Effects of an extra $U(1)$ axial condensate on the radiative decay $\eta' \to \gamma\gamma$ at finite temperature

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Supported by recent lattice results, we consider a scenario in which a $U(1)$-breaking condensate survives across the chiral transition in QCD. This scenario has important consequences for the pseudoscalar-meson sector, which can be studied using an effective Lagrangian model. In particular, generalizing the results obtained in a previous paper (where the zero-temperature case was considered), we study the effects of this $U(1)$ chiral condensate on the radiative decay $\eta' \to \gamma\gamma$ at finite temperature.

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I. INTRODUCTION

There is evidence from some lattice results [1–3] that a new $U(1)$-breaking condensate survives across the chiral transition at T_{ch}, staying different from zero up to a temperature $T_{U(1)} > T_{ch}$. $T_{U(1)}$ is, therefore, the temperature at which the $U(1)$ axial symmetry is (effectively) restored, meaning that, for $T > T_{U(1)}$, there are no $U(1)$-breaking condensates. This scenario has important consequences for the pseudoscalar-meson sector, which can be studied using an effective Lagrangian model [4–7], including also the new $U(1)$ chiral condensate. This one has the form $C_{U(1)} = \langle \mathcal{O}_{U(1)} \rangle$, where, for a theory with L light quark flavors, $\mathcal{O}_{U(1)}$ is a $2L$-fermion local operator that has the chiral transformation properties of [8]:[1]

$$\mathcal{O}_{U(1)} \sim \det_{st}(\overline{q}_{sR} q_{tL}) + \det_{st}(\overline{q}_{sL} q_{tR}), \tag{1.1}$$

where $s,t = 1, \ldots, L$ are flavor indices; the color indices [not explicitly indicated in Eq. (1.1)] are arranged in such a way that (i) $\mathcal{O}_{U(1)}$ is a color singlet and (ii) $C_{U(1)} = \langle \mathcal{O}_{U(1)} \rangle$ is a *genuine* $2L$-fermion condensate; i.e., it has no *disconnected* part proportional to some power of the quark-antiquark chiral condensate $\langle \overline{q}q \rangle$ (see Refs. [6,7,9]).

The low-energy dynamics of the pseudoscalar mesons, including the effects due to the anomaly, the $q\overline{q}$ chiral condensate, and the new $U(1)$ chiral condensate, can be described, in the limit of large number N_c of colors, and expanding to the first order in the light quark masses, by an effective Lagrangian written in terms of the topological charge density Q, the mesonic field $U_{ij} \sim \overline{q}_{jR} q_{iL}$ (up to a multiplicative constant), and the new field variable $X \sim \det(\overline{q}_{sR} q_{tL})$ (up to a multiplicative constant), associated with the new $U(1)$ condensate [4–7]:

$$\begin{aligned}\mathcal{L}(U,U^\dagger,X,X^\dagger,Q) &= \frac{1}{2}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{2}\partial_\mu X \partial^\mu X^\dagger - V(U,U^\dagger,X,X^\dagger) \\ &\quad + \frac{i}{2}\omega_1 Q\, \mathrm{Tr}(\ln U - \ln U^\dagger) \\ &\quad + \frac{i}{2}(1-\omega_1) Q (\ln X - \ln X^\dagger) + \frac{1}{2A} Q^2,\end{aligned} \tag{1.2}$$

where the potential term $V(U,U^\dagger,X,X^\dagger)$ has the form

$$\begin{aligned}V(U,U^\dagger,X,X^\dagger) &= \frac{\lambda_\pi^2}{4}\mathrm{Tr}[(U^\dagger U - \rho_\pi \mathbf{I})^2] + \frac{\lambda_X^2}{4}(X^\dagger X - \rho_X)^2 \\ &\quad - \frac{B_m}{2\sqrt{2}}\mathrm{Tr}(MU + M^\dagger U^\dagger) \\ &\quad - \frac{c_1}{2\sqrt{2}}[\det(U) X^\dagger + \det(U^\dagger) X].\end{aligned} \tag{1.3}$$

$M = \mathrm{diag}(m_1, \ldots, m_L)$ is the quark mass matrix and A is the topological susceptibility in the pure Yang-Mills (YM) theory. (This Lagrangian generalizes the one originally proposed in Refs. [10], which included only the effects due to the anomaly and the $q\overline{q}$ chiral condensate.) All the parameters appearing in the Lagrangian must be considered as functions of the physical temperature T. In particular, the parameters ρ_π and ρ_X determine the expectation values $\langle U \rangle$ and $\langle X \rangle$ and so they are responsible respectively for the behavior of the theory across the $SU(L) \otimes SU(L)$ and the $U(1)$ chiral phase transitions, as follows:

$$\rho_\pi|_{T<T_{ch}} \equiv \frac{1}{2}F_\pi^2 > 0, \quad \rho_\pi|_{T>T_{ch}} < 0,$$

$$\rho_X|_{T<T_{U(1)}} \equiv \frac{1}{2}F_X^2 > 0, \quad \rho_X|_{T>T_{U(1)}} < 0. \tag{1.4}$$

The parameter F_π is the well-known pion decay constant, while the parameter F_X is related to the new $U(1)$ axial condensate. Indeed, from Eq. (1.4), $\rho_X = \frac{1}{2}F_X^2 > 0$ for

[1]Throughout this paper we use the following notation for the left-handed and right-handed quark fields: $q_{L,R} \equiv (1 \pm \gamma_5) q/2$, with $\gamma_5 \equiv -i\gamma^0\gamma^1\gamma^2\gamma^3$.

$T<T_{U(1)}$, and therefore, from Eq. (1.3), $\langle X\rangle = F_X/\sqrt{2}\neq 0$. Remembering that $X\sim\det(\overline{q}_{sR}q_{tL})$, up to a multiplicative constant, we find that F_X is proportional to the new $2L$-fermion condensate $C_{U(1)}=\langle\mathcal{O}_{U(1)}\rangle$ introduced above.

In the same way, the pion decay constant F_π, which controls the breaking of the $SU(L)\otimes SU(L)$ symmetry, is related to the $q\overline{q}$ chiral condensate by a simple and well-known proportionality relation (see Refs. [4,7] and references therein): $\langle\overline{q}_i q_i\rangle_{T<T_{ch}}\simeq -\frac{1}{2}B_m F_\pi$. (Moreover, in the simple case of L light quarks with the same mass m, $m_{NS}^2 = mB_m/F_\pi$ is the squared mass of the nonsinglet pseudoscalar mesons and one gets the well-known Gell-Mann–Oakes–Renner relation: $m_{NS}^2F_\pi^2\simeq -2m\langle\overline{q}_i q_i\rangle_{T<T_{ch}}$.)

It is not possible to find, in a simple way, the analogous relation between F_X and the new condensate $C_{U(1)} = \langle\mathcal{O}_{U(1)}\rangle$.

However, as we have shown in a previous paper [11], information on the quantity F_X [i.e., on the new $U(1)$ chiral condensate, to which it is related] can be derived, in the realistic case of $L=3$ light quarks with nonzero masses m_u, m_d, and m_s, from the study of the radiative decays of the pseudoscalar mesons η and η' in two photons. In Ref. [11] only the zero-temperature case ($T=0$) has been considered and a first comparison of our results with the experimental data has been performed: the results are encouraging, pointing toward some evidence of a nonzero $U(1)$ axial condensate.

In this paper, generalizing the results obtained in Ref. [11], we study the effects of the $U(1)$ chiral condensate on the radiative decay $\eta'\to\gamma\gamma$ at finite temperature ($T\neq 0$), so opening the possibility of a comparison with future heavy-ion experiments. In Sec. II we first rediscuss the radiative decays of the pseudoscalar mesons at $T=0$, considering a *more general* electromagnetic anomaly interaction term, obtained by adding a *new* electromagnetic interaction term to the original electromagnetic anomaly term adopted in Ref. [11] [see Eqs. (2.8)–(2.10) below]. As we shall see, the inclusion of this new electromagnetic interaction term does not modify for $T=0$ (or, more generally, for $T<T_{ch}$) the decay amplitudes for the processes $\pi^0\to\gamma\gamma$, $\eta\to\gamma\gamma$, and $\eta'\to\gamma\gamma$: therefore, all the results (both analytical and numerical) obtained in Ref. [11] concerning these processes remain unaffected. However, the new electromagnetic interaction term will prove to be crucial in the discussion of the $\eta'\to\gamma\gamma$ radiative decay at finite temperature (in particular for $T>T_{ch}$), which will be studied in detail in Sec. III.

II. RADIATIVE DECAYS OF THE PSEUDOSCALAR MESONS AT $T=0$

In order to study the radiative decays of the pseudoscalar mesons to two photons, we have to introduce the electromagnetic interaction in our effective model (1.2). Under *local* $U(1)$ electromagnetic transformations

$$q\to q' = e^{i\theta e\mathbf{Q}}q, \quad A_\mu\to A'_\mu = A_\mu - \partial_\mu\theta, \tag{2.1}$$

the fields U and X transform as follows:

$$U\to U' = e^{i\theta e\mathbf{Q}}Ue^{-i\theta e\mathbf{Q}}, \quad X\to X' = X. \tag{2.2}$$

Therefore, we have to replace the derivative of the fields $\partial_\mu U$ and $\partial_\mu X$ with the corresponding *covariant* derivatives:

$$D_\mu U = \partial_\mu U + ieA_\mu[\mathbf{Q},U], \quad D_\mu X = \partial_\mu X. \tag{2.3}$$

Here $\mathbf{Q}$ is the quark charge matrix (in units of e, the absolute value of the electron charge):

$$\mathbf{Q} = \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3}\end{pmatrix}. \tag{2.4}$$

In addition, we have to reproduce the effects of the electromagnetic anomaly, whose contribution to the four-divergence of the $U(1)$ axial current $J_{5,\mu} = \overline{q}\gamma_\mu\gamma_5 q$ and of the $SU(3)$ axial currents $A_\mu^a = \overline{q}\gamma_\mu\gamma_5(\tau_a/\sqrt{2})q$ [the matrices τ_a, with $a=1,\ldots,8$, are the generators of the algebra of $SU(3)$ in the fundamental representation, with normalization $\mathrm{Tr}(\tau_a\tau_b)=\delta_{ab}$], is given by

$$(\partial^\mu J_{5,\mu})^{e.m.}_{anomaly} = 2\,\mathrm{Tr}(\mathbf{Q}^2)G,$$
$$(\partial^\mu A^a_\mu)^{e.m.}_{anomaly} = 2\,\mathrm{Tr}\left(\mathbf{Q}^2\frac{\tau_a}{\sqrt{2}}\right)G, \tag{2.5}$$

where $G\equiv(e^2N_c/32\pi^2)\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ [$F_{\mu\nu}$ being the electromagnetic field strength tensor], thus breaking the corresponding chiral symmetries. We observe that $\mathrm{Tr}(\mathbf{Q}^2\tau_a)\neq 0$ only for $a=3$ or $a=8$.

We must look for an interaction term $\mathcal{L}_I$ (constructed with the chiral Lagrangian fields and the electromagnetic operator G) which, under a $U(1)$ axial transformation $q\to q' = e^{-i\alpha\gamma_5}q$, transforms as

$$U(1)_A: \quad \mathcal{L}_I\to\mathcal{L}_I + 2\alpha\,\mathrm{Tr}(\mathbf{Q}^2)G, \tag{2.6}$$

while, under $SU(3)$ axial transformations of the type $q\to q' = e^{-i\beta\gamma_5\tau_a/\sqrt{2}}q$ (with $a=3,8$), it transforms as

$$SU(3)_A: \quad \mathcal{L}_I\to\mathcal{L}_I + 2\beta\,\mathrm{Tr}\left(\mathbf{Q}^2\frac{\tau_a}{\sqrt{2}}\right)G. \tag{2.7}$$

By virtue of the transformation properties of the fields U and X under a $U(3)\otimes U(3)$ chiral transformation [$q_L\to V_L q_L$, $q_R\to V_R q_R \Rightarrow U\to V_L U V_R^\dagger$ and $X\to\det(V_L)\det(V_R)^* X$, where V_L and V_R are arbitrary 3×3 unitary matrices [4,7]], one can see that the simplest term describing the electromagnetic anomaly interaction term is the following:

$$\mathcal{L}_I = \frac{i}{2}G\,\mathrm{Tr}[\mathbf{Q}^2(\ln U - \ln U^\dagger)], \tag{2.8}$$

which is exactly the one originally proposed in Ref. [12] and also adopted in Ref. [11]. However, the presence of the new meson field X allows us to construct also another electromagnetic interaction term, still proportional to the pseudoscalar operator G, but totally *invariant* under $U(3)\otimes U(3)$ chiral transformations:

$$\Delta\mathcal{L}_I = f_\Delta \frac{i}{6} G\,\mathrm{Tr}(\mathbf{Q}^2)[\ln(X\det U^\dagger)-\ln(X^\dagger \det U)], \tag{2.9}$$

where f_Δ is an (up to now) arbitrary real parameter (the coefficient 1/6 has been introduced for convenience; see Sec. III). We can thus add the two expressions (2.8) and (2.9) to form a new (more general) electromagnetic anomaly interaction term $\overline{\mathcal{L}_I}$, which, of course, satisfies both the transformation properties (2.6) and (2.7), exactly as $\mathcal{L}_I$:

$$\overline{\mathcal{L}_I} = \mathcal{L}_I + \Delta\mathcal{L}_I = \frac{i}{2} G\,\mathrm{Tr}[\mathbf{Q}^2(\ln U - \ln U^\dagger)] + f_\Delta \frac{i}{6} G\,\mathrm{Tr}(\mathbf{Q}^2) \times[\ln(X\det U^\dagger)-\ln(X^\dagger \det U)]. \tag{2.10}$$

Therefore, we shall consider the following effective chiral Lagrangian, which includes the new electromagnetic interaction terms described above:

$$\begin{aligned}\mathcal{L}(U,U^\dagger,X,X^\dagger,Q,A^\mu) &= \frac{1}{2}\mathrm{Tr}(D_\mu U D^\mu U^\dagger) + \frac{1}{2}\partial_\mu X \partial^\mu X^\dagger - V(U,U^\dagger,X,X^\dagger) \\ &\quad + \frac{i}{2}\omega_1 Q\,\mathrm{Tr}(\ln U - \ln U^\dagger) + \frac{i}{2}(1-\omega_1) \\ &\quad \times Q(\ln X - \ln X^\dagger) + \frac{1}{2A}Q^2 + \overline{\mathcal{L}_I} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},\end{aligned} \tag{2.11}$$

where the potential term $V(U,U^\dagger,X,X^\dagger)$ is the one written in Eq. (1.3).

The decay amplitude of the generic process "meson $\rightarrow\gamma\gamma$" is entirely due to the electromagnetic anomaly interaction term $\overline{\mathcal{L}_I}$, which can be written more explicitly in terms of the meson fields $\pi_a(a=1,\ldots,8)$, S_π, and S_X, defined as follows [4,6,7]:

$$U = \frac{F_\pi}{\sqrt{2}}\exp\left[\frac{i\sqrt{2}}{F_\pi}\left(\sum_{a=1}^{8}\pi_a\tau_a + \frac{S_\pi}{\sqrt{3}}\mathbf{I}\right)\right],$$

$$X = \frac{F_X}{\sqrt{2}}\exp\left(\frac{i\sqrt{2}}{F_X}S_X\right). \tag{2.12}$$

The π_a are the self-Hermitian fields describing the octet pseudoscalar mesons; S_π is the usual "quark-antiquark" $SU(3)$-singlet meson field associated with U, while S_X is the "exotic" six-fermion meson field associated with X [4,6,7].

Inserting the expressions (2.12) into Eq. (2.10), one finds that

$$\overline{\mathcal{L}_I} = -G\frac{1}{3F_\pi}\left[\pi_3 + \frac{1}{\sqrt{3}}\pi_8 + \frac{2\sqrt{2}}{\sqrt{3}}S_\pi - f_\Delta\frac{2\sqrt{2}}{3F_X}(\sqrt{3}F_X S_\pi - F_\pi S_X)\right]. \tag{2.13}$$

The fields π_3,π_8,S_π, and S_X mix together, while the remaining π_a are already diagonal [6]. However, neglecting the experimentally small mass difference between the quarks up and down [i.e., neglecting the experimentally small violations of the $SU(2)$ isotopic spin], π_3 also becomes diagonal and can be identified with the physical state π^0. The fields (π_8,S_π,S_X) can be written in terms of the eigenstates (η,η',η_X) as follows:

$$\begin{pmatrix}\pi_8\\ S_\pi\\ S_X\end{pmatrix} = \mathbf{C}\begin{pmatrix}\eta\\ \eta'\\ \eta_X\end{pmatrix}, \tag{2.14}$$

where $\mathbf{C}$ is the following 3×3 orthogonal matrix [11]:

$$\mathbf{C} = \begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{pmatrix} = \begin{pmatrix}\cos\tilde{\varphi} & -\sin\tilde{\varphi} & 0\\ \sin\tilde{\varphi}\dfrac{F_\pi}{F_{\eta'}} & \cos\tilde{\varphi}\dfrac{F_\pi}{F_{\eta'}} & \dfrac{\sqrt{3}F_X}{F_{\eta'}}\\ \sin\tilde{\varphi}\dfrac{\sqrt{3}F_X}{F_{\eta'}} & \cos\tilde{\varphi}\dfrac{\sqrt{3}F_X}{F_{\eta'}} & -\dfrac{F_\pi}{F_{\eta'}}\end{pmatrix}. \tag{2.15}$$

Here $F_{\eta'}$ is defined as follows [11]:

$$F_{\eta'} \equiv \sqrt{F_\pi^2 + 3F_X^2}, \tag{2.16}$$

and can be identified with the η' decay constant in the chiral limit of zero quark masses. Moreover, $\tilde{\varphi}$ is a mixing angle, which can be related to the masses of the quarks m_u, m_d, m_s, and therefore to the masses of the octet mesons, by the following relation [11]:

$$\tan\tilde{\varphi} = \frac{F_\pi F_{\eta'}}{6\sqrt{2}A}(m_\eta^2 - m_\pi^2), \tag{2.17}$$

where $m_\pi^2 = 2B\tilde{m}$ and $m_\eta^2 = \frac{2}{3}B(\tilde{m}+2m_s)$, with $B\equiv B_m/2F_\pi$ and $\tilde{m}\equiv(m_u+m_d)/2$.

Concerning the masses of the two singlet states, we recall that [4–7] the field η' has a "light" mass, in the sense of the

$N_c \to \infty$ limit, being, in the chiral limit of zero quark masses[2]

$$m_{\eta'}^2 = \frac{6A}{F_{\eta}'^2} = \frac{6A}{F_\pi^2 + 3F_X^2} = \mathcal{O}\left(\frac{1}{N_c}\right). \quad (2.18)$$

[If we put $F_X = 0$, Eq. (2.18), or the corresponding expression including the light-quark masses [6] reported in footnote 2, reduces to the well-known Witten-Veneziano relation for the η' mass [13].] On the contrary, the field η_X has a sort of "heavy hadronic" mass of order $\mathcal{O}(N_c^0)$ in the large-N_c limit. Both η' and η_X have the same quantum numbers (spin, parity, and so on), but they have a different quark content: one is mostly $S_\pi \sim i(\overline{q}_L q_R - \overline{q}_R q_L)$, while the other is mostly $S_X \sim i[\det(\overline{q}_{sL} q_{tR}) - \det(\overline{q}_{sR} q_{tL})]$, as one can see from Eqs. (2.14), (2.15).

The interaction Lagrangian (2.13), written in terms of the physical fields π^0, η, η', and η_X, reads as follows:

$$\overline{\mathcal{L}_I} \equiv -G \frac{1}{3F_\pi} (\pi^0 + a_1 \eta + a_2 \eta' + \overline{a_3} \eta_X), \quad (2.19)$$

where $a_i = (\alpha_i + 2\sqrt{2}\beta_i)/\sqrt{3}$ (or $i = 1,2,3$), so that

$$a_1 = \sqrt{\frac{1}{3}} \left(\cos\tilde{\varphi} + 2\sqrt{2} \sin\tilde{\varphi} \frac{F_\pi}{F_{\eta'}} \right), \quad (2.20)$$

$$a_2 = \sqrt{\frac{1}{3}} \left(2\sqrt{2} \cos\tilde{\varphi} \frac{F_\pi}{F_{\eta'}} - \sin\tilde{\varphi} \right), \quad (2.21)$$

$$a_3 = 2\sqrt{2} \left(\frac{F_X}{F_{\eta'}} \right), \quad (2.22)$$

and, moreover,

$$\overline{a_3} = a_3 + \Delta a_3 \quad \text{with} \quad \Delta a_3 = -f_\Delta \frac{2\sqrt{2} F_{\eta'}}{3F_X}. \quad (2.23)$$

The values of the coefficients a_1, a_2, and a_3 are exactly the same as were calculated in Ref. [11]: therefore, the inclusion of the new electromagnetic interaction term (2.9) in the expression for the electromagnetic anomaly interaction term (2.10) modifies only (for $T=0$ or, more generally, for $T < T_{ch}$; see the discussion in the next section) the decay amplitude for the process $\eta_X \to \gamma\gamma$, while leaving unchanged the other decay amplitudes for the processes $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, and $\eta' \to \gamma\gamma$. Indeed, from Eqs. (2.14) and (2.15) we derive that

$$\eta_X = \frac{1}{F_\eta'} (\sqrt{3} F_X S_\pi - F_\pi S_X), \quad (2.24)$$

and thus we immediately see that the term proportional to f_Δ in Eq. (2.13) is simply equal to

$$\Delta \mathcal{L}_I = -G \frac{1}{3F_\pi} \left(-f_\Delta \frac{2\sqrt{2} F_{\eta'}}{3F_X} \right) \eta_X = -G \frac{1}{3F_\pi} \Delta a_3 \eta_X. \quad (2.25)$$

The expressions for the decay amplitudes are

$$A(\pi^0 \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi} I, \quad (2.26)$$

$$A(\eta \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi} \sqrt{\frac{1}{3}} \left(\cos\tilde{\varphi} + 2\sqrt{2} \sin\tilde{\varphi} \frac{F_\pi}{F_{\eta'}} \right) I, \quad (2.27)$$

$$A(\eta' \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi} \sqrt{\frac{1}{3}} \left(2\sqrt{2} \cos\tilde{\varphi} \frac{F_\pi}{F_{\eta'}} - \sin\tilde{\varphi} \right) I, \quad (2.28)$$

$$A(\eta_X \to \gamma\gamma) = \frac{e^2 N_c}{12\pi^2 F_\pi} 2\sqrt{2} \left(\frac{F_X}{F_{\eta'}} - f_\Delta \frac{F_{\eta'}}{3F_X} \right) I, \quad (2.29)$$

where $I \equiv \varepsilon_{\mu\nu\rho\sigma} k_1^\mu \epsilon_1^{\nu *} k_2^\rho \epsilon_2^{\sigma *}$ (k_1, k_2 being the four-momenta of the two final photons and ϵ_1, ϵ_2 their polarizations). Consequently, the following decay rates (in the real case $N_c = 3$) are derived:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_\pi^3}{64\pi^3 F_\pi^2}, \quad (2.30)$$

$$\Gamma(\eta \to \gamma\gamma) = \frac{\alpha^2 m_\eta^3}{192\pi^3 F_\pi^2} \left(\cos\tilde{\varphi} + 2\sqrt{2} \sin\tilde{\varphi} \frac{F_\pi}{F_{\eta'}} \right)^2, \quad (2.31)$$

$$\Gamma(\eta' \to \gamma\gamma) = \frac{\alpha^2 m_{\eta'}^3}{192\pi^3 F_\pi^2} \left(2\sqrt{2} \cos\tilde{\varphi} \frac{F_\pi}{F_{\eta'}} - \sin\tilde{\varphi} \right)^2, \quad (2.32)$$

$$\Gamma(\eta_X \to \gamma\gamma) = \frac{\alpha^2 m_{\eta_X}^3}{8\pi^3 F_\pi^2} \left(\frac{F_X}{F_{\eta'}} - f_\Delta \frac{F_{\eta'}}{3F_X} \right)^2, \quad (2.33)$$

where $\alpha = e^2/4\pi \simeq 1/137$ is the fine-structure constant.

The results (2.30)–(2.32) are exactly the same as were found in Ref. [11]. [If we put $F_X = 0$, i.e., if we neglect the new $U(1)$ chiral condensate, the expressions written above reduce to the corresponding ones derived in Ref. [12] using an effective Lagrangian which includes only the usual $\overline{q}q$ chiral condensate.] Therefore also the numerical results obtained in Ref. [11] concerning the processes $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ remain unaffected. In particular, using the experimental values for the various quantities which appear in Eqs. (2.31) and (2.32), i.e.,

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[2]The expression for the η' mass, when including the light-quark masses, reads as follows [6]: $(1 + 3F_X^2/F_\pi^2) m_{\eta}'^2 + m_\eta^2 - 2m_K^2 = 6A/F_\pi^2$, with $m_K^2 = B(\tilde{m} + m_s)$.

$$F_\pi = 92.4(4) \ \text{MeV},$$

$$m_\eta = 547.30(12) \ \text{MeV},$$

$$m_{\eta'} = 957.78(14) \ \text{MeV},$$

$$\Gamma(\eta \to \gamma\gamma) = 0.46(4) \ \text{keV},$$

$$\Gamma(\eta' \to \gamma\gamma) = 4.26(19) \ \text{keV}, \tag{2.34}$$

we can extract the following values for the quantity F_X and for the mixing angle $\tilde{\varphi}$ [11]:

$$F_X = 27(9) \ \text{MeV}, \quad \tilde{\varphi} = 16(3)°, \tag{2.35}$$

and these values are perfectly consistent with the relation (2.17) for the mixing angle, if we use for the pure YM topological susceptibility the estimate $A = (180 \pm 5 \ \text{MeV})^4$, obtained from lattice simulations [14].

Nevertheless, the new electromagnetic interaction term will play a crucial role in the discussion of the $\eta' \to \gamma\gamma$ radiative decay at finite temperature, in particular for $T > T_{ch}$; this will be studied in detail in the next section.

III. RADIATIVE DECAYS OF THE PSEUDOSCALAR MESONS AT $T \neq 0$

We want now to address the finite-temperature case ($T \neq 0$). As already said in the Introduction, this will be done (using a sort of mean-field approximation) simply by considering all the parameters appearing in the Lagrangian as functions of the physical temperature T. In such a way, the results obtained in the previous section can be extended to the whole region of temperatures below the chiral transition ($T < T_{ch}$), provided that the T dependence is included in all the parameters appearing in Eqs. (2.30)–(2.33).

What happens when approaching the chiral transition temperature T_{ch} from below ($T \to T_{ch}-$)? We know that $F_\pi(T) \to 0$ when $T \to T_{ch}-$. Let us consider, for simplicity, the chiral limit of zero quark masses. From Eq. (2.18) we see that $m^2_{\eta'} \to 2A(T_{ch})/F^2_X(T_{ch})$ when $T \to T_{ch}-$ and, from Eqs. (2.14), (2.15), we derive

$$\eta' = \frac{1}{F'_\eta}(F_\pi S_\pi + \sqrt{3} F_X S_X), \tag{3.1}$$

so that $\eta' \to S_X$ when $T \to T_{ch}-$. In this same limit, the η' decay rate (2.32) tends to the value

$$\Gamma(\eta' \to \gamma\gamma) \underset{T \to T_{ch-}}{\to} \frac{\alpha^2 m^3_{\eta'}(T_{ch})}{72\pi^3 F^2_X(T_{ch})}. \tag{3.2}$$

What happens, instead, in the region of temperatures $T_{ch} < T < T_{U(1)}$, above the chiral phase transition [where the $SU(3) \otimes SU(3)$ chiral symmetry is restored, while the $U(1)$ chiral condensate is still present]? First of all, we observe that we have continuity in the mass spectrum of the theory through the chiral phase transition at $T = T_{ch}$. In fact, if we study the mass spectrum of the theory in the region of temperatures $T_{ch} < T < T_{U(1)}$ [4,6,7], we find that the singlet meson field S_X, associated with the field X in the chiral Lagrangian, according to the second Eq. (2.12) [instead, the first Eq. (2.12) is no longer valid in this region of temperatures], has a squared mass given by (in the chiral limit) $m^2_{S_X} = 2A/F^2_X$. This is nothing but the *would-be* Goldstone particle coming from the breaking of the $U(1)$ chiral symmetry, i.e., the η', which, for $T > T_{ch}$, is a sort of "exotic" matter field of the form $S_X \sim i[\det(\overline{q}_{sL} q_{tR}) - \det(\overline{q}_{sR} q_{tL})]$. Its existence could be proved perhaps in the near future by heavy-ion experiments.

And what about the η' radiative decay rate in the region of temperatures $T_{ch} < T < T_{U(1)}$? Since $\eta' = S_X$ above T_{ch}, the electromagnetic anomaly interaction term describing the process $\eta' \to \gamma\gamma$ for $T > T_{ch}$ is only the part of $\overline{\mathcal{L}_I}$, written in Eq. (2.10), which depends on the field X:

$$\Delta \mathcal{L}_{S_X \gamma\gamma} = f_\Delta \frac{i}{6} G \, \text{Tr}(\mathbf{Q}^2)(\ln X - \ln X^\dagger) = -f_\Delta \frac{2\sqrt{2}}{9F_X} G S_X. \tag{3.3}$$

From this equation we easily derive the following expression for the $\eta' \to \gamma\gamma$ decay amplitude above T_{ch}:

$$A(\eta' \to \gamma\gamma)|_{T > T_{ch}} = f_\Delta \frac{e^2 N_c \sqrt{2}}{18\pi^2 F_X} I, \tag{3.4}$$

and, consequently, the following expression for the $\eta' \to \gamma\gamma$ decay rate (in the real case $N_c = 3$) above T_{ch}:

$$\Gamma(\eta' \to \gamma\gamma)|_{T > T_{ch}} = f_\Delta \frac{\alpha^2 m^3_{\eta'}}{72\pi^3 F^2_X}. \tag{3.5}$$

If we require that $\Gamma(\eta' \to \gamma\gamma)$ is a continuous function of T across the chiral transition at T_{ch}, then from Eqs. (3.2) and (3.5) we obtain the following condition for f_Δ:

$$f_\Delta(T_{ch}) = 1. \tag{3.6}$$

This means that

$$\Gamma(\eta' \to \gamma\gamma)|_{T = T_{ch}} = \frac{\alpha^2 m^3_{\eta'}(T_{ch})}{72\pi^3 F^2_X(T_{ch})}. \tag{3.7}$$

The decay rates and the masses at finite temperature could be determined in the near future heavy-ion experiments and then Eq. (3.7) will provide an estimate for the value of F_X at $T = T_{ch}$. Vice versa, if we were able to determine the value of F_X in some other independent way (e.g., by lattice simulations; see Ref. [11]), then Eq. (3.7) would give a theoretical estimate of the ratio $\Gamma(\eta' \to \gamma\gamma)/m^3_{\eta'}$ at $T = T_{ch}$, which could be compared with the experimental results. For example, if we make the (very plausible indeed) assumption that the value of F_X does not change very much going from $T = 0$ up to $T = T_{ch}$ (it will vanish at a temperature $T_{U(1)}$ above T_{ch}), i.e., $F_X(T_{ch}) \simeq F_X(0)$, and if we take for $F_X(0)$ the value reported in Eq. (2.35), then Eq. (3.7) furnishes the following estimate:

$$\Gamma(\eta' \to \gamma\gamma)|_{T=T_{ch}}/m^3_{\eta'}(T_{ch}) = \frac{\alpha^2}{72\pi^3 F_X^2(T_{ch})} \simeq 3.3^{+4.1}_{-1.4} \times 10^{-11} \ \text{MeV}^{-2}. \quad (3.8)$$

In other words, comparing with the corresponding quantities at $T=0$, reported in Eq. (2.34), one gets that

$$\frac{\Gamma(\eta' \to \gamma\gamma)|_{T=T_{ch}}/m^3_{\eta'}(T_{ch})}{\Gamma(\eta' \to \gamma\gamma)|_{T=0}/m^3_{\eta'}(0)} \simeq 7^{+8}_{-3}. \quad (3.9)$$

Thus, even with very large errors, due to our poor knowledge of the value of F_X, there is a quite definite prediction that the ratio $\Gamma(\eta' \to \gamma\gamma)/m^3_{\eta'}$ should have a sharp increase on approaching the chiral transition temperature T_{ch}. [Of course, a smaller value of F_X would result in a larger value for the ratio in Eq. (3.9), and this case seems indeed to be favored from the upper limit $F_X \lesssim 20$ MeV obtained from the *generalized* Witten-Veneziano formula for the η' mass [6].] One could also argue that it is physically plausible that the η' mass (of the order of 1 GeV) remains practically unchanged when going from $T=0$ up to T_{ch} (which, from lattice simulations, is known to be of the order of 170 MeV: see, e.g., Ref. [2]); in that case, Eq. (3.9) would give an estimate for the ratio between the η' decay rates at $T=T_{ch}$ and $T=0$. However, we want to stress that our result (3.9) is more general and does not rely on any given assumption on the behavior of $m_{\eta'}(T)$ with the temperature T.

IV. CONCLUSIONS

There is evidence from some lattice results that a new $U(1)$-breaking condensate survives across the chiral transition at T_{ch}, staying different from zero up to $T_{U(1)} > T_{ch}$. This fact has important consequences for the pseudoscalar-meson sector, which can be studied using an effective Lagrangian model, including also the new $U(1)$ chiral condensate. This model could perhaps be verified in the near future by heavy-ion experiments, by analyzing the pseudoscalar-meson spectrum in the singlet sector.

In Ref. [11] we have also investigated the effects of the new $U(1)$ chiral condensate on the radiative decays, at $T=0$, of the pseudoscalar mesons η and η' to two photons. A first comparison of our results with the experimental data has been performed: the results are encouraging, pointing toward some evidence of a nonzero $U(1)$ axial condensate. In this paper, generalizing the results obtained in Ref. [11], we have studied the effects of the $U(1)$ chiral condensate on the radiative decay $\eta' \to \gamma\gamma$ at finite temperature ($T \neq 0$). In particular, we have been able to get a quite definite theoretical prediction [see Eq. (3.9)] for the ratio between the $\eta' \to \gamma\gamma$ decay rate and the third power of the η' mass in the proximity of the chiral transition temperature T_{ch} (which, from lattice simulations, is expected to be of the order of 170 MeV); this prediction could in principle be tested in future heavy-ion experiments.

However, as we have already stressed in the conclusions of Ref. [11], one should keep in mind that our results have been derived from a very simplified model, obtained by doing a first-order expansion in $1/N_c$ and in the quark masses. We expect that such a model can furnish only qualitative or, at most, ''semiquantitative'' predictions. When going beyond the leading order in $1/N_c$, it becomes necessary to take into account questions of renormalization-group behavior of the various quantities and operators involved in our theoretical analysis. This issue has been widely discussed in the literature, both in relation to the proton-spin crisis problem [15], and also in relation to the study of η, η' radiative decays [16]. Further studies are therefore necessary in order to continue this analysis from a more quantitative point of view. We expect that some progress will be made along this line in the near future.

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