nucl. Phys. **B212**, 29 (1983).
[38] R.L. Jaffe and X. Ji, Nucl. Phys. **B375**, 527 (1992).
[39] A. Airapetian *et al.* (HERMES), Phys. Rev. Lett. **84**, 4047 (2000).
[40] A. Airapetian *et al.* (HERMES), Phys. Rev. D **64**, 097101 (2001).
[41] A. Airapetian *et al.* (HERMES), Phys. Lett. B **562**, 182 (2003).
[42] K.A. Oganessyan, H.R. Avakian, N. Bianchi, and A.M. Kotzinian, hep-ph/9808368.
[43] A.V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. D **67**, 114014 (2003).
[44] R.L. Jaffe and X. Ji, Phys. Rev. Lett. **67**, 552 (1991).
[45] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
[46] T.P. Cheng and R.F. Dashen, Phys. Rev. Lett. **26**, 594 (1971).
[47] L.S. Brown, W.J. Pardee, and R.D. Peccei, Phys. Rev. D **4**, 2801 (1971).
[48] J. Gasser, H. Leutwyler, and M.E. Sainio, Phys. Lett. B **253**, 252 (1991).
[49] M.G. Olsson and W.B. Kaufmann, PiN Newslett. **16**, 382 (2002).
[50] D.B. Leinweber, A.W. Thomas, and R.D. Young, hep-lat/0302020.
[51] H. Avakian *et al.* (CLAS), hep-ex/0301005.
[52] F. Yuan, hep-ph/0310279.
[53] L.P. Gamberg, D.S. Hwang, and K.A. Oganessyan, Phys. Lett. B **584**, 276 (2004).
[54] J.C. Collins and D.E. Soper, Nucl. Phys. **B194**, 445 (1982).